Direct-current Josephson effect in SNS junctions of anisotropic superconductors

Yasuhiro Asano*
Department of Applied Physics, Hokkaido University, Sapporo 060-8628, Japan
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We derive a formula of the dc Josephson current between two superconductors with anisotropic pairing symmetry. One of the basic characters in the junctions of the anisotropic superconductors is the formation of the zero-energy bound states at the junction interfaces, which leads to the low-temperature anomaly of the Josephson current. The contribution of the zero-energy states to the Josephson current is taken into account in the present formalism.

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I. INTRODUCTION

The discovery of high-T_c superconductors[1] has stimulated intensive research in this field. The symmetry of the Cooper pair is important for understanding the mechanism of high-T_c superconductivity. The Josephson effect in anisotropic superconductors has attracted much attention in recent years because the high-T_c superconductors might have d-wave pairing symmetry.[2,3] So far, transport properties in various junctions of d-wave superconductors have been discussed in a number of studies.[4–20] In anisotropic superconductors, the sign of the pair potential depends on the direction of a quasiparticle's motion. As a consequence, the zero-energy states[21] (ZES's) are formed at the normal-metal/superconductor (NS) interface when the potential barrier at the interface is large enough. The ZES's are clearly observed in the conductance spectra of N/I/d-wave superconductor junctions,[22,23] where I denotes the insulator. It is known that the ZES's are responsible for the low-temperature anomaly of the Josephson current in superconductor-insulator-superconductor(SIS) junctions of the d-wave superconductor.[9,10]

The anisotropic superconductivity itself has been an important topic in condensed-matter physics since unconventional superconductivity was found in heavy-fermion materials such as $CeCu_2Si_2$, UBe_{13}, and UPt_3.[24–27] In a recent study, anisotropic superconductivity was found in a layered perovskite, Sr_2RuO_4.[28] Some interesting effects of the anisotropy in the pairing symmetry on Josephson currents are revealed in previous works.[29–31] However, in order to study the contribution of the ZES to the Josephson current, we have to pay careful attention to the boundary condition of the wave function at the junction interface.[32]

In this paper, we derive a formula of the dc Josephson current between the two anisotropic superconductors with spin-singlet and spin-triplet Cooper pairs. The results are an extended version of the Furusaki-Tsukada formula for s-wave superconductor junctions.[33] The influence of the ZES on the Josephson current is naturally taken into account in the obtained formula because the Josephson current is expressed by the Andreev reflection[34] coefficients (ARC's) of the junction. The low-temperature anomaly is described by the dependence of the ARC's on the temperature. Throughout this paper, we take the units of $\hbar = k_B = 1$, where k_B is the Boltzmann constant.

This paper is organized as follows. In Sec. II, we derive the Josephson current formula based on the mean-field theory of superconductivity. In Sec. III, the Josephson current is expressed for the superconductors with spin-singlet and spin-triplet Cooper pairs. The conclusion is given in Sec. IV.

II. JOSEPHSON CURRENT FORMULA I

Let us consider the superconductor-normal-metal-superconductor(SNS) junction as shown in Fig. 1, where the length of the normal metal is L_N and the cross section of the junction is S_J. The Hamiltonian in the mean-field approximation reads

$$\begin{aligned} H_{MF} = \frac{1}{2}\int d\mathbf{r}\int d\mathbf{r}' [\tilde{c}^{\dagger}(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}')\hat{h}_0(\mathbf{r}')\tilde{c}(\mathbf{r}') \\ -\tilde{c}^{t}(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}')\hat{h}_0^{*}(\mathbf{r}')\{\tilde{c}^{\dagger}(\mathbf{r}')\}^{t} \\ +\tilde{c}^{\dagger}(\mathbf{r})\hat{\Delta}(\mathbf{r}-\mathbf{r}')\{\tilde{c}^{\dagger}(\mathbf{r}')\}^{t} - \tilde{c}^{t}(\mathbf{r})\hat{\Delta}^{*}(\mathbf{r}-\mathbf{r}')\tilde{c}(\mathbf{r}')], \end{aligned} \tag{1}$$

$$\hat{h}_0(\mathbf{r}) = \left[-\frac{\nabla^2}{2m} + V_0(\mathbf{r}) - \mu_F\right]\hat{\sigma}_0 + \mathbf{V}(\mathbf{r})\cdot\hat{\boldsymbol{\sigma}}, \tag{2}$$

$$\tilde{c}(\mathbf{r}) \equiv \begin{pmatrix} c_{\uparrow}(\mathbf{r}) \\ c_{\downarrow}(\mathbf{r}) \end{pmatrix}, \tag{3}$$

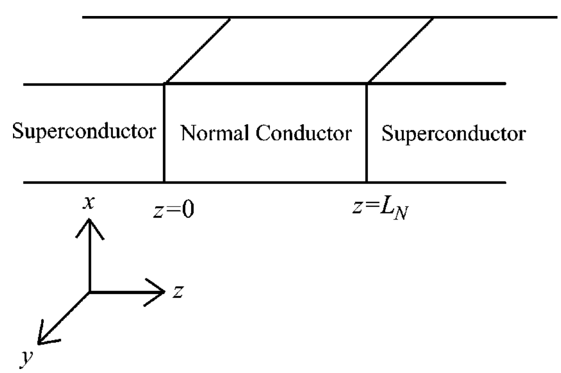

FIG. 1. The SNS junction of the anisotropic superconductors. The phase of the pair potential on the left (right) superconductor is $\varphi_L(\varphi_R)$.

where $c_\sigma(\mathbf{r})$ is the annihilation operator of an electron at $\mathbf{r}$ with spin $\sigma=\uparrow$ or $\downarrow$, $\{\tilde{c}(\mathbf{r})\}^t$ is the transposition of Eq. (3), $\hat{\sigma}_0$ is the unit matrix of 2×2, μ_F is the Fermi energy, and $V_0(\mathbf{r})$ denotes the spin-independent potential which includes the barrier potential at the two NS interfaces $V_b\{\delta(z)+\delta(z-L_N)\}$. The spin-orbit scattering in the normal metal is denoted by $\mathbf{V}(\mathbf{r})\cdot\hat{\boldsymbol{\sigma}}$. The pair potential between an electron with $(\sigma,\mathbf{r})$ and that with $(\sigma',\mathbf{r}')$ is given by $\Delta_{\sigma,\sigma'}(\mathbf{r}-\mathbf{r}')$. In the normal segment $(0<z<L_N)$, the pair potential is taken to be zero. In what follows, we use $\widehat{\cdots}$ for 2×2 matrices. The pair potential is given by

$$\hat{\Delta}(\mathbf{r})=\begin{cases} id_0(\mathbf{r})\hat{\sigma}_2 & (\text{singlet}),\\ i[\mathbf{d}(\mathbf{r})\cdot\hat{\boldsymbol{\sigma}}]\hat{\sigma}_2 & (\text{triplet}),\end{cases} \tag{4}$$

where $\hat{\sigma}_j$ with $j=1,2$, and 3 are the Pauli's matrices. The pair potential satisfies a relation

$$-\hat{\Delta}^t(\mathbf{r}'-\mathbf{r})=\hat{\Delta}(\mathbf{r}-\mathbf{r}'). \tag{5}$$

