Decoupling of superconducting layers in the magnetic superconductor $RuSr_2GdCu_2O_8$

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(Received 21 March 2001; published 5 October 2001)

We propose the model for magnetic properties of the magnetic superconductor $RuSr_2GdCu_2O_8$, that incorporates the theory of the superconducting/ferromagnetic multilayers. The transition line $T_d(h)$, on which the Josephson coupled superconducting planes are decoupled, i.e., $j_c(T_d)=0$, is calculated as a function of the exchange energy h. As the result of this decoupling a nonmonotonic behavior of magnetic properties, such as the lower critical field H_{c1}, Josephson plasma frequency, etc., is realized near (or by crossing) the $T_d(h)$ line. The obtained results are used in analyzing the newly discovered antiferromagnetic ruthenocuprate $RuSr_2GdCu_2O_8$ with possible weak ferromagnetic order in the RuO_2 planes.

DOI: 10.1103/PhysRevB.64.184501 PACS number(s): 74.80.Dm, 74.70.Pq

I. INTRODUCTION

The physics of magnetic superconductors is interesting due to competition of magnetic order and singlet superconductivity in bulk materials. The problem of their coexistence was first set up theoretically in the pioneering work by Ginzburg[1] in 1956, while the experimental progress in the field began after the discovery of ternary rare-earth (RE) compounds $(RE)Rh_4B_4$ and $(RE)Mo_6X_8$ ($X=S,Se$) (Ref. 2) with a regular distribution of localized RE magnetic moments. It turned out that in many of these systems, superconductivity (with the critical temperature T_s) coexists rather easily with antiferromagnetic (AF) order (with the critical Néel temperature T_N), where usually the situation with $T_N<T_s$ is realized.[2] Due to their antagonistic characters, singlet superconductivity and ferromagnetic order cannot coexist in bulk samples with realistic physical parameters. However, under certain conditions the ferromagnetic order is transformed, in the presence of superconductivity, into a spiral or domainlike structure—depending on the type and strength of magnetic anisotropy in the system.[3] As the result of this competition, these two orderings coexist in a limited temperature interval $T_{s2}<T<T_m$ (the reentrant behavior) in $ErRh_4B_4$ and $HoMo_6S_8$, or even down to $T=0$ K in $HoMo_6Se_8$, where T_m is the critical temperature for the existence of the inhomogeneous magnetic order. The coexistence region in $ErRh_4B_4$ is narrow where $T_s=8.7$ K, $T_m\approx0.8$ K, and $T_{s2}\approx0.7$ K, while for $HoMo_6S_8$ it is even narrower with $T_s=1.8$ K, $T_m\approx0.74$ K, and $T_{s2}\approx0.7$ K—see Refs. 2 and 3. In most of the new quaternary rare-earth compounds $(RE)Ni_2B_2C$ the antiferromagnetic order and superconductivity coexist up to $T=0$ K,[4] while in $HoNi_2B_2C$ an additional oscillatory magnetic structure is realized in a limited temperature interval. This oscillatory magnetic structure competes strongly with superconductivity giving rise to reentrant behavior in this compound.[5] Recently Pobell's group in Bayreuth[6] made a remarkable discovery of the coexistence of superconductivity and nuclear magnetic order in $AuIn_2$ with $T_s=0{,}207$ K and $T_m=35$ μK. This exciting phenomenon was explained in Ref. 7 where it is argued that superconductivity can coexist either with spiral or domainlike nuclear magnetic ordering only, depending on the strength of magnetic anisotropy in this cubic system. Important contribution to the physics of magnetic superconductors has been made in Ref. 8, where for the first time the coexistence of weak ferromagnetism and superconductivity was proposed. In such a case the spontaneous vortex state due to weak ferromagnetism is also possible.

We point out that in the above-cited magnetic superconductors the exchange interaction (between localized magnetic moments and conduction electrons) influences superconductivity much stronger than the electromagnetic interaction. The latter is due to the localized magnetic moments that create dipolar magnetic field, thus affecting the orbital motion of superconducting electrons.

