Quantum theory of a nematic Fermi fluid

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We develop a microscopic theory of the electronic nematic phase proximate to an isotropic Fermi liquid in both two and three dimensions. Explicit expressions are obtained for the small amplitude collective excitations in the ordered state; remarkably, the nematic Goldstone mode (the director wave) is overdamped except along special directions dictated by symmetry. At the quantum critical point we find a dynamical exponent of $z=3$, implying stability of the Gaussian fixed point. The leading perturbative effect of the overdamped Goldstone modes leads to a breakdown of Fermi-liquid theory in the nematic phase and to strongly angle-dependent electronic self energies around the Fermi surface. Other metallic liquid-crystal phases, e.g., a quantum hexatic, behave analogously.

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There is a growing body of both experimental and theoretical evidence for the relevance of inhomogeneous and/or anisotropic metallic phases in a wide array of highly correlated electronic systems. Quasi-one-dimensional (stripe or ''electronic smectic'') phases have been observed in a large variety of transition-metal oxides.[1] More recently,[2,3] the dramatic discovery of a metallic phase with a strongly anisotropic resistivity tensor for a range of magnetic fields in ultraclean heterojunctions has provided clear evidence of the existence of a ''quantum Hall nematic'' phase. In parallel, theoretical work[4,5] has been carried out on electronic liquid-crystal phases. These ground-state phases are classified, based on broken symmetries, by analogy with classical liquid crystals. So far, these studies have focused primarily on the smectic, which is a unidirectional density wave with broken translational symmetry in only one direction, but which supports liquidlike electron flow,[6,7] and to a lesser extent on the nematic, which is uniform but anisotropic (breaks rotational symmetry).[8,9] A nematic state in the proximity to the smectic state can be visualized most naturally as a melted smectic, i.e.,, a smectic with dislocations. However, a theory of the nematic phase based on this picture has yet to be satisfactorily formalized.

In this paper we approach the nematic metal via a complimentary route, from the isotropic and weakly correlated side. In this limit, the zero-temperature isotropic to nematic transition is a Fermi-surface instability. The director order parameter which characterizes the broken symmetry of the nematic state is a rank two symmetric traceless tensor which is even under time reversal. In two dimensions, but not in three, the order parameter is odd under 90° spatial rotation. For simplicity we first consider spinless fermions in two dimensions with full rotational symmetry, deferring until later any discussion of spin and the symmetry-breaking effects of the crystal fields which are inevitable in actual solids. While many microscopic definitions of this order parameter are possible, we shall see that the natural one in the present context is the quadrupole density,

$$\hat{\mathbf{Q}}(x) \equiv -\frac{1}{k_F^2}\Psi^\dagger(\vec{r})\begin{pmatrix} \partial_x^2-\partial_y^2 & 2\partial_x\partial_y \\ 2\partial_x\partial_y & \partial_y^2-\partial_x^2 \end{pmatrix}\Psi(\vec{r}), \qquad (1)$$

where k_F is the Fermi wave number. The order parameter $\mathbf{Q}\equiv\langle\hat{\mathbf{Q}}\rangle$ can be expressed in terms of an amplitude and a phase, $Qe^{2i\theta}=Q_{11}+iQ_{12}$. Thus, in the broken symmetry state the Fermi surface is elliptical (i.e., the Fermi momentum varies around the Fermi surface), with eccentricity proportional to Q along a major axis at an azimuthal angle $\pm\theta$.

At finite temperature, T, where the long-distance physics is purely classical, an electronic liquid crystal has much the same character as a conventional liquid crystal. In the case of the two-dimensional nematic, this means that there is no true long-range order, and a Kosterlitz-Thouless transition at a critical temperature to the disordered (high-temperature) state. It is a remarkable feature of the hydrodynamics of classical nematics (in both two and three dimensions) that the director wave (i.e., the finite-temperature Goldstone mode) is overdamped.[10,11]