The Hamiltonian in Eq. (1) is diagonalized by the Bogoliubov transformation,

$$\begin{pmatrix}\tilde{c}(\mathbf{r})\\ \{\tilde{c}^\dagger(\mathbf{r})\}^t\end{pmatrix}=\sum_\lambda\begin{pmatrix}\hat{u}_\lambda(\mathbf{r}) & \hat{v}^*_\lambda(\mathbf{r})\\ \hat{v}_\lambda(\mathbf{r}) & \hat{u}^*_\lambda(\mathbf{r})\end{pmatrix}\begin{pmatrix}\tilde{\alpha}_\lambda\\ \{\tilde{\alpha}^\dagger_\lambda\}^t\end{pmatrix}, \tag{6}$$

$$H_{MF}=\sum_\lambda \tilde{\alpha}^\dagger\hat{E}_\lambda\tilde{\alpha}_\lambda, \tag{7}$$

$$\hat{E}_\lambda=\begin{pmatrix}E_{\lambda,1} & 0\\ 0 & E_{\lambda,2}\end{pmatrix}, \tag{8}$$

where

$$\tilde{\alpha}_\lambda\equiv\begin{pmatrix}\alpha_{\lambda,\uparrow}\\ \alpha_{\lambda,\downarrow}\end{pmatrix} \tag{9}$$

denotes the annihilation operator of a Bogoliubov quasiparticle. The wave functions satisfy the Bogoliubov-de Gennes (BdG) equation,[35]

$$\int d\mathbf{r}'\begin{bmatrix}\delta(\mathbf{r}-\mathbf{r}')\hat{h}_0(\mathbf{r}') & \hat{\Delta}(\mathbf{r}-\mathbf{r}')\\ -\hat{\Delta}^*(\mathbf{r}-\mathbf{r}') & -\delta(\mathbf{r}-\mathbf{r}')\hat{h}^*_0(\mathbf{r}')\end{bmatrix}\begin{bmatrix}\hat{u}_\lambda(\mathbf{r}')\\ \hat{v}_\lambda(\mathbf{r}')\end{bmatrix}=\begin{bmatrix}\hat{u}_\lambda(\mathbf{r})\\ \hat{v}_\lambda(\mathbf{r})\end{bmatrix}\hat{E}_\lambda. \tag{10}$$

When the wave function

$$\begin{bmatrix}\hat{u}_\lambda(\mathbf{r})\\ \hat{v}_\lambda(\mathbf{r})\end{bmatrix} \tag{11}$$

belongs to an eigenvalue $\hat{E}_\lambda$, the wave function

$$\begin{bmatrix}\hat{v}^*_\lambda(\mathbf{r})\\ \hat{u}^*_\lambda(\mathbf{r})\end{bmatrix} \tag{12}$$

belongs to $-\hat{E}_\lambda$. They satisfy the following relations:

$$\int d\mathbf{r}\{\hat{u}^\dagger_\lambda(\mathbf{r})\hat{u}_{\lambda'}(\mathbf{r})+\hat{v}^\dagger_\lambda(\mathbf{r})\hat{v}_{\lambda'}(\mathbf{r})\}=\delta_{\lambda,\lambda'}\hat{\sigma}_0, \tag{13}$$

$$\int d\mathbf{r}\{\hat{u}^\dagger_\lambda(\mathbf{r})\hat{v}^*_{\lambda'}(\mathbf{r})+\hat{v}^\dagger_\lambda(\mathbf{r})\hat{u}^*_{\lambda'}(\mathbf{r})\}=\hat{0}, \tag{14}$$

$$\sum_\lambda{}' \{\hat{u}_\lambda(\mathbf{r})\hat{u}^\dagger_\lambda(\mathbf{r}')+\hat{v}^*_\lambda(\mathbf{r})\hat{v}^t_\lambda(\mathbf{r}')\}=\delta(\mathbf{r}-\mathbf{r}')\hat{\sigma}_0, \tag{15}$$

$$\sum_\lambda{}' \{\hat{u}_\lambda(\mathbf{r})\hat{v}^\dagger_\lambda(\mathbf{r}')+\hat{v}^*_\lambda(\mathbf{r})\hat{u}^t_\lambda(\mathbf{r}')\}=\hat{0}. \tag{16}$$

Here Σ'_λ is a summation over λ with $E_\lambda>0$. The local charge density is defined by

$$P(\mathbf{r},\tilde{t})=-e\,\tilde{c}^\dagger(\mathbf{r},\tilde{t})\tilde{c}(\mathbf{r},\tilde{t}), \tag{17}$$

where $\tilde{t}$ is the time. The current conservation law implies

$$\frac{\partial}{\partial\tilde{t}}P(\mathbf{r},\tilde{t})+\nabla\cdot\mathbf{J}(\mathbf{r},\tilde{t})=0. \tag{18}$$

The dc Josephson current between the two superconductors is given by the expectation value of Eq. (18),

$$\mathbf{J}(\mathbf{r})=\frac{e}{4mi}\lim_{\mathbf{r}'\to\mathbf{r}}(\nabla_{\mathbf{r}'}-\nabla_{\mathbf{r}})T\sum_{\omega_n}\mathrm{Tr}\,\check{\mathcal{G}}_{\omega_n}(\mathbf{r},\mathbf{r}'), \tag{19}$$

$$\check{\mathcal{G}}_{\omega_n}(\mathbf{r},\mathbf{r}')=\sum_\lambda{}'\left[\begin{pmatrix}\hat{u}_\lambda(\mathbf{r})\\ \hat{v}_\lambda(\mathbf{r})\end{pmatrix}(i\omega_n\hat{\sigma}_0-\hat{E}_\lambda)^{-1}\begin{pmatrix}\hat{u}_\lambda(\mathbf{r}')\\ \hat{v}_\lambda(\mathbf{r}')\end{pmatrix}^\dagger+\begin{pmatrix}\hat{v}^*_\lambda(\mathbf{r})\\ \hat{u}^*_\lambda(\mathbf{r})\end{pmatrix}(i\omega_n\hat{\sigma}_0+\hat{E}_\lambda)^{-1}\begin{pmatrix}\hat{v}^*_\lambda(\mathbf{r}')\\ \hat{u}^*_\lambda(\mathbf{r}')\end{pmatrix}^\dagger\right], \tag{20}$$

where T is the temperature, $\check{\mathcal{G}}_{\omega_n}(\mathbf{r},\mathbf{r}')$ is the Matsubara Green function of the SNS junctions, and $\smile$ indicates 4×4 matrices. On the derivation of Eq. (19), we have assumed that the amplitude of the pair potential is much smaller than the Fermi energy μ_F.

In the superconductors, we assume that all potentials are uniform. The BdG equation in Eq. (10) is given in the Fourier representation

$$\begin{bmatrix}\xi_{\mathbf{k}}\hat{\sigma}_0 & \hat{\Delta}(\mathbf{k})\\ -\hat{\Delta}^*(-\mathbf{k}) & -\xi_{\mathbf{k}}\hat{\sigma}_0\end{bmatrix}\begin{bmatrix}\hat{u}_{\mathbf{k}}\\ \hat{v}_{\mathbf{k}}\end{bmatrix}=\begin{bmatrix}\hat{u}_{\mathbf{k}}\\ \hat{v}_{\mathbf{k}}\end{bmatrix}\hat{E}_{\mathbf{k}}, \tag{21}$$

where $\xi_{\mathbf{k}}=\mathbf{k}^2/(2m)-\mu_F$ and

$$\hat{\Delta}(\mathbf{r}-\mathbf{r}')=\sum_{\mathbf{k}}\hat{\Delta}(\mathbf{k})e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}, \tag{22}$$

$$\hat{\Delta}(\mathbf{k})=\begin{cases} id_0(\mathbf{k})\hat{\sigma}_2 & (\text{singlet}),\\ i[\mathbf{d}(\mathbf{k})\cdot\hat{\boldsymbol{\sigma}}]\hat{\sigma}_2 & (\text{triplet}).\end{cases} \tag{23}$$

