Some remarks on pseudogap behavior of nearly antiferromagnetic metals

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In the antiferromagnetically ordered phase of a metal, gaps open on parts of the Fermi surface if the Fermi volume is sufficiently large. We discuss simple qualitative and heuristic arguments under what conditions precursor effects, i.e., pseudogaps, are expected in the *paramagnetic* phase of a metal close to an antiferromagnetic quantum phase transition. At least for weak interactions, we do not expect the formation of pseudogaps in a three-dimensional material. According to our arguments, the upper critical dimension d_c for the formation of pseudogaps is $d_c=2$. However, at the present stage we cannot rule out a higher upper critical dimension, $2\leqslant d_c\leqslant 3$. We also discuss briefly the role of statistical interactions in pseudogap phases.

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Experiments on metals close to an antiferromagnetic quantum critical point (QCP) show clearly that these systems cannot be described by standard Fermi-liquid theory. This is not very surprising, as at the QCP magnetic fluctuations dominate and electronic quasiparticles scatter from spin fluctuations characterized by a diverging correlation length. Indeed, a theory of quantum critical fluctuations interacting weakly with Fermi-liquid quasiparticles[1,2] can explain a substantial part of the experiments, at least if effects like weak impurity scattering are properly taken into account.[3] However, a number of experiments seems to contradict the standard spin-fluctuation scenario, presently the best studied example for this is probably $CeCu_{6-x}Au_x$.[4–6] It has been speculated that this might be due to anomalous two-dimensional spin fluctuation[5] or a partial breakdown of the Kondo effect.[6]

In this paper we discuss a different route which can lead to a breakdown of the theory of weakly interacting spin fluctuations, proposed by Hertz.[1,2] The general idea[7] is the following: close to the QCP, the behavior of the system is dominated by large antiferromagnetic domains of size ξ, slowly fluctuating on the time scale $\tau_\xi\sim\xi^{z_{\rm op}}$ where $z_{\rm op}$ is the dynamical critical exponent of the order parