Local quasiparticle density of states in ferromagnet/superconductor nanostructures

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We study the proximity effect in superconductor/ferromagnet (S/F) systems and propose a detailed theoretical description of the damped-oscillatory behavior of the quasiparticle local density of states in a ferromagnet. It is demonstrated that impurities play a very important role in determining the amplitude and the shape of spatial and energy dependance of the density of states. Bearing in mind the possible comparison with experiments, we investigate different types of S/F structures as well as temperature variation of the local density of states.

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I. INTRODUCTION

Usually a superconducting transition occurs in a state where the phase of the superconducting order parameter is constant all over the system. There are only a few examples when the phase of the superconducting order parameter can vary. One of those examples is the Fulde and Ferrell,[1] and Larkin and Ovchinnikov[2] superconducting-state predicted more than thirty years ago. The authors[1,2] suggested that the superconducting order parameter may be modulated in real space by an exchange field acting on the electron spins. Until now, however, there are no unambiguous experimental evidences of this modulated-state formation. A superconductor (S) in contact with a ferromagnet[3–7] (F) presents another possibility when the modulation of the phase of the superconducting order parameter should appear. Indeed, in the case of an S/F bilayer, a Cooper pair entering a ferromagnet will feel the magnetic exchange field h and as a result, it will produce a phase modulation with the characteristic period of the order of ξ_f, the coherence length scale in a ferromagnet. In a clean ferromagnet $\xi_f \sim v_F/h$,[3] while in the dirty limit $\xi_f \sim \sqrt{l v_F/h}$,[4,5] where l is the electron mean-free path. It has been predicted that such a phase-modulation effect must result in the oscillatory behavior of the critical temperature T_c of S/F multilayers[4,5] and of the perpendicular critical current[3,6] as a function of F-layer thickness. These oscillations may be interpreted in terms of π-phase shift of the superconducting order parameter in adjacent S layers. Oscillations of the critical temperature in S/F multilayers have been found experimentally in Nb/Gd multilayers[8] and very recently the most direct proof of π-phase formation has been obtained by critical-current measurements in Nb/Cu-Ni/Nb Josephson junctions.[9]

The oscillatory damping of induced superconducting correlation in S/F systems leads also to the similar behavior of the local quasiparticle density of states. In the framework of diffusive approximation this effect has been predicted in Ref. 10 and found experimentally in Ref. 11, where the density of states induced in a ferromagnet by a S-layer has been measured by planar-tunneling spectroscopy. The theoretical analysis of the proximity effect in S/F structure in a quasi-ballistic regime made in Ref. 12 has also revealed the oscillatory behavior of the density of states as a function of F-layer thickness. Note that in this case the effective electronic mean-free path is of the order magnitude of the thickness of the F layer. The purpose of this work is to present the detailed analysis of the modification of density of states in a ferromagnetic layer due to the proximity effect. The general formalism is briefly overviewed in Sec. II. In Sec. III, we study the clean limit and the influence of weak-impurity scattering. The dirty limit is studied in Sec. IV, which allows us to describe recent experimental data.[11]

II. GENERAL FORMALISM

The very convenient set of equations describing inhomogeneous superconductivity has been elaborated by Eilenberger.[13] They are transportlike equations for the energy-integrated Green's functions f and g, assuming that relevant length scales are much larger than atomic length scales. For simplicity, we restrict ourselves to the case when all quantities depend only on one coordinate x, the distance from the interface. If we consider the Cooper pairing of electrons in the presence of the ferromagnetic exchange field h acting on the electron spins, the Eilenberger equations take the form (see for example Ref. 14)

$$\left(\omega + ih + \frac{1}{2\tau}\bar{g}(x,\omega)\right) f(x,\theta,\omega) + \frac{1}{2} v_F \cos\theta \frac{\partial f(x,\theta,\omega)}{\partial x} = \left(\boldsymbol{\Delta} + \frac{1}{2\tau}\bar{f}(x,\omega)\right) g(x,\theta,\omega),$$

$$\bar{g}(x,\omega) = \int \frac{d\Omega}{4\pi} g(x,\theta,\omega), \quad \bar{f}(x,\omega) = \int \frac{d\Omega}{4\pi} f(x,\theta,\omega),$$

$$f(x,\theta,\omega) f^{+}(x,\theta,\omega) + g^2(x,\theta,\omega) = 1, \tag{1}$$

where $f^{+} = f^{*}(v_F \to -v_F, h \to -h)$ and $\tau = l/v_F$, elastic-scattering time. The Eilenberger Green's functions f and g depend on Matsubara frequencies $\omega \to \omega_n = \pi T(2n+1)$ at temperature T, coordinate x, and on θ, the angle between the x axis and the direction of the Fermi velocities v_F. In the following, we will consider that Cooper pairing is always absent in the F layer and then the corresponding superconducting order parameter Δ is equal to zero in the ferromagnetic region. The geometry of the system we consider is presented in Fig. 1 (a superconducting electrode connected to

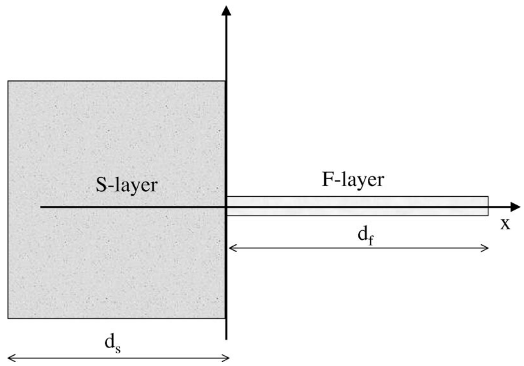

FIG. 1. Geometry of the S/F system.

a thin ferromagnetic rod) enables us to suppose that the proximity of the ferromagnetic metal affects only slightly the superconductivity in the S layer. In fact the similar situation is realized in the usual S/F bilayer geometry if the parameter γ, which characterizes the strength of the proximity effect[15] is small, see also Eq. (13) below. Thus, the superconducting order parameter in the S layer may be taken equal to its unperturbed value, $\mathbf{\Delta}=\text{const}$.