Since relations

$$d_0(-\mathbf{k})=d_0(\mathbf{k}), \tag{24}$$

$$\mathbf{d}(-\mathbf{k})=-\mathbf{d}(\mathbf{k}) \tag{25}$$

are satisfied in the momentum space, we find

$$-\hat{\Delta}^t(-\mathbf{k})=\hat{\Delta}(\mathbf{k}). \tag{26}$$

When $z<z'\leqslant 0$, the Green function can be calculated to be

$$\begin{aligned}\check{\mathcal{G}}_{\omega_n}(\mathbf{r},\mathbf{r}')=&-im\omega_n\sum_{\mathbf{p}}\chi_{\mathbf{p}}(\boldsymbol{\rho})\chi^*_{\mathbf{p}}(\boldsymbol{\rho}')\check{\Phi}_L\\ &\times\Bigg[\Bigg\{\begin{pmatrix}\hat{u}^e_+\\ \hat{v}^e_+\end{pmatrix}\hat{K}_p(k^e_+,z)+\begin{pmatrix}\hat{u}^h_+\\ \hat{v}^h_+\end{pmatrix}\hat{K}_p(k^h_+,z)\hat{a}_1\\ &+\begin{pmatrix}\hat{u}^e_-\\ \hat{v}^e_-\end{pmatrix}\hat{K}_p(-k^e_-,z)\hat{b}_1\Bigg\}\hat{K}_p(-k^e_+,z')\\ &\times\begin{pmatrix}k^e_{1,+} & 0\\ 0 & k^e_{2,+}\end{pmatrix}^{-1}\hat{\Omega}^{-1}_+\begin{pmatrix}\hat{u}^e_+\\ \hat{v}^e_+\end{pmatrix}^\dagger\\ &+\Bigg\{\begin{pmatrix}\hat{u}^h_-\\ \hat{v}^h_-\end{pmatrix}\hat{K}_p(-k^h_-,z)+\begin{pmatrix}\hat{u}^e_-\\ \hat{v}^e_-\end{pmatrix}\hat{K}_p(-k^e_-,z)\hat{a}_2\\ &+\begin{pmatrix}\hat{u}^h_+\\ \hat{v}^h_+\end{pmatrix}\hat{K}_p(k^h_+,z)\hat{b}_2\Bigg\}\\ &\times\hat{K}_p(k^h_-,z')\begin{pmatrix}k^h_{1,-} & 0\\ 0 & k^h_{2,-}\end{pmatrix}^{-1}\\ &\times\hat{\Omega}^{-1}_-\begin{pmatrix}\hat{u}^h_-\\ \hat{v}^h_-\end{pmatrix}^\dagger\Bigg]\check{\Phi}^*_L,\end{aligned} \tag{27}$$

with

$$k^e_{l,\pm}=\sqrt{2m[\mu_F-\epsilon(\mathbf{p})+i\Omega_{l,\pm}]}, \tag{28}$$

$$k^h_{l,\pm}=\sqrt{2m[\mu_F-\epsilon(\mathbf{p})-i\Omega_{l,\pm}]}, \tag{29}$$

$$\Omega_{l,\pm}=\sqrt{\omega_n^2+|\Delta_{l,\pm}|^2}, \tag{30}$$

$$\epsilon(\mathbf{p})=\frac{\mathbf{p}^2}{2m}, \tag{31}$$

$$d_{0,\pm}=d_0(\mathbf{p},\pm k_z), \tag{32}$$

$$\mathbf{d}_\pm=\mathbf{d}(\mathbf{p},\pm k_z), \tag{33}$$

$$\hat{\Delta}_\pm=\begin{cases} id_{0,\pm}\hat{\sigma}_2 & \text{(singlet)},\\ i(\mathbf{d}_\pm\cdot\hat{\boldsymbol{\sigma}})\hat{\sigma}_2 & \text{(triplet)},\end{cases} \tag{34}$$

$$\mathbf{q}_\pm=i\mathbf{d}_\pm\times\mathbf{d}^*_\pm, \tag{35}$$

$$\hat{K}_p(k,z)=\begin{pmatrix}e^{ik_1z} & 0\\ 0 & e^{ik_2z}\end{pmatrix}, \tag{36}$$

$$\hat{\Omega}_\pm=\begin{pmatrix}\Omega_{1,\pm} & 0\\ 0 & \Omega_{2,\pm}\end{pmatrix}, \tag{37}$$

$$\check{\Phi}_j=\begin{pmatrix}e^{i\varphi_j}\hat{\sigma}_0 & 0\\ 0 & e^{-i\varphi_j}\hat{\sigma}_0\end{pmatrix}, \tag{38}$$

$$\chi_{\mathbf{p}}(\boldsymbol{\rho})=\frac{\exp(i\mathbf{p}\cdot\boldsymbol{\rho})}{\sqrt{S_J}}, \tag{39}$$

where φ_j for $j=L$ or R is the phase of the superconductor, $\mathbf{p}=(k_x,k_y)$, and $\boldsymbol{\rho}=(x,y)$. The amplitude of the pair potential for unitary states is defined by

$$|\Delta_{l,\pm}|=|\Delta_\pm|=\begin{cases}|d_{0,\pm}| & \text{(singlet)},\\ |\mathbf{d}_\pm| & \text{(triplet)}.\end{cases} \tag{40}$$

In the unitary states, these amplitudes are independent of l, where l represents the spin configuration of a quasiparticle. The amplitude of the pair potential depends on the spin configuration of a quasiparticle in nonunitary states,

$$|\Delta_{l,\pm}|=\begin{cases}\sqrt{|\mathbf{d}_\pm|^2+|\mathbf{q}_\pm|} & (l=1),\\ \sqrt{|\mathbf{d}_\pm|^2-|\mathbf{q}_\pm|} & (l=2).\end{cases} \tag{41}$$

In Eqs. (28) and (29), $k^{e(h)}_{l,\pm}$ is the wave number in the electron (hole) branch for lth spin channel. In the following, we approximately describe that $k^{e(h)}_{l,\pm}\approx k_z=\sqrt{2m[\mu_F-\epsilon(\mathbf{p})]}$ as shown in Eqs. (32) and (33), where $(\mathbf{p},\pm k_z)$ is the wave number on the Fermi surface. The lth column of

$$\begin{pmatrix}\hat{u}^{e(h)}_\pm\\ \hat{v}^{e(h)}_\pm\end{pmatrix} \tag{42}$$

corresponds to the wave function of the lth spin state in the electron (hole) branch. The reflection coefficients from the left superconductor to the left superconductor are defined in the matrix form[33]

$$\hat{a}_{j=1,2}=\begin{pmatrix}a_j(1,1) & a_j(1,2)\\ a_j(2,1) & a_j(2,2)\end{pmatrix}, \tag{43}$$

$$\hat{b}_{j=1,2}=\begin{pmatrix}b_j(1,1) & b_j(1,2)\\ b_j(2,1) & b_j(2,2)\end{pmatrix}. \tag{44}$$

The ARC from the lth spin state in the electron (hole) branch to the l'th spin state in the hole (electron) branch is denoted by $\hat{a}_1(l',l)[\hat{a}_2(l',l)]$. In the same way, $\hat{b}_1(l',l)[\hat{b}_2(l',l)]$ is the normal reflection coefficient from the lth spin state in the electron (hole) branch to the l'th spin state in the electron (hole) branch. These reflection coefficients are the function of $\mathbf{p}$ which indicates the propagating channel at the left NS interface. By substituting Eq. (27) into Eq. (19), the Josephson current becomes

$$J=\frac{ie}{2}\sum_{\mathbf{p}} T\sum_{\omega_n} \mathrm{Tr}\ \omega_n\left[\begin{pmatrix}\hat{u}_+^h\\ \hat{v}_+^h\end{pmatrix}\hat{a}_1\hat{\Omega}_+^{-1}\begin{pmatrix}\hat{u}_+^e\\ \hat{v}_+^e\end{pmatrix}^\dagger -\begin{pmatrix}\hat{u}_-^e\\ \hat{v}_-^e\end{pmatrix}\hat{a}_2\hat{\Omega}_-^{-1}\begin{pmatrix}\hat{u}_-^h\\ \hat{v}_-^h\end{pmatrix}^\dagger\right]. \tag{45}$$

The expression of the Josephson current in Eq. (45) corresponds to the Furusaki-Tsukada formula.[33]

