PHYSICAL REVIEW B, VOLUME 64, 201318(R)

# Modulation origin of 1/$f$ noise in two-dimensional hopping

V. Ya. Pokrovskii,[1,2] A. K. Savchenko,[1] W. R. Tribe,[3] and E. H. Linfield[3]
[1]School of Physics, University of Exeter, Exeter EX4 4QL, United Kingdom
[2]Institute of Radio Engineering and Electronics, 103907 Moscow, Russia
[3]Cavendish Laboratory, University of Cambridge, Cambridge CB3 0HE, United Kingdom

(Received 17 July 2001; published 5 November 2001)

We show that 1/$f$ noise in a two-dimensional electron gas with hopping conduction can be explained by the modulation of conducting paths by fluctuating occupancy of nonconducting states. The noise is sensitive to the structure of the critical hopping network, which is varied by changing electron concentration, sample size and temperature. With increasing temperature, it clearly reveals the crossover between different hopping regimes.

DOI: 10.1103/PhysRevB.64.201318 PACS number(s): 72.70.+m, 72.20.Ee

The problem of 1/$f$ noise in hopping conduction has been attracting much attention for nearly 20 years, with several models proposed to explain the conductance fluctuations.[1–3] In the single-electron approach the hopping conduction is determined by the critical network (percolation cluster) connecting the pairs of localized states.[4] In the earlier model,[1] developed for nearest-neighbor hopping (NNH), it was suggested that 1/$f$ noise originates from fluctuations in the electron concentration on the cluster, caused by slow electron exchange with the states in its pores. This theory predicts a saturation in the noise spectrum at low frequencies, which has not been seen so far in experiment. In an alternative approach,[2] resistance fluctuations come from the change in the hopping paths themselves, due to fluctuations of the energy levels of the localized states in the cluster. These fluctuations are caused by the random electric field generated by the states outside the cluster, due to the random occupancy of the states. To explain the absence of the saturation predicted in Ref. 1, a configuration model was proposed.[3] In this model 1/$f$ noise is treated as a many-electron problem and is explained by transitions between metastable states of the interacting hopping system.

On the experimental side, it was shown on Si inversion layers that 1/$f$ noise is an intrinsic property of hopping conduction.[5] In experiments on mesoscopic $n$-GaAs samples,[6] an indication was obtained that the energy level fluctuations are important in the origin of hopping noise. Large random telegraph signals (RTSs) were observed, and it was suggested that they are produced by the random field due to low-frequency, long-distance hops between states outside the cluster. The modulation theory of noise[2] has considered the integral effect of all modulators in large samples. Here, the temperature dependence of the resistance noise $S_R/R^2(T)$ has been derived, although the experiments performed on 3D samples[7,8] did not show a unanimous agreement with the theory. While a weak temperature dependence of 1/$f$ noise in the NNH regime[7] was used in Ref. 2, as support for the modulation theory, the measurements of noise in variable-range hopping (VRH)[8] have shown a temperature dependence of opposite sign to that predicted in Ref. 2, and were interpreted as a consequence of electron-electron interaction.

In this work we study 1/$f$ noise in 2D hopping of an electron channel in a MESFET structure, in a broad range of temperatures, from 4.2 to 60 K. In this versatile transistor system, we perform experiments at different electron concentrations, varied by the gate voltage. This allows us to vary the resistance and topology of the critical cluster in the same sample, and see how these changes affect the noise. Additional control of the conducting network is achieved by changing the sample length: in the shortest sample with $L \simeq 0.2$ $\mu$m the cluster is reduced down to a set of well separated linear chains.[9] Our results show that in the VRH regime the character of the cluster is reflected in 1/$f$ noise, and it can be well described in terms of the single-electron modulation approach. Also, the noise properties appear to be very sensitive to the transition between different hopping regimes, observed when the temperature is increased.

