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# Theory of neutron channeling in the resonant layer of multilayer systems

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A wave theory of neutron channeling is presented. An experiment to check the theory is considered.

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## I. INTRODUCTION

We consider theoretically neutron channeling in a planar system like the one shown in Fig. 1 (it will be considered in more detail in the next section).

This system consists of two layers on a substrate, with the optical potential of the system shown in Fig. 2 (without Cd). The channeling takes place in the second ("resonant") layer, which has low optical potential. In this figure it is Ti, which has a negative optical potential, or potential well of depth $u_2$ ($u_{\mathrm{Ti}}=-50.81$ neV) (Fig. 2). The substrate ("reflector") has a positive optical potential $u_3$, here Cu($u_{\mathrm{Cu}}=171$ neV), and is usually thick enough for the waves channeled in the resonant layer to be totally reflected from it. Above Ti there is a "tunneling" layer. It also has a positive optical potential ($u_1$ in Fig. 2) to hold the channeled wave inside the resonant layer. This potential may be even higher than that of the substrate ($u_1>u_3$), as for example, is shown in Fig. 1, where the tunneling layer is Ni ($u_{\mathrm{Ni}}=245$ neV). However, it must be thin enough for the outside wave to penetrate (tunnel) into the resonant Ti layer. Above the system a Cd layer is shown, which absorbs neutrons, and prevents the feeding of the channel along its entire length from the outside.

Such multilayer systems are well known (see, for instance, Ref. 1), however, there is no good understanding how the external wave is coupled to the channeling one. It is understood what happens if the channel is a microwaveguide, in which the wave is an eigenmode. In this case the wave is totally reflected from both sides, and the waveguide is fed from the outside through either its entrance orifice at the edge surface or through a special wedge.

In our case the channel is not a perfect waveguide because its upper side is slightly permeable. Thus rather than eigenmodes we have resonant ones, with the channel fed only through the upper side. In this case the wave function in the system under the illuminated part of the upper surface (to the left of Cd in Fig. 1) is well known, but we do not know what the channeling wave is, how it propagates under the nonilluminated area, and how it is matched at the interface between the illuminated and nonilluminated parts. These are the main questions we address in this paper.

Microwaveguides are well known for x rays[2–4] where they are used for microfocusing, which is applied to microdiffraction, microimaging, and microspectroscopy. Research in this field has been highly stimulated by the advent of modern powerful synchrotron sources.

Neutron sources are considerably less powerful, and a single microwaveguide, if fed only through the entrance, gives an intensity too low to be useful because of the small area of the entrance hole. Thus, it is interesting to look for the channeling in a geometry like the one shown in Fig. 1.

The first observation of neutron channeling[5] in thin films showed that such experiments are feasible and deserves further theoretical and experimental consideration.[6–10]

Experiments on neutron channeling in thin-film neutron waveguides have been discussed in several papers.[11–15] In this paper we present a different approach. We show how to relate the extinction of neutron flux along the channel to the tunneling through the first layer, and how to find the distribution of neutrons exiting the channel.[11,13]

The problems of channeling in multilayer resonant (MR) systems are closely related to the problems of reflection from them, which are studied by reflectometry. The resonant structure of MR systems shows itself in an interference pattern of reflectivity even in the subcritical reflection region, when the normal-to-surface component $E_z$ of the total energy of the incident beam is in the range $0<E_z<u_3$. This pattern has well-pronounced resonant minima only when the system has

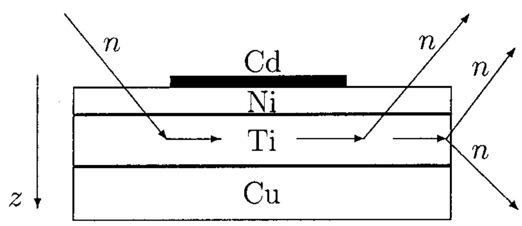


FIG. 1. A mirror for measuring resonant neutron channeling.

