Beyond the random phase approximation in the Singwi-Sjölander theory of the half-filled Landau level

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(Received 3 May 2001; published 15 October 2001)

We study the $\nu=1/2$ Chern-Simons system and consider a self-consistent field theory of the Singwi-Sjölander type which goes beyond the random phase approximation (RPA). By considering the Heisenberg equation of motion for the longitudinal momentum operator, we are able to show that the zero-frequency density-density response function vanishes linearly in the long-wavelength limit independent of any approximation. From this analysis, we derive a consistency condition for a decoupling of the equal-time density-density and density-momentum correlation functions. By using the Heisenberg equation of motion of the Wigner distribution function with a decoupling of the correlation functions which respects this consistency condition, we calculate the response functions of the $\nu=1/2$ system. In our scheme, we get a density-density response function which vanishes linearly in the Coulomb case for zero frequency in the long-wavelength limit. Furthermore, we derive the compressibility, and the Landau energy as well as the Coulomb energy. These energies are in better agreement with numerical and exact results, respectively, than the energies calculated in the RPA.

DOI: 10.1103/PhysRevB.64.195307 PACS number(s): 71.10.Pm, 73.43.−f, 71.27.+a

I. INTRODUCTION

The combination of an electronic interaction and a strong magnetic field in a two-dimensional electron system yields a rich variety of phases. These are best classified by the filling factor ν, which is the electron density divided by the density of a completely filled Landau level. In the case of $\nu\cong 1/2$, the behavior of the system resembles that of a Fermi liquid in the absence of a magnetic field or at small magnetic fields. This effect can be explained with a new sort of quasiparticles: at $\nu=1/2$, each electron combines with two flux quanta of the magnetic field to form a composite fermion; these composite fermions then move in an effective magnetic field which is zero on the average. The interpretation of many experiments supports this picture. We mention transport experiments with antidots, in which features of the resistivity are related to closed loops of the composite fermions around the dots,[1] and also focusing experiments.[2] An overview over further experiments can be found in Ref. 3. A field theoretical formulation of this composite fermion picture was first established by Halperin, Lee, and Read[4] (HLR) as well as Kalmeyer and Zhang.[5] They formulated the Hamiltonian in terms of Chern-Simons (CS) transformed electrons and studied within the random phase approximation (RPA) many physical quantities. Besides the theory of HLR there are other alternative formulations of the composite fermionic picture which are mainly based on a gauge transformation of the CS Hamiltonian.[6]

We used in Ref. 7 the theory of HLR for a determination of the ground-state energy of the $\nu=1/2$ system in the RPA. There we found an infrared diverging Landau energy. This problem was solved in Refs. 8–10 by taking into account the correct normal ordering of the operators. We obtained a Landau as well as a Coulomb energy in the RPA which is in satisfactory agreement with the exact and numerical results, respectively.[9,10] In Ref. 11, Conti and Chakraborty (CC) improved the calculation of the Coulomb energy by taking into account dynamical correlations through the formalism of Singwi, Tosi, Land, and Sjölander[12] (STLS), known as the Singwi-Sjölander theory and established first in the calculation of the structure factor of the Coulomb theory. This method is a generalization of the RPA. In comparison to the RPA this method results in a Coulomb energy which is a better approximation to the Coulomb energy obtained earlier by numerical simulations of interacting electrons in the lowest Landau level by Morf and d'Ambrumenil[13] and by Girlich.[14] CC did not calculate the Landau energy of the $\nu=1/2$ system which would also be a very interesting quantity to be compared with the RPA as well as the true result $\omega_c/2$, the Landau energy per electron. In their theory the resulting zero-frequency density-density response function vanishes as the square of the wave vector in the long-wavelength limit. This is in contradiction to the RPA result where it vanishes linearly for the Coulomb interaction.[4] In their paper, CC mentioned that the behavior of the zero-frequency response function of their theory is similar to the zero-frequency response functions of the alternative formulations of the CS theory[6] cited above. But later on, Halperin and co-workers[15] showed that the zero-frequency response function of these theories also vanishes linearly in the wave vector for the Coulomb interaction if one expands the approximation to the RPA.

In this paper, we will show that the quadratic behavior of the zero-frequency response function in the theory of CC results from the decoupling of the equal-time density-density and density-momentum correlation functions. In CS theories, this needs a careful treatment as will be shown by considering the Heisenberg equation of motion for the longitudinal momentum operator. With the help of this equation, we will get a zero-frequency response function of the $\nu=1/2$ system which vanishes linearly for the Coulomb interaction in the long-wavelength limit. It can be also learned from this pro-

cedure which relation in the decoupling approximations of density-density and density-momentum correlation functions in CS theories should be fulfilled. In the STLS theory it is the Heisenberg equation of motion of the Wigner distribution function which is used to get the response functions of the system. In CS theories, one has to decouple simultaneously a density-density correlation function and a density-momentum correlation function. We will show that the consistency relation for the decoupling of these two functions are not fulfilled in the theory of CC. This is the reason for the quadratic behavior of the long-wavelength zero-frequency density-density response function in their theory. We will suggest a STLS-type theory of the CS system, in which the decouplings respect the consistency condition. With the help of this decoupling, we will calculate the density-density response function, the compressibility, the dynamic structure factor, the static structure factor, and the Coulomb energy, as well as the Landau energy.

The paper is organized as follows: We will introduce in Sec. II the CS Hamiltonian and calculate the Heisenberg equation of motion for the longitudinal momentum. Then we get from this equation the static density-density response function in the long-wavelength limit and discuss the consistency relation for a decoupling of the equal-time density-density and density-momentum correlation functions. We will study in Sec. III the equation of motion of the Wigner distribution function and suggest a decoupling that respects this relation.