We have studied three samples, with lengths $L=0.2$, 0.5, and 1 $\mu$m and width $W=5$ $\mu$m arranged as individual transistors on a uniformly doped, 1500-Å-thick GaAs layer with donor concentration $\simeq 10^{17}$ cm$^{-3}$. At zero gate voltage $V_g$, the channel has metallic conduction, and with its depletion a transition to strong localization occurs.[10] The thickness $t$ of the channel in the depleted regime is close to the mean donor separation ($\approx 200$ Å) and satisfies the condition of 2D hopping: $t \leqslant r$, where $r$ is the typical hopping distance. The resistance noise is measured as fluctuations of the current in the voltage-controlled regime. For a fixed $V_g$, averaged fast-Fourier spectra, $S_I(f)$, are obtained in the range from 0.1 to 50 Hz for different voltages $V$ across the sample, not exceeding 10 mV to satisfy linear $I-V$. It has been established that the spectral density $S_I(f)$ is proportional to the square of the dc current, so that the spectral density of resistance fluctuations $S_R$ is found from the relation $S_R(f)/R^2 \equiv S_I(f)/I^2$.

In Fig. 1 the temperature dependence of the resistance is shown for two representative samples of different length: (a) $L=1$ $\mu$m, and (b) $L=0.2$ $\mu$m (the properties of the sample with length 0.5 $\mu$m are very similar to those of the 1 $\mu$m sample). In both samples, with decreasing concentration there is a transition to hopping transport where $R(T)$ becomes strongly temperature dependent. At $T \lesssim 13$ K, the resistance of the large sample is described by two-dimensional VRH: $R_\square = R_0 \exp(T_0/T)^{1/3}$, where $R_\square$ is resistance "per square" and $R_0 \simeq 40$ kOhm. For $T$ above 13 K $R(T)$ is described by a simple exponential dependence $R_\square \propto \exp(\Delta E/k_B T)$, which can suggest the NNH regime. One can

 

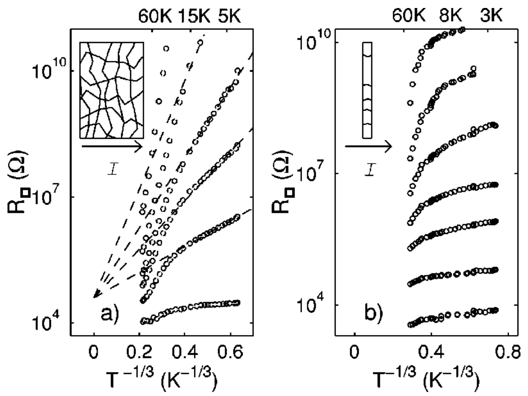


FIG. 1. Temperature dependence of resistance at different gate voltages for, from bottom to top: (a) long sample, $V_g=-0.7$, $-1.24$, $-1.34$, $-1.39$, $-1.45$, $-1.52$ V, and (b) short sample $V_g=-0.22$, $-0.78$, $-1.25$, $-1.38$, $-1.46$, $-1.52$, $-1.58$ V. Dashed lines in (a) show agreement with 2D variable-range hopping. The insets illustrate the structure of the hopping paths—a cluster in the large sample and chains in the small sample. Arrow $I$ shows the direction of the current.

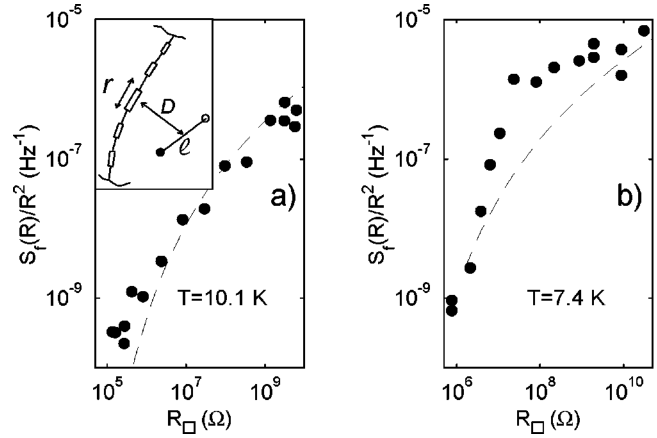


FIG. 2. Dependence of the noise power (at $f=1$ Hz) on resistance varied by $V_g$: (a) long sample; (b) short sample. The broken lines show fits by the relation $S_R(f)/R^2 \propto \ln^{5.66}(R/R_0)$, with $R_0=40$ k$\Omega$. Inset: a diagram of a two-site modulator acting on the critical hop; $r$ is the length of the critical hop with resistance $\rho_{ij}$, $l$ is the separation of the two states of the modulator, $D$ is its distance from the critical hop.

see that at low temperatures the behavior of the small sample in Fig. 1(b) is quite different: The gradient of the temperature dependence is much smaller and it only slightly increases with decreasing concentration.