Throughout this paper, we use the representation

$$\begin{pmatrix}\hat{u}_\pm^e\\ \hat{v}_\pm^e\end{pmatrix}=\begin{pmatrix}u_\pm\hat{\sigma}_0\\ v_\pm\dfrac{\hat{\Delta}_\pm^\dagger}{|\Delta_\pm|}\end{pmatrix}, \tag{46}$$

$$\begin{pmatrix}\hat{u}_\pm^h\\ \hat{v}_\pm^h\end{pmatrix}=\begin{pmatrix}v_\pm\dfrac{\hat{\Delta}_\pm}{|\Delta_\pm|}\\ u_\pm\hat{\sigma}_0\end{pmatrix}, \tag{47}$$

$$u_\pm=\sqrt{\frac{1}{2}\left(1+\frac{\Omega_\pm}{\omega_n}\right)}, \tag{48}$$

$$v_\pm=\sqrt{\frac{1}{2}\left(1-\frac{\Omega_\pm}{\omega_n}\right)} \tag{49}$$

for unitary states. In these states, $\Delta_{l,\pm}$ is independent of l because $\mathbf{q}=0$. For nonunitary states,[31] we use

$$\begin{pmatrix}\hat{u}_\pm^e\\ \hat{v}_\pm^e\end{pmatrix}=\begin{pmatrix}\hat{u}_\pm\\ \hat{\Delta}_\pm^\dagger\hat{v}_\pm\end{pmatrix}, \tag{50}$$

$$\begin{pmatrix}\hat{u}_\pm^h\\ \hat{v}_\pm^h\end{pmatrix}=\begin{pmatrix}\hat{\Delta}_\pm\hat{v}_\pm^*\\ \hat{u}_\pm^*\end{pmatrix}, \tag{51}$$

$$\hat{u}_\pm=Q_\pm\sum_{l=1}^{2} u_{l,\pm}\hat{S}_{l,\pm}, \tag{52}$$

$$\hat{v}_\pm=Q_\pm\sum_{l=1}^{2} v_{l,\pm}\frac{\hat{S}_{l,\pm}}{|\Delta_{l,\pm}|}, \tag{53}$$

$$u_{l,\pm}=\sqrt{\frac{1}{2}\left(1+\frac{\Omega_{l,\pm}}{\omega_n}\right)}, \tag{54}$$

$$v_{l,\pm}=\sqrt{\frac{1}{2}\left(1-\frac{\Omega_{l,\pm}}{\omega_n}\right)}, \tag{55}$$

$$(Q_\pm)^{-2}=8|\mathbf{q}_\pm|(|\mathbf{q}_\pm|+q_{3,\pm}), \tag{56}$$

$$\hat{S}_{l,\pm}=\hat{P}_{l,\pm}\cdot\hat{t}_l, \tag{57}$$

$$\hat{P}_{1,\pm}=|\mathbf{q}_\pm|\hat{\sigma}_0+\mathbf{q}_\pm\cdot\hat{\boldsymbol{\sigma}}, \tag{58}$$

$$\hat{P}_{2,\pm}=|\mathbf{q}_\pm|\hat{\sigma}_0-\mathbf{q}_\pm\cdot\hat{\boldsymbol{\sigma}}, \tag{59}$$

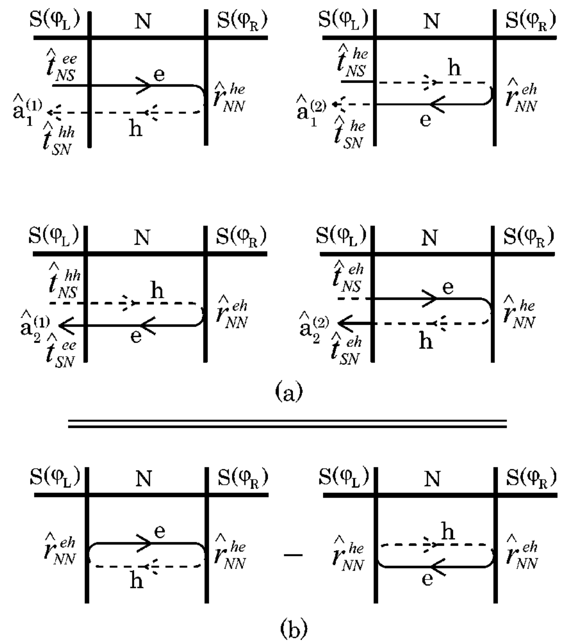

FIG. 2. The reflection coefficients in (a) contribute to the Josephson current. In this paper, we neglect the higher-order processes involving multiple Andreev reflection more than twice. The Josephson current calculated from the four reflection processes in (a) is summarized in the reflection processes in (b).

$$\hat{t}_1=\hat{\sigma}_0+\hat{\sigma}_3, \tag{60}$$

$$\hat{t}_2=\hat{\sigma}_0-\hat{\sigma}_3. \tag{61}$$

In this paper, we consider the four reflection processes to calculate $\hat{a}_1$ and $\hat{a}_2$ as shown in Fig. 2(a) and neglect the higher-order terms. This approximation is justified when the potential barrier at the NS interfaces is large enough and/or the transmission probability in the normal segment is low enough. Thus we consider the insulators and the dirty normal metals in the normal segment. In order to estimate $\hat{a}_1$ and $\hat{a}_2$, we calculate the transmission and the reflection coefficients at the single NS interface for fixed $\mathbf{p}$ as shown in Appendix A. The 64 coefficients are obtained from the continuity condition of the wave function at the NS interface since there are eight incoming and eight outgoing channels for each $\mathbf{p}$. The ARC's in Fig. 2(a) are given by

$$\hat{a}_1^{(1)}(\mathbf{p})=\sum_{\mathbf{p}'}\hat{t}_{SN}^{hh}(\mathbf{p},L)\cdot\hat{t}_{\mathbf{p},\mathbf{p}'}^{h} \times\hat{r}_{NN}^{he}(\mathbf{p}',R)\cdot\hat{t}_{\mathbf{p}',\mathbf{p}}^{e}\cdot\hat{t}_{NS}^{ee}(\mathbf{p},L), \tag{62}$$

$$\hat{a}_1^{(2)}(\mathbf{p})=\sum_{\mathbf{p}'}\hat{t}_{SN}^{he}(\mathbf{p},L)\cdot\hat{t}_{\mathbf{p},\mathbf{p}'}^{e} \times\hat{r}_{NN}^{eh}(\mathbf{p}',R)\cdot\hat{t}_{\mathbf{p}',\mathbf{p}}^{h}\cdot\hat{t}_{NS}^{he}(\mathbf{p},L), \tag{63}$$

$$\hat{a}_2^{(1)}(\mathbf{p})=\sum_{\mathbf{p}'} \hat{t}_{SN}^{ee}(\mathbf{p},L)\cdot\hat{t}^e_{\mathbf{p},\mathbf{p}} \times\hat{r}_{NN}^{eh}(\mathbf{p}',R)\cdot\hat{t}^h_{\mathbf{p}',\mathbf{p}}\cdot\hat{t}_{NS}^{hh}(\mathbf{p},L), \tag{64}$$

$$\hat{a}_2^{(2)}(\mathbf{p})=\sum_{\mathbf{p}'} \hat{t}_{SN}^{eh}(\mathbf{p},L)\cdot\hat{t}^h_{\mathbf{p},\mathbf{p}'}\cdot\hat{r}_{NN}^{he}(\mathbf{p}',R)\cdot\hat{t}^e_{\mathbf{p}',\mathbf{p}}\cdot\hat{t}_{NS}^{eh}(\mathbf{p},L), \tag{65}$$

where $\hat{t}^{e(h)}_{\mathbf{p}',\mathbf{p}}$ is the transmission coefficient of the electronlike (holelike) quasiparticle in the normal conductor, and $\mathbf{p}'$ indicates the propagating channel at the right NS interface. The transmission coefficients in the normal metal are described by

$$\hat{t}^e_{\mathbf{p}',\mathbf{p}}=iv_{\mathbf{p}}e^{-ik_z'L_N}\int d\boldsymbol{\rho}\int d\boldsymbol{\rho}' \times\hat{\mathcal{G}}^{N,e}_{\omega_n}(\boldsymbol{\rho}',L_N;\boldsymbol{\rho},0)\chi^*_{\mathbf{p}'}(\boldsymbol{\rho}')\chi_{\mathbf{p}}(\boldsymbol{\rho}), \tag{66}$$

$$\hat{t}^h_{\mathbf{p},\mathbf{p}'}=iv_{\mathbf{p}'}e^{ik_z'L_N}\int d\boldsymbol{\rho}\int d\boldsymbol{\rho}' \times\hat{\mathcal{G}}^{N,h}_{\omega_n}(\boldsymbol{\rho},0;\boldsymbol{\rho}',L_N)\chi^*_{\mathbf{p}}(\boldsymbol{\rho})\chi_{\mathbf{p}'}(\boldsymbol{\rho}'), \tag{67}$$

where $\hat{\mathcal{G}}^{N,e(h)}_{\omega_n}(\mathbf{r},\mathbf{r}')$ is the Green function of the normal conductor in the electron (hole) branch and $v_{\mathbf{p}}$ is the velocity in the z direction of a quasiparticle belonging to the propagating channel $\mathbf{p}$.[36] We assume that the NS interface is sufficiently clean so that $\mathbf{p}$ is conserved while at the transmission and the reflection at the interface. In $\hat{a}_1^{(1)}$ in Eq. (62), a quasiparticle wave is initially incident into the normal segment from the left superconductor through the channel specified by $\mathbf{p}$. After the Andreev reflection at the right NS interface, we assume that the reflected wave transmits to the left superconductor through the initial channel of $\mathbf{p}$ because of the retroactive property of a quasiparticle under time-reversal symmetry in the normal segment.[37] The two ARC's in Eq. (45) are given by $\hat{a}_1=\hat{a}_1^{(1)}+\hat{a}_1^{(2)}$ and $\hat{a}_2=\hat{a}_2^{(1)}+\hat{a}_2^{(2)}$, respectively.