Recently, a new class of magnetic superconductors based on layered perovskite ruthenocuprate compound $RuSr_2GdCu_2O_8$ comprising CuO_2 bilayers and RuO_2 monolayers has been synthesized.[9] This compound belongs also to the class of high-T_s superconductors (HTS). A subsequent study of transport and magnetic properties has revealed that it exhibits some kind of ferromagnetic order at the critical temperature $T_N=133$–137 K. The polarized neutron scattering measurements[10] show that the magnetic structure (which appears at T_N) is predominantly antiferromagnetic with a Ru magnetic moment $\mu_{Ru}\approx1.18\mu_B$ along the c axis at low temperature. The same measurements put an upper limit $\sim0.1\mu_B$ to any net ferromagnetic zero-field Ru moment. Concerning the last point, the important results came from magnetization measurements first reported in Ref. 9, which show a hysteresis loop and remanent magnetization. The latter hints to existence of a ferromagnetic component in the system. Recent magnetization measurements on $RuSr_2EuCu_2O_8$ (Ref. 11) give evidence for a small ferromagnetic component, which lies probably parallel to the RuO_2 plane, with the magnetic moment (per Ru) $\sim0.05\mu_B$ at 5 K consistent with the neutron scattering data.[10] Note that the smaller value of magnetic moment $(0.05\mu_B)$ in this compound tells us that in the Gd compound some admixture of the large Gd moment might take place. This conclusion is also confirmed by the zero-field muon spin rotation (ZF-μSR) measurements,[12] which provide important evidence that the magnetic order is homogeneous on a microscopic scale and accounts for most of the sample volume. At lower temperatures the superconductivity sets in at $T_s=35$–45 K without affecting the AF order,[10,12] notably. This fact means that superconductivity that is realized predominantly in the CuO_2 planes, and magnetic order that is present only in the RuO_2 planes, interact rather weakly, i.e., these two orders are separated spatially. Recently, it was reported[13] that in $Ru_{1-x}Sr_2GdCu_{2+x}O_{8-y}$ the highest superconducting critical temperature reaches 72 K for $x=0.3$–0.4, while there is no sign of the weak ferromagnetic (WF) component in the RuO_2 planes.

It seems that $RuSr_2GdCu_2O_8$ has very interesting magnetic properties, which might result in the absence of Meissner phase in some samples,[14,15] while in some others it is realized.[9,16] (This problem will be briefly discussed in Sec. IV.)

In this paper we propose a model of layered magnetic superconductor with weak-ferromagnetism, which might be relevant for the $RuSr_2GdCu_2O_8$ compound—the SWF (superconducting WF) model. This model, studied in Sec. II, assumes the existence of S/F multilayers with small hopping parameter t between S (superconducting) and F (ferromagnetic) planes along the c axis , i.e., $t<T_s$. As a result the small t gives rise to an effective Josephson coupling current j_c between superconducting planes. It turns out that j_c is suppressed by the exchange field present in the F plane only, which causes drastic changes in magnetic properties. The Gibbs free energy $\mathcal{G}$ of such a magnetic superconductor with both AF and WF orderings in external magnetic field $\mathbf{H}$ is formulated in Sec. III. Based on it the lower critical field H_{c1} is also studied there. The estimation of theoretical parameters of the SWF model from the experimental results in $RuSr_2GdCu_2O_8$ is done in Sec. IV, where the obtained results are discussed too.

II. MODEL FOR S/F ATOMIC MULTILAYER AND JOSEPHSON CURRENT

As was mentioned above we consider the magnetic superconductor $RuSr_2GdCu_2O_8$ as a prototype for S/F atomic multilayers by assuming good conduction in CuO_2 planes—with the quasiparticle spectrum $\xi_S(\mathbf{p})$, and a small hopping parameter $t<T_s$ between the S and F planes (i.e., along the c axis). The second assumption is related to the existence of WF order (with the magnetization $\mathbf{M}$ lying in the RuO_2 planes), which gives rise to an effective exchange field parameter $\mathbf{h}=h\mathbf{e}_{ab}$. The latter affects spins of conduction electrons with the dispersion $\xi_F(\mathbf{p})$ moving in the normal conducting RuO_2 planes. (The a-b plane is sometimes labeled by the x-y plane.) The parameter h can be related to an effective spontaneous spin S_{eff} (magnetization normalized to saturation magnetization) in the a-b plane, i.e., $h=J^{ab}S_{\text{eff}}$—see also Sec. IV.

The electronic part of the SWF model is similar to the model in Ref. 17 and in what follows the same notation is used. According to this model the elementary cell of the superlattice consists of one superconducting and one ferromagnetic layer that are both metallic. For simplicity it is supposed here that both layers have similar quasiparticle energy spectra, i.e., $\xi(\mathbf{p})\equiv[\xi_S(\mathbf{p})\approx\xi_F(\mathbf{p})]$. It is also assumed that the superconductivity is realized in S planes (CuO_2 planes) with pairing coupling $g(\mathbf{p})$ (having in mind application to the HTS compound $RuSr_2GdCu_2O_8$ the clean limit, $\xi_0\ll l$, is supposed). The Hamiltonian of the system is given by

$$H=\sum_{\mathbf{p},n,i,\sigma}\xi(\mathbf{p})a^{\dagger}_{n,i,\sigma}(\mathbf{p})a_{n,i,\sigma}(\mathbf{p})+t[a^{\dagger}_{n,1,\sigma}(\mathbf{p})a_{n,-1,\sigma}(\mathbf{p})+a^{\dagger}_{n+1,-1,\sigma}(\mathbf{p})a_{n,1,\sigma}(\mathbf{p})+\text{H.c.}]+H_{int1}+H_{int2},$$