At zero temperature, the nematic phase possesses a true broken symmetry. Many features of this state follow as a direct consequence of symmetry breaking, independent of microscopic considerations. The quantum transition between the nematic and isotropic states can be studied at mean-field level by considering the Landau expansion of the ground-state energy as

$$E(\mathbf{Q}) = E(\mathbf{0}) + \frac{A}{4}\mathrm{Tr}[\mathbf{Q}^2] + \frac{B}{8}\mathrm{Tr}[\mathbf{Q}^4] + \cdots. \qquad (2)$$

That only even terms appear in this expansion is a consequence of the odd parity of $\mathbf{Q}$ under 90° rotations. At the transition point, A changes from positive in the isotropic phase to negative in the nematic phase, in which $Q=\sqrt{|A|/B}+\cdots$. The elastic theory which governs any long wavelength, static variations of the order parameter is directly inherited from the classical theory,[10,11] leading to an energy density functional of the form

$$\mathcal{V}[\mathbf{Q}] = E(\mathbf{Q}) - \frac{\tilde{\kappa}}{4}\mathrm{Tr}[\mathbf{QDQ}] - \frac{\tilde{\kappa}'}{4}\mathrm{Tr}[\mathbf{Q}^2\mathbf{DQ}] + \cdots, \quad (3)$$

where $D_{i,j}\equiv\partial_i\partial_j$ (neglecting total derivative terms). In the ordered phase, Eq. 3 leads to two elastic moduli (Frank

constants), but since they are interchanged by a 90° rotation, the difference between the two is proportional to Q, and so is small so long as Q is small. The dynamics of the collective modes, as well as explicit expressions for the various coefficients which enter the theory, must be derived from microscopic considerations.

I. THE MODEL

We take as our model

$$H=\int d\vec{r}\Psi^{\dagger}(\vec{r})\epsilon(\vec{\nabla})\Psi(\vec{r}) + \frac{1}{4}\int d\vec{r}\int d\vec{r}' F_2(\vec{r}-\vec{r}')\mathrm{Tr}[\hat{\mathbf{Q}}(\vec{r})\hat{\mathbf{Q}}(\vec{r}')]. \quad (4)$$

Here $\epsilon(\vec{k})$ is the single-particle energy and we have ignored all density-density interactions other than the essential ones, for present purposes, involving the quadrapolar density.[12] The single-particle energy can be linearized about the Fermi surface, but for later convenience we keep one further term in the expansion,[13] $\epsilon(\vec{k})=v_F q[1+a(q/k_F)^2]$ with $q\equiv|\vec{k}|-k_F$. To be explicit, we take the interaction to be the Fourier transform of a simple Lorentzian, $F_2(\vec{r})=(2\pi)^{-2}\int d\vec{k}e^{i\vec{q}\cdot\vec{r}}F_2/[1+\kappa F_2 q^2]$, where F_2 is the appropriate Landau parameter. Our results are not qualitatively sensitive to any of these details.

Landau parameters in a strongly correlated fluid are notoriously difficult to deduce from microscopic considerations, but they can be large. In He_3, for instance, $N_F F_0$ is found to vary[14] from 10 to 80 as a function of pressure between 0 and 27 atm. At the simplest level, one might hope to express these parameters in terms of the Fourier transform of an effective density-density interaction as

$$F_n=V(|\vec{q}|=0)\,\delta_{n,0}-\int_0^{2\pi}\frac{d\theta}{2\pi}\cos(n\theta)V[2k_F\sin(\theta/2)]. \quad (5)$$

Clearly, it is possible to obtain a large negative F_2 and positive (or small) F_0 and F_1 from this expression, even if V is positive, especially if V is peaked at a momentum transfer of order $2k_F$. Such structure will occur in any fluid with a large degree of local crystallinity.

To analyze the collective properties of this system, we introduce a Hubbard-Stratanovich[15,16] field $\mathbf{n}$ to decouple the four-fermion interaction, and then integrate out the fermions formally to obtain the effective action, $S_{\mathrm{eff}}[\mathbf{n}]$. Although $\mathbf{n}$ is also a traceless symmetric tensor, it is convenient to introduce a vectorial notation in which $\mathbf{n}=n_1\boldsymbol{\sigma}_z+n_{-1}\boldsymbol{\sigma}_x$ where $\boldsymbol{\sigma}_\alpha$ are the Paul matrices. While S_{eff} is very complicated, we can readily find the saddle-point solutions, $\overline{\mathbf{n}}$, which are extrema of S_{eff}, and can then obtain explicit expressions for S_{eff} in powers of $\delta\mathbf{n}\equiv\mathbf{n}-\overline{\mathbf{n}}$. In the limit $\kappa\to\infty$, the saddle-point (Fermi-liquid) approximation becomes exact, and more generally this approach can be viewed as an expansion based on the small parameter, $1/\kappa$.