By using Eqs. (45) and (62)–(65), we can derive the general expression of the Josephson current,

$$J=ie\sum_{\mathbf{p}}\sum_{\mathbf{p}'}T\sum_{\omega_n}\mathrm{Tr}[\hat{r}_{NN}^{eh}(\mathbf{p},L)\cdot\hat{t}^h_{\mathbf{p},\mathbf{p}'}\cdot\hat{r}_{NN}^{he}(\mathbf{p}',R)\cdot\hat{t}^e_{\mathbf{p}',\mathbf{p}} -\hat{r}_{NN}^{he}(\mathbf{p},L)\cdot\hat{t}^e_{\mathbf{p},\mathbf{p}'}\cdot\hat{r}_{NN}^{eh}(\mathbf{p}',R)\cdot\hat{t}^h_{\mathbf{p}',\mathbf{p}}], \tag{68}$$

without further approximations. The reflection processes in Eq. (68) are summarized in Fig. 2(b). Since the relations

$$\hat{t}^e_{-\mathbf{p},-\mathbf{p}'}=\{\hat{t}^h_{\mathbf{p},\mathbf{p}'}\}^*, \tag{69}$$

$$\hat{t}^h_{-\mathbf{p}',-\mathbf{p}}=\{\hat{t}^e_{\mathbf{p}',\mathbf{p}}\}^*, \tag{70}$$

$$\hat{r}_{NN}^{eh}(-\mathbf{p},R)=\{\hat{r}_{NN}^{he}(\mathbf{p},R)\}^*, \tag{71}$$

$$\hat{r}_{NN}^{he}(-\mathbf{p},L)=\{\hat{r}_{NN}^{eh}(\mathbf{p},L)\}^* \tag{72}$$

are satisfied (see Appendices A and B), the Josephson current results in

$$J=-2e\,\mathrm{Im}\sum_{\mathbf{p}}\sum_{\mathbf{p}'}T\sum_{\omega_n}\mathrm{Tr} \times[\hat{r}_{NN}^{eh}(\mathbf{p},L)\cdot\hat{t}^h_{\mathbf{p},\mathbf{p}'}\cdot\hat{r}_{NN}^{he}(\mathbf{p}',R)\cdot\hat{t}^e_{\mathbf{p}',\mathbf{p}}]. \tag{73}$$

The formula in Eq. (73) can be applied to various Josephson junctions. For instance, it is possible to calculate the Josephson current in clean SIS junctions by substituting $\hat{t}^{e(h)}_{\mathbf{p},\mathbf{p}'}\propto\delta_{\mathbf{p},\mathbf{p}'}\hat{\sigma}_0$. We also note that the two superconductors are not necessarily identical to each other.

III. JOSEPHSON CURRENT FORMULA II

Since Josephson current is described by the ARC's at the NS interface in Eq. (73),we represent ARC's of the spin-singlet, the spin-triplet unitary, and the spin-triplet nonunitary superconductors in the following.

When the superconductor has spin-singlet Cooper pairs, the coefficients are given by

$$\hat{r}_{NN}^{eh}(\mathbf{p},L)=-i\hat{\Gamma}_{su}(\mathbf{p},L)e^{i\varphi_L}, \tag{74}$$

$$\hat{r}_{NN}^{he}(\mathbf{p},R)=-i\hat{\Gamma}^\dagger_{su}(\mathbf{p},R)e^{-i\varphi_R}, \tag{75}$$

$$\hat{\Gamma}_{su}(\mathbf{p},j)=i\Gamma_{su}(\mathbf{p},j)\hat{\sigma}_2, \tag{76}$$

$$\Gamma_{su}(\mathbf{p},j)=\frac{\overline{k_z^2}K_+d_{0,-}}{\Xi_{su}}\Big|_j, \tag{77}$$

$$\Xi_{su}=(H^2+\overline{k_z^2})d_{0,+}d_{0,-}+H^2K_+K_-, \tag{78}$$

$$K_\pm=\Omega_\pm-|\omega_n|, \tag{79}$$

$$\overline{k_z}=k_z/k_F, \tag{80}$$

where $H=mV_b/k_F$ represents the potential barrier height at the NS interface and $j=L$ or R symbolically denote the character of the superconductors such as the symmetry of the pair potential and orientation angle.

When the superconductor is in spin-triplet unitary states, the ARC's are given by

$$\hat{r}_{NN}^{eh}(\mathbf{p},L)=-i\hat{\Gamma}_{tu}(\mathbf{p},L)e^{i\varphi_L}, \tag{81}$$

$$\hat{r}_{NN}^{he}(\mathbf{p},R)=-i\hat{\Gamma}^\dagger_{tu}(\mathbf{p},R)e^{-i\varphi_R}, \tag{82}$$

$$\hat{\Gamma}_{tu}(\mathbf{p},j)=i\boldsymbol{\Gamma}_{tu}(\mathbf{p},j)\cdot\hat{\boldsymbol{\sigma}}\hat{\sigma}_2, \tag{83}$$

$$\boldsymbol{\Gamma}_{tu}(\mathbf{p},j)=\overline{k_z^2}K_+\frac{\Xi_{tu}\mathbf{d}_- -(H^2+\overline{k_z^2})(\mathbf{d}_+^*\times\mathbf{d}_-)\times\mathbf{d}_-}{\Xi_{tu}^2-\mathbf{D}_{tu}\cdot\mathbf{D}_{tu}}\Bigg|_j, \tag{84}$$

$$\Xi_{tu}=(H^2+\overline{k_z^2})\mathbf{d}_+^*\cdot\mathbf{d}_-+H^2K_+K_-, \tag{85}$$

$$\mathbf{D}_{tu}=-i(H^2+\overline{k_z^2})(\mathbf{d}_+^*\times\mathbf{d}_-). \tag{86}$$

In the unitary states, $\mathbf{d}_\pm$ often has a single component. In such a case, we find

$$\mathbf{\Gamma}_{tu}(\mathbf{p},j)=\overline{k_z^2}K_+\frac{\mathbf{d}_-}{\Xi_{tu}}\bigg|_j, \tag{87}$$

because $\mathbf{d}_+^*\times\mathbf{d}_-=0$.

Finally we show the ARC's in the nonunitary states,

$$\hat{r}_{NN}^{eh}(\mathbf{p},L)=-i\hat{\Gamma}_{nu}(\mathbf{p},L)e^{i\varphi_L}, \tag{88}$$

$$\hat{r}_{NN}^{eh}(\mathbf{p},R)=-i\hat{\Gamma}_{nu}^\dagger(\mathbf{p},R)e^{-i\varphi_R}, \tag{89}$$

$$\hat{\Gamma}_{nu}=i\mathbf{\Gamma}_{nu}(\mathbf{p},j)\cdot\hat{\boldsymbol{\sigma}}\hat{\sigma}_2, \tag{90}$$

$$\mathbf{\Gamma}_{nu}(\mathbf{p},j)=\overline{k_z^2}\frac{\mathbf{D}_{nu}}{\mathbf{D}_{nu}\cdot\mathbf{D}_{nu}}\bigg|_j, \tag{91}$$

$$\mathbf{D}_{nu}=(H^2+\overline{k_z^2})\left\{\frac{1}{2}\sum_l\frac{\mathbf{D}_{l,+}}{K_{l,+}}\right\}+H^2\left\{\frac{1}{2}\sum_l\frac{K_{l,-}\mathbf{D}_{l,+}}{|\Delta_{l,-}|^2}\right\}, \tag{92}$$