II. STATIC RESPONSE FUNCTION OF THE $\nu=1/2$ CS SYSTEM

At first, we review the main steps of the HLR approach of the $\nu=1/2$ system. Here, we follow in the main the notation of CC.[11] The CS transformation for spinless fermions is defined by[4]

$$\Psi^+(\vec{r})=\Psi_e^+(\vec{r})\exp\left[2i\int d\vec{r}'\arg(\vec{r}-\vec{r}')\rho(\vec{r}')\right], \qquad (1)$$

where $\Psi_e^+(\vec{r})$ is the electron creation operator, $\Psi^+(\vec{r})$ is the transformed fermion operator, $\rho(\vec{r})$ is the density operator of the fermions, and $\arg(\vec{r})$ is the angle that $\vec{r}$ forms with the x axis. The kinetic part of the Hamiltonian is given after the transformation as

$$H_{\rm kin}=\frac{1}{2m_b}\int d^2r\Psi^+(\vec{r})[-i\vec{\nabla}+\delta\vec{A}(\vec{r})]^2\Psi(\vec{r}), \qquad (2)$$

where m_b is the electron band mass and

$$\delta A_i(\vec{r})=\int d\vec{r}'\,\phi_i(\vec{r}-\vec{r}')[\rho(\vec{r}')-\rho_0] \qquad (3)$$

is the fluctuation of the CS vector potential. ρ_0 is the mean density of the $\nu=1/2$ system and $\vec{\phi}(\vec{r})=2\vec{\nabla}\arg(\vec{r})=2\vec{e}_z\times\vec{r}/r^2$. We used the convention $\hbar=1$ and $c=1$ in the above formula (2). By expanding the Hamiltonian in Eq. (2) and keeping only terms up to second order in the density fluctuations, one gets in the momentum space

$$H=\sum_{\vec{k}}\frac{k^2}{2m_b}a_{\vec{k}}^\dagger a_{\vec{k}}+\sum_{\vec{k}\neq 0}i\frac{v_1(k)}{m_b}\left[:\left(\frac{\vec{k}}{k}\times\vec{\pi}(\vec{k})\right)\rho(-\vec{k}):\right.$$
$$\left.+\frac{1}{2}[v_0(k)+v_2(k)]:\rho(\vec{k})\rho(-\vec{k}):\right]. \qquad (4)$$

Here $\vec{\pi}(\vec{k})$ is the Fourier transformed momentum operator $\vec{\pi}(\vec{r})=-i\Psi(\vec{r})^+\vec{\nabla}\Psi(\vec{r})$, $a_{\vec{k}}^\dagger$ creates a CS fermion with momentum $\vec{k}$, and $v_0(k)=2\pi\epsilon^2/k$ is the Coulomb interaction where $\epsilon^2=e^2/\varepsilon$. e is the charge of the electron and ε is the dielectric constant of the background. $v_1(k)=4\pi/k$ and $v_2(k)=(4\pi)^2\rho_0/(m_bk^2)$ are CS potentials. We denote by $:A:$ the normal ordering of the operator A. With the help of the CS transformation of the electronic Hamiltonian, we thus get a CS Hamiltonian which does not contain a magnetic field.

First, we will discuss the asymptotics for small wave vectors of the zero-frequency density-density response function by a method which allows us to study the decoupling of correlation functions in a Singwi-Sjölander theory of the half-filled lowest Landau level. Singwi and Tosi used in Ref. 16 a method to obtain the compressibility sum rule for the Coulomb system from the Heisenberg equation of motion of the longitudinal momentum operator $\vec{q}/q\cdot\vec{\pi}(\vec{q})$. The commutator CH of $\vec{q}/q\cdot\vec{\pi}(\vec{q})$ with the Hamiltonian (4) is

$$\mathrm{CH}(\vec{q},t)=\sum_{\vec{k}'}\frac{1}{m_b}\frac{(\vec{k}\vec{q})^2}{q}a_{\vec{k}+\vec{q}/2}^\dagger(t)a_{\vec{k}-\vec{q}/2}(t)+\sum_{\vec{q}'\neq 0}[v_0(q')+v_2(q')]:\rho(\vec{q}',t)\rho(\vec{q}-\vec{q}',t):\left(\frac{\vec{q}'\vec{q}}{q}\right)$$
$$+i\sum_{\vec{q}'}\frac{v_1(q')}{m_b}:\left(\frac{\vec{q}'}{q'}\times\vec{\pi}(\vec{q}',t)\right)\rho(\vec{q}-\vec{q}',t):\left(\frac{\vec{q}'\vec{q}}{q}\right)-i\sum_{\vec{q}'\neq 0}\frac{v_1(q')}{m_b}\left[:\left(\frac{\vec{q}'}{q'}\times\vec{\pi}(\vec{q}+\vec{q}',t)\right)\rho(-\vec{q}',t):\left(\frac{\vec{q}'\vec{q}}{q}\right)\right.$$
$$\left.+:\left(\frac{\vec{q}}{q}\cdot\vec{\pi}(\vec{q}+\vec{q}',t)\right)\rho(-\vec{q}',t):\left(\frac{\vec{q}'}{q'}\times\vec{q}\right)\right]. \qquad (5)$$

We consider in addition a coupling to an external potential $V^{\rm ext}(\vec{q},t)$. Since we want to study the adiabatic limit we neglect the time derivative in the Heisenberg equation of motion

$$\sum_{\vec{q}'} V^{\rm ext}(\vec{q},t)\rho(\vec{q}-\vec{q}',t)\left(\frac{\vec{q}'\vec{q}}{q}\right) = -{\rm CH}(\vec{q},t). \qquad (6)$$

We take the expectation value of this equation of motion with respect to the ground state of the system (ground state for $V^{\rm ext}=0$) and get the following equation which is valid to linear order in $V^{\rm ext}$:

$$V^{\rm ext}(\vec{q},t)\rho_0 = -\langle {\rm CH}(\vec{q},t)\rangle_c \frac{1}{q} - \rho_0(v_0(q)+v_2(q))\langle\rho(\vec{q},t)\rangle - i\rho_0\frac{v_1(q)}{m_b}\left(\frac{\vec{q}}{q}\times\langle\vec{\pi}(\vec{q},t)\rangle\right). \qquad (7)$$

$\langle\cdot\rangle_c$ is the cumulant part of the expectation value $\langle AB\rangle_c = \langle AB\rangle - \langle A\rangle\langle B\rangle$, where A, B are the operators $\vec{\pi}$ or ρ.