From now on we will work with the Hubbard-Stratanovich field $\mathbf{n}$ rather than with the order-parameter field $\mathbf{Q}$. They are related by a Legendre transform, and symmetry constrains the form of the effective actions in similar ways.[15] In particular, the effective action for the $\mathbf{n}$ field has elastic terms similar to those of Eq. (3) with new elastic moduli κ and κ' which are proportional to $\tilde{\kappa}$ and $\tilde{\kappa}'$.

The saddle-point equations are obtained by minimizing an expression of the form of Eq. (2), with $\overline{\mathbf{n}}$ replacing $\mathbf{Q}$ and $A=1/(2N_F)+F_2$ where N_F is the density of states at the Fermi surface. Clearly, the isotropic phase is stable[17] so long as $2N_F F_2>-1$, while the nematic phase occurs where this inequality is violated. The quartic term B is determined by couplings that are formally irrelevant (in the renormalization-group sense) at the isotropic Fermi-liquid fixed point; for the explicit Hamiltonian considered above, $B=(3aN_F|F_2|^3)/(8E_F^2)$ where $E_F\equiv v_F k_F$ is the Fermi energy.

The effective action which governs the fluctuations about the saddle point can be computed to quadratic order in $\delta\mathbf{n}$,

$$S_{\mathrm{eff}}[\mathbf{n}]=S_{\mathrm{eff}}[\overline{\mathbf{n}}]+\frac{1}{2N_F}\sum_{a,b=\pm1}\int\frac{d\omega}{2\pi}\frac{d^2q}{(2\pi)^2} \times\delta n_a^*\mathcal{L}_{a,b}(\vec{q},\omega;\overline{\mathbf{n}})\delta n_b+\cdots, \quad (6)$$

where $\mathcal{L}_{a,b}$ is the inverse propagator of the collective modes, and the collective mode dispersion relation, ω_q, is determined from the solution of the implicit equation

$$\det[\mathcal{L}_{a,b}(\vec{q},\omega)]=0. \quad (7)$$

It is convenient to express $\mathcal{L}_{a,b}$ as the sum of a static and dynamical piece: $\mathcal{L}_{a,b}(\vec{q},\omega)\equiv\mathcal{L}_{a,b}(\vec{q},0)+\tilde{\mathcal{M}}_{a,b}(\vec{q},\omega)$. Symmetry strongly constrains the transverse component of $\mathcal{L}_{a,b}(\vec{q},0)\sim q^2$ in the ordered phase. In other words, being essentially the static/classical deformation energy, its form is determined by the expression in Eq. (3). However, as we will see, symmetry considerations do not fully determine the dynamical piece.

II. THE ISOTROPIC PHASE

We warm up by analyzing the fluctuation spectrum in the isotropic phase for $2N_F F_2>-1$. By explicitly computing the fermionic response functions, with $\delta\equiv-\frac{1}{2}-1/(N_F F_2)$ and $s\equiv\omega/v_F q$, we find

$$\mathcal{L}_{a,b}(\vec{q},\omega)=\delta_{a,b}(\kappa q^2+\delta)+\tilde{\mathcal{M}}_{a,b}(\vec{q},\omega), \quad (8)$$

$$\tilde{\mathcal{M}}_{a,b}(\vec{q},\omega)=\frac{s}{2}\int_0^{2\pi}\frac{d\theta}{2\pi}\frac{\mathcal{P}_{a,b}(\theta)}{s-\cos(\theta-\phi)}, \quad (9)$$

$$\mathcal{P}_{a,b}(\theta)=\begin{pmatrix}2\cos^2 2\theta & \sin 4\theta\\ \sin 4\theta & 2\sin^2 2\theta\end{pmatrix} \quad (10)$$