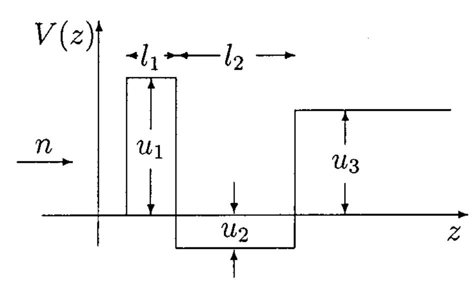


FIG. 2. Stepwise potential of a multilayer resonant system. The $z$ axis is directed along normal to the surface of the multilayer system.

 

high absorption in the tunneling, resonant or/and reflector layer.

Experiments on subcritical ($E_z < u_3$) reflection were reported in Refs. 16–20. The depth of the minima in the reflectivity curve (the curve that shows the dependence of reflectivity on neutron momentum transfer) was enhanced through either a special choice of material with high absorption for layer $u_1$, or by embedding a thin absorbing layer into the resonant one. Experiments were also performed with polarized neutrons in magnetic layers where spin rotation played the role of absorption.[21,22]

In the following section we consider the reflectivity of resonant systems and channeling in them.

## II. WAVE FUNCTION IN A MULTILAYER SYSTEM

We shall treat the multilayers through the method of multiple reflections presented in Refs. 23–25, which is applicable to scalar and magnetic systems. This method is an analytical one, and it is more appropriate for our analysis than the other methods such as those in Refs. 26–28, which, in our opinion, are more appropriate for numerical calculations.[27]

We denote $x$ as the coordinate along the mirror and $z$ as the coordinate along its normal. The wave function in the middle layer $u_2$ under the area illuminated by the incident plane wave $\exp(ik_zx+ik_zz)$ is equal to

$$\psi(z,x)=A(k_z)[\exp(ik_{2z}z)+\exp(ik_{2z}l_2)\rho_{23}(k_{2z}) \times\exp\{-ik_{2z}(z-l_2)\}]\exp(ik_xx), \tag{1}$$

where $k_{2z}=\sqrt{k_z^2+u_2}$, $k_z$ and $k_x$ are the components of the neutron wave vector, and $\rho_{23}(k_{2z})$ is the reflection amplitude from the totally reflecting layer $u_3$ inside the layer $u_2$. The factor $A(k_z)$ is determined by the equation

$$A(k_z)=\tau_{12}+\rho_{21}\rho_{23}\exp(2ik_{2z}l_2)A(k_z), \tag{2}$$

which shows that the wave propagating towards the substrate is the sum of the wave penetrating through the tunneling layer form the outside (the first term), and the wave reflected from both sides in the tunneling layer (the second term). The term $\tau_{12}$ is the transmission amplitude of the wave through the barrier $u_1$ from vacuum into the layer $u_2$, and $\rho_{21}$ is the reflection amplitude in layer $u_2$ from the barrier $u_1$. From Eq. (2) it follows that

$$A(k_z)=\frac{\tau_{12}}{1-\rho_{21}\rho_{23}\exp(2ik_{2z}l_2)}. \tag{3}$$

### A. Neutron resonances in MR systems

Since reflection from $u_3$ is total, the amplitude $\rho_{23}$ is of the form $\rho_{23}=\exp(i\phi_{23})$. After substitution of this $\rho_{23}$ and $\rho_{21}=|\rho_{21}|\exp(i\phi_{21})$ into Eq. (1) the denominator becomes

$$1-\rho_{21}\rho_{23}\exp(2ik_{2z}l_2) =1-|\rho_{21}|\exp(2ik_{2z}l_2+i\phi_{23}+i\phi_{21}). \tag{4}$$

It has minima at points $k_z$, which satisfy the equation

$$2k_{2z}l_2+\phi_{23}(k_{2z})+\phi_{21}(k_{2z})=2\pi n \tag{5}$$

with integer $n$. At these points, which correspond to resonances, the amplitude $A(k_z)$ of the wave function (1) has maxima. The magnitudes of these maxima are larger the closer $|\rho_{21}|$ is to unity, or the smaller is the transmission amplitude $|\tau_{21}|$.