In the following we will discuss the leading order in q of the terms in Eq. (7). By considering only the linear order in $V^{\rm ext}$ we get for $q\to 0$ the following relation (compare Ref. 16):

$$\langle O\rangle = \left[\frac{\partial}{\partial\rho_0}\langle O\rangle^{V^{\rm ext}=0}\right]\langle\rho(q,t)\rangle, \qquad (8)$$

for an operator O. Here $\langle\cdot\rangle^{V^{\rm ext}=0}$ is given by the expectation value for $V^{\rm ext}=0$. We obtain that $\langle {\rm CH}(\vec{q},t)\rangle_c/q$ has the asymptotic behavior $O(q^0)\langle\rho(\vec{q})\rangle$ for $q\to 0$. For deriving this asymptotics we take the $q\to 0$ limit in every cumulant expectation value in Eq. (5). Every additive term in Eq. (5) contains either a term proportional to q or has a term linear in $\vec{q}'\cdot\vec{q}/q$ (the terms with a quadratic q' in the numerator cancel). We now discuss the leading terms in Eq. (7). We begin with the third term of the right-hand side of Eq. (7) and retransform it with the help of Eq. (1) from the CS fermions to the electrons (CS retransformation). Denoting the expectation value with respect to the electronic ground state by $\langle\cdot\rangle_e$ we obtain

$$\langle\vec{\pi}(\vec{q},t)\rangle = m_b\langle\vec{J}(q,t)\rangle_e - i\sum_{\vec{q}'\neq 0}\vec{e}_z \times\frac{\vec{q}'}{q'^2}\langle:\rho(\vec{q}'+\vec{q})\rho(-\vec{q}'):\rangle_e. \qquad (9)$$

$J(\vec{q},t)$ is the electron current operator $J(\vec{q},t) = \Sigma_{\vec{k}}(\vec{k}/m_b)\,a^\dagger_{e,\vec{k}+\vec{q}/2}(t)\,a_{e,\vec{k}-\vec{q}/2}(t) + (1/m_b)\,\Sigma_{\vec{q}'}\vec{A}(\vec{q}')\,\rho(\vec{q}-\vec{q}',t)$. Here $\vec{A}(\vec{q}')$ is the external vector potential of the $\nu=1/2$ system. Now we use Eq. (8) to calculate the first term of the right-hand side of Eq. (9). Since $\langle\vec{J}(\vec{r},t)\rangle_e^{V^{\rm ext}=0}=0$ for all electron densities of the system, one gets zero for this term. The cumulant expectation value of the second term of Eq. (9) is of order $O(q^0)\langle\rho(\vec{q})\rangle$. Thus, the third term of Eq. (7) is given $(q\to 0)$by the noncumulant part

$$i\rho_0\frac{v_1(q)}{m_b}\left(\frac{\vec{q}}{q}\times\langle\vec{\pi}(\vec{q},t)\rangle\right) = -\rho_0 v_2(q)\langle\rho(\vec{q},t)\rangle + O(q^0)\langle\rho(\vec{q},t)\rangle. \qquad (10)$$

By inserting this equation into Eq. (7) we get for the static linear response in the limit $q\to 0$

$$\langle\rho(\vec{q},t)\rangle = -\frac{1}{v_0(q)+O(q^0)}V^{\rm ext}(\vec{q},t). \qquad (11)$$

Thus the static density-density response function vanishes as $v_0(q)^{-1}$, i.e., linearly for $q\to 0$.

The above results are rigorous consequences of the equation of motion in the limit $q\to 0$. We consider this result as a consistency condition for an approximative calculation of the correlation functions: An approximative density-density correlation function and the corresponding density-momentum correlation function have to satisfy the above analysis. Especially we have seen that the $1/q^2$ singularity of the noncumulant part of the commutator in Eq. (7), due to the commutation with the v_2 term of H [second term in Eq. (5)], is canceled by the $1/q^2$ singularity of the noncumulant part, due to the commutation with the v_1 term of H [third term in Eq. (5)]. Thus there is no $1/q^2$ singularity in the denominator of Eq. (11). It is clear from the structure of the operators and from the derivation above that the first term gets this $1/q^2$ singularity by averaging with respect to every state not only the CS ground state. This is no longer true for the second term. In this term one gets the $1/q^2$ singularity by averaging over the CS ground state. This ground state is reached by the dynamics of the CS system. Thus the consistency relation is written for $q\to 0$ as

$$\sum_{\vec{q}'\neq 0} v_2(q')\langle:\rho(\vec{q}',t)\rho(\vec{q}-\vec{q}',t):\rangle\left(\frac{\vec{q}'\vec{q}}{q}\right) + i\sum_{\vec{q}'}\frac{v_1(q')}{m_b}\left\langle:\left(\frac{\vec{q}'}{q'}\times\vec{\pi}(\vec{q}',t)\right)\rho(\vec{q}-\vec{q}',t):\right\rangle\left(\frac{\vec{q}'\vec{q}}{q}\right) - i\sum_{\vec{q}'\neq 0}\frac{v_1(q')}{m_b}\left\langle:\left(\frac{\vec{q}'}{q'}\times\vec{\pi}(\vec{q}+\vec{q}',t)\right)\rho(-\vec{q}',t):\right\rangle \times\left(\frac{\vec{q}'\vec{q}}{q}\right) = O(q). \qquad (12)$$

Here, the third term cancels the cumulant part of the second term to order $O(q)$.

We add a remark concerning the Hamiltonian (4). The Hamiltonian (4) is truncated by keeping only terms up to second order in the quadratic density fluctuations. We can show by the same methods as above that an analysis of the full problem with the Hamiltonian (2) leads to the same results.

In the next section we formulate a Singwi-Sjölander theory of the half-filled Landau level, where we make an

approximation of the momentum-density and density-density correlation functions that respects the consistency condition (12) derived above.

III. CS RESPONSE FUNCTION WHICH INCLUDES DYNAMICAL CORRELATIONS

In this section, we will calculate response functions of the $\nu=1/2$ CS system that include correlations beyond the RPA. As mentioned earlier, one transforms the Hamiltonian of electrons in a magnetic field to a CS Hamiltonian (4) at zero magnetic field. Thus one can calculate response functions of the $\nu=1/2$ system with approximative methods that were developed for the Coulomb system earlier. In this section, we will apply the theory of STLS (Ref. 12) to the CS system in the spirit of CC.[11] The response function matrix χ relates the density $\rho(\vec{k},\omega)$ and transverse momentum response $\pi_{\mathrm{T}}(\vec{k},\omega)=\vec{k}/k\times\vec{\pi}(\vec{k},\omega)$ to an external perturbation a scalar potential V^{ext} and a transverse vector potential $A_{\mathrm{T}}^{\mathrm{ext}}$ via

$$\begin{pmatrix}\rho(\vec{k},\omega)\\ \pi_{\mathrm{T}}(\vec{k},\omega)\end{pmatrix}=[\chi(\vec{k},\omega)]\begin{pmatrix}V^{\mathrm{ext}}(\vec{k},\omega)\\ A_{\mathrm{T}}^{\mathrm{ext}}(\vec{k},\omega)\end{pmatrix}. \tag{13}$$