III. CLEAN LIMIT AND INFLUENCE OF WEAK-IMPURITY SCATTERING

Let us study first the density of states in ferromagnetic region in the limit $\tau\to\infty$. In this clean limit, the Eilenberger equations in a ferromagnet can be written as

$$(\omega+ih)f(x,\theta,\omega)+\frac{1}{2}v_{Fx}\frac{\partial f(x,\theta,\omega)}{\partial x}=0,$$

$$(\omega+ih)f^{+}(x,\theta,\omega)-\frac{1}{2}v_{Fx}\frac{\partial f^{+}(x,\theta,\omega)}{\partial x}=0, \tag{2}$$

$$f(x,\theta,\omega)f^{+}(x,\theta,\omega)+g^{2}(x,\theta,\omega)=1. \tag{3}$$

Supposing the length of the ferromagnetic rod to be much larger than the characteristic distance of S/F proximity effect, we may use the condition $f(x,\theta,\omega)\to 0$ as $x\to\infty$ and the condition of continuity of the Eilenberger Green functions at the S/F interface demonstrated in Ref. 16. Thus for a weak proximity effect, the solution of Eilenberger equations can be readily found in S and F regions (see for example Ref. 17) and for $v_{Fx}>0$ we have

$$\begin{array}{cc} x<0, & x>0, \\ f_0=\dfrac{\Delta}{\Omega}\left[1+\dfrac{\Omega-\omega}{\Omega+\omega}\exp\left(\dfrac{2\Omega x}{v_{Fx}}\right)\right], & f_0=\dfrac{2\Delta}{\Omega+\omega}\exp\left(-2\dfrac{\omega+ih}{v_{Fx}}x\right), \\ f_0^{+}=\dfrac{\Delta}{\Omega}\left[1-\exp\left(\dfrac{2\Omega x}{v_{Fx}}\right)\right], & f_0^{+}=0, \end{array} \tag{4}$$

while for $v_{Fx}<0$

$$\begin{array}{cc} x<0, & x>0, \\ f_0=\dfrac{\Delta}{\Omega}\left[1-\exp\left(\dfrac{-2\Omega x}{v_{Fx}}\right)\right], & f_0=0, \\ f_0^{+}=\dfrac{\Delta}{\Omega}\left[1+\dfrac{\Omega-\omega}{\Omega+\omega}\exp\left(\dfrac{-2\Omega x}{v_{Fx}}\right)\right], & f_0^{+}=\dfrac{2\Delta}{\Omega+\omega}\exp\left(2\dfrac{\omega+ih}{v_{Fx}}x\right), \end{array} \tag{5}$$

where $\Omega=\sqrt{\Delta^2+\omega^2}$. These results permit us to conclude that in the ferromagnet, the normal Eilenberger function $g(x,\theta,\omega)=\sqrt{1-f(x,\theta,\omega)f^{+}(x,\theta,\omega)}=\text{sgn}(\omega_n)$ does not have any dependance on the spatial coordinate x. Note that similar behavior has been reported for the pure S/N case treated in Ref. 18. Consequently, in pure limit, the density of states does not present any spatial variation (and even energy dependence) in spite of oscillating decaying behavior of f and f^{+} functions. This is a somewhat artificial situation because in reality the impurity scattering is always present and plays an important role in S/F proximity effect. To obtain the first-order correction (on the parameter $1/h\tau\ll 1$) to the density of states it is useful to use the following exact equation

$$v_{Fx}\frac{\partial}{\partial x}(ff^{+})=\tau^{-1}(\overline{f}f^{+}-\overline{f^{+}}f)g, \tag{6}$$

which can be readily derived from Eq. (1). Further on we may substitute on the right-hand side f and f^{+} by the solutions f_0 and f_0^{+} of the system of Eilenberger equations (2) in the clean limit and put $g=\text{sgn}(\omega_n)$. Finally, we obtain two first-order equations for different orientations of the Fermi velocities with respect to the S/F interface (and for positive Matsubara frequencies)