$$H_{int1}=\frac{1}{2}\sum_{\mathbf{p}_1,\mathbf{p}_2,n,\sigma}g(\mathbf{p}_1-\mathbf{p}_2)a^{\dagger}_{n,1,\sigma}(\mathbf{p}_1)a^{\dagger}_{n,1,-\sigma}(-\mathbf{p}_1)a_{n,1,-\sigma}(-\mathbf{p}_2)a_{n,1,\sigma}(\mathbf{p}_2),$$

$$H_{int2}=-\sum_{\mathbf{p},n,\sigma}h\sigma a^{\dagger}_{n,-1,\sigma}(\mathbf{p})a_{n,-1,\sigma}(\mathbf{p}),$$

where $a^{\dagger}_{n,i,\sigma}(\mathbf{p})$ is the creation operator of an electron with spin σ (the quantization axis is parallel to the a-b plane) in the nth elementary cell and momentum $\mathbf{p}$ in the layer i is parallel to the a-b plane, where $i=1$ for the S layer, and $i=-1$ for the F layer. Since the obtained results below are qualitatively similar for s- and d-wave pairing, the calculations were done for s-wave pairing where $g(\mathbf{p})=g_0$ is constant, while quantitative changes due to d-wave pairing are discussed below and in Sec. IV.

By assuming that the order parameter changes from cell to cell in the manner $\Delta_n=|\Delta|e^{i\varphi_n}$ (with $\varphi_n=kn$ in absence of orbital effects) the quasiparticle Green's functions are obtained in the standard way.[17] The self-consistency equation for the order parameter $|\Delta|$ reads[17]

$$\frac{1}{\Lambda}=T\sum_{\omega}\int_{-\infty}^{\infty}d\xi\int_0^{2\pi}\frac{dq}{2\pi}\frac{\tilde{\omega}_+\tilde{\omega}_-}{|\Delta|^2\tilde{\omega}_+\tilde{\omega}_--(\omega_-\tilde{\omega}_--|\mathcal{T}_{q+k}|^2)(\omega_+\tilde{\omega}_+-|\mathcal{T}_q|^2)},\qquad(1)$$

where $\Lambda=g_0\rho(0)$, $\rho(0)=m_{\|}/2\pi$ is the electron density of states at the Fermi level in the normal state, and $\omega_{\pm}=i\omega\pm\xi(p)$, $\tilde{\omega}_{\pm}=\omega_{\pm}+h$, $\omega=(2n+1)\pi T$ are Matsubara frequencies at temperature T. The quasimomentum q lies in the direction perpendicular to the layers, and $\mathcal{T}_q=2t\cos(q/2)e^{iq/2}$ and $\mathcal{T}_{q+k}=2t\cos[(q+k)/2]e^{i(q+k)/2}$.

The free energy $\mathcal{F}$ in the superconducting state is obtained by using the following relation:

$$\frac{\partial \mathcal{F}}{\partial|\triangle|}=\frac{|\triangle|}{g_0}-\frac{T\rho(0)}{2\pi}\sum_{\omega}\int_{-\infty}^{\infty}d\xi\int_0^{2\pi}dqF_{11}^{\dagger}, \tag{2}$$

where the expression for $F_{11}^{\dagger}$ is obtained in Ref. 17.

In order to study transport and magnetic properties in magnetic field we need to know the supercurrent j_z flowing across the layers (along the c axis in $RuSr_2GdCu_2O_8$). In this case, the vector potential $\mathbf{A}_z=A_z\mathbf{e}_z$ enters the Hamiltonian through the substitution $t\rightarrow te^{\pm iedA_z/c}$, where d is S-F interlayer distance, and the part of the Hamiltonian depending on A_z is given by

$$H_A=\sum_{\mathbf{p},n,i,\sigma}t[a_{n,1,\sigma}^{\dagger}(\mathbf{p})a_{n,-1,\sigma}(\mathbf{p})e^{iedA_z/c}+a_{n+1,-1,\sigma}^{\dagger}(\mathbf{p})a_{n,1,\sigma}(\mathbf{p})e^{iedA_z/c}+\text{H.c.}]. \tag{3}$$

The supercurrent across the planes is obtained by the standard procedure

$$j_z=\frac{c}{2d}\,\frac{\delta H_A}{\delta A_z}. \tag{4}$$

Note that the Josephson supercurrent in the S/F superlattice is carried by Andreev bound states, similarly to the S/N case. In S layer the supercurrent is carried by Cooper pairs, but in N layer it flows via quasiparticles, which recondense in the next S layer; bound states represent this process.[18]