Following the original derivation of STLS,[12] we start from the equation of motion for the one-body Wigner distribution function,

$$f^{(1)}(\vec{r},\vec{p};t)=\sum_{\vec{k}} e^{i\vec{k}\cdot\vec{r}}\langle a^{\dagger}_{\vec{p}-\vec{k}/2}(t)a_{\vec{p}+\vec{k}/2}(t)\rangle, \tag{14}$$

which determines the density $\rho(\vec{r},t)=\Sigma_{\vec{p}}f^{(1)}(\vec{r},\vec{p};t)$ and the momentum $\vec{\pi}(\vec{r},t)=\Sigma_{\vec{p}}\vec{p}f^{(1)}(\vec{r},\vec{p};t)$. The Heisenberg equation of motion for $f^{(1)}(\vec{r},\vec{p};t)$ is

$$\begin{aligned}\frac{\partial}{\partial t}f^{(1)}(\vec{r},\vec{p};t)=&\frac{\vec{p}\cdot\vec{\nabla}_{\vec{r}}}{m_b}f^{(1)}(\vec{r},\vec{p};t)\\ &+\int d^2r'\sum_{\vec{p}'}\left[\frac{(\vec{p}-\vec{p}')_j}{m_b}(\nabla_{r,i}\phi_j)(\vec{r}-\vec{r}')\nabla_{p,i}\right.\\ &\left.+[\nabla_{r,i}(v_0+v_2)](\vec{r}-\vec{r}')\nabla_{p,i}\right]\\ &\times f^{(2)}(\vec{r},\vec{p};\vec{r}',\vec{p}';t)+\nabla_{p,i}f^{(1)}\nabla_{r,i}V^{\mathrm{ext}}(\vec{r},t)\\ &+\frac{p_j}{m_b}\nabla_{p,i}f^{(1)}\nabla_{r,i}A_j^{\mathrm{ext}}(\vec{r},t),\end{aligned} \tag{15}$$

where $\nabla_{p,i}=\partial/\partial p_i$ and

$$\begin{aligned}f^{(2)}(\vec{r},\vec{p};\vec{r}',\vec{p}';t)=&\sum_{\vec{k},\vec{k}'}e^{i\vec{k}'\cdot\vec{r}'}e^{i\vec{k}\cdot\vec{r}}\langle:a^{\dagger}_{\vec{p}-\vec{k}/2}(t)a_{\vec{p}+\vec{k}/2}(t)\\ &\times a^{\dagger}_{\vec{p}'-\vec{k}'/2}(t)a_{\vec{p}'+\vec{k}'/2}(t):\rangle\end{aligned} \tag{16}$$

is the two-body distribution function.

Now we have to decouple the correlation function $f^{(2)}(\vec{r},\vec{p};\vec{r}',\vec{p}';t)$ in Eq. (15). For the Coulomb theory STLS uses the following decoupling:

$$f^{(2)}(\vec{r},\vec{p};\vec{r}',\vec{p}';t)\approx f^{(1)}(\vec{r},\vec{p};t)f^{(1)}(\vec{r}',\vec{p}';t)g(\vec{r}-\vec{r}'). \tag{17}$$

If one uses the decoupling function $g(\vec{r}-\vec{r}')=1$ in Eq. (17), it is easily seen that with the help of this decoupling one gets the density-density response function in the RPA from the equation of motion (15). Such a decoupling does not respect the small-distance correlations. STLS take into account these correlations by taking for $g(\vec{r}-\vec{r}')$ the equilibrium, static pair correlation function. This could also be motivated as follows. One can determine $g(\vec{r}-\vec{r}')$ by summing Eq. (17) over $\vec{p}$ and $\vec{p}'$ such that both sides of the approximation (17) coincide for $V^{\mathrm{ext}}=A_{\mathrm{T}}^{\mathrm{ext}}=0$:

$$g(\vec{r}-\vec{r}')=\frac{\langle:\rho(\vec{r})\rho(\vec{r}'):\rangle}{\langle\rho(\vec{r})\rangle\langle\rho(\vec{r}')\rangle}. \tag{18}$$

The ansatz (17), (18) is a decoupling, specific for the Coulomb theory because the interaction of the Coulomb Hamiltonian consists only of a density-density vertex. In the CS theory the Hamiltonian (4) has in addition to the density-density vertex a density-momentum vertex. The effect of this is given by the first term in the square brackets on the right-hand side of Eq. (15). It is not clear whether the decoupling (17), (18) is a good approximation for this term. In their approximation CC (Ref. 11) used the decoupling (17), (18) for the CS theory.

In the following, we want to check if this decoupling respects the consistency condition (12). For this, we have to determine the density-density correlation function and the density-momentum correlation function from the decoupled two-body Wigner distribution function. By a summation over $\vec{p}$, $\vec{p}'$ we get from Eq. (17)

$$\langle:\rho(\vec{r}',t)\rho(\vec{r},t):\rangle\approx\langle\rho(\vec{r}',t)\rangle\langle\rho(\vec{r},t)\rangle g(\vec{r}'-\vec{r}), \tag{19}$$

$$\langle:\vec{\pi}(\vec{r}',t)\rho(\vec{r},t):\rangle\approx\langle\vec{\pi}(\vec{r}',t)\rangle\langle\rho(\vec{r},t)\rangle g(\vec{r}'-\vec{r}). \tag{20}$$

By inserting these decouplings into Eq. (12) we get for the first summand $\rho_0 v_2(q)q\langle\rho(\vec{q},t)\rangle+O(q)$. The third summand is zero. The second summand of Eq. (12) is given by a similar discussion as in the last section [see especially the discussion below Eq. (9)]:

$$\begin{aligned}i\rho_0\sum_{\vec{k}}\frac{v_1(\vec{q}+\vec{k})}{m_b}&\left(\frac{(\vec{q}+\vec{k})}{|\vec{q}+\vec{k}|}\times\langle\vec{\pi}(\vec{q},t)\rangle\right)(S(k)-1)\\ &\times\frac{(\vec{q}+\vec{k})\vec{q}}{q}-\rho_0 v_2(q)q\langle\rho(\vec{q},t)\rangle+O(q),\end{aligned} \tag{21}$$

where $S(k)-1$ is the Fourier transformation of $g(r)-1$. With the help of $\Sigma_{\vec{k}}[S(k)-1]=-1$ we get for Eq. (21) $-(1/2)\rho_0 v_2(q)q\langle\rho(\vec{q},t)\rangle+O(q)$. Thus we obtain that the decoupling of CC does not fulfill the consistency condition (12). That is the reason why CC obtained in the scheme of their decoupling a zero-frequency density-density response function which vanishes as the square of the wave vector in the long-wavelength limit.