Let us consider how large these maxima are. If we neglect losses we have the law of energy or flux conservation, which is the same as unitarity condition represented by the relation

$$k_{2z}|\rho_{21}|^2+|\tau_{21}|^2k_z=k_{2z}, \tag{6}$$

where $\tau_{21}$ is the transmission amplitude through barrier $u_1$ from the layer $u_2$ into the vacuum, and $k_z, k_{2z}$ are the normal-to-surface components of the neutron wave vector in vacuum and inside the $u_2$ layer. The relation (6) leads to

$$|\rho_{21}|^2=\sqrt{1-|\tau_{21}|^2\frac{k_z}{k_{2z}}}\approx 1-\frac{1}{2}|\tau_{21}|^2\frac{k_z}{k_{2z}}, \tag{7}$$

for $|\tau_{21}|\ll 1$.

From detailed balance theorem it follows that the probability of transmission from vacuum into $u_2$ layer, and the reverse, are equal. It gives

$$|\tau_{12}|^2k_{2z}/k_z=|\tau_{21}|^2k_z/k_{2z}. \tag{8}$$

This leads to $\tau_{21}=(k_{2z}/k_z)\tau_{12}$. Substitution into Eq. (7) and into Eq. (3) with account of Eq. (5) gives

$$|A(k_z)|=\frac{|\tau_{12}|}{1-|\rho_{21}|}\approx\frac{2}{|\tau_{21}|}\gg 1. \tag{9}$$

Thus for $|\tau_{21}|\approx 0.1$ we have at resonance $|A(k_z)|^2\approx 400$.

### B. Mechanism of particle accumulation in the resonant layer

We can easily understand the mechanism of wave-function enhancement in the $u_2$ layer at resonance. Indeed, at resonances the wave $\exp(ik_{2z}l_2)\rho_{23}(k_{2z})\exp(-ik_{2z}[z-l_2])$ in Eq. (1) going from the substrate towards the $u_1$ layer after reflection at the point $z=0$ becomes equal to $\exp(2ik_{2z}l_2)\rho_{23}(k_{2z})\rho_{21}(k_{2z})\exp(ik_{2z}z)=|\rho_{21}|\exp(ik_{2z}z)$ because of Eq. (5), that is it positively interferes with the previous wave $\exp(ik_{2z}z)$ and enhances it.

Such enhancement facilitates penetration of the external wave into the $u_2$ layer. It suggests that the reflection amplitude of the external wave decreases, and we want to prove that this is indeed so.

Let us consider the resulting amplitude of the outgoing wave as consisting of two parts: the directly reflected incident wave with amplitude $\rho_{12}$, and the part that is transmitted into the $u_2$ layer, then reflected from substrate and transmitted back into vacuum again. The amplitude of the second wave is equal to $\tau_{12}\tau_{21}\rho_{23}(k_{2z})\exp(2ik_{2z}l_2)$. Thus the amplitude of the resulting reflection is given by

$$\tilde{\rho}_{12}=\rho_{12}+\tau_{12}\tau_{21}\rho_{23}(k_{2z})\exp(2ik_{2z}l_2) =\rho_{12}\left[1+\frac{\tau_{21}\tau_{12}}{\rho_{21}\rho_{12}}|\rho_{21}|\right], \tag{10}$$

where in the last equality we took into account Eq. (5). It is easy to show[24,29] that

$$\rho_{12}=\frac{r_{01}-r_{21}\exp(-2k_{1z}l_1)}{1-r_{01}r_{21}\exp(-2k_{1z}l_1)},$$

$$\rho_{21}=\frac{r_{21}-r_{01}\exp(-2k_{1z}l_1)}{1-r_{01}r_{21}\exp(-2k_{1z}l_1)},\quad r_{01}=\frac{k_z-ik_{1z}}{k_z+ik_{1z}},\quad (11)$$

$$\tau_{12}=\frac{t_{01}t_{12}\exp(-k_{1z}l_1)}{1-r_{01}r_{21}\exp(-2k_{1z}l_1)},$$

$$\tau_{21}=\frac{t_{21}t_{10}\exp(-k_{1z}l_1)}{1-\tau_{01}\tau_{21}\exp(-2k_{1z}l_1)},\quad r_{21}=\frac{k_{2z}-ik_{1z}}{k_{2z}+ik_{1z}},\quad (12)$$