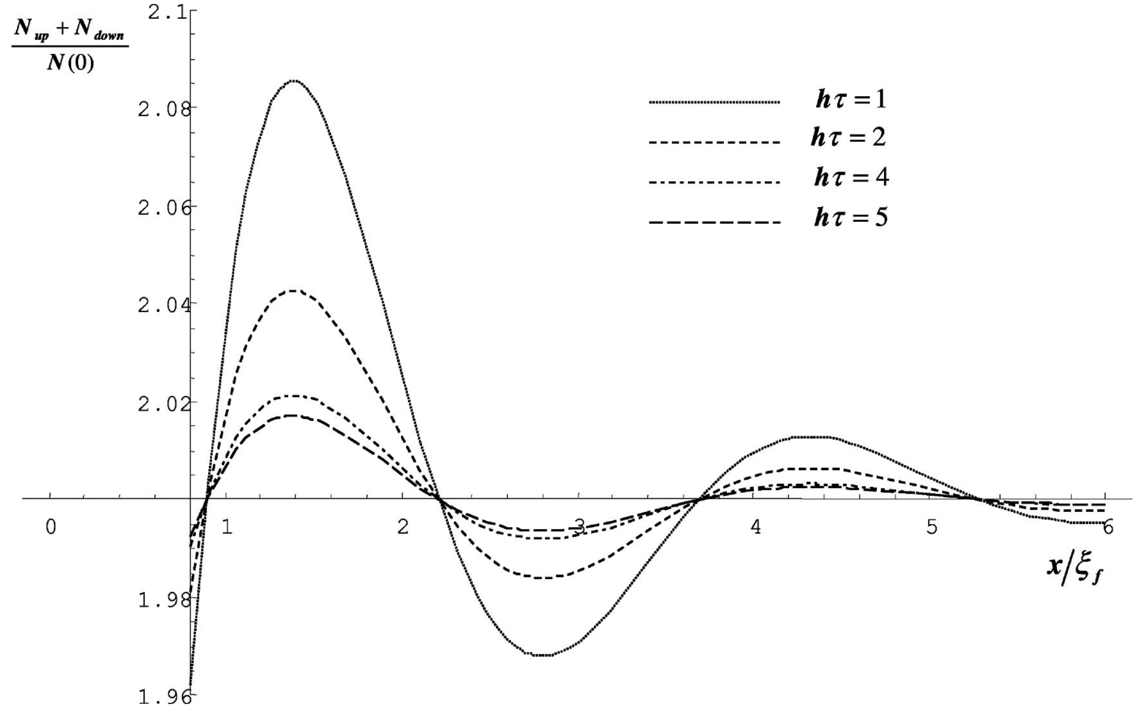

FIG. 2. Variation of the normalized density of states at low energy ($\omega=0.1h$) in the ferromagnet as a function of the normalized distance x/ξ_f to the S/F interface. The different curves correspond to different values of the parameter $h\tau$ ($h\tau=1, h\tau=2, h\tau=3, h\tau=4, h\tau=5$). The parameter $\tilde{\boldsymbol{\Delta}}=\boldsymbol{\Delta}/h$ is chosen to be 0.05.

$$v_{Fx}>0, \qquad\qquad v_{Fx}<0,$$

$$\tau v_{Fx}\frac{\partial}{\partial x}(ff^{+})=-\overline{f_0^{+}}f_0, \quad \tau v_{Fx}\frac{\partial}{\partial x}(ff^{+})=\overline{f_0}f_0^{+}. \tag{7}$$

Solving Eq. (7), we find the following analytical expression for the product ff^{+}

$$ff^{+}(x,\omega)=\frac{2\Delta^2\exp\left(-\frac{4(\omega+ih)x}{v_F}\right)}{\tau(\omega+ih)(\Omega+\omega)^2}\times\left[\frac{2(\omega+ih)x}{v_F}\mathrm{Ei}\left(\frac{2(\omega+ih)x}{v_F}\right)\times\exp\left(\frac{2(\omega+ih)x}{v_F}\right)-1\right]^2, \tag{8}$$

where Ei is the exponential-integral function $\mathrm{Ei}(x)=\int_1^{+\infty}[\exp(-xt)/t]dt$. Using the relation $g(x,\theta,\omega)=\sqrt{1-f(x,\theta,\omega)f^{+}(x,\theta,\omega)}$ and performing analytical continuation over Matsubara frequency $\omega_n\to i\omega$ we directly obtain the density of states for the spin-up orientation $N_{\uparrow}=\mathrm{Re}[g(x,\omega\to i\omega)]$. The density of states for spin-down orientation follows from the substitution $h\to -h$. We have plotted in Fig. 2 the total density of states as a function of the distance from the interface for different values of the parameter $h\tau$. We obviously see that when the cleanliness of the sample increases (increasing $h\tau$), the amplitude of the oscillations of the density of states goes to zero in accordance with the oscillations disappearance in the clean limit. We see that the predicted damped oscillations have a period of the order of magnitude of v_F/h, the coherence length in the ferromagnet in the clean limit. Note also that far away from the interface, the total density of states decreases as $\sin(x)/x^2$, compared to $\exp(-x)\sin(x)$ dependence in the dirty-limit case[10] and see Sec. IV.

IV. DIRTY LIMIT

The quasiclassical equations describing a dirty superconductor has been derived by Usadel[19] starting from the Eilenberger equations (1) and supposing that the electronic mean-free path of the electrons is short enough compared to the coherence length to produce an isotropization of movement of electrons. Previously, the density of states in the S/N bilayer in the dirty limit has been calculated in Ref. 20 using Usadel equations and proved to give a good description of relevant experimental data.[21] The analysis of the density-of-states oscillations in S/F system based on Usadel equations has been performed in Ref. 10. In this section we extend this approach taking into account the modification of diffusion coefficient due to relatively strong exchange field in ferromagnet ($h\gg T_c$). In case of strong impurity-scattering Eilenberger functions can be written as $f(x,\theta,\omega)=F(x,\omega)+f_1$ and $g(x,\theta,\omega)=G(x,\omega)+g_1$, where the angular dependence is present in f_1 and g_1 functions only, $G=\overline{g}$, $F=\overline{f}$, and the conditions $g_1\ll G$, $f_1\ll F$ hold. Similar to the standard derivation of Usadel equations, we may perform the averaging (1) over the angle $\theta[\int d(\cos\theta)]$

$$(\omega+ih)F+\frac{1}{2}\overline{v_F\cos\theta\frac{\partial f_1}{\partial x}}=\boldsymbol{\Delta}G. \tag{9}$$

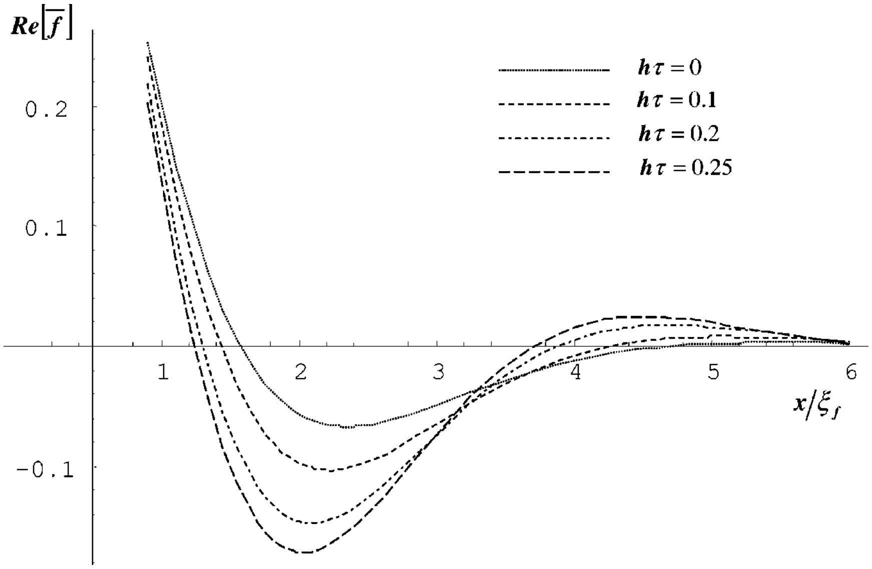

FIG. 3. Variation of the real part of the anomalous Green function $\overline{f}$ at zero energy as a function of the normalized distance to the S/F interface. The different curves are obtained for different values of the parameter $h\tau$ ($h\tau=0, h\tau=0.1, h\tau=0.2, h\tau=0.25$).