The evolution of $R(T)$ with decreasing concentration in a transistor structure is caused by an increase of the typical hopping length, when the Fermi level moves down the tail of the localized states. In the Miller–Abrahams approach the resistance of an electron hop between sites $i$ and $j$ is determined by the hopping distance $r_{ij}$ and the effective activation energy $\Delta E_{ij}$:[4]

$$\rho_{ij} \simeq \rho_0 \exp\left(\frac{2r_{ij}}{a} + \frac{\Delta E_{ij}}{k_B T}\right). \qquad (1)$$

In the VRH regime the typical hopping distance $r \simeq a\xi$, where $\xi=(T_0/T)^{1/3}$, $T_0=13.5/(ga^2)$ and $g$ is the density of states at the Fermi level. Hence, $r$ increases with decreasing temperature and concentration. The correlation length of the network $L_c$, which is a measure of an individual link in the cluster shown in the inset to Fig. 1, also increases as $L_c \simeq r\xi^{\nu}$, where $\nu=1.33$ in 2D.[4] In the NNH regime, expected at higher temperatures when $2r_{ij}/a > \Delta E_{ij}/(k_B T)$, the hopping distance is determined by the nearest localized states and does not depend on temperature, so that the cluster density also becomes temperature independent.

The weak temperature dependence for the small sample [Fig. 1(b)] can be explained by the fact that its conducting network is reduced into a set of parallel hopping chains,[11,12] Fig. 1(b) inset. The chain conductance is larger than that of the cluster and is less temperature dependent. The evolution of a 2D cluster into a set of 1D chains occurs when $L_c$ becomes larger than the sample length $L$.[11] The hopping exponent $\xi$, estimated for the large sample, in the range $|V_g| = 1.2$–$1.35$ V is $\xi = 2.5$–$5$, which gives $L_c \approx a\xi^{1+\nu} = 0.1$–$0.5$ $\mu$m for localization length $a \simeq a_B \simeq 100$ Å. This satisfies the condition $L \leqslant L_c$ for the majority of concentrations in the small sample.

With this understanding of the specifics of electron hopping in the two samples, let us turn to noise measurements to see how the difference in the topology of the hopping paths is reflected in $1/f$ noise. We have established that in both samples the frequency dependence of $S_R(f)/R^2$ is close to $1/f$ in the whole range of $V_g$ and $T$. Similar to Ref. 6, on top of $1/f$ noise some individual fluctuators can be revealed as maxima in the frequency dependence of $[f \times S_R(f)/R^2]$, each indicating a contribution of a Lorentzian spectrum. In the small sample the amplitude of the RTSs is seen to be larger and can reach $\sim 10\%$ of the sample resistance. (Assuming that the modulator affects only the chains positioned nearby, the total number of chains in the small sample is small–about 10.)

The noise power increases with decreasing concentration in the whole temperature range in both samples. In Fig. 2 we show examples of their normalized noise power $S_R(f)/R^2$ at frequency $f=1$ Hz presented as a function of resistance varied by $V_g$. (A similar experiment, although near the metal-to-insulator transition and not in hopping, was performed on $In_xO_{1-x}$ films where noise power showed saturation with increasing resistance.[13]) In the small sample the growth is weaker at large resistances, where the formation of hopping chains is expected.

Comparison of the temperature dependences of noise and resistance in the large sample in the whole temperature range shows an interesting tendency, Fig. 3. With increasing $T$ and approaching the transition from the VRH to the activation regime, the dependence $R(T)$ becomes stronger, while the temperature dependence of $S_R(f)/R^2(T)$ weakens. (The short sample has shown a similar tendency, but with a weaker temperature dependence of noise at lower temperatures.)