$$t_{01}=\frac{2k_z}{k_z+ik_{1z}},\quad t_{21}=\frac{2k_{2z}}{k_{2z}+ik_{1z}},\quad t_{10}=\frac{2ik_{1z}}{k_z+ik_{1z}},$$

$$t_{12}=\frac{2ik_{1z}}{k_{2z}+ik_{1z}},\quad k_{1z}=\sqrt{u_1-k_z^2},\quad (13)$$

where $\tau_{ik}$ denotes the reflection, and $t_{ik}$ the transmission, amplitudes for a potential step from level $u_i$ to level $u_k$, and $l_1$ is the width of the $u_1$ layer.

For real potentials $u_1,u_2$ we have $\tau_{21}\tau_{12}/\rho_{21}\rho_{12}=-|\tau_{21}\tau_{12}|/|\rho_{21}\rho_{12}|$. Substitution into Eq. (10) gives

$$\tilde{\rho}_{21}=\rho_{12}\left[1-\frac{|\tau_{12}\tau_{21}|}{|\rho_{12}\rho_{21}|}|\rho_{21}|\right],\quad (14)$$

or $|\tilde{\rho}_{12}|<|\rho_{12}|$. It shows that indeed the wave passed through the resonance layer compensates a little bit the directly reflected one.

However, at energies $E_z<u_3$ reflection is always total, independent of the $E_z$, thus the amplitude in the resonant layer should be accumulated to such a level that the wave transmitted through $u_1$ into vacuum would overcompensate the directly reflected one up to the point at which the modulus of the resulting reflected amplitude is unity. Indeed, if we multiply the second term in the brackets of expression (14) by the resonant factor $1/(1-|\rho_{21}|)$, we obtain

$$\rho_r=\rho_{12}\left[1-\left|\frac{\tau_{21}\tau_{12}}{\rho_{21}\rho_{12}}\right|\frac{|\rho_{21}|}{1-|\rho_{21}|}\right]$$
$$=\frac{\rho_{12}}{|\rho_{12}\rho_{21}|}\,\frac{|\rho_{12}\rho_{21}|-|\rho_{21}|(|\rho_{12}\rho_{21}|+|\tau_{12}\tau_{21}|)}{1-|\rho_{21}|}=-\frac{\rho_{12}}{|\rho_{12}|},\quad (15)$$

where we used the relations $|\rho_{12}\rho_{21}|+|\tau_{12}\tau_{21}|=1$, and $|\rho_{12}|=|\rho_{21}|$, which follow directly from Eqs. (11) and (12). Expression (13) then shows that the resulting reflection is indeed total.

### C. Channeling in the middle layer

To begin our discussion of ideas for the channeling in the resonant layer, let us consider a system slightly different from that shown in Fig. 1. It will be simpler if illumination is stopped by absorbers arranged as shown in Fig. 3. This is easier as the boundary conditions under the nonilluminated area remain the same as under the illuminated one, which means that the resonant conditions remain the same. In the case of Fig. 1, reflectivity in the resonant layer from the upper side of the nonilluminated area is different from that under the illuminated one, and the positions of the resonances should be slightly shifted. This leads to some complications in matching at the interface between the illuminated and nonilluminated parts.

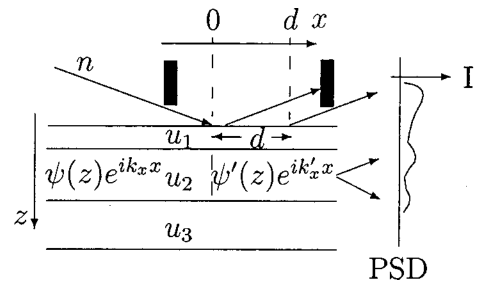


FIG. 3. Matching the wave function at $x=0$ in two parts of a multilayer mirror, and distribution of outgoing neutrons over position sensitive detector PSD.

In the simplified geometry the front of the incident wave is limited by an absorber, and the position sensitive detector (PSD) does not see the illuminated part of the mirror.