$$\mathbf{D}_{1,\pm}=\mathbf{d}_\pm^*+i\frac{\mathbf{d}_\pm^*\times\mathbf{q}_\pm}{|\mathbf{q}_\pm|}, \tag{93}$$

$$\mathbf{D}_{2,\pm}=\mathbf{d}_\pm^*-i\frac{\mathbf{d}_\pm^*\times\mathbf{q}_\pm}{|\mathbf{q}_\pm|}, \tag{94}$$

$$K_{l,\pm}=\Omega_{l,\pm}-|\omega_n|. \tag{95}$$

The detail of the calculation is shown in Appendix A. The expression of the ARC's in nonunitary states is rather more complicated than that in the unitary states. But if the relations

$$\mathbf{d}=\mathbf{d}_+=\nu\mathbf{d}_-, \tag{96}$$

$$\nu=1 \quad \text{or} \quad -1 \tag{97}$$

are satisfied, the ARC can be reduced to a simple expression

$$\hat{r}_{NN}^{eh}(\mathbf{p},L)=-i\left\{\frac{\overline{k_z^2}}{2|\mathbf{q}|}\sum_{l=1}^{2}\frac{\hat{P}_l}{\Xi_{nu}(l)}\right\}\hat{\Delta}\bigg|_L e^{i\varphi_L}, \tag{98}$$

$$\hat{r}_{NN}^{he}(\mathbf{p},R)=-i\hat{\Delta}^\dagger\left\{\frac{\overline{k_z^2}}{2|\mathbf{q}|}\sum_{l=1}^{2}\frac{\hat{P}_l}{\Xi_{nu}(l)}\right\}\bigg|_R e^{-i\varphi_R}, \tag{99}$$

$$\Xi_{nu}(l)=H^2\{(1-\nu)|\omega_n|+(1+\nu)\Omega_l\}+\overline{k_z^2}(|\omega_n|+\Omega_l). \tag{100}$$

The effects of the ZES's on the ARC's can be easily confirmed in Eqs. (78), (85), and (100). For instance in Eq. (100), we find in the limit of $H>>1$ and $\omega_n\to 0$,

$$\Xi_{nu}(l)\to\begin{cases}2H^2|\Delta_l| & (\nu=1),\\ \overline{k_z^2}|\Delta_l| & (\nu=-1).\end{cases} \tag{101}$$

In the absence of the ZES ($\nu=1$), the reflection coefficients are proportional to $1/H^2$. On the other hand in the presence of the ZES ($\nu=-1$), the reflection coefficients are independent of the barrier height. Thus the ARC describes the low-temperature anomaly of the Josephson current.

In the normal metal, the Green function in Eqs. (66) and (67) satisfy a relation as shown in Appendix B,

$$\mathcal{G}_{\omega_n}^{N,h}(\mathbf{r}',\mathbf{r})=-\hat{\sigma}_2[\hat{\mathcal{G}}_{\omega_n}^{N,e}(\mathbf{r},\mathbf{r}')]^\dagger\hat{\sigma}_2, \tag{102}$$

because of time-reversal symmetry. The transmission coefficients can be parametrized by

$$\begin{aligned}\hat{\tau}(\mathbf{p}',\mathbf{p})&\equiv\tau_0(\mathbf{p}',\mathbf{p})\hat{\sigma}_0+\boldsymbol{\tau}(\mathbf{p}',\mathbf{p})\cdot\hat{\boldsymbol{\sigma}}\\ &=\sqrt{v_{\mathbf{p}}v_{\mathbf{p}'}}\int d\boldsymbol{\rho}\int d\boldsymbol{\rho}'\chi_{\mathbf{p}'}^*(\boldsymbol{\rho}')\chi_{\mathbf{p}}(\boldsymbol{\rho})\\ &\quad\times\hat{\mathcal{G}}_{\omega_n}^{N,e}(\boldsymbol{\rho}',L_N;\boldsymbol{\rho},0).\end{aligned} \tag{103}$$

Since the amplitude of the spin-orbit scattering is much smaller than that of the spin-independent transmission probability, we assume that

$$|\tau_0|\gg|\boldsymbol{\tau}|. \tag{104}$$

The conductance of the normal metal at $T=0$ is given by

$$\begin{aligned}G_N&=\lim_{\omega_n\to 0}\frac{e^2}{h}\mathrm{Tr}\sum_{\mathbf{p},\mathbf{p}'}\hat{\tau}(\mathbf{p}',\mathbf{p})\hat{\tau}^\dagger(\mathbf{p}',\mathbf{p})\\ &\simeq\lim_{\omega_n\to 0}\frac{2e^2}{h}\sum_{\mathbf{p},\mathbf{p}'}|\tau_0(\mathbf{p}',\mathbf{p})|^2.\end{aligned} \tag{105}$$

By using Eq. (103), the Josephson current is rewritten to be

$$\begin{aligned}J=&-2e\,\mathrm{Im}\sum_{\mathbf{p}}\sum_{\mathbf{p}'}T\sum_{\omega_n}\mathrm{Tr}[\hat{r}_{NN}^{eh}(\mathbf{p},L)\cdot\hat{\sigma}_2\{\tau_0^*\hat{\sigma}_0+\boldsymbol{\tau}^*\cdot\hat{\boldsymbol{\sigma}}\}\\ &\times\hat{\sigma}_2\cdot\hat{r}_{NN}^{he}(\mathbf{p}',R)\cdot\{\tau_0\hat{\sigma}_0+\boldsymbol{\tau}\cdot\hat{\boldsymbol{\sigma}}\}].\end{aligned} \tag{106}$$

At first we consider the Josephson junction where the two superconductors have spin-singlet Cooper pairs. The Josephson current is given by

$$J_{SS}=4e\sin\varphi T\sum_{\omega_n}\sum_{\mathbf{p},\mathbf{p}'}\Gamma_{su}(\mathbf{p}',R)|\tau_0(\mathbf{p}',\mathbf{p})|^2\Gamma_{su}(\mathbf{p},L), \tag{107}$$

where $\varphi=\varphi_L-\varphi_R$.

Second, we consider the junction where the spin-triplet and spin-singlet superconductors are on the left- and right-hand sides, respectively. The Josephson current results in

$$J_{TS}=4eT\sum_{\omega_n}\sum_{\mathbf{p},\mathbf{p}'}\mathrm{Im}[e^{i\varphi}\Gamma_{su}(\mathbf{p}',R)\mathbf{W}(\mathbf{p}',\mathbf{p})\cdot\mathbf{\Gamma}_t(\mathbf{p},L)], \tag{108}$$

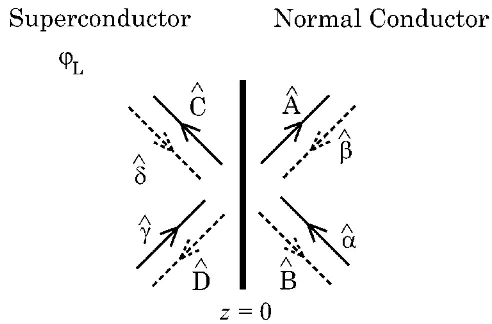

FIG. 3. The transmission and the reflection coefficients at the left NS interface. There are eight incoming and outgoing channels.

$$\mathbf{W}(\mathbf{p}',\mathbf{p})=(\tau_0^*\boldsymbol{\tau}+\tau_0\boldsymbol{\tau}^*+i\boldsymbol{\tau}^*\times\boldsymbol{\tau})(\mathbf{p}',\mathbf{p}), \qquad (109)$$

where $\boldsymbol{\Gamma}_t$ represents $\boldsymbol{\Gamma}_{tu}$ in Eq. (84) or $\boldsymbol{\Gamma}_{nu}$ in Eq. (91). As shown in Eq. (109), the J_{TS} vanishes when the spin-orbit scattering does not occur in the normal metal.[29–31]

Finally when the two superconductors have spin-triplet Cooper pairs, the Josephson current can be expressed to be

$$J_{TT}=4eT\sum_{\omega_n}\sum_{\mathbf{p},\mathbf{p}'}\mathrm{Im}[e^{i\varphi}\boldsymbol{\Gamma}_t(\mathbf{p},L)\cdot\boldsymbol{\Gamma}_t^*(\mathbf{p}',R)|\tau_0(\mathbf{p}',\mathbf{p})|^2]. \qquad (110)$$

The obtained formulas in Eqs. (107), (108), and (110) are essentially the same as those in the previous results.[31] However in the presence of the ZES's at the NS interfaces, the dependence of the Josephson current on the temperature is drastically different from that in the previous results. The ARC's ($\boldsymbol{\Gamma}_{su}$, $\boldsymbol{\Gamma}_{tu}$, and $\boldsymbol{\Gamma}_{nu}$) describe the low-temperature anomaly of the Josephson current in the SNS junctions of the anisotropic superconductors.