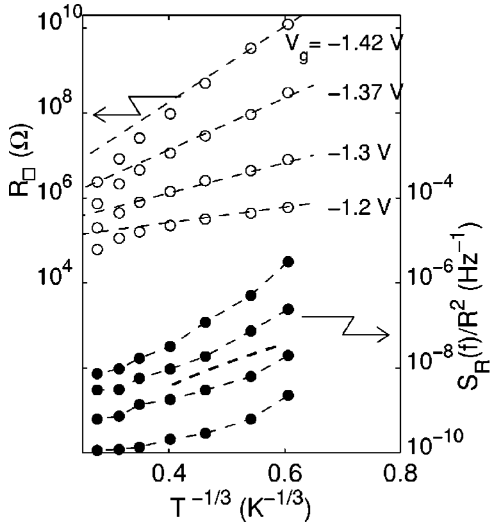


FIG. 3. Temperature dependence of resistance $R_\square$ (open circles) for the long sample, with dashed lines $R(T) \propto \exp(T_0/T)^{1/3}$, and its normalized noise power (closed circles) at $f=1$ Hz, where the dashed lines through the points are guides to the eye. The bold dashed line is plotted using Eq. (4).

We can explain these results within the modulation approach, which we modify here for the case of 2D hopping. According to model,[2] the noise power is determined by two factors: (a) the number of links in the cluster $N$, i.e., the cluster density; (b) the modulation intensity of the critical (dominant) hop in the link. To find the integral noise, random fluctuations $(\delta\rho_i, \delta\rho_i)_\omega$ of the critical resistances $\rho_i$ are averaged over $N$ links of the cluster, to give

$$\frac{S_R(f)}{R^2} \sim \frac{1}{N^2}\Sigma_i \frac{(\delta\rho_i, \delta\rho_i)_\omega}{\rho_i^2} \sim \frac{1}{N}\frac{(\delta\rho_i)_\omega^2}{\rho_i^2}. \qquad (2)$$

The modulation of resistance $\rho_i$ is caused by the shift $\delta V$ of the energy $\Delta E_{ij}$ in the critical hop, Eq. (1), so that $|\delta\rho_i/\rho_i| \sim \min(\delta V/k_B T, 1)$. (The maximum possible modulation of $\sim$100% corresponds to large energy change, $\delta V/k_B T > 1$, when the critical path itself is altered.) The increase of noise in Fig. 2 with decreasing concentration can then be viewed as being primarily due to a decrease in the number of fluctuating elements $N=(L/L_c)^2$, when, with increasing hopping length $r$, the cluster becomes less dense. Slow increase of noise in the small sample indicates that the number of statistically chosen, most conductive, chains does not vary significantly when electron concentration is decreased.[11]

The temperature effect on noise in Fig. 3 can also be understood in terms of its effect on the critical cluster density. In VRH, the number of links $N$ rapidly increases when the temperature goes up, and this is why the noise drops. However, in the activation regime, the cluster size no longer increases and the noise dependence on $T$ weakens.

To analyze the temperature dependence of the noise power quantitatively, one also has to take into account the temperature dependence of the modulation intensity $M(T)$. Introducing the modulator density $P(E,\tau)$, where $E$ is the energy of the modulator level and $\tau$ is the relaxation time, and assuming that $P(\tau)$ to be exponentially broad to produce the 1/$f$ spectrum, one can write in 2D:

$$M = \frac{(\delta\rho_i)_\omega^2}{\rho_i^2} \sim \frac{P(T)T}{\omega}\int \min\left(\frac{[\delta V(D)]^2}{T^2}, 1\right) d^2R$$
$$\sim \frac{P(T)T}{\omega} D_T^2(T). \qquad (3)$$

In this expression $\delta V(D,r)$ is the potential variation on the length $r$ of the critical hop, $\delta V \sim r(\partial V/\partial D)$, which is caused by an electron transition between the two states of the modulator positioned at distance $D$ from the critical hop, inset to Fig. 2(a). The factor $T$ in this expression describes the number of active modulators whose occupancy fluctuates at a given temperature. The area $D_T^2(T)$ gives the number of the strongest ($\sim$100%) modulators that are able to alter the critical hop. Such modulators have to be positioned within the distance $D_T$ to the hop, such that $\delta V(D_T) \simeq k_B T$. The distance $D_T(T)$ depends on the details of the potential $V(D)$ produced by the modulator. Namely, for a dipole potential, when the separation $l$ of the two states of the modulator is smaller than the distance $D$ from the modulator to the critical hop, $\delta V \sim r(\partial V/\partial D) \propto rl/D^3$ and $D_T^3 \sim e^2 rl(\omega)/(\chi k_B T)$. Then the efficiency $M(T)$ increases with temperature in VRH as $T(r/T)^{2/3} \propto T^{1/9}$ [taking into account that $r \sim a\xi$ and $\xi = (T_0/T)^{1/3}$]. In the opposite case of large $l$, the potential of the modulator is determined by the Coulomb potential of the state closest to the critical hop, so that $\delta V \propto r/D^2$ and $D_T^2 \sim e^2 r/(\chi k_B T)$. In this case the modulation efficiency $M(T) \propto T^{-1/3}$ decreases with increasing temperature.

To get information about the temperature dependence $M(T)$ in VRH of the large sample, we have performed noise measurements at the condition when, at different temperatures, the resistance of the sample is kept constant by adjusting $V_g$. For VRH this corresponds to the same density of the cluster, so that the temperature dependence of noise is only due to the variation of the modulation efficiency. Assuming $P$ =const and $\xi$=const in this experiment, one can get $M(T) \propto T^{1/3}$ for the dipole potential and $M(T) \propto$const for the Coulomb potential. The results are shown in Fig. 4. The increase of noise with decreasing temperature at $T<13$ K is an indication that the modulator potential is not of a dipole type but probably produced by a Coulomb potential of a single state. However, at larger resistances the $M(T)$-dependence appears to be stronger than expected in this case, which requires further consideration.

One can see that with increasing $T$ and transition to the exponential regime, the temperature dependence of noise at a fixed sample resistance changes its sign, Fig. 4. This results from the fact that in this regime the noise measured at a fixed $V_g$ only weakly decreases with increasing $T$, while the resistance drops significantly. In addition, the noise strongly depends on resistance, Fig. 2(a). Therefore, when $T$ is increased and $V_g$ is then adjusted to keep the resistance constant, the measured noise increases. In this regime the cluster becomes temperature independent, and the noise can be caused by

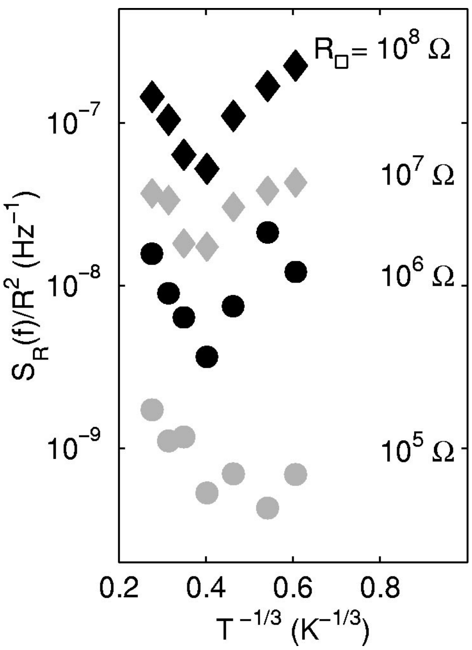


FIG. 4. Temperature dependence of noise at $f = 1$ Hz when the resistance is kept constant by adjusting $V_g$. Different symbols correspond to different values of $R$ indicated in the plot.

both the modulation[2] and fluctuation of concentration.[1] There is a difficulty, however, in analyzing this regime quantitatively, as in our case it is not conventional NNH. This is seen in Fig. 1(a) where the activation energy of $R(T)$ increases with decreasing concentration, which means that not all the states of the impurity band are involved in hopping and their number depends on the Fermi level position. This can be due to the fact that at high $T$ one should take into account the energy dependence of the density of states in the tail of the impurity band, which modifies the character of hopping.[14]