Under the illuminated part of the mirror ($x<0$), the wave function inside the layer $u_2$ is equal to

$$\psi(x,z)=A(k_z)[\exp(ik_{2z}z)+\exp(ik_{2z}l_2)\rho_{23}(k_{2z})$$
$$\times\exp\{-ik_{2z}(z-l_2)\}]\exp(ik_x x),\quad (16)$$

where $k_x$ is the component along the $x$ axis of the wave vector in the incident wave above the mirror, and the coefficients $A,\rho_{23}$, and wave number $k_{2z}$ are the same as in Eq. (1).

The wave function under the nonilluminated surface is a solution of the Schrödinger equation, which we can represent in the form

$$\psi(x,z)=A[\exp(ik'_{2z}z)+\exp(ik'_{2z}l_2)\rho_{23}(k'_{2z})$$
$$\times\exp\{-ik'_{2z}(z-l_2)\}]\exp(ik'_x x),\quad (17)$$

where $A=A(k_z)$, and $k'^2_{2z}+k'^2_x=k^2+u_2$. If we take $k'_{2z}=k_{2z}-i\kappa$, where $k_{2z}=\sqrt{k_z^2+u_2}$, then $k'_x=\sqrt{k_x^2+2i\kappa k_{2z}}\approx k_x+i\kappa k_{2z}/k_x$, where $k_x^2=k^2-k_z^2$. The imaginary part, $\kappa>0$, should be chosen in such a way that

$$\rho_{21}(k'_{2z})\rho_{23}(k'_{2z})\exp(2ik'_{2z}l_2)=1.\quad (18)$$

This form matches well the wave function (17) to the right and wave function (16) to the left of the section where $x=0$. Some mismatch because of $\kappa$ gives some reflected wave going to the left from this section. However, for small $\kappa$ (later we shall see, when it is small) the reflected wave is also small, so we can neglect it.

The equation (18) is the same as Eq. (2) but with $\tau_{12}$ omitted. When it is satisfied, the wave function has the same dependence on $z$ at every point $x$, and only its amplitude $|A|$

contains a factor, which, because of imaginary part of $k_x'$, exponentially decays along $x$ axis,

$$\exp(ik_x'x)=\exp(ik_xx)\exp\left(-\frac{x}{2x_e}\right),\quad \frac{1}{2x_e}=\kappa\frac{k_{2z}}{k_x}. \tag{19}$$

In the case when $1-|\rho_{21}|\ll 1$, the magnitude of $\kappa$ is small, and we can neglect it in amplitudes $\rho_{23}$ and $\rho_{21}$. Then Eq. (18) is represented as

$$|\rho_{21}(k_z)|\exp(2ik_{2z}l_2+i\phi_{23}+i\phi_{21}+2\kappa l_2)=1. \tag{20}$$

For $k_{2z}$ satisfying condition (5), we obtain $|\rho_{21}|\exp(2\kappa l_2)=1$, which gives the magnitude of $\kappa$,

$$\kappa=-\frac{1}{2l_2}\ln(|\rho_{21}(k_{2z})|)\approx\frac{k_z}{4l_2k_{2z}}|\tau_{21}(k_{2z})|^2$$
$$=\frac{k_{2z}}{4l_2k_z}|\tau_{12}(k_z)|^2, \tag{21}$$

where relations (7),(8) were taken into account. Thus $\kappa\ll k_{2z}\approx\pi/l_2$, when $|\tau_{12}|^2\ll 4\pi k_z/k_{2z}$, which is always satisfied, so our approximation is always very well justified.

Let us estimate the range $x_e\equiv k_x/2k_{2z}\kappa=2l_2k_x/k_z|\tau_{21}|^2$ of exponentially decaying factor in Eq. (19). For neutrons with wavelength 4 Å, in particular, the grazing angle of incidence is small (we need total reflection from substrate), so $k_z/k_x\approx 10^{-3}$. Thus $x_e=2\times 10^3 l_2/|\tau_{21}|^2$. For $|\tau_{21}|\approx|\tau_{12}|\approx 0.1$ and $l_2=2000$ Å we have $x_e=4$ cm.