In the case of small hopping parameter $t\ll T_s$ the Josephson current along the c axis is obtained in leading order (proportional to t^4) by standard perturbation theory. After the integration over the energy ξ it reads

$$j_z=4e\pi|\Delta|^2t^4\rho(0)T\sum_{\omega>0}\left\{2\omega\frac{5h^4+6h^2|\Delta|^2+|\Delta|^4-4h^2\omega^2}{R^2(\omega)(\omega^2+h^2)}-\frac{|\Delta|^2+h^2}{R(\omega)\Omega^3(\omega)}-2\frac{(|\Delta|^2+h^2)^2-4h^2\omega^2}{R^2(\omega)\Omega(\omega)}\right\}\sin(k)=j_c\sin k, \tag{5}$$

where $R(\omega)=(|\Delta|^2+h^2)^2+4h^2\omega^2$ and $\Omega(\omega)=\sqrt{\omega^2+|\Delta|^2}$. As in the standard Josephson effect, the supercurrent j_z is proportional to $\sin k$, k being the phase difference between nth and $(n+1)$-th S layers.

In what follows we calculate numerically the critical current j_c [in Eq. (5)] at any point of the phase diagram (T,h) by replacing $|\Delta|\rightarrow\Delta_0(T)$, where $\Delta_0(T)$ is given by the BCS theory. The latter is correct due to the smallness of t, in which case T_s is practically unaffected by the exchange field, i.e., $T_s\approx T_{s0}$ up to the second order terms in t/T_{s0}. Here, T_{s0} is the critical temperature of bare S layers.

From Eq. (5) it comes out in particular, that near T_s and for $h=0$ one has $j_c>0$, while $j_c<0$ for $h\gg T_s$. The change of sign of j_c (near T_s), which corresponds to the transition from $k=0$ to $k=\pi$ in the ground state, occurs at $h_c=3.77T_{s0}$, in accordance with the calculation in Ref. 17. At low temperatures, $T\rightarrow 0$, j_c goes to zero at $h/\Delta_0(0)\simeq 1/2$, which just corresponds to $h_{c0}=0.87T_{s0}$ at $T=0$, again in accordance with Ref. 17. Note that the same approach if applied to d-wave pairing[19] gives $h_{c0}^{(d)}=0.6T_{s0}$ at $T=0$. The sign change of j_c is related to the transition from the "0" phase to "π" phase. This transition goes smoothly if we take into account the higher-order term ($\sim t^8\cos 2k$) in the free energy, in fact it means that the width of the region Δh where the transition from "0" phase to "π" phase occurs is of the order of $\Delta h\sim t^4/T_{s0}^3$. In the case of weak hopping $t\ll T_{s0}$ this region is very narrow and we may define the decoupling line $j_c(T_d,h)=0$, which results in the (T_d,h) phase diagram shown in Fig. 1.

The temperature dependence of the Josephson penetration depth $\lambda_J=\sqrt{c\phi_0/8\pi^2|j_c|(2d)}$,[20] where ϕ_0 is the flux quantum, is shown in Fig. 2 for various $h\neq 0$. Here $2d$ is the period of the multilayer. One should note its nonmonotonic behavior if $h\neq 0$, particularly when $h\sim T_s$. Based on these results one can analyze some magnetic properties, like the lower critical field H_{c1} in the a-b plane.

III. GIBBS ENERGY AND IN-PLANE CRITICAL FIELD H_{C1}

A. Gibbs energy

In order to calculate the lower critical field H_{c1} (and the possible absence of Meissner phase[14,15]) we need the Gibbs energy functional $\mathcal{G}$. Having in mind the application to the $RuSr_2GdCu_2O_8$ we assume, according to the neutron scattering data,[10] that in the magnetic subsystem (F layers coinciding with RuO_2 planes) AF order with spins along the c axis is realized at $T_N\gg T_s$. The AF order parameter is $\mathbf{L}=L_z\mathbf{e}_z$. The

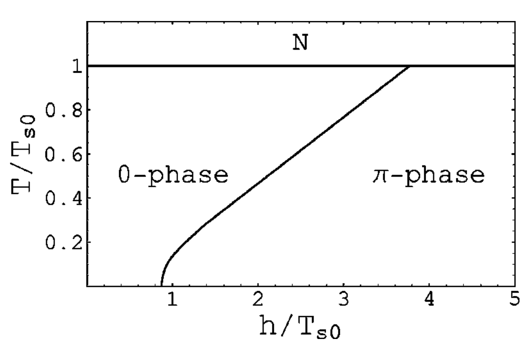

FIG. 1. The (T,h)-phase diagram for the case $t\ll T_{s0}$. $j_c(T,h)=0$ on the black line.