After multiplying Eq. (1) by $v_F \cos\theta$ and averaging once again over angle we obtain

$$\left(\omega+ih+\frac{G}{2\tau}\right)\overline{v_F\cos\theta f_1}+\frac{1}{6}v_F^2\frac{\partial F}{\partial x}=\left(\mathbf{\Delta}+\frac{F}{2\tau}\right)\overline{g_1 v_F\cos\theta}. \tag{10}$$

Now for simplicity we may restrict ourself to the situation near the critical superconducting temperature where we may put $G=\mathrm{sgn}(\omega_n)$, and the combination of Eqs. (9) and (10) leads immediately to

$$\overline{v_F\cos\theta\frac{\partial f_1}{\partial x}}=-\frac{v_F^2}{6\left(\omega+ih+\frac{1}{2\tau}\right)}\frac{\partial^2 F}{\partial x^2}, \tag{11}$$

and finally we obtain the following Usadel equation

$$(\omega+ih)F-\frac{D_f(1-2ih\tau)}{2}\frac{\partial^2 F}{\partial x^2}=\mathbf{\Delta}, \tag{12}$$

with the diffusion coefficient $D_f=(\tau v_F^2/3)$. More generally we may demonstrate that in the dirty limit in strong ferromagnets, the standard Usadel equations[19] are applicable with the renormalization $D_f\to D_f(1-2ih\tau)$ and $\omega\to\omega+ih$. Note that in the usual situation the condition of applicability of Usadel equations is $\tau T_c\ll 1$, but in our case in the presence of strong exchange field it becomes more restrictive $h\tau\ll 1$ [or $l\ll\xi_f=\sqrt{D_f/(2h)}$]. Due to condition $h\gg T_c$ we have kept in Eq. (12) only the most-important corrections of the order $h\tau$ and neglected the terms proportional to $\omega\tau$. As in our model the superconducting order parameter is equal to zero in the F layer, the anomalous Green function F is the solution of the Eq. (12) with $\Delta=0$. General boundary conditions in the dirty limit for Usadel equations have been derived by Kuprianov and Lukichev[15]

$$\xi_f\gamma\frac{\partial F_f}{\partial x}=\xi_s\frac{\partial F_s}{\partial x}, \tag{13}$$

$$F_s=F_f+\xi_f\gamma_b\frac{\partial F_f}{\partial x}, \tag{14}$$

where $\gamma=(\sigma_s\xi_f)/(\sigma_f\xi_s)$, $\sigma_f(\sigma_s)$ is the conductivity of the F layer (S layer above T_c), $\xi_f=\sqrt{D_f/(2h)}$, $\xi_s=\sqrt{D_s/(2T_c)}$ is the superconducting coherence length of the S layer, the parameter $\gamma_b=(R_b\sigma_f)/\xi_f$, where R_b is the S/F boundary resistance. The parameter γ_b is directly related to the transparency of the interface $T=1/(1+\gamma_b)$.[22] The condition $T=0$ ($\gamma_b=\infty$) corresponds to a vanishingly small-boundary transparency, and the condition $T=1$ ($\gamma_b=0$) corresponds to a perfectly transparent interface. In our geometry, the proximity effect is weak, consequently close to the interface on distances of the order of magnitude ξ_s the variations of the Green function F_s are small. Thus, considering a weak proximity effect is equivalent to taking a limit $\gamma\ll 1$ in Eq. (13). In this part, we only consider perfectly transparent interfaces and infinite S/F layers to avoid unnecessary mathematical complications. The corresponding generalization for the case of arbitrary transparency is straightforward. Thus the anomalous Green function may be written directly as

$$F(\tilde{x},\tilde{\omega})=\frac{\tilde{\Delta}}{\sqrt{\tilde{\Delta}^2+\tilde{\omega}^2}}\exp[-\tilde{x}(1+i)(1+ih\tau)], \tag{15}$$

if we introduce the dimensionless coordinate $\tilde{x}=x/\xi_f$, and parameters $\tilde{\omega}=\omega/h$ and $\tilde{\Delta}=\Delta/h$. The real part of the anomalous Green function in the ferromagnet, which plays in some sense the role of superconducting order-parameter induced in a ferromagnet by proximity effect has the following form

$$\mathrm{Re}[F(\tilde{x},\tilde{\omega})]=\frac{\tilde{\Delta}}{\sqrt{\tilde{\Delta}^2+\tilde{\omega}^2}}\exp\left[-\frac{\tilde{x}}{1+h\tau}\right]\cos\left[\frac{\tilde{x}}{1-h\tau}\right]. \tag{16}$$

For illustration, we present in Fig. 3 its damped-oscillatory behavior for different scattering rates τ. We see that the spatial period of the oscillations $\xi_f(1-h\tau)$ and the damping length of the oscillations $\xi_f(1+h\tau)$ are both of the order of magnitude of the superconducting coherence length in the ferromagnet. However, the spatial period decreases with increasing $h\tau$, whereas the damping length increases with increasing $h\tau$. The similar behavior is characteristic for spatial dependance of the density of states . In the strong dirty limit ($h\tau\rightarrow 0$), we get the simple damped oscillatory behavior $\mathrm{Re}[F(\tilde{x},\tilde{\omega})]\propto \exp(-\tilde{x})\cos(\tilde{x})$ already found in Ref. 10