### D. Calculation of intensity distribution over nonilluminated surface

All the above formulas were obtained for a single incident neutron, with its flux over unit area of the entrance surface proportional to $k_z$. For the intensity $I_0$ we need to renormalize the coefficient $A$ by a factor $\sqrt{I_0/k}$. Then the number of neutrons crossing the cross area of the resonant layer per unit time, at some point $x$ under the nonilluminated surface of the mirror, is

$$J(x)=\frac{dN(x)}{dt}=\frac{I_0}{k}\int_0^{l_2}wdz|\psi(z)|^2k_x\exp(-x/x_e), \tag{22}$$

where $w$ is the width of the mirror surface along the $y$ axis.

From Eq. (17) it follows that

$$|\psi(z)|^2\approx 4|A|^2\cos^2(\{k_{2z}[z-l_2]-\phi_{23}\}/2). \tag{23}$$

Substitution into Eq. (22) gives the integral

$$k_{2z}\int_0^{l_2}\cos^2(\{k_{2z}[z-l_2]-\phi_{23}\}/2)dz$$
$$\approx\int_0^{n\pi}\cos^2(\chi)d\chi=\frac{n\pi}{2},\ \text{where}\ \ \chi\approx k_{2z}z,$$

and $n$ is the integer of the resonance $k_{2z}l_2=n\pi$. Thus Eq. (22) is

$$J(x)=2I_0\frac{k_x}{k}wl_2|A|^2\exp(-x/x_e). \tag{24}$$

From Eq. (24) it follows that the total number of neutrons crossing per unit time the section of the channel $u_2$ at the point $x=0$ is

$$J(0)=2I_0\frac{k_x}{k}wl_2|A|^2. \tag{25}$$

At resonance, when Eq. (9) is valid, for $w=1$ cm, $|A|^2=400$, and $l_2=2\times 10^{-5}$ cm we have $J(0)\approx 10^{-2}I_0$.

Now we can calculate the total number of neutrons that go out through an element $wdx$ of the nonilluminated surface at point $x$,

$$dJ(x)=-\frac{dJ(x)}{dx}dx=\frac{I_0}{k}wdx|A|^2|\tau_{21}|^2k_z\exp(-x/x_e)$$
$$\equiv\frac{J(0)}{x_e}e^{-x/x_e}dx. \tag{26}$$

If the nonilluminated area is infinite in $x$ direction, then the total number of outgoing neutrons is equal to the integral over $dx$ from 0 to $\infty$, which is naturally equal to $J(0)$.

The above result is valid for monochromatic and collimated neutrons with precisely resonant normal component $k_z^2$ of the neutron energy. If the spectrum is not monochromatic, we should replace Eq. (25) by the expression

$$J(0)=2\frac{dI_0(k_z^2)}{dk_z^2}\Gamma\frac{k_z}{k}wl_2|A|^2, \tag{27}$$

where $\Gamma$ is the resonance width, calculated as follows. Let us denote phase (5) as $\Phi(k_z)$, then the resonance condition (5) is $\Phi(k_z)=2\pi n$, where $n$ is an integer $(n\geqslant 0)$ called the resonance order. Near the $n$th resonance the $\exp(i\Phi)$ in the denominator (4) can be approximated as

$$\exp(i\Phi)\approx 1+i\frac{d\Phi}{dE}(E-E_n)\approx 1+i\frac{l_2}{k_{2z}}(E-E_n), \tag{28}$$

where $E=k_z^2$. Substitution into Eq. (3) gives

$$A(k_z)=\frac{iC}{E-E_n+i\Gamma}, \tag{29}$$

where

$$\Gamma=\frac{k_{2z}}{l_2}\frac{1-|\rho_{21}|}{|\rho_{21}|}\approx\frac{|\tau_{21}|^2k_z}{2l_2}\approx\frac{2k_z}{l_2|A|^2},\quad C=\tau_{12}\frac{k_{2z}}{l_2|\rho_{21}|}, \tag{30}$$

and we approximated $d\Phi/dE$ as $2l_2(dk_{2z}/dE)=l_2/k_{2z}$, because of the weak dependence of the phases $\phi_{23}$ and $\phi_{21}$ on energy.