IV. CONCLUSION

We derive a formula of the dc Josephson current between the two anisotropic superconductors based on mean-field theory of superconductivity. The Josephson current is expressed by the Andreev reflection coefficients at the junction interfaces. The contribution of the zero-energy bound states formed at the NS interfaces to the Josephson current is taken into account. The formula can be applied to SIS and SNS junctions of the anisotropic superconductors with spin-singlet and spin-triplet Cooper pairs.

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APPENDIX A: TRANSMISSION AND REFLECTION COEFFICIENTS AT THE NS INTERFACE

We derive the transmission and reflection coefficients of the NS interface at $z=0$ for a spin-triplet nonunitary superconductor as shown in Fig. 3. In what follows, we calculate the coefficients after analytic continuation ($E\to i\omega_n$) for $\omega_n>0$. In the normal metal, the wave function of the quasiparticle can be described by

$$\boldsymbol{\Psi}_{\mathbf{p}}^N(\boldsymbol{\rho},z)=\left[\begin{pmatrix}\hat{\alpha}\\ \hat{0}\end{pmatrix}e^{-ik_zz}+\begin{pmatrix}\hat{0}\\ \hat{\beta}\end{pmatrix}e^{ik_zz}+\begin{pmatrix}\hat{A}\\ \hat{0}\end{pmatrix}e^{ik_zz}+\begin{pmatrix}\hat{0}\\ \hat{B}\end{pmatrix}e^{-ik_zz}\right]\chi_{\mathbf{p}}(\boldsymbol{\rho}), \qquad (A1)$$

where $\hat{\alpha}$ and $\hat{\beta}$ ($\hat{A}$ and $\hat{B}$) are the amplitudes of the incoming (outgoing) waves in the electron and hole channels, respectively. In the same way, the wave function in the superconductor is given by

$$\boldsymbol{\Psi}_{\mathbf{p}}^S(\boldsymbol{\rho},z)=\check{\Phi}_L\left[\begin{pmatrix}\hat{u}_+\\ \hat{\Delta}_+^\dagger\hat{v}_+\end{pmatrix}e^{ik_zz}\hat{\gamma}+\begin{pmatrix}\hat{\Delta}_-\hat{v}_-^*\\ \hat{u}_-^*\end{pmatrix}e^{-ik_zz}\hat{\delta}+\begin{pmatrix}\hat{u}_-\\ \hat{\Delta}_-^\dagger\hat{v}_-\end{pmatrix}e^{-ik_zz}\hat{C}+\begin{pmatrix}\hat{\Delta}_+\hat{v}_+^*\\ \hat{u}_+^*\end{pmatrix}e^{ik_zz}\hat{D}\right]\chi_{\mathbf{p}}(\boldsymbol{\rho}), \qquad (A2)$$

where $\hat{\gamma}$ and $\hat{\delta}$ ($\hat{C}$ and $\hat{D}$) are the amplitudes of incoming (outgoing) waves in the electron and hole channels, respectively.

The two wave functions satisfy the continuity condition at the left NS interface (i.e., $z=0$),

$$\boldsymbol{\Psi}_{\mathbf{p}}^N(\boldsymbol{\rho},0)=\boldsymbol{\Psi}_{\mathbf{p}}^S(\boldsymbol{\rho},0), \qquad (A3)$$

$$\left.\frac{\partial}{\partial z}\boldsymbol{\Psi}_{\mathbf{p}}^N(\boldsymbol{\rho},z)\right|_{z=0}-2mV_b\boldsymbol{\Psi}_{\mathbf{p}}^N(\boldsymbol{\rho},0)=\left.\frac{\partial}{\partial z}\boldsymbol{\Psi}_{\mathbf{p}}^S(\boldsymbol{\rho},z)\right|_{z=0}. \qquad (A4)$$

From Eqs. (A3) and (A4), we obtain the transmission and reflection coefficients,

$$\hat{t}_{SN}^{ee}(\mathbf{p},L)=\overline{k_z}\,\boldsymbol{\kappa}\,\hat{u}_-^{-1}\,\hat{Z}_2^\dagger\,\hat{\xi}_{2,+}^\dagger\,e^{-i\varphi_L/2}, \qquad (A5)$$

$$\hat{t}_{SN}^{eh}(\mathbf{p},L)=\overline{k_z}\,H\,\hat{u}_-^{-1}\,\hat{Z}_2^\dagger\,e^{i\varphi_L/2}, \qquad (A6)$$

$$\hat{t}_{SN}^{he}(\mathbf{p},L)=-\overline{k_z}\,H(\hat{u}_+^*)^{-1}\,\hat{Z}_1\,e^{-i\varphi_L/2}, \qquad (A7)$$

$$\hat{t}_{SN}^{hh}(\mathbf{p},L)=\overline{k_z}\,\boldsymbol{\kappa}^*(\hat{u}_+^*)^{-1}\,\hat{Z}_1\hat{\xi}_{2,-}\,e^{i\varphi_L/2}, \qquad (A8)$$

$$\hat{r}_{SS}^{eh}(\mathbf{p},L)=-\hat{u}_-^{-1}\hat{\Delta}_-\,\hat{v}_-^*-i\frac{H^2}{\omega_n}\hat{u}_-^{-1}\,\hat{Z}_2^\dagger(\hat{u}_-^t)^{-1}\hat{\Omega}_-, \qquad (A9)$$

$$\hat{r}_{SS}^{he}(\mathbf{p},L)=-(\hat{u}_+^*)^{-1}\hat{\Delta}_+^\dagger\hat{v}_+-i\frac{H^2}{\omega_n}(\hat{u}_+^*)^{-1}\hat{Z}_1(\hat{u}_-^\dagger)^{-1}\hat{\Omega}_+, \qquad (A10)$$

$$\hat{r}_{NN}^{eh}(\mathbf{p},L)=-i\overline{k_z^2}\,\hat{Z}_2^\dagger\,e^{i\varphi_L}, \qquad (A11)$$

$$\hat{r}_{NN}^{he}(\mathbf{p},L)=-i\overline{k_z^2}\,\hat{Z}_1\,e^{-i\varphi_L}, \qquad (A12)$$

$$\hat{t}_{NS}^{ee}(\mathbf{p},L)=\frac{\overline{k_z\kappa}}{\omega_n}\hat{Z}_2^{\dagger}\,\hat{\xi}_{2,+}^{\dagger}\,(\hat{u}_+^{\dagger})^{-1}\hat{\Omega}_+\,e^{i\varphi_L/2}, \tag{A13}$$

$$\hat{t}_{NS}^{eh}(\mathbf{p},L)=-\frac{\overline{k_z H}}{\omega_n}\hat{Z}_2^{\dagger}\,(\hat{u}_-^{t})^{-1}\hat{\Omega}_-\,e^{i\varphi_L/2}, \tag{A14}$$

$$\hat{t}_{NS}^{he}(\mathbf{p},L)=\frac{\overline{k_z H}}{\omega_n}\hat{Z}_1(\hat{u}_+^{\dagger})^{-1}\hat{\Omega}_+\,e^{-i\varphi_L/2}, \tag{A15}$$

$$\hat{t}_{NS}^{hh}(\mathbf{p},L)=\frac{\overline{k_z\kappa^*}}{\omega_n}\hat{Z}_1\,\hat{\xi}_{2,-}(\hat{u}_+^{t})^{-1}\hat{\Omega}_-\,e^{-i\varphi_L/2}, \tag{A16}$$

$$\hat{r}_{NN}^{he}(-\mathbf{p},L)=[\hat{r}_{NN}^{eh}(\mathbf{p},L)]^*. \tag{A17}$$