where ϕ is the polar angle subtended by $\vec{q}$. Without loss of generality, we can set $\phi=0$, in which case the two components of the pseudovector correspond to the longitudinal (n_1) and transverse (n_{-1}) polarizations of the quadrupolar wave, and $\mathcal{L}_{a,b}$ is diagonal:

$$\tilde{\mathcal{M}}_{a,b}(\vec{q},\omega)=\delta_{a,b}\mathcal{M}_b(s), \quad \text{with} \quad s\equiv\omega/v_F q, \quad (11)$$

$$\mathcal{M}_{\pm 1}(s)=\frac{s}{2\sqrt{s^2-1}}[1\pm(-s+\sqrt{s^2-1})^4]. \quad (12)$$

For $F_2>0$ there exist soundlike propagating modes with $\omega_q>v_F q(s>1)$, i.e., the quadrupolar analogs of zero sound (one for each polarization). They are undamped because they lie outside of the particle-hole continuum. Closer to the nematic phase boundary, the evolution of the quadrupolar oscillations of the isotropic Fermi liquid deviates markedly from that of simple zero sound. The dynamics clearly distinguishes the two polarizations and there are underdamped modes even when $F_2<0$.

III. THE QUANTUM CRITICAL REGIME

The difference in the dynamics of the two polarizations becomes more pronounced as the quantum critical point is approached. As $\delta\to 0^+$ and for $\omega\ll v_F q$

$$\mathcal{L}_{++}(\omega,q)=\kappa q^2+\delta-i\frac{\omega}{v_F q}+\cdots; \quad (13)$$

$$\mathcal{L}_{--}(\omega,q)=\kappa q^2+\delta-\left(\frac{\omega}{v_F q}\right)^2-2i\left(\frac{\omega}{v_F q}\right)^3+\cdots. \quad (14)$$

The transverse mode becomes more and more weakly damped with $\omega_-\approx i\sqrt{\delta}v_F q+v_f\sqrt{\kappa}q^2$ while the longitudinal mode remains overdamped, $\omega_+\sim iq^3$. This behavior should be compared and contrasted with the behavior of the paramagnon collective mode near the Fermi-liquid to ferromagnetic Fermi liquid transition where, in the disordered phase and at the critical point, all three polarizations are identical, and indeed have dispersions similar to that of the longitudinal ("++") quadrupolar mode. The difference arises from the fact that, unlike in the nematic, in the ferromagnet (in the absence of spin-orbit coupling), the broken symmetry is unrelated to any spatial symmetries. Nevertheless, as with the ferromagnet, a straightforward scaling analysis of the effective action implies a dynamical critical exponent $z=3$ at the quantum critical point. Remarkably, at the critical point, the transverse mode has higher characteristic energy ($\omega_-\sim q^2\sim\omega_+^{2/3}$) than the overdamped critical mode; it plays no role in the critical theory, and indeed the ω^2/q^2 term in the inverse propagator which makes it dynamical is irrelevant for $z=3$ scaling, $\omega^2/(v_F q)^2\sim q^4$. Moreover, $z=3$ also implies that interaction terms of order $\mathbf{n}^4$ and higher in $S_{\rm eff}$ are irrelevant at the quantum critical point. Thus, according to the standard lore,[18,19] the critical behavior is fully captured by the Gaussian theory.

IV. THE NEMATIC PHASE

In the ordered phase, but close to the quantum critical point, the order parameter is small. Here we can, to good approximation, ignore the dependence of the $\vec{q}$ and ω dependent terms in the effective action on the ordered moment,[20] $\mathcal{L}_{a,b}(\vec{q},\omega;\overline{\mathbf{n}})=\mathcal{L}_{a,b}(\vec{0},0;\overline{\mathbf{n}})+\mathcal{L}_{a,b}(\vec{q},\omega;\mathbf{0})+\cdots$. The higher-order terms are, among other things, responsible for the difference between the two Frank constants in the elastic energy of the nematic; as they complicate the normal-mode analysis, and make little important qualitative difference, we will defer considering them until later. Without loss of generality, we can choose the principal axis of the nematic state to lie along $\hat{x}$, so that $\overline{\mathbf{n}}=\bar{n}\boldsymbol{\sigma}_{\mathbf{z}}$ (where $\boldsymbol{\sigma}_{\mathbf{z}}$ is the Pauli matrix). In this case, fluctuations of the amplitude of the order parameter are associated with the $+1$ component of $\delta\mathbf{n}$, and -1 are the phase (i.e., orientational) deformations. Inside the nematic phase, as for phonons in a crystal, the longitudinal and tranverse modes are mixed unless $\vec{q}$ lies along a symmetry axis.