A. Infinite ferromagnet length

In this section, we examine the behavior of the density of states in the infinite ferromagnet at arbitrary temperature, assuming that the dirty-limit conditions are held. We may use the complete set of Usadel equations

$$-\tilde{D}_f\vec{\nabla}[G(x,\omega,h)\vec{\nabla}F(x,\omega,h)-F(x,\omega,h)\vec{\nabla}G(x,\omega,h)] +2[\omega+ih(x)]F(x,\omega,h)=0, \tag{17}$$

$$G^2(x,\omega,h)+F(x,\omega,h)F^*(x,-h,\omega)=1, \tag{18}$$

where $\tilde{D}_f=D_f(1-2ih\tau)$ is the renormalized diffusion coefficient in the ferromagnetic region. However, due to the condition $h\tau\ll 1$ of applicability of Usadel equations this renormalization is small and we will omit it further in this paper (as it has been demonstrated before it only weakly modifies the ratio between decaying length and period of oscillation). Equations (17) and (18) naturally suggest the parametrization of F and G by a function $\Theta(x,\omega)$, such as $F(x,\omega,h)=\sin\Theta(x,\omega,h)$ and $G(x,\omega,h)=\cos\Theta(x,\omega,h)$. Under this parametrization the Eq. (17) may be written as

$$-\frac{\partial^2\Theta(\tilde{x},\tilde{\omega})}{\partial\tilde{x}^2}+2(\tilde{\omega}+i)\sin\Theta(\tilde{x},\tilde{\omega})=0, \tag{19}$$

with $\tilde{x}=x/\xi_f$ and $\tilde{\omega}=\omega/h$. For infinite F and S layers, the solution of this Sine-Gordon equation (19) is $\Theta(\tilde{x},\tilde{\omega})=4\arctan[\tan(\theta_0/4)\exp(-\tilde{x}\sqrt{2}\sqrt{i+\tilde{\omega}})]$.[23] In the case of perfectly transparent interfaces the normal Green function can be readily written as

$$G(\tilde{x},\tilde{\omega})=\cos\left\{4\arctan\left[\tan\left(\frac{1}{4}\arcsin\frac{1}{\sqrt{1+(\tilde{\omega}/\tilde{\Delta})^2}}\right)\times\exp(-\tilde{x}\sqrt{2}\sqrt{i+\tilde{\omega}})\right]\right\}. \tag{20}$$

Note that in the limit T close to T_c ($\Delta\rightarrow 0$), this expression gives the same formula for the normal Green function $G(\tilde{x},\tilde{\omega})$ as in Ref. 10. To calculate the density of states $N_\uparrow(\tilde{x},\tilde{\omega})$, we perform, as usual the analytical continuation $\omega_n\rightarrow i\omega$ of the normal Green function G, and the density of states for the spin-down orientation is found by substituting h by $-h$ in the expression giving $N_\uparrow(\tilde{x},\tilde{\omega})$. Note that the period of these oscillations is always of the order of magnitude of the coherence length ξ_f whereas the amplitude of these oscillations strongly depends on the ratio Δ/ω. The amplitude of the oscillations increases when the ratio Δ/ω get closer to one and decreases when the ratio Δ/ω tends to zero or to infinity. In Fig. 4, we present the normalized density of states as a function of temperature for different distances from the S/F interface.

B. Finite ferromagnet length

In the experiments[11] the superconducting density of states has been measured in a thin ferromagnetic film with thickness d_f in the range of 50–100 Å. To describe the situation[11] we may take into account the usual boundary conditions at a vacuum interface $(\partial F/\partial x)(x=d_f)=0$. It is possible to present the explicit analytical results near the superconducting transition temperature. The solution of linearized Usadel equation fitting the general boundary condition at the S/F boundary (14) and vacuum-interface condition is

$$F(\tilde{x})=\frac{1}{\sqrt{1+\frac{\tilde{\omega}^2}{\tilde{\Delta}^2}}}\frac{\cosh[\sqrt{2}\sqrt{i+\tilde{\omega}}(\tilde{x}-\tilde{d}_f)]}{\cosh[\sqrt{2}\sqrt{i+\tilde{\omega}}\tilde{d}_f]-\sqrt{2}\sqrt{i+\tilde{\omega}}\gamma_b\sinh[\sqrt{2}\sqrt{i+\tilde{\omega}}\tilde{d}_f]}, \tag{21}$$

where the following dimensionless parameters are used, $\tilde{\Delta}=\Delta/h,\tilde{\omega}=\omega/h,\tilde{x}=x/\xi_f$, and $\tilde{d}_f=df/\xi_f$. As usual, the density of states for the spin-up direction is found by performing the analytical continuation $\omega_n\rightarrow i\omega$ of the normal Green function G

$$N_\uparrow(\tilde{x},\tilde{\omega})=N(0)\mathrm{Re}\sqrt{1-\frac{1}{1-\frac{\tilde{\omega}^2}{\tilde{\Delta}^2}}\frac{\cosh^2[(1+i)\sqrt{1+\tilde{\omega}}(\tilde{x}-\tilde{d}_f)]}{\{\cosh[(1+i)\sqrt{1+\tilde{\omega}}\tilde{d}_f]-(1+i)\sqrt{1+\tilde{\omega}}\gamma_b\sinh[(1+i)\sqrt{1+\tilde{\omega}}\tilde{d}_f]\}^2}}, \tag{22}$$