Substitution of Eqs. (30) and (9) into Eq. (27) gives

$$J(0)=4\frac{dI_0(k_z^2)}{dk_z^2}wk_z. \tag{31}$$

For estimation we can replace $dI_0/dk_z^2 = I_0/k_z^2$, $k_z = \pi/l_2$, and accepting $w=1$ cm and $l_2 = 2\times10^{-5}$ cm, we obtain

$$J(0)=4I_0\frac{w}{k_z}\approx\frac{4}{\pi}I_0wl_2\approx2\times10^{-5}I_0, \tag{32}$$

which means that the experiment is feasible for $I_0 \propto 10^7\ n/\text{sec/cm}^2$.

For the distribution of exit neutrons over the nonilluminated surface in the experiment with the geometry shown in Fig. 3 we, according to Eq. (26), can expect the count rate

$$\frac{dJ(x)}{dx}=\frac{\Gamma}{k_z^2}\frac{J(0)}{x_e}\exp(-x/x_e)\approx\frac{4}{\pi x_e}I_0wl_2\exp(-x/x_e), \tag{33}$$

and for the above parameters we get $dJ/dx \approx 0.5 \times 10^{-5} I_0 \exp(-x/x_e)\ \text{sec}^{-1}\,\text{cm}^{-1}$.

For the experiment with the geometry shown in Fig. 1, we can get a similar estimate, although the matching under the Cd layer is worse because of the change of the reflection amplitude $\rho_{21}\to\rho'_{21}$ at the upper surface of the layer $u_2$, and consequently some shift of the resonant values of $k_{2z}$. This matching will lead to excitation of several modes under Cd and reflection from the section from the point $x=0$ to the left.

The above considerations can be generalized to the case when absorption in the layers and scattering due to inhomogeneities and interface roughnesses are not negligible. In that case all the formulas must be multiplied by the ratio

$$\frac{\Gamma^2}{(\Gamma+\Gamma_l)^2},$$

and the extinction length $x_e$ should be changed to $X_e$, where

$$\frac{1}{X_e}=\frac{1}{x_e}+\frac{1}{x_l},$$

$\Gamma_l$ and $x_l$ describe the losses due to absorption and scattering on roughness and inhomogeneities.

We can also easily generalize the above considerations to magnetic multilayer systems with noncollinear magnetization of nearby layers. See, for example, Refs. 25 and 27.

## III. SUMMARY

We considered neutron channeling in a multilayer system. We applied simple analytical algebra to it, and have shown how to match the illuminated and nonilluminated areas in a resonant layer. For matching we used the ansatz (17), which represents the wave function of a channeled neutron under the nonilluminated area. This wave function precisely satisfies the Schrödinger equation in the resonant layer, is almost identical to the wave function on the other side of the matching cross area, and determines the extinction rate of the channeling function along the channel in the absence of losses. It is trivial to add losses, if they are present, and we leave it to the reader. Our estimations show that a neutron channeling experiment is feasible. For cold neutrons with nonmonochromatic intensity $I_0$ we can expect count rates of the order $10^{-5}I_0$, which are measurable in sufficiently good background conditions. For x rays the situation on the one hand is better because of the high luminosity of x ray sources, but on the other hand is worse because of the high absorption of x rays in matter. These two factors should cancel each other, however, so that for x rays we can also expect good feasibility.