Because we have neglected the difference between Frank constants, the purely elastic energy is a sum of independent contributions from the phase and amplitude modes,

$$\mathcal{L}_{++}(\vec{q},0;\overline{\mathbf{n}})=2|\delta|+\kappa q^2,$$

$$\mathcal{L}_{--}(\vec{q},0;\overline{\mathbf{n}})=\kappa q^2, \quad (15)$$

and $\mathcal{L}_{-+}(\vec{q},0;\mathbf{0})=0$. However, even at this level of accuracy, the dynamics unavoidably mixes the phase and amplitude modes. Explicitly, the dynamical matrix is a function of the scaled variable $s=\omega/v_F q$ *and* the angle, ϕ, between the principal axis of the nematic order and the wave vector $\vec{q}$,

$$\mathcal{M}(s,\phi)=\frac{s}{2}\begin{pmatrix} B(s)+A(s)\cos 4\phi & A(s)\sin 4\phi \\ A^*(s)\sin 4\phi & B(s)-A(s)\cos 4\phi \end{pmatrix}, \quad (16)$$

with $B(s)=1/\sqrt{s^2-1}$ and $A(s)=B(s)(\sqrt{s^2-1}-s)^4$. [$A^*(s)$ is the complex conjugate of $A(s)$.] Only if $\vec{q}$ is along a symmetry direction, $\phi=0,\pm\pi/4,\pm\pi/2$, etc., is $\mathcal{M}$ diagonal. That the longitudinal and transverse modes are mixed for all other propagation directions, even in the absence of terms which depend explicitly on $\overline{\mathbf{n}}$ is, at first sight, curious, and is a direct consequence of the fact that the nematic order parameter breaks spatial rotational symmetry, not an internal symmetry (such as spin).

As in the isotropic phase, we diagonalize $\mathcal{L}_{a,b}$ to obtain the collective-mode spectrum. An excitation is called "transverse" if for $\vec{q}\to 0$ it is polarized perpendicular to the prin-

cipal axis of the nematic and ''longitudinal'' otherwise; the corresponding eigenvalues of the inverse propagator are $\mathcal{L}_\perp$ and $\mathcal{L}_\parallel$.

For general ϕ and as $s\to 0$, the inverse propagator for the transverse (Goldstone) mode is

$$\mathcal{L}_\perp = \kappa q^2 - \frac{is}{2}\sin^2 2\phi - s^2\left(\cos 4\phi + \frac{\sin^2 4\phi}{128|\delta|}\right) + \mathcal{O}(s^3). \tag{17}$$

This result is remarkable and, to our knowledge, unprecedented: Landau damping dominates the dynamics of the Goldstone boson, making it overdamped (the last term is irrelevant) over most of phase space. Formally, Eq. (17) is reminiscent of those studied in conjunction with quantum criticality in metals,[18,19] or when a transverse gauge field is coupled to a Fermi liquid.[21,22] In all three cases Landau damping results in a soft boson with dissipative dynamics. Naturally, the origin of the softness is different in each case; while the softness of the quantum critical propagator is the result of fine tuning to criticality, the softness of the gauge and the director-wave propagators stems, respectively, from gauge invariance and broken rotational symmetry. Nevertheless, here, as in the other two problems (and in two dimensions), Eq. (17) implies a specific-heat vanishing as $T^{2/3}$ at low temperatures.