To get a better insight into the violation of the consistency condition (12), we calculate the Fourier transformation of the first two terms in Eq. (12) multiplied by q. This is given by

$$-\int d^2r'\nabla_{r,i}\{\langle:[\rho(\vec{r}',t)-\rho_0]\rho(\vec{r},t):\rangle \times\nabla_{r,i}v_2(\vec{r}'-\vec{r})\}-\int d^2r'\nabla_{r,i}\Bigg\{\langle:[\vec{\nabla}_{r'} \times\vec{\pi}(\vec{r}',t)]\rho(\vec{r},t):\rangle\frac{\nabla_{r,i}v_2(\vec{r}'-\vec{r})}{4\pi\rho_0}\Bigg\}. \tag{22}$$

Because of the appearance of the $\vec{\pi}$ in the second term in Eq. (22), we proceed similarly as in the discussion below Eq. (9) by retransforming to the electronic system. Then, the term of quadratic density fluctuations cancels the first term in Eq. (22). This is no longer true by using the decouplings (19), (20) in Eq. (22). The operator $\vec{\nabla}_{r'}$ in the second term in Eq. (22) acts in this decoupling on $\langle\pi(\vec{r}')\rangle\langle\rho(\vec{r})\rangle$ as well as on $g(\vec{r}'-\vec{r})$. This results in two summands. When we take the quadratic density fluctuation part after a CS retransformation of these two terms we get that the first term is canceled by the first term in Eq. (22) [with the help of $\vec{\nabla}_{r'}\times\vec{\phi}(\vec{r}'-\vec{r})=2\,\delta(\vec{r}'-\vec{r})$]. The term with $\vec{\nabla}_{r'}$ acting on $g(\vec{r}'-\vec{r})$ is the reason for the violation of the consistency condition. In other words, doing the CS retransformation first and then the decoupling of the correlation function, or vice versa, leads to different results. We require that these two actions commute and that fixes the density-momentum decoupling in the case of a given decoupling of the density-density correlation function. For the given density-density decoupling of STLS, Eq. (19), we get by this requirement the decoupling

$$\langle:\vec{\nabla}_{r'}\times\pi(\vec{r}',t)\rho(\vec{r},t):\rangle\approx g(\vec{r}'-\vec{r})\langle[\vec{\nabla}_{r'}\times\vec{\pi}(\vec{r}',t)]\rangle \times\langle\rho(\vec{r},t)\rangle. \tag{23}$$

Thus we get that $g(\vec{r}'-\vec{r})$ should not be differentiated. In the decoupling of the equation of motion (15), we get for the second term in the square brackets after a Fourier transformation with respect to $\vec{r}$

$$\int d^2rd^2r'\sum_{\vec{p}'} e^{i\vec{q}\cdot\vec{r}}[\nabla_{r,i}v(\vec{r}-\vec{r}')]\nabla_{p,i}f^{(2)}(\vec{r},\vec{p};\vec{r}',\vec{p}';t) \approx\int d^2rd^2r'\sum_{\vec{p}'} e^{i\vec{q}\cdot\vec{r}}[\nabla_{r,i}v(\vec{r}-\vec{r}')]\nabla_{p,i}f^{(1)}(\vec{r},\vec{p};t)f^{1}(\vec{r}',\vec{p}';t)g(\vec{r}-\vec{r}'), \tag{24}$$

where $v=v_0+v_2$. The first term in the square brackets should be decoupled by

$$\int d^2rd^2r'\sum_{\vec{p}'} e^{i\vec{q}\cdot\vec{r}}\frac{(\vec{p}-\vec{p}')_j}{m_b}(\nabla_{r,i}\phi_j)(\vec{r}-\vec{r}')\nabla_{\vec{p},i}f^{(2)}(\vec{r},\vec{p};\vec{r}',\vec{p}';t) =\int d^2rd^2r'\sum_{\vec{p}'} e^{i\vec{q}\cdot\vec{r}}[\nabla_{r,i}v_2(\vec{r}-\vec{r}')]\frac{1}{4\pi\rho_0}(\vec{\nabla}_r\times\vec{p}+\vec{\nabla}_{r'}\times\vec{p}'+i\vec{q}\times\vec{p})\nabla_{\vec{p},i}f^{(2)}(\vec{r},\vec{p};\vec{r}',\vec{p}';t)$$
$$\approx\int d^2rd^2r'\sum_{\vec{p}'} e^{i\vec{q}\cdot\vec{r}}[\nabla_{r,i}v_2(\vec{r}-\vec{r}')]\frac{1}{4\pi\rho_0}g(\vec{r}-\vec{r}')(\vec{\nabla}_r\times\vec{p}+\vec{\nabla}_{r'}\times\vec{p}'+i\vec{q}\times\vec{p})\nabla_{\vec{p},i}f^{(1)}(\vec{r},\vec{p};t)f^{(1)}(\vec{r}',\vec{p}';t), \tag{25}$$

where $g(\vec{r}-\vec{r}')$ is not differentiated.

With the help of the Heisenberg equation of motion (15) and the approximations (24) and (25) it is possible to calculate the response matrix (13) [similar to the calculations of CC (Ref. 11)]. By doing this we get

$$\chi=\chi^0[1-U\chi^0]^{-1}, \tag{26}$$

where

$$\chi^0=\begin{pmatrix}\chi^0_{\rho\rho} & 0\\ 0 & \chi^0_T\end{pmatrix}. \tag{27}$$

$\chi^0_{\rho\rho}$ is the ideal gas density-density response and χ^0_T is the corresponding transversal momentum-momentum response. These response functions are known analytically.[7] The matrix of the effective potentials is

$$U=\begin{pmatrix}w_0(k)+w_2(k) & iw_1(k)\\ -iw_1(k) & 0\end{pmatrix}, \tag{28}$$

where $w_\alpha(k)=[1-G_\alpha(k)]v_\alpha(k)$ and the local field factors $G_\alpha(k)$ are given by

$$G_\alpha(k)=\frac{1}{\rho_0}\sum_{\vec{p}}[1-S(p)]\frac{[\vec{k}\cdot(\vec{k}-\vec{p})]^{(a_\alpha+b_\alpha)/2}}{k^{a_\alpha}|\vec{k}-\vec{p}|^{b_\alpha}}. \tag{29}$$

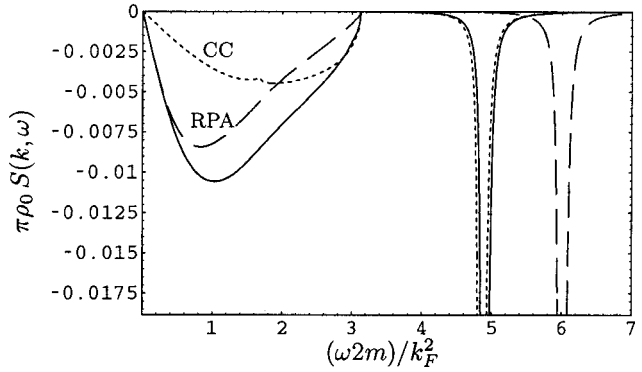

FIG. 1. The dynamic structure factor $S(k,\omega)$ times $\pi\rho_0$ for $k=0.6k_F$ and $r_S=6$, as a function of $\omega 2m/k_F^2$ for the RPA, CC, and our decoupling (solid line). The δ-function peak corresponding to the inter-Landau-level mode has been artificially broadened for clarity and contains most of the spectral strength.