Here we define

$$\hat{\xi}_{1,\pm}=\left(\frac{1}{2|\mathbf{q}_\pm|}\sum_{l=1}^{2}\frac{K_{l,\pm}}{|\Delta_{l,\pm}|^2}\hat{P}_{l,\pm}\right)\hat{\Delta}_\pm, \tag{A18}$$

$$\hat{\xi}_{2,\pm}=\left(\frac{1}{2|\mathbf{q}_\pm|}\sum_{l=1}^{2}\frac{\hat{P}_{l,\pm}}{K_{l,\pm}}\right)\hat{\Delta}_\pm, \tag{A19}$$

$$\hat{Z}_1=[H^2\hat{\xi}_{1,+}+|\kappa|^2\hat{\xi}_{2,-}]^{-1}, \tag{A20}$$

$$\hat{Z}_2=[H^2\hat{\xi}_{1,-}+|\kappa|^2\hat{\xi}_{2,+}]^{-1}, \tag{A21}$$

$$\kappa=\overline{k_z}+iH. \tag{A22}$$

In the same way, the ARC's at the right NS interface are given by

$$\hat{r}_{NN}^{he}(\mathbf{p},R)=-i\overline{k_z^2}\,\hat{Z}_2\,e^{-i\varphi_R}, \tag{A23}$$

$$\hat{r}_{NN}^{eh}(-\mathbf{p},R)=[\hat{r}_{NN}^{he}(\mathbf{p},R)]^*. \tag{A24}$$

On the derivation, we use identities,

$$\hat{S}_{l,\pm}^{\dagger}\cdot\hat{S}_{l',\pm}=\frac{\hat{t}_l}{2Q_\pm^2}\,\delta_{l,l'}, \tag{A25}$$

$$\hat{S}_{l,\pm}\cdot\hat{S}_{l',\pm}^{\dagger}=\frac{\hat{P}_{l,\pm}}{2|\mathbf{q}_\pm|Q_\pm^2}\,\delta_{l,l'}, \tag{A26}$$

$$\hat{P}_{l,\pm}\cdot\hat{P}_{l',\pm}=2|\mathbf{q}_\pm|\hat{P}_{l,\pm}\,\delta_{l,l'}, \tag{A27}$$

$$\hat{S}_{l,\pm}^{\dagger}\cdot\hat{\Delta}_\pm\cdot\hat{\Delta}_\pm^{\dagger}\cdot\hat{S}_{l',\pm}=\frac{|\Delta_{l,\pm}|^2}{2Q_\pm^2}\hat{t}_l\delta_{l,l'}, \tag{A28}$$

$$\hat{\Delta}_\pm^{\dagger}\cdot\hat{S}_{l,\pm}\cdot\hat{S}_{l',\pm}^{\dagger}\cdot\hat{\Delta}_\pm=\frac{|\Delta_{l,\pm}|^2}{2|\mathbf{q}_\pm|Q_\pm^2}\hat{P}_{l,\pm}^*\,\delta_{l,l'}, \tag{A29}$$

$$\hat{\Delta}_\pm\cdot\hat{P}_{l,\pm}^*=\hat{P}_{l,\pm}\cdot\hat{\Delta}_\pm, \tag{A30}$$

$$\hat{P}_{l,\pm}\cdot\hat{\Delta}_\pm\cdot\hat{\Delta}_\pm^{\dagger}=|\Delta_{l,\pm}|^2\hat{P}_{l,\pm}. \tag{A31}$$

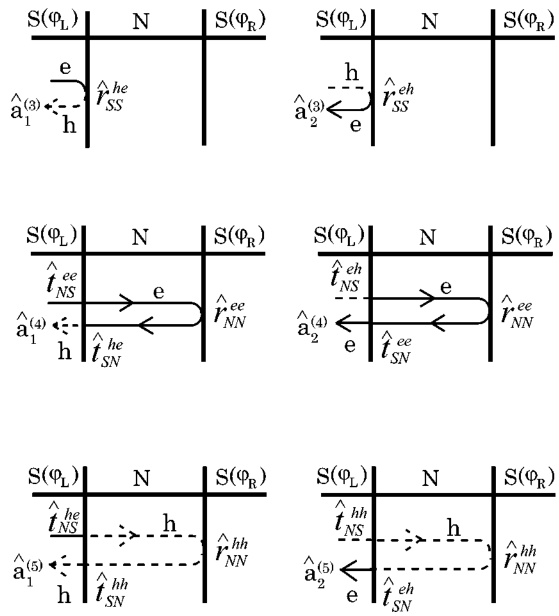

FIG. 4. The reflection processes included in the coefficients $\hat{a}_1$ and $\hat{a}_2$. These processes, however, do not contribute to the Josephson current.

The ARC's of superconductors in the unitary states can be calculated in the same way. The derivation of the transmission and reflection coefficients in the unitary states is much simpler than that in the nonunitary states.

In addition to the four reflection processes shown in Fig. 2(a), six reflection processes can be considered for $\hat{a}_1$ and $\hat{a}_2$ as shown in Fig. 4. By using the coefficients in Eqs. (A5)–(A16), it is possible to show that these six processes do not contribute to the Josephson current.

APPENDIX B: TRANSMISSION COEFFICIENTS IN THE NORMAL METAL

Since the amplitude of the pair potential in the normal metal is taken to be zero, the BdG equation in Eq. (10) is decoupled into the two equations,

$$\hat{h}_0(\mathbf{r})\hat{u}_\lambda=\hat{u}_\lambda\hat{E}_\lambda, \tag{B1}$$

$$-\hat{h}_0^*(\mathbf{r})\hat{v}_\lambda=\hat{v}_\lambda\hat{E}_\lambda. \tag{B2}$$

The Green function in the normal metal obeys the equation,

$$[i\omega_n\hat{\sigma}_0-\hat{h}_0(\mathbf{r})]\hat{\mathcal{G}}_{\omega_n}^{N,e}(\mathbf{r},\mathbf{r}')=\delta(\mathbf{r}-\mathbf{r}')\hat{\sigma}_0, \tag{B3}$$

$$[i\omega_n\hat{\sigma}_0+\hat{h}_0^*(\mathbf{r})]\hat{\mathcal{G}}_{\omega_n}^{N,h}(\mathbf{r},\mathbf{r}')=\delta(\mathbf{r}-\mathbf{r}')\hat{\sigma}_0. \tag{B4}$$

The Green function in the two branches is represented by

$$\hat{\mathcal{G}}_{\omega_n}^{N,e}(\mathbf{r},\mathbf{r}')=\sum_\lambda\,\hat{u}_\lambda(\mathbf{r})[i\omega_n\hat{\sigma}_0-\hat{E}_\lambda]^{-1}\hat{u}_\lambda^{\dagger}(\mathbf{r}'), \tag{B5}$$

$$\hat{\mathcal{G}}_{\omega_n}^{N,h}(\mathbf{r},\mathbf{r}')=-[\hat{\mathcal{G}}_{\omega_n}^{N,e}(\mathbf{r},\mathbf{r}')]^*, \tag{B6}$$

where we use the complex conjugate of Eq. (B1) for the Green function in the hole branch. By using Eqs. (66) and (67), we can show the relations

$$\hat{t}^{e}_{-\mathbf{p},-\mathbf{p}'}=[\hat{t}^{h}_{\mathbf{p},\mathbf{p}'}]^{*}, \tag{B7}$$

$$\hat{t}^{h}_{-\mathbf{p}',-\mathbf{p}}=[\hat{t}^{e}_{\mathbf{p}',\mathbf{p}}]^{*}. \tag{B8}$$

When time-reversal symmetry holds in the normal metal, we find

$$\hat{h}_0^{*}(\mathbf{r})i\hat{\sigma}_2\hat{u}_\lambda=i\hat{\sigma}_2\hat{u}_\lambda\hat{E}_\lambda. \tag{B9}$$

The Green function in the hole branch is described by that in the electron branch,

$$\hat{\mathcal{G}}^{N,h}_{\omega_n}(\mathbf{r}',\mathbf{r})=-\hat{\sigma}_2[\hat{\mathcal{G}}^{N,e}_{\omega_n}(\mathbf{r},\mathbf{r}')]^{\dagger}\hat{\sigma}_2. \tag{B10}$$

*Email address: asano@eng.hokudai.ac.jp

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