The angular -dependent damping term in Eq. (17) is native to the nematic state; along symmetry directions, when it vanishes, a propagating (i.e., undamped as $q\to 0$) mode results. For $\phi=0$ and $\pi/2$, and for $\omega \ll v_F q$,

$$\mathcal{L}_\perp^{0,\pi/2} = \kappa q^2 - s^2 - is^3 + \mathcal{O}(s^4), \tag{18}$$

implying a dispersion according to $\omega_q = v_F\sqrt{\kappa} q^2$. An additional propagating mode with a soundlike spectrum $\omega_q = v_F q/\sqrt{2}$ exists for $\phi=\pi/4$. These long-lived modes are somewhat peculiar: they are already present at the quantum critical point (with precursors even in the isotropic phase!), where indeed they propagate in all directions, although they appear as higher-energy excitations which do not directly enter the critical phenomena. These collective phenomena can be summarized pictorially (see Fig. 1).

Finally the inverse propagator of the amplitude mode is

$$\mathcal{L}_\parallel = 2|\delta| - \frac{is}{2}\cos^2 2\phi + \mathcal{O}(s^2). \tag{19}$$

The apparent fourfold symmetry in these expressions is, in part, inherited from the special symmetry of the two-dimensional director: $\bar{\mathbf{n}}$ is odd under rotation by 90° so quantities even in $\bar{\mathbf{n}}$ will be *four*fold symmetric, even inside the nematic state. However, the precise fourfold symmetry is an artifact of our neglecting the $\bar{\mathbf{n}}$ dependence of the $\vec{q}$ and ω dependent terms in $\mathcal{L}$. For instance, κ should rightly be replaced by the two distinct Franc constants $\kappa_\perp$ and $\kappa_\parallel$ in Eq. (17), respectively, where from Eq. (3), $\kappa_\perp - \kappa_\parallel \sim \kappa' |\bar{\mathbf{n}}| + \cdots$. At the level of the Gaussian theory constructed thus far the corrections can be analyzed perturbatively (in $\bar{\mathbf{n}}$),

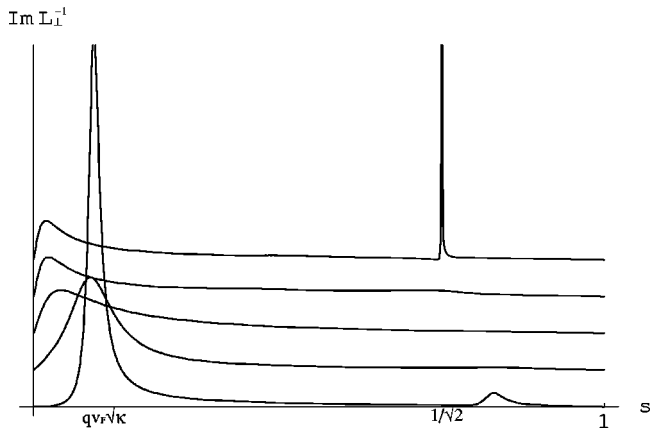

FIG. 1. The spectral function, $\mathrm{Im}\chi = \mathrm{Im}\mathcal{L}_\perp^{-1}$, of the nematic Goldstone modes as a function of $s=\omega/v_F q$ (at fixed small q) for different angles of propagation from (top to bottom) $\phi=\pi/4$, $\phi=3\pi/16$, $\phi=\pi/8$, $\phi=\pi/16$ and $\phi=0$, respectively. The curves are plotted on the same scale, with vertical offset, for $\delta=0.1$, $\kappa=1$, $v_F=1$, and $q=0.15$. At $\phi=0$ there is a sharp propagating mode [see Eq. (18)]. Otherwise the spectral function is linear as $\omega\to 0$, signifying a diffusive peak ($\mathrm{Im}\chi(\omega)/\omega$ is peaked at $\omega=0$). At $\phi=\pi/4$ the spectrum contains an additional long-lived soundlike mode.

leading neither to qualitative modifications of modes' dispersions nor to spoiling of the fourfold symmetry [e.g., in the damping term in Eq. (17)]. More generally, because the nematic order does not gap any section of the Fermi surface, but only distorts its shape, the dynamical consequences of the fermionic particle-hole continuum are preserved, even far from the critical point. In other words, the characteristic scale for the frequency dependence is still $v_F q$ (i.e., vanishing at long wavelengths) in the broken-symmetry phase, regardless of the magnitude of the Fermi-surface distortion.