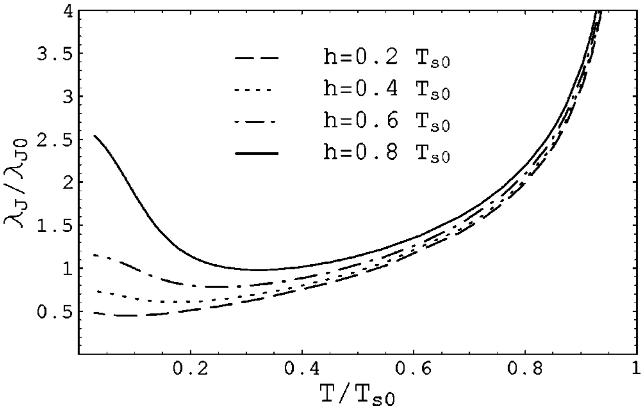

FIG. 2. The T dependence of the Josephson penetration depth $\lambda_J(T,h)$ for various h. We defined $\Delta_0=1.76T_{s0}$, $j_{c0}=e\rho(0)t^4/\Delta_0^2$, and $\lambda_{J0}=\sqrt{c\phi_0/16\pi^2 d j_{c0}}$.

magnetization measurements[9,11] imply WF order with the magnetization lying (most probably) parallel to the a-b planes and with the effective moment, $|\mathbf{M}|/n_{\mathrm{Ru}}=\mu_{\mathrm{eff}}<0.1\mu_B$, where n_{Ru} is the density of the magnetic Ru ions. In order to construct the magnetic free energy it is necessary to know the symmetry of the system as well as orientations of the easy axes in different sublattices. So, for instance, if the magnetic anisotropy energy on different sublattices are unequal, then one expects the WF order to be realized. However, at present there are no sufficient experimental data on the local lattice distortion that might favor WF order and accordingly the preferred direction of $\mathbf{M}$. The above-discussed experiments[10,12,9,11] suggest only that $\mathbf{M}$ is in the a-b plane, i.e., $\mathbf{M}=M_\eta \mathbf{e}_\eta$, where $\mathbf{e}_\eta$ is in the a-b plane. In the following we define the x axis by $\mathbf{e}_x\equiv\mathbf{e}_\eta$. As the result of this analysis the SWF model contains the following order parameters: L_z for the AF order, M_x for the WF order, and $\Delta_n(x,y)$ for the S order.

In the applied magnetic field $\mathbf{H}$ the Gibbs energy of the layered magnetic superconductor reads (see also Refs. 3 and 8),

$$\mathcal{G}[\Delta_n,\mathbf{L},\mathbf{M},\mathbf{B};\mathbf{H}]=\int dV\left(\mathcal{F}_M[\mathbf{L},\mathbf{M}]+\frac{(\mathbf{B}-4\pi\mathbf{M})^2}{8\pi}-\frac{\mathbf{BH}}{4\pi}\right)+\sum_n (2d)\int dx dy \mathcal{F}_S[\Delta_n,\mathbf{A}], \tag{6}$$

where $\Delta_n\equiv\Delta_n(x,y)$, and $\mathbf{L}$, $\mathbf{M}$, $\mathbf{B}$ are also coordinate dependent. The magnetic field $\mathbf{B}=\mathrm{rot}\,\mathbf{A}$ is due to the dipolar field created by the magnetic moments, the external magnetic field, and the superconducting screening current. The vector potential $\mathbf{A}=\mathbf{A}_{ab}+\mathbf{A}_c=A_{ab}\mathbf{e}_{ab}+A_z\mathbf{e}_z$ contains the component A_{ab} in the a-b plane, and A_z along the c axis.

The magnetic free-energy density functional $\mathcal{F}_M[\mathbf{L},\mathbf{M}]$, which mimics the experimental results in $RuSr_2GdCu_2O_8$ with $\mathbf{L}=L_z\mathbf{e}_z$ and $\mathbf{M}=M_x\mathbf{e}_x$, is given by the following phenomenological expression:

$$\mathcal{F}_M[\mathbf{L},\mathbf{M}]=\frac{\alpha}{2}\mathbf{L}^2+\frac{\beta}{4}\mathbf{L}^4+\frac{\delta}{2}\mathbf{M}^2-\gamma L_z M_x+F_a(\mathbf{L},\mathbf{M})+F_g(\nabla\mathbf{L},\nabla\mathbf{M})+\cdots. \tag{7}$$

Although this expression is quantitatively correct near the AF transition at T_N it is also suitable for semiquantitative analysis even below superconducting transition temperature T_s, due to the smallness of $\mathbf{M}$ and γ. The first two terms describe the AF order $[\alpha=\alpha'(T-T_N)<0,\beta>0]$, whereas the third $(\delta>0)$ and fourth $(\sim\gamma)$ terms describe the induced WF by the AF order. The parameters α and δ are due to the exchange interaction (between Ru spins in $RuSr_2GdCu_2O_8$), where one has $\alpha'\sim 1/\theta_{em}$ and $\delta\sim T_N/\theta_{em}$ with $\theta_{em}=2\pi\mu_B^2\sim 1$ K. The unknown anisotropy term $\mathcal{F}_a$ fixes the direction of $\mathbf{L}$ and $\mathbf{M}$, i.e., L_z, M_x.