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[1] S. B. Pogossian and H. Le Gall, Opt. Commun. **114**, 235 (1995).
[2] V. I. Glebov, E. I. Denisou, and N. K. Zhevago [Sov. Phys. Tech. Phys. **36**, 11 (1991)].
[3] D. H. Bilderback, S. A. Hoffman, and D. J. Thiel, Science **263**, 201 (1994).
[4] Y. P. Feng, S. K. Sinha, E. E. Fullerton, G. Grubel, D. Abernathy, D. P. Siddons, and J. B. Hastings, Appl. Phys. Lett. **67**, 3647 (1995).
[5] K. A. Steinhauser, A. Steyerl, H. Scheckenhofer, and S. S. Malik, Phys. Rev. Lett. **44**, 1306 (1980).
[6] Huai Zhang, S. K. Satija, P. D. Gallagher, J. A. Dura, K. Ritly, C. P. Flynn, and J. F. Ankner, Phys. Rev. B **52**, 17 501 (1995).
[7] M. Hino, N. Achiwa, S. Tasaki, T. Ebisawa, T. Kawai, and D. Yamazaki, Phys. Rev. A **61**, 013607 (1999).
[8] M. Maaza, B. Bardo, and F. Bridou, Nucl. Instrum. Methods Phys. Res. A **326**, 531 (1993).
[9] I. V. Bondarenko, V. I. Bodnarchuk, S. N. Balashov, P. Geltenbort, A. G. Klein, A. V. Kozlov, D. A. Korneev, S. V. Masalovich, V. G. Nosov, A. I. Frank, P. Hoghoj, and A. Cimmino, Phys. At. Nucl. **62**, 721 (1999).
[10] V. L. Aksenov and Yu. V. Nikitenko, Physica B **268**, 313 (1999).
[11] Y. P. Feng, C. F. Majkrzak, S. K. Sinha, D. G. Wiesler, H. Zhang, and H. W. Deckman, Phys. Rev. B **49**, 10 814 (1994).
[12] A. Menelle, S. P. Pogossian, H. Le Gall, J. M. Desvignes, and J. Ben Youssef, Physica B **234–236**, 510 (1997).
[13] S. P. Pogossian, A. Menelle, H. Le Gall, J. M. Desvignes, and M. Artinian, Phys. Rev. B **53**, 14 359 (1996).
[14] S. P. Pogossian, J. Ben Youssef, H. Le Gall, J. M. Desvignes, and A. Menelle, J. Appl. Phys. **81**, 4281 (1997).
[15] S. P. Pogossian, A. Menelle, H. Le Gall, J. Ben Youssef, and J. M. Desvignes, J. Appl. Phys. **83**, 1159 (1988).
[16] A. Maaza, B. Pardo, J. P. Chauvineau, A. Raynal, A. Menelle, and F. Bridou, Phys. Lett. A **223**, 145 (1996).
[17] A. Maaza, B. Pardo, J. P. Chauvineau, A. Menelle, A. Raynal, and F. Bridou, Phys. Lett. A **235**, 19 (1997).
[18] F. Radu and V. K. Ignatovich, Physica B **292**, 160 (2000).

[19] V. L. Aksenov, Yu. V. Nikitenko, F. Radu, Yu. M. Gledenov, and P. V. Sedyshev, Physica B **276–278**, 946 (2000).

[20] Huai Zhang, P. D. Gallagher, S. K. Satija, R. M. Lindstrom, R. L. Paul, T. P. Russell, P. Lambooy, and E. J. Kramer, Phys. Rev. Lett. **72**, 3044 (1996).

[21] V. L. Aksenov and Yu. V. Nikitenko, Physica B **297**, 101 (2002).

[22] F. Radu, M. Etzkorn, R. Siebrecht, A. Schreyer, K. Westerholt, and H. Zabel, J. Magn. Magn. Mater (to be published 2002).

[23] V. K. Ignatovich, Physica B **175**, 33 (1991).

[24] V. K. Ignatovich, Am. J. Phys. **57**, 873 (1989).

[25] D. A. Korneev, V. I. Bodnarchuk, and V. K. Ignatovich, Pis'ma Zh. Eksp. Teor. Fiz. **63**, 900 (1996) [JETP Lett. **63**, 944 (1996)]; Vladimir K. Ignatovich, in *Proceedings of the International symposium on Advance in Neutron Optics and Related Research Facilities (Neutron Optics in Kumatori '96)*, edited by M. Utsuro, S. Kawano, T. Kawai, and A. Kawaguchi [J. Phys. Soc. Jpn. **65**, Suppl. A 7 (1996)].

[26] S. J. Blundell and J. A. C. Bland, Phys. Rev. B **46**, 3391 (1992).

[27] F. Radu and V. K. Ignatovich, Physica B **267**, 175 (1999).

[28] A. Ruhm, B. P. Toperverg, and H. Dosch, Phys. Rev. B **60**, 16 073 (1999).

[29] V. K. Ignatovich, *The Physics of Ultracold Neutrons* (Clarendon, Oxford, 1990).