Clearly, interactions among the collective modes can be treated perturbatively in $1/\kappa$. What the effects of these interactions are away from the critical point is presently unknown.

It is important to recall that any symmetric traceless tensor can serve as the order parameter for the nematic state. While we have chosen the deformation of the Fermi surface, alternate choices include the inverse mass tensor which defines the Drude weight of the optical conductivity, and the resistivity tensor itself. They can be interrelated explicitly in our case (the corrections to this first order result are cubic),

$$\frac{\rho_{xx}-\rho_{yy}}{\rho_{xx}+\rho_{yy}} = \frac{1}{2}\frac{m_y - m_x}{m_y + m_x} = \frac{n_{11}}{E_F} + \mathcal{O}(|\bar{n}|^3), \tag{20}$$

where m_x and m_y are the effective masses of the quasiparticles in the nematic state, as determined by the spontaneous anisotropy of the Fermi surface.

V. SINGLE-PARTICLE PROPERTIES

We will now consider the effect of the collective modes on the single-particle self-energy. To one-loop order, the

imaginary part of the self-energy (the scattering rate) is given by

$$\Sigma''(\epsilon,\vec{k}) = \frac{\pi}{\sqrt{3}} \frac{(\kappa k_F^2)^{1/3}}{\kappa N_F} \left| \frac{k_x k_y}{k_F^2} \right|^{4/3} \left| \frac{\epsilon}{2 v_F k_F} \right|^{2/3} + \cdots, \quad (21)$$

where $\vec{k}$ lies on the Fermi surface and $\cdots$ signifies subleading terms in powers of ϵ. The strong angular dependence of Σ'' is a startling consequence of the symmetry of the nematic. Along the symmetry directions, $\vec{k}=(k_x,0)$ and $(0,k_y)$, the scattering rate (at the Fermi surface) has a different energy dependence indicative of a long lived quasiparticle:[23]

$$\Sigma''(\epsilon) = \frac{\pi}{3 N_F \kappa} \frac{1}{(\kappa k_F^2)^{1/4}} \left| \frac{\epsilon}{v_F k_F} \right|^{3/2} + \cdots. \quad (22)$$

Although perturbative, our results unambiguously signal the breakdown of Fermi-liquid theory, i.e., the spectral function no longer has a quasiparticle pole over most of the Fermi surface.[23] For fixed ϵ, the perturbation theory is arbitrarily accurate for large enough κ, but for fixed κ, perturbation theory may well break down at small ϵ. We have not determined yet whether this indicates a phase transition to a new, ordered state, e.g., a superconducting state, or the occurrence of a genuine, two-dimensional (2D) non-Fermi-liquid phase. With the exception of the discrete set of symmetry-related points where the quasiparticle survives collective damping, the frequency dependence of our result is identical to the comparable perturbative result for fermions coupled to (Landau damped) gauge or quantum critical fields.[18,19,21,22] However, it is important to emphasize that formal resemblance among these three problems need not persist beyond the lowest order in perturbation theory.

VI. EXTENSIONS AND SPECULATIONS

Thus far we have considered a rotationally invariant system. Since the electron fluid is typically realized in a solid-state context, it is important to consider the effects of explicit rotational symmetry breaking by the underlying lattice. These are relevant operators in the nematic phase, and their main effect is to gap out the Goldstone bosons. In the presence of such explicit symmetry breaking, the system behaves, at sufficiently low energy, like an anisotropic metal with Fermi-liquid properties. However, an analysis of their effects on the quasiparticle self-energies indicates a crossover from non-Fermi-liquid to Fermi-liquid behavior below an energy scale $\epsilon^* \approx (v_F \lambda^{3/2})/\sqrt{\kappa}$ where λ is a dimensionless measure of the lattice-induced anisotropy on the bare quasiparticle energy. In many cases, λ is small so ϵ^* is very small.