Since in the following we analyze the lower critical field along the a-b plane, with characteristic length scales λ_{ab}, $\lambda_J\gg\xi_{ab},d_{\text{Ru-Ru}}$, where ξ_{ab} is the coherence length in S layers and $d_{\text{Ru-Ru}}$ is the Ru-Ru distance, it is justified to omit the gradient term $\mathcal{F}_g(\nabla\mathbf{L},\nabla\mathbf{M})$. By minimizing $\mathcal{F}_M[\mathbf{L},\mathbf{M}]$ with respect to L_z and M_x one gets (at temperatures $T_s<T<T_N$)

$$M_x^0=\frac{\gamma}{\delta}L_z. \tag{8}$$

The neutron scattering and magnetization measurements give limits for $(L_z/n_{\mathrm{Ru}})\sim(1\text{–}2)\mu_B$, and $(M_x^0/n_{\mathrm{Ru}})<(0.05\text{–}0.1)\mu_B$, which implies an upper limit for γ, i.e., $(\gamma/\delta)\lesssim(0.05\text{–}0.1)$.

According to experiments[10,12] the AF (and WF) ordering is practically unaffected by the appearance of superconductivity, then it is reasonable to neglect the effect of the exchange (Ruderman-Kittel-Kasuya-Yosida) interaction in $\mathcal{F}_S[\Delta_n,\mathbf{A}]$. Therefore we keep in $\mathcal{F}_S[\Delta_n,\mathbf{A}]$ the electromagnetic interaction between superconducting electrons and magnetic order only

$$\mathcal{F}_S[\Delta_n,\mathbf{A}]=\mathcal{F}_S[|\Delta_n|,0]+\frac{2\pi\lambda_{ab}^2}{c^2}\mathbf{j}_{ab}^2+\frac{j_c\phi_0}{2\pi c(2d)}(1-\cos\chi_{n,n+1}), \tag{9}$$

where $F_S[|\Delta_n|,0]$ is the condensation energy and $\Delta_n=|\Delta_n|\exp(i\varphi_n)$. The current in the a-b plane $\mathbf{j}_{ab}$ reads

$$\mathbf{j}_{ab}=-\frac{c}{4\pi\lambda_{ab}^2}\left(\mathbf{A}_{ab}-\frac{\phi_0}{2\pi}\nabla_{ab}\varphi_n\right), \tag{10}$$

where λ_{ab} is the bulk London penetration depth in the a-b superconducting layers (we assumed $\lambda_a=\lambda_b\equiv\lambda_{ab}$). The last term depends on the gauge invariant phase $\chi_{n,n+1}$,

$$\chi_{n,n+1}=\varphi_{n+1}-\varphi_n-\frac{4\pi A_z d}{\phi_0}, \tag{11}$$

which characterizes the effective Josephson coupling between two neighboring S planes with the distance $2d$. It is due to the hopping between S and F planes. The exchange interaction between conduction electrons and localized Ru moments affects superconductivity by renormalizing j_c, which is a function of h and is determined by Eq. (5).

B. Lower critical field H_{c1}

Let us calculate the lower critical field H_{c1}^{ab} for the case when the magnetic field $\mathbf{H}$ and the single vortex are along the magnetization $\mathbf{M}=M_x\mathbf{e}_x$, i.e., $\mathbf{H}=H_x\mathbf{e}_x$ and $\mathbf{B}=B_x\mathbf{e}_x$. By the standard minimization procedure of the Gibbs free energy $\mathcal{G}[\Delta_n,\mathbf{L},\mathbf{M},\mathbf{B};\mathbf{H}]$ with respect to $\Delta_n,\mathbf{L},\mathbf{M},\mathbf{B}$, and by assuming the continuum limit,[20] one gets the complete set of equations for these quantities as well as the Gibbs free energy of the vortex $\mathcal{G}_v$—see also Refs. 3 and 8.

$$(\delta+4\pi)M_x-\gamma L_z-B_x=0. \tag{12}$$

The Maxwell equation for the magnetic field $\mathbf{B}$ reads

$$\mathrm{rot}(\mathbf{B}-4\pi\mathbf{M})=\frac{4\pi}{c}\mathbf{j}_s, \tag{13}$$

where

$$\mathbf{j}_s=\mathbf{j}_{ab}+\mathbf{j}_z. \tag{14}$$

The in-plane current $\mathbf{j}_{ab}$ is given by Eq. (10) while $\mathbf{j}_z$ is the Josephson current between planes

$$\mathbf{j}_z=j_c\mathbf{e}_z\sin\chi_{n,n+1}. \tag{15}$$

The phase $\chi_{n,n+1}$ is given by Eq. (11). In the following we assume that the vortex axis $\mathbf{B}$ and the external field $\mathbf{H}$ are along the x axis. By the standard procedure we get the equation for the single vortex (centered on the origin)

$$\lambda_{ab}^2\frac{\partial^2 B_x}{\partial z^2}+\lambda_J^2\frac{\partial^2 B_x}{\partial y^2}-\frac{B_x}{p}=0. \tag{16}$$