Here $a_0=b_0=1$, $a_1=0$, $b_1=2$ and $a_2=0$, $b_2=2$.

In the following, we denote the quantities which are calculated by the means of CC with an upper index CC, for comparison. The structure factor $S(k)$ can be calculated with the help of the fluctuation-dissipation theorem[19]

$$S(k)=-\frac{1}{\rho_0\pi}\int_0^\infty d\omega\,\mathrm{Im}[\chi_{\rho\rho}(k,\omega)], \tag{30}$$

where $\chi_{\rho\rho}$ is density-density part of the response matrix χ in Eq. (13):

$$\chi_{\rho\rho}(k,\omega)=\frac{\chi^0_{\rho\rho}(k,\omega)}{1-\chi^0_{\rho\rho}[w_0(k)+w_2(k)+w_1(k)^2\chi^0_T]}. \tag{31}$$

By using their decoupling, CC calculated a similar response as in Eqs. (28) and (29) with $a_0^{\mathrm{CC}}=b_0^{\mathrm{CC}}=1$, $a_1^{\mathrm{CC}}=1$, $b_1^{\mathrm{CC}}=1$ and $a_2^{\mathrm{CC}}=0$, $b_2^{\mathrm{CC}}=2$. Using the rotational invariance of $S(k)$ it is easy to show that $G_0(k)=G_0^{\mathrm{CC}}(k)$ is linear in k. Further $G_1(k)=G_2^{\mathrm{CC}}(k)=G_2(k)$ is quadratic in k and $G_1^{\mathrm{CC}}(0)=1/2$. Thus we obtain for our decoupling a zero-frequency density-density response function which vanishes linearly in the long-wavelength limit. For the CC decoupling it vanishes as the square of the wave vector in the long-wavelength limit. Since the only difference in the G terms of our decoupling and the decoupling of CC is given by G_1, we will compare G_1 for these two decouplings. One gets from the integral (29) that $G_1(k)$ approaches to $G_1^{\mathrm{CC}}(k)$ for $k\gtrsim k_F$. Both functions are equal for $k\to\infty$. As mentioned above this is not the case for $k\to 0$.

In the following, we will calculate a solution of the (integral) equations (29)–(31) by a numerical iteration method. As a starting point we use $G_\alpha=0$ for all α, corresponding to the RPA. For doing this one has to choose the dimensionless coupling strength $r_s=1/(a_0\sqrt{\pi\rho_0})$, where a_0 is the Bohr radius $a_0=\epsilon^2 m_b$. Since the results show little variation with this coupling strength (in the region $1\leqslant r_s\leqslant 10$), we will choose for definiteness $r_s=6$ in the following figures. This special choice of r_s could be motivated through calculations of the effective mass of the $\nu=1/2$ system.[11] In the following

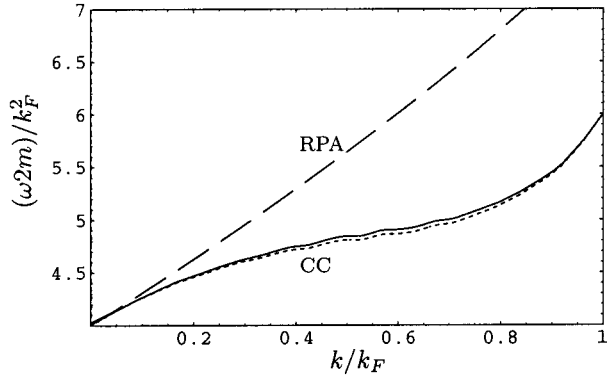

FIG. 2. The cyclotron peak of the RPA, CC, and our decoupling (solid line) for $r_s=6$. It is determined through the singularity of $\mathrm{Im}[\chi_{\rho\rho}]$.

we present and discuss our results for various quantities derived from the density-density response function $\chi_{\rho\rho}$. In Fig. 1 we show $(\pi\rho_0)S(k,\omega)$, where

$$S(k,\omega)=-\frac{1}{\pi\rho_0}\mathrm{Im}[\chi_{\rho\rho}(k,\omega)] \tag{32}$$

is the dynamical structure factor. With the help of $\chi^0_{\rho\rho}=\rho_0 k^2/(m_b\omega^2)$ and $\chi^0_T=2\pi\rho_0^2k^2/(m_b^3\omega^2)$ which is valid for $k/k_F\ll 1$, $kk_F/m_b\ll\omega$, we get from Eq. (31) that $S(k,\omega)$ has a pole at the cyclotron frequency $\omega=k_F^2/m_b$ describing the inter-Landau-level excitation, which is unaffected at $k=0$ by correlations and is in agreement with Kohn's theorem. For finite values of k we get from Fig. 2 that the cyclotron mode [the pole of $S(k,\omega)$] of CC and our decoupling are given by a smaller frequency than in the RPA. From Fig. 1 we obtain that $S(k,\omega)\approx S^{\mathrm{CC}}(k,\omega)$ for $\omega\gg kk_F/m_b$. This can be understood by the asymptotic form of $\chi^0_{\rho\rho}$ and χ^0_T in this range and the similarity of $G_0(k)$ and $G_2(k)$ for CC and our decoupling for $k\ll k_F$. It is also seen from Fig. 1 that $S(k,\omega)\approx S^{\mathrm{RPA}}(k,\omega)$ for small values of $\omega 2m/k_F^2$. This can be also seen in Fig. 3, where we show the function $\chi_{\rho\rho}(k,0)$. One sees from this figure that $\chi_{\rho\rho}(k,0)\approx\chi^{\mathrm{RPA}}_{\rho\rho}(k,0)$ for $k\lesssim k_F$ and $\chi_{\rho\rho}(k,0)\approx\chi^{\mathrm{CC}}_{\rho\rho}(k,0)$ for $k\gtrsim k_F$. This is understandable by