There are many obvious and interesting generalizations of these ideas. With very little difference, we can consider the theory of the metallic hexatic, which is similarly non-Fermi liquid. Such a state would be triggered by a large, negative F_6. While at first that might sound artificial, if we view the quantum hexactic as a melted Wigner crystal, it is a natural state to consider. This may have relevance to the long-standing problem of apparently metallic states in two dimensions. In the presence of electron spin, a host of new and interesting states become possible which intertwine spin and spatial ordering. For example, one could imagine a state triggered by a large and repulsive F_2^a (i.e., in the triplet channel) in which the nematic order parameters of the spin-up and spin-down electrons are rotated relative to each other. Finally, deeper inside the nematic phase the Fermi surface gets increasingly distorted, leading to nearly nested segments; it is thus natural to consider a further instability, triggered by "backscattering," to a stripe ordered electronic smectic phase.

The three-dimensional nematic can be analyzed similarly. Here, the director is a 3×3 rank two symmetric traceless tensor. On symmetry grounds one anticipates that, since a cubic invariant is allowed in the free energy, the isotropic to nematic transition will generally be first order. We have repeated the calculation of the ground state energy Eq. (2)] in three dimensions. In contrast to two dimensions, we indeed find a cubic term which usually[24] favors a uniaxial nematic over a biaxial one and its sign determines whether the nematic Fermi surface is oblate or prolate. The Goldstone mode is overdamped as in two dimensions; its contribution to the single-particle scattering rate is of the "marginal Fermi-liquid" form,[25] $\Sigma''(\epsilon) \propto |\epsilon|$, but strongly angle dependent.

The considerations presented in this paper may serve as a starting point for a microscopic theory of a quantum Hall nematic fluid. In particular, it was proposed in Refs. 5 and 8 that the observed large anisotropy in Landau levels $N \geqslant 2$ could be explained by a nematic fluid phase. The results that we have derived in this paper indicate that this is a very concrete possibility. Moreover, the relation between anisotropic transport and a nematic order parameter proposed on symmetry grounds in Ref. 8 follows clearly from the spontaneous effective-mass anisotropy in the nematic state (discussed above) by means of simple Boltzmann transport arguments. A candidate ground-state wave function for a quantum Hall nematic at filling factor $\nu=1/2$ can be constructed in the spirit of the Jain wave function for the composite Fermi sea[26,27] as

$$\Psi_{CNFL}(z_i) = \mathcal{P} \prod_{i>j} (z_i - z_j)^2 e^{-\sum_j (|z_j|^2/2)} \Psi_{NFL}, \quad (23)$$

where Ψ_{NFL} denotes the ground-state wave function of the 2D nematic we have considered here, z_i are the complex coordinates of the electrons (with the magnetic length set to 1), and $\mathcal{P}$ is the Landau-level projection operator. However, for a subtle gapless state like this it is not clear that simple wave functions are able to fully capture the physics.

Finally, the picture of the nematic phase that emerges from our results is strikingly reminiscent of the behavior of the so-called "normal phase" of high-temperature superconductors. In particular the fermion spectral function that we find in the nematic phase has a behavior that is qualitatively similar to the electron spectral function measured by angle-resolved photoemission spectroscopy (ARPES) in BSCCO.[28–30] In fact, Ioffe and Millis[31] have proposed a phenomenological form for the quasiparticle scattering rate with

the angular dependence $|k_x - k_y|$ and a $|\epsilon|$ marginal Fermi-liquid energy dependence. It is tempting to speculate that a possible origin of this sort of behavior may be a nematic Fermi fluid state setting in at temperatures close to the pseudogap T^*. In fact, in Ref. 4, two of us suggested that as a strongly correlated electron system is cooled down from a high-temperature isotropic fluid phase, the first stage of charge ordering should be precisely a nematic phase.[32] As stressed in Ref. 4, this is a Kosterlitz-Thouless thermodynamic phase transition, which is reduced to an Ising transition in by fourfold lattice anisotropy or rounded in an orthorhombic crystal. Incidentally, spontaneous breaking of rotational symmetry in the two-dimensional Hubbard model has also been studied numerically.[34] Recent light-scattering experiments by Rubhausen and coworkers[33] have found evidence of a rounded nematic transition at the charge-ordering temperature of the manganite $Bi_{1-x}Ca_xMnO_3$.

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