The parameter $p=\delta/(\delta+4\pi)$ takes into account the additional screening due to the appearance of the WF order. After straightforward transformations the Gibbs energy of the vortex per unit length, $\mathcal{G}_v$, has the form

$$\mathcal{G}_v=\frac{p^2}{8\pi}\int dxdy\left\{\frac{B_x^2}{p}+\lambda_{ab}^2\left(\frac{\partial B_x}{\partial z}\right)^2+\lambda_J^2\left(\frac{\partial B_x}{\partial y}\right)^2\right\}-\frac{\phi_0\tilde{H}_{c1}}{4\pi}, \tag{17}$$

where

$$\tilde{H}_{c1}=H_{ext}+4\pi pM_x^0.$$

M_x^0 is approximately given by Eq. (8).

The solution of Eq. (16) is

$$B_x(y,z)=\frac{\phi_0}{2\pi p\lambda_{ab}\lambda_J}K_0\left(\frac{R}{\sqrt{p}}\right), \tag{18}$$

where $R=\sqrt{y^2/\lambda_J^2+z^2/\lambda_{ab}^2}$ and K_0 is the Bessel function of the zero order of an imaginary argument. Inserting such a solution into Eq. (17) a straightforward calculation gives the lower critical field $\tilde{H}_{c1}$ from the condition $\mathcal{G}_v=0$,

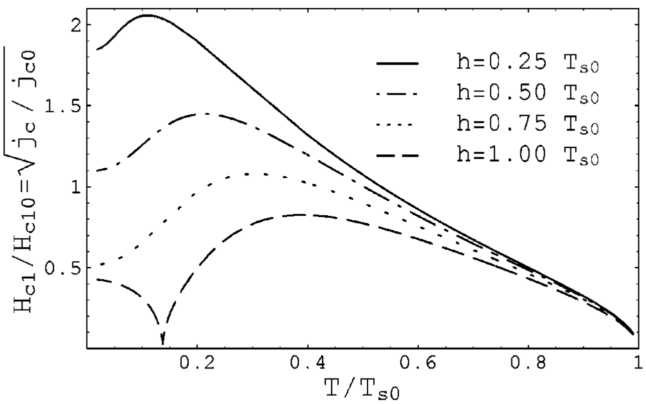

FIG. 3. The T dependence of the lower critical field $\tilde{H}_{c1}(T,h)$ in the a-b plane for various h. We defined $\tilde{H}_{c10}=(\phi_0/4\pi\lambda_{J0}\lambda_{ab})\ln(\lambda_{ab}\sqrt{p}/d)$.

$$H_{ext}+4\pi pM_x^0=\tilde{H}_{c1}\approx\frac{\phi_0}{4\pi\lambda_J\lambda_{ab}}\ln\frac{\lambda_{ab}\sqrt{p}}{d}, \tag{19}$$

where $M_x^0=(\gamma/\delta)L_z$. We stress that the logarithmic factor in Eq. (19) is due to the nonlinear core effects of the Josephson vortex.[20] Note that in systems with $T_N\gg\theta_{em}$, like in $RuSr_2GdCu_2O_8$, one has $p\sim 1$.

From Eq. (19) it is seen that for

$$M_x^0>\frac{p\phi_0}{16\pi^2\lambda_J\lambda_{ab}}\ln\frac{\lambda_{ab}\sqrt{p}}{d} \tag{20}$$

spontaneous vortices appear in the system. This condition is more easily realized near the 0-to-π transition (decoupling) line $T_d(h)$, i.e., when λ_J is significantly increased. This means that in systems where the exchange parameter fulfils the condition $0.87T_{s0}<h<3.77T_{s0}$ for s wave pairing, while for d-wave pairing $0.87T_{s0}$ is replaced by $0.6T_{s0}$, then by lowering the temperature the $\tilde{H}_{c1}(T,h)$ shows pronounced nonmonotonic behavior reaching minimum at the 0-π boundary line as it is seen in Fig. 3.

IV. COMPARISON WITH THE EXPERIMENT AND DISCUSSION

Let us discuss some relevant points related to the interpretation of the obtained results on $RuSr_2GdCu_2O_8$.