Let us come back to the low-temperature VRH where our results show the domination of the modulation source of noise. We can now combine the information about the modulation efficiency $M(T)$ in the VRH regime with the number of links $N(T) = (L/r\xi^\nu)^2$, and get the temperature dependence of the noise power in VRH of the large sample:

$$\frac{S_R(f)}{R^2} \sim \frac{1}{N(T)} M(T) \sim \left(\frac{r\xi^\nu}{L}\right)^2 \frac{P_0 T}{\omega} \left(\frac{e^2 r}{\chi k_B T}\right) \propto \xi^{2\nu+3} \propto T^{-1.89}. \tag{4}$$

This result is in satisfactory agreement with $S_R(T)/R^2$ presented in Fig. 3 for smaller resistances within the experimental error in the measured noise power. Using Eq. (4) and relation $\xi = \ln[R(T)/R_0]$ we can also obtain a relationship between the noise power and the resistance: $S_R(f)/R^2 \propto \xi^{2\nu+3} = \ln^{5.66}(R_\square/R_0)$. Comparison of this dependence with data in Fig. 2(a) also shows reasonable agreement.

In conclusion, we have demonstrated that the site-energy modulation model accounts for the properties of $1/f$ noise in the VRH regime of 2D hopping in an $n$-GaAs channel. We have been able to separate different components of $1/f$ modulation noise that are responsible for its temperature and resistance dependence. It has also been shown that the noise measurements are sensitive to the transition between different hopping regimes, as well as to the difference in the topology of the hopping cluster in large and small samples.

We are grateful to S. M. Kogan, M. E. Raikh, B. I. Shklovskii, and V. I. Kozub for helpful discussions, and the Royal Society, EPSRC, the RFBR (Grant No. 99-02-17387), and MNTP "Physics of Solid State Nanostructures" (Grant No. 97-1052) for financial support.

[1] B.I. Shklovskii, Solid State Commun. **33**, 273 (1980); Sh.M. Kogan, B.I. Shklovskii, Fiz. Tekhn. Poluprovodn **15**, 1049 (1981) [Sov. Phys. Semicond. **15**, 605 (1981)].
[2] V.I. Kozub, Solid State Commun. **97**, 843 (1996).
[3] Sh. Kogan, Phys. Rev. B **57**, 9736 (1998).
[4] B.I. Shklovskii and A. L. Efros, *Electronic Properties of Doped Semiconductors* (Springer, Berlin, 1984).
[5] R.F. Voss, J. Phys. C **11**, L923 (1978).
[6] V.V. Kuznetsov, E.I. Laiko, and A.K. Savchenko, JETP Lett. **49**, 453 (1989).
[7] I. Shlimak, Y. Kraftmakher, R. Ussyshkin, and K. Zilberberg, Solid State Commun. **93**, 829 (1995).
[8] J.G. Massey and M. Lee, Phys. Rev. Lett. **79**, 3986 (1997).
[9] E.I. Laiko *et al.*, Sov. Phys. JETP **66**, 1258 (1987); A.O. Orlov, M.E. Raikh, I.M. Ruzin, and A.K. Savchenko, Solid State Commun. **72**, 169 (1989).
[10] D.J. Newson, M. Pepper, and T.J. Thornton, Phil. Mag. B, **56**, 775 (1987).
[11] M. E. Raikh and I. M. Ruzin, in *Mesoscopic Phenomena in Solids*, edited by B. L. Altshuler, P. A. Lee, and R. A. Webb (Elsevier Science, New York, 1991), p. 315.
[12] L.I. Glazman and K.A. Matveev, Zh. Eksp. Teor. Fiz. **94**, 332 (1988) [Sov. Phys. JETP **67**, 1276 (1988)].
[13] O. Cohen and Z. Ovadyahu, Phys. Rev. B **50**, 10 442 (1994).
[14] F.R. Shapiro and D. Adler, Journal of Non-Crystalline Solids **74**, 189 (1985); D. Monroe, Phys. Rev. Lett. **54**, 146 (1985).