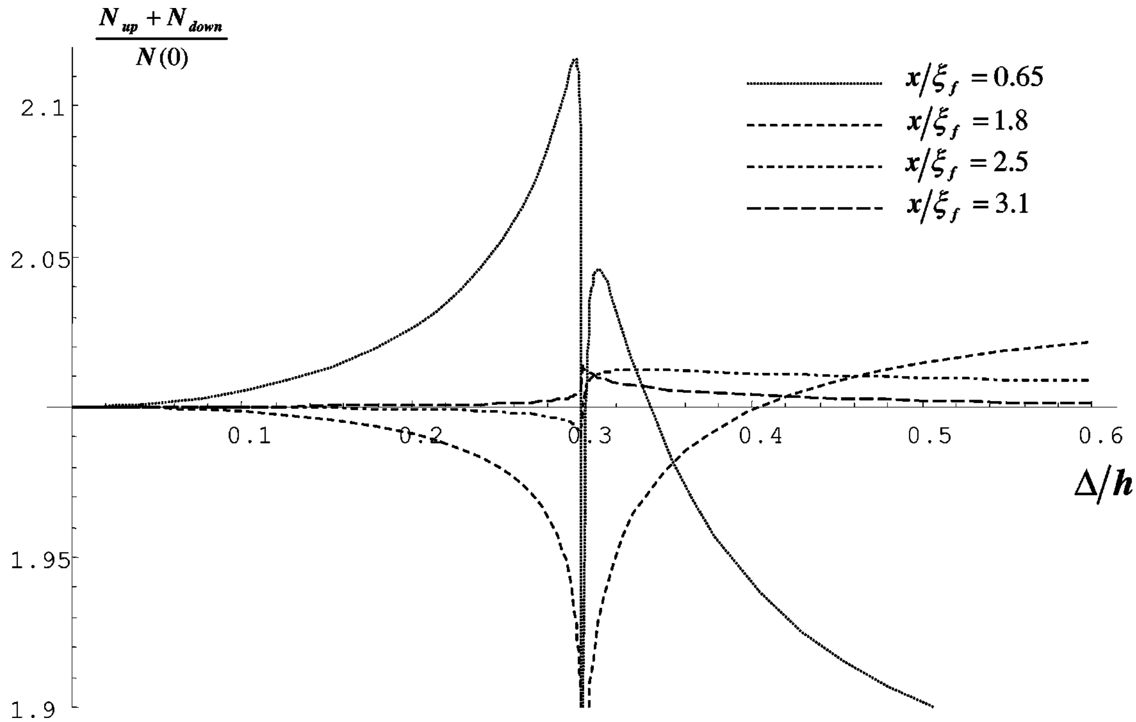

FIG. 4. Variation of the normalized local density of states (N_{tot}) for $\omega = 0.3h$ as a function of the ratio $\tilde{\mathbf{\Delta}} = \mathbf{\Delta}/h$ that gives an idea of the density-of-states variation with the temperature. We suppose that the only temperature-dependant parameter is $\mathbf{\Delta}(T)$ while the exchange field h is constant. The different curves are obtained for different values of the distance x/ξ_f from the interface ($x=0.5\xi_f$ and $x=1.7\xi_f$).

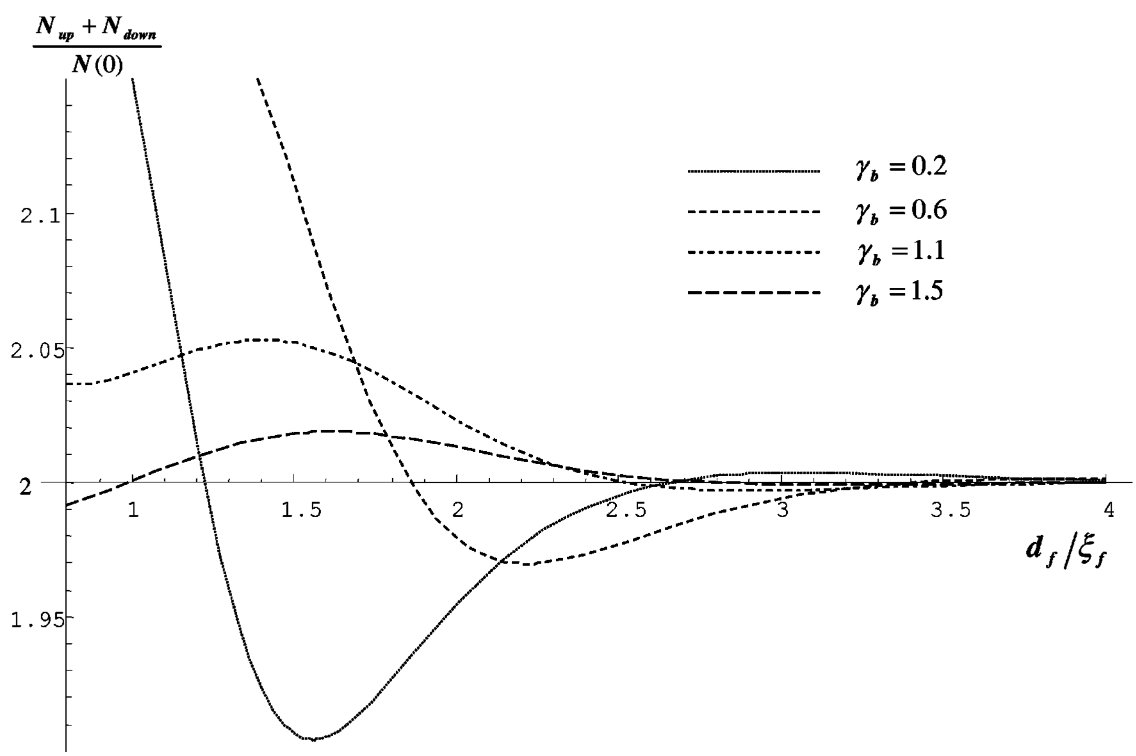

FIG. 5. Variation of the normalized local density of states (N_{tot}) at zero energy as a function of the F layer thickness normalized by the coherence length ξ_f. The different curves are obtained for different values of the transparency coefficient γ_b ($\gamma_b = 0.2$, $\gamma_b = 0.6, \gamma_b = 1.1, \gamma_b = 1.5$).