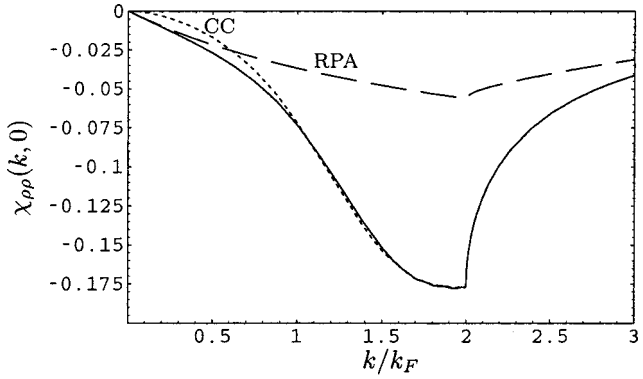

FIG. 3. Static density-density response function $\chi_{\rho\rho}(k,0)$ as a-function of k/k_F for the RPA, CC, and our (solid line) decoupling scheme for $r_s=6$. The discontinuity in the derivative at $k=2k_F$ is due to the Fermi surface.

TABLE I. The Coulomb interaction energy $\langle e^{\mathrm{Coul}}(r_s)\rangle$ per particle.

r_s	This work	CC	RPA
$\to 0$	$-1.00\epsilon^2/(a_0 r_s)$	$-1.00\epsilon^2/(a_0 r_s)$	$-1.19\epsilon^2/(a_0 r_s)$
6	$-1.04\epsilon^2/(a_0 r_s)$	$-1.04\epsilon^2/(a_0 r_s)$	$-1.76\epsilon^2/(a_0 r_s)$

the asymptotic forms of the $G(k)$'s (see the discussion above). One also obtains from this figure that $\chi^{\mathrm{CC}}(k,0)$ has the asymptotic $O(k^2)$ for $k\to 0$.

By using the compressibility sum rule[17,19]

$$K=-\frac{1}{\rho_0^2}\lim_{k\to 0}\left(\frac{\chi_{\rho\rho}(k,0)}{1+2\pi\dfrac{\epsilon^2}{k}\chi_{\rho\rho}(k,0)}\right), \tag{33}$$

we can calculate the compressibility K of the $\nu=1/2$ system. The denominator is due to the fact that by changing the area of the system the positive background has to be changed also to conserve neutrality. With the help of $\chi^0_{\rho\rho}(k,0)=-m_b/(2\pi)$, $\chi^0_T(k,0)=-\rho_0/m_b$, and Eq. (29) this leads to

$$K=\frac{m_b}{2\pi\rho_0^2}\frac{1}{\dfrac{16}{3}+\dfrac{m_b}{2\pi\rho_0}\langle e^{\mathrm{Coul}}(r_s)\rangle}. \tag{34}$$

We call $\langle e^{\mathrm{Coul}}(r_s)\rangle=1/2\Sigma_{\vec{k}}v_0(k)[S(k)-1]$ the Coulomb interaction energy per particle following Ref. 19. For a comparison of the compressibility with the other theories we mention that $K^{\mathrm{RPA}}=(3/4)(m_b/2\pi\rho_0^2)$ for the RPA and $K^{\mathrm{CC}}=0$ for CC. Now we calculate $\langle e^{\mathrm{Coul}}(r_s)\rangle$ for the RPA, CC, and our decoupling. The Coulomb interaction energies for $r_s\to 0$ and $r_s=6$ are shown in Table I. We see from this table that the Coulomb interaction energies of our theory and CC are equal. By calculating the full Coulomb energy per particle via coupling constant integration, $u^{\mathrm{Coul}}(r_s)=1/(a_0 r_s^2)\int_0^{r_s}dr_s'(a_0 r_s')\langle e^{\mathrm{Coul}}(r_s')\rangle$, we get for CC and our theory $u^{\mathrm{Coul}}(r_s)\approx -1.02\epsilon^2/(a_0 r_s)$. The Coulomb energy of electrons in the lowest Landau level was calculated earlier by Morf and d'Ambrumenil[13] and by Girlich[14] by numerical diagonalization methods. Within this method the Coulomb energy per electron is given by $u^{\mathrm{Coul}}_{\mathrm{num}}(r_s)\approx -0.88\epsilon^2/(a_0 r_s)$. This energy has to be compared with the $r_s\to 0$ Coulomb energy of the RPA, CC, and our method. We obtain that the $r_s\to 0$ energy of the STLS-type methods is in better agreement with the numerical results than the Coulomb energy of the RPA. (The formula for the coupling constant integration leads for small r_s to the equality of the Coulomb interaction energy with the Coulomb energy.) Furthermore, we see from Table I that in the STLS-type theories the Coulomb energy of the lowest Landau level is a very good approximation to the total Coulomb energy including higher Landau levels. This is not the case for the RPA. To get the reason for the equality of the Coloumb energies of our method and CC we show in Fig. 4 for $r_s=6$ that $S(k)=S^{\mathrm{CC}}(k)$ for almost all k/k_F. We verify that all curves in the figure obey the leading $(k/k_F)^2/2$ behavior for small k/k_F, required by the Kohn theorem. Furthermore, we find for our method that the ω momenta of the dynamical structure factor $S(k,\omega)$ for small k (excluding the cyclotron contribution) coincides with that of the RPA (Refs. 4 and 18) because for small k our effective potentials coincide with that of the RPA.

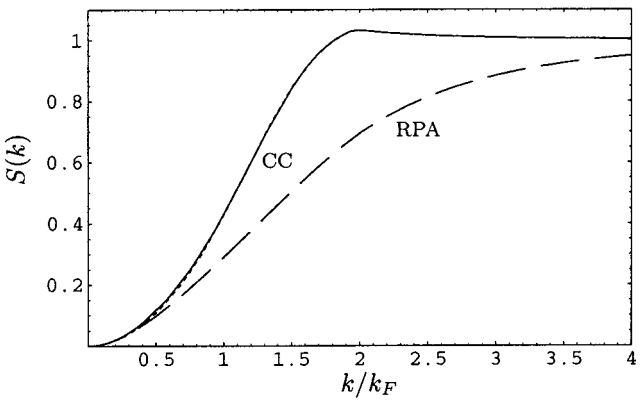

FIG. 4. Static structure factor $S(k)$ as a function of k/k_F in the RPA, CC, and our (solid line) scheme for $r_s=6$.