(i) In order to analyze the magnetic properties the value of the exchange field parameter $h=J^{ab}S_{\mathrm{eff}}$ is needed. If one takes the experimental value $S_{\mathrm{eff}}\sim 0.1$ one gets $\gamma/\delta\sim 0.1$ since $S_{\mathrm{eff}}\approx\gamma/\delta$. However, at present we do not know the relation between J^{ab} and T_N. As $d_{\text{Ru-Ru}}^{ab}\ll d_{\text{Ru-Ru}}^{c}$ one expects that the coupling of spins along the c direction, J^c, is much smaller than along the a-b plane, J^{ab}. In such a situation one has $T_N\sim J^{ab}/\ln(J^{ab}/J^c)$. Then, $J^{ab}>T_N$ and $h\sim 10$–20 K. We pay attention to the fact that there are evidences that in the underdoped HTS materials d wave pairing is realized.[21] In that case the point on the phase diagram $j_c(T=0,h_{c0})=0$ is realized for $h_{c0}^{(d)}=0.6T_{s0}$. According to the specific-heat measurements[12] in $RuSr_2GdCu_2O_8$ with $T_s=30$–40 K this compound behaves like an underdoped HTS material. If it is so it gives $h_{c0}^{(d)}\sim 20$ K, i.e., h is near h_{c0} and a nonmonotonic behavior of $\tilde{H}_{c1}$ is expected as shown in Fig. 3.

(ii) If M_x^0 fulfils Eq. (20) then there is a spontaneous vortex state and the Meissner effect is absent. In opposite case the Meissner state is realized.

(iii) It may happen that $M_x^0<\tilde{H}_{c1}$ in some temperature intervals and $M_x^0>\tilde{H}_{c1}$ in the interval between, which case corresponds to a reentrant behavior.

(iv) At present the origin and the magnitude of the parameter γ in Eqs. (7, 8) is unknown. However, it may also happen that in polycrystalline samples strains induce additional changes of this quantity. A drastic case might be realized if the symmetry of the crystal implies that $\gamma=0$. Even in that case strains in samples, for instance the component σ_{xy}, can induce a magnetic moment in piezomagnetic systems, i.e., $M_x^0\sim\sigma_{xy}L_z$ thus producing weak ferromagnetism. If strains in a sample are such that $M_x^0>\tilde{H}_{c1}$ then the Meissner phase is not realized as reported in Refs. 14 and 15. In such a way one could reconcile the opposite claims on existence[9,16] and nonexistence,[14,15] of the Meissner phase in differently prepared samples of $RuSr_2GdCu_2O_8$.

(v) Based on the above analysis one expects that dynamical properties of such a system are very exotic. For systems near the decoupling line $j_c(T_d,h)=0$ there is a significant reduction of the Josephson plasma frequency $\omega_{0,SF}\sim\sqrt{j_c}\ll\omega_{0,JJ}$ (Ref. 23) (compared to standard Josephson junction with $\omega_{0,JJ}$) for the waves propagating along the x-y planes in the S/F superlattice

$$\omega_{SF}^2=\omega_{0,SF}^2+v_{SF}^2q^2.$$

Due to the microscopic character of the S/F superlattice one expects that $v_{SF}^2\gg v_{JJ}^2$ where v_{JJ}, is the phase velocity for the Josephson junction made from bulk superconductors. This means that in a S/F superlattice, like for instance in $RuSr_2GdCu_2O_8$, it is possible to tune $\omega_{0,SF}^2$ nonmonotonically and also to extract the radiation with much higher intensity than in the single Josephson junction.

(vi) Concerning the exchange interaction between conduction electrons in RuO_2 planes and Ru spins, it is unimportant in which direction the total spin (that is the magnetization **M**) points. In particular, the location of 0-to-π phase transition only depends on the intensity of the exchange field h. Also, the possibility for F layers to be polarized ferromagnetically in each layer but antiferromagnetically from one layer to the next one was considered in Ref. 22, where the phase diagram was proved to be sligthly modified (at leading order t^4/T_{s0}^4) compared to the case we consider in the present paper. However, the magnetic properties like H_{c1} strongly depend on the precise orientation of the magnetization since the superconducting screening is strongly anisotropic. Thus, the results we obtain are specific to the magnetic model described above.

In conclusion, we have shown that in a S/F superlattice with the exchange field $h\sim T_s$ acting in F planes only a nontrivial and nonmonotonic behavior of magnetic properties, like the lower critical field H_{c1}, is realized. This property is due to the decrease of the effective Josephson coupling between S planes by increasing h. In the present paper we only discuss the case when $T_s\ll T_N$ and the intensity of the exchange field h can be considered constant. A very interesting situation arises when $T_s\gtrsim T_N$ and the temperature dependence of h should now be taken into account. In particular, it would become possible to drive the 0-to-π phase transition more easily by varying the temperature. Indeed it would take place as soon as the saturated value for the exchange field is higher than h_{c0}. Such a situation should be explored in more details in the future.

ACKNOWLEDGMENTS

One of us (M.L.K.) thanks H. F. Braun, C. Bernhard, and B. Keimer for useful discussion of their experimental results. M.L.K. acknowledges the support of the Deutsche Forschungsgemeinschaft through the Forschergruppe ''Transportphänomene in Supraleitern und Suprafluiden.'' This work was also supported by the ESF ''Vortex'' Program and the CEA (Accord Cadre n°12 M).

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