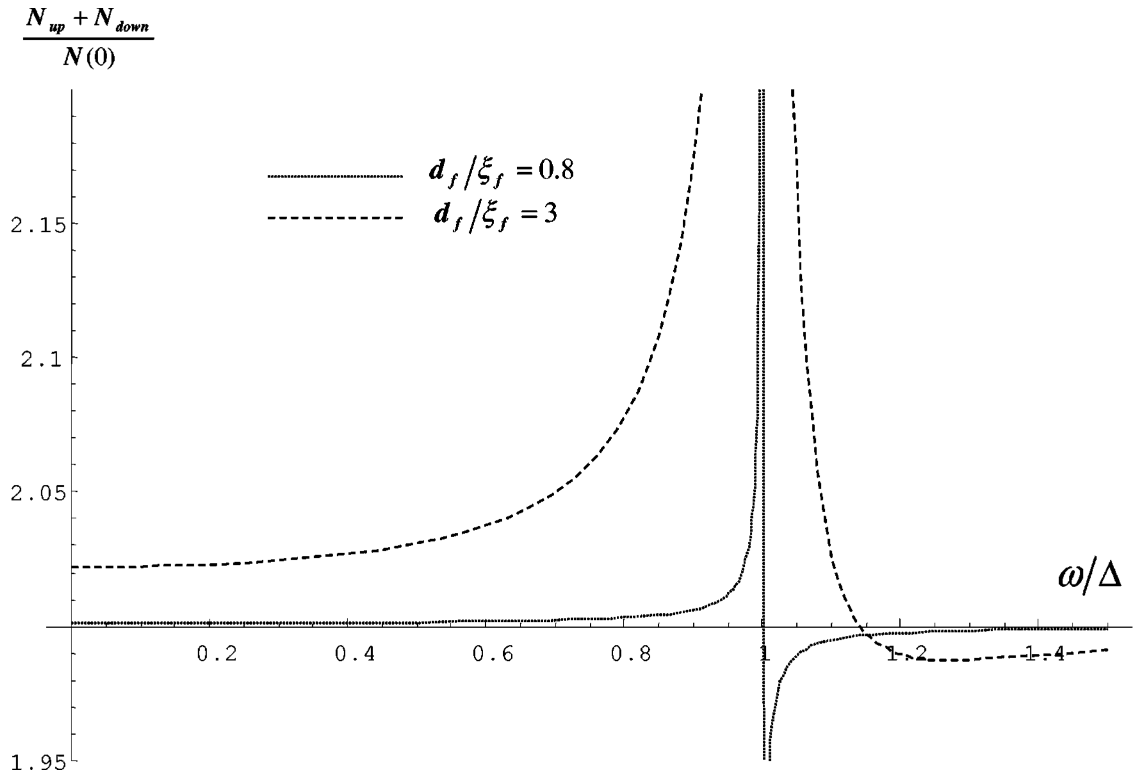

FIG. 6. Low-energy dependance (plotted as a function of ω/Δ) of the normalized local density of states (N_{tot}) for $\gamma_b = 1.5$ and for different values of the F layer thickness $\tilde{d}_f = 0.8$, and $\tilde{d}_f = 3$. The parameter $\tilde{\boldsymbol{\Delta}}$ is chosen to be 0.1.

where $N(0)$ is the density of states per spin in the normal state. The density of states for the opposite direction of the spin $N_\downarrow$ is simply found by substituting h by $-h$ in Eq. (22). The characteristic spatial dependance of $N_{tot}(\tilde{d}_f, \tilde{\omega})$, for $\tilde{\omega} \ll 1$ (see Fig. 5), is a damped-oscillatory behavior with a characteristic length ξ_f (much smaller than the superconducting coherence length ξ_s). This characteristic length is almost constant for all values of the transparency parameter, whereas the amplitude of the oscillations is very sensitive to the value of γ_b. The low-energy ($\omega \sim \Delta$) dependance of $N_{tot}(\tilde{d}_f, \omega/\Delta)$ is presented in Fig. 6. One should notice that the shape of the density of states in the region close to the

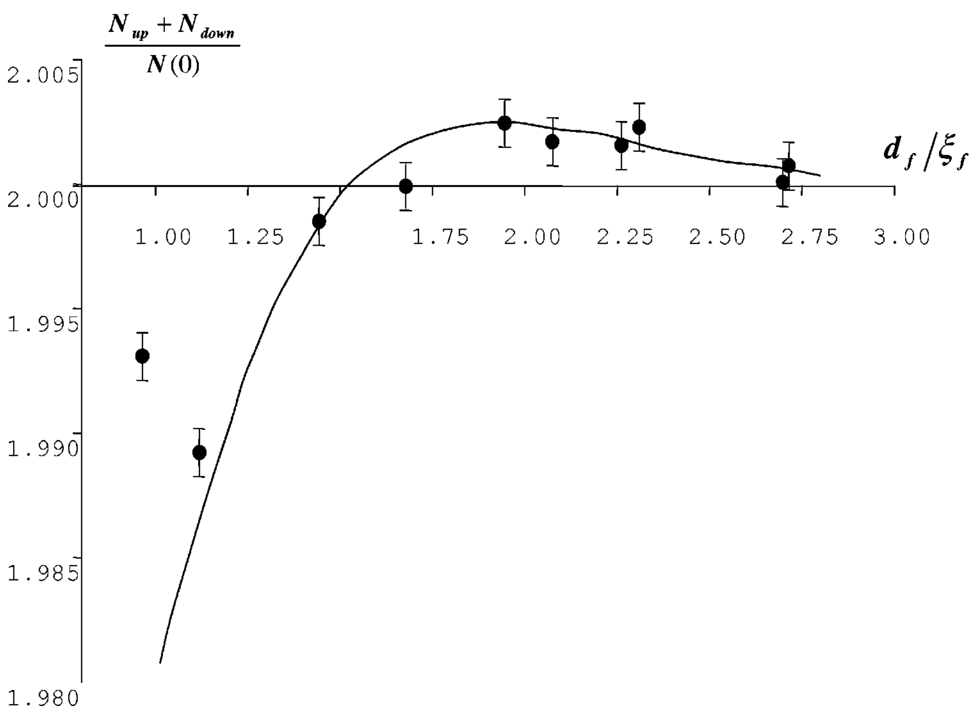

FIG. 7. The experimental points correspond to the measurement of the tunneling conductance, done by Kontos *et al.*,[11] at zero energy vs the PdNi thickness normalized by the coherence length ξ_f. The theoretical curve is the best fit obtained by using formula (22), with transparency coefficient $\gamma_b = 1.5$. The resistivity ρ_f of the 50-Å thick PdNi layer is of the order of magnitude of 50 $\mu\Omega$ cm, the coherence length is of the order of magnitude of 50 Å and the sample has a surface of area 100 μm$\times$100 μm. So our fit gives an interface resistance of the order of magnitude of $4\times10^{-7}\,\Omega$, which is compatible with the low-energy dissipation measurement of Nb/PdNi/Nb junctions.