At last we calculate the Landau energy. As in the case of the calculation of the Landau energy in the RPA,[7] one obtains with the help of a coupling constant integration for the Landau energy in our decoupling scheme an infrared diverging energy. This is caused by the simplification in considering only quadratic density fluctuations in the derivation of the Hamiltonian (4) from the exact CS Hamiltonian (2) (compare the remark at the end of Sec. II). The Hamiltonian (4) has two defects: (i) The restriction to the quadratic density fluctuations leads to an ultraviolet divergence. (ii) The order of the operators in the product of the two ρ is changed to normal order which eliminates the ultraviolet divergence but results in an infrared divergence.[8] A solution of this problem for the energy in the RPA was given in Ref. 9 by considering for the energy calculation all maximal divergent diagrams (RPA) together with all first-order diagrams of the Hamiltonian (2). The first-order energy (per particle) of the CS Hamiltonian (2) is[20] $uf^{\mathrm{mag}}=4.00/[m_b(a_0 r_s)^2]$. In the following, we make a similar calculation for our decoupling scheme. Thus, we have to calculate the Landau energy per particle, $u^{\mathrm{mag}}=ud^{\mathrm{mag}}+uf^{\mathrm{mag}}$, of the Hamiltonian (4) but to extend the first-order terms to those of the Hamiltonian (2) with the full three-particle interaction. To this end we have to multiply $v_1(k)$ and $v_2(k)$ in Eq. (28) by a parameter λ. Then the solution for χ of the (integral) equations (29)–(31) depends on λ. From the Hamiltonian (4) we get that for determining ud^{mag} one has to calculate two terms [one contains the density-density response function $\chi_{\rho\rho}(k,\omega;\lambda)$, and the other contains the density-momentum reponse function $\chi_{\rho\pi}(k,\omega;\lambda)$]. After some algebra one gets for ud^{mag} through coupling constant integration

$$ud^{\mathrm{mag}}=-\frac{1}{\rho_0\pi}\int_0^1 d\lambda\sum_{\vec{k}}\int_0^\infty d\omega \times\left[\frac{1}{2}v_2(k)\,\mathrm{Im}[\chi_{\rho\rho}(k,\omega;\lambda)-\chi^0_{\rho\rho}(k,\omega)] +v_1(k)w_1(k)\,\mathrm{Im}[\chi^0_T(k,\omega)\chi_{\rho\rho}(k,\omega;\lambda)]\right]. \tag{35}$$

With the help of a numerical integration of Eq. (35) we get $ud^{\mathrm{mag}}=-2.16/[m_b(a_0r_s)^2]$. The total Landau energy per particle u^{mag} is then given by

$$u^{\mathrm{mag}}=uf^{\mathrm{mag}}+ud^{\mathrm{mag}}=1.84\frac{1}{m_b(a_0r_s)^2}. \tag{36}$$

By doing a similar calculation for the total Landau energy per particle within the RPA,[9] we get $u^{\mathrm{mag}}_{\mathrm{RPA}}=1.60/[m_b(a_0r_s)^2]$. The Landau energy can be compared to the exact Landau energy $\omega_c/2$ of the $\nu=1/2$ system. This is given by $u^{\mathrm{mag}}_{\mathrm{ex}}=2.00/[m_b(a_0r_s)^2]$. Thus we obtain that, in comparison with the RPA, the Landau energy and the Coulomb energy of the STLS-type theories are in better agreement with the exact Landau energy and the Coulomb energy (from numerical diagonalization).

IV. CONCLUSION

In this paper, we consider an approximation of the response function of the $\nu=1/2$ system which goes beyond the RPA. The method we used is the STLS theory, first established for the Coulomb system.[12] Recently this theory was applied to the CS system by CC.[11] In their theory, CC obtain a density-density response function which vanishes as the square of the wave vector in the long-wavelength limit. We show in this paper that the zero-frequency density-density response function vanishes linearly in the long-wavelength limit independent of any approximation. We obtain this result by considering the Heisenberg equation of motion for the longitudinal momentum operator. From this equation of motion, we derive a consistency condition for a decoupling of the equal-time density-density and density-momentum correlation functions. We show that this consistency condition is not fulfilled in the theory of CC and that is the reason for the quadratic behavior of the zero-frequency long-wavelength limit of the density-density response. Based on the functional form of the Heisenberg equation of motion of the Wigner distribution function (with external potentials), we suggest a decoupling of the correlation functions in this equation which respects the consistency condition.

We solve the decoupled Heisenberg equation of motion by numerical iterations and get the response functions of the theory. In contrast to the theory of CC, we obtain a density-density response function which vanishes linearly in the long-wavelength limit for zero frequency. We get agreement for the density-density response function with the theory of CC for momenta $k\gtrsim k_F$. For $k\lesssim k_F$ we get agreement with the density-density response function of CC for frequencies $\omega\gtrsim kk_F/m_b$ and to the RPA for $\omega\lesssim kk_F/m_b$. Further, we calculate the compressibility of the theory by using the compressibility sum rule. We obtain a Coulomb correction to the compressibility not contained in the RPA. With the help of the response functions, we calculate the static structure factor, the excitation spectrum, and the Landau as well as the Coulomb energies. As in the theory of CC, we get density excitations which are lower in their frequencies as a function of the wave vector than the excitations calculated with the help of the RPA. The obtained excitation spectrum is almost identical to the spectrum of CC. The same holds for the static structure factor. This is the reason for the agreement of the lowest-Landau-level Coulomb energy as well as the full Coulomb energy to the energies calculated by CC. The lowest-Landau-level Coulomb energy fits better to the Coulomb energy calculated by numerical methods[13,14] ($\approx$114% of the numerical lowest-Landau-level Coulomb energy) than the RPA. We remark that the relative part of the Coulomb energy in the lowest Landau level, i.e., linear in ϵ^2, is much enhanced as compared to the RPA. Finally, we calculate the Landau energy of the system. We obtain a much better approximation of the exact Landau energy ($\approx$92% of the exact Landau energy of the $\nu=1/2$ system) than the RPA.

In summary, a consistent decoupling of the Wigner function in the Heisenberg equation of motion leads to results which are in better agreement with known numerical and exact results, respectively, than the RPA.

ACKNOWLEDGMENTS

We would like to thank S. Conti, M. Hellmund, and W. Apel for many helpful discussions during the course of this work. Further we acknowledge the support of the Graduiertenkolleg ''Quantenfeldtheorie'' at the University of Leipzig and the DFG Schwerpunktprogramm ''Quanten-Hall-Systeme.''

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