S/F interface (at $d_f \ll \xi_f$), is similar to the density of states of a conventional superconductor. There are divergencies of the density of states at $\omega = \pm\Delta$ and also at much-higher energies, when $\omega = \pm h$. It should be underlined that these divergencies would be in fact limited by the inelastic processes that have not been considered in our model. In addition, in tunneling measurements at finite temperatures, the fine structure of density of states always contributes in some "thermally averaged" form and the peak at $\omega = \pm h$ is so narrow that it certainly exceeds actual experimental resolution.

In the Kontos *et al.* experiment,[11] an S layer of Niobium and an F layer of PdNi have been used. The typical gap energy is $\Delta_{Nb} = 1,\ 40 \times 10^{-3}$ eV, the measurement of the conductance of the S/F junction has been made at $T = 0.3$ K. So the resolution of this experiment is of the order of magnitude of 10^{-4} eV$= \Delta_{Nb}/10$, which is sufficient to give an idea of the shape of the density of states at $\omega \sim \Delta_{Nb}$. We have fitted their measurement of the tunneling conductance at zero energy of the PdNi/Nb tunnel junction with our theoretical expression of the density of states. The only fitting parameter is the finite-transparency parameter γ_b. The best fit is obtained for $\gamma_b \simeq 1.5$ (see Fig. 7) and it reproduces well the experimental data for thicknesses larger than the coherence length. Indeed, close to the interface, the experimental density of states can differ from our theoretical predictions for several reasons. First for experimental reasons, a magnetically dead layer can appear and it may enhance artificially the coherence length. The presence of some oxides layers that can be formed during the fabrication of the bilayer may also play some role. Finally note that in our theoretical approach we have neglected the influence of the ferromagnet on the S layer. If the corresponding conditions are not fulfilled in the experiment a notable modification of the superconducting order parameter near the interface can be produced.

V. CONCLUSION

We have calculated the superconducting density-of-states induced in a ferromagnet by the proximity effect in a rich variety of situations that could be useful for further experimental studies of peculiar proximity effect in S/F structures. Our results clearly show that the appearing damping density-of-states oscillations are quite robust and disappear only in the nonrealistic extremely clean limit. The characteristic period of these oscillations is much shorter than the superconducting coherence length and its precise form depends on a large number of parameters, such as the exchange field h, the transparency of the interface, and the thickness of the ferromagnetic layer. The existence of these oscillations under almost all conditions is quite important and means that the experimental conditions needed to fabricate Josephson junctions with a π-phase shift, as suggested in Ref. 24, are probably much-less restrictive than it was previously supposed.

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[1] P. Fulde and R.A. Ferrell, Phys. Rev. **135**, 1550 (1964).
[2] A.I. Larkin and Y.N. Ovchinnikov, Zh. Éksp. Teor. Fiz. **47**, 1136 (1964) [Sov. Phys. JETP **20**, 762 (1965)].
[3] A. Buzdin, L.N. Bulaevskii, and S.V. Panyukov, JETP Lett. **35**, 178 (1982).
[4] A.I. Buzdin and M.V. Kuprianov, JETP Lett. **52**, 487 (1990).
[5] Z. Radovic, M. Ledvij, L. Dobrosavljevic-Grujic, A.I. Buzdin, and J.R. Clem, Phys. Rev. B **44**, 759 (1991).
[6] A.I. Buzdin and M.V. Kuprianov, JETP Lett. **53**, 321 (1991).
[7] E.A. Demler, G.B. Arnold, and M.R. Beasley, Phys. Rev. B **55**, 15 174 (1997).
[8] J.S. Jiang, D. Davidovic, D.H. Reich, and C.L. Chien, Phys. Rev. Lett. **74**, 314 (1995).
[9] V.V. Ryazanov, V.A. Oboznov, A. Yu. Rusanov, A.V. Veretennikov, A.A. Golubov, and J. Aarts, Phys. Rev. Lett. **86**, 2427 (2001).
[10] A. Buzdin, Phys. Rev. B **62**, 11 377 (2000).
[11] T. Kontos, M. Aprili, J. Lesueur, and X. Grison, Phys. Rev. Lett. **86**, 304 (2001).
[12] M. Zareyan, W. Belzig, and Yu.V. Nazarov, Phys. Rev. Lett. **86**, 308 (2001).
[13] G. Eilenberger, Z. Phys. **214**, 195 (1968).
[14] L.N. Bulaevskii, A.I. Buzdin, M.L. Kulic, and S.V. Panyukov, Adv. Phys. **34**, 175 (1985).
[15] M.Y. Kuprianov and V.F. Lukichev, Zh. Éksp. Teor. Fiz. **94**, 139 (1988) [Sov. Phys. JETP **67**, 1163 (1988)].
[16] Z.G. Ivanov, M. Yu. Kuprianov, K.K. Likharev, S.V. Meriakri, and O.V. Snigirev, Fiz. Nizk. Temp. **7**, 560 (1981) [Sov. J. Low Temp. Phys. **7**, 274 (1981)].
[17] M.V. Baranov, A.I. Buzdin, and L.N. Bulaevskii, Zh. Éksp. Teor. Fiz. **91**, 1063 (1986) [Sov. Phys. JETP **64**, 628 (1986)].
[18] A.D. Zaikin, Solid State Commun. **41**, 533 (1982).
[19] L. Usadel, Phys. Rev. Lett. **95**, 507 (1970).
[20] W. Belzig, C. Bruder, and G. Schön, Phys. Rev. B **54**, 9443 (1996).
[21] S. Guéron, H. Pothier, N.O. Birge, D. Esteve, and M.H. Devoret, Phys. Rev. Lett. **77**, 3025 (1996).
[22] J. Aarts, J.M.E. Geers, E. Brück, A.A. Golubov, and R. Coehoorn, Phys. Rev. B **56**, 2779 (1997).
[23] M. Thinkham, *Introduction to Superconductivity* (McGraw-Hill, New York, 1996).
[24] A.I. Buzdin, B. Bujicic, and M. Yu. Kupriyanov, Zh. Éksp. Teor. Fiz. **101**, 231 (1992) [Sov. Phys. JETP **74**, 124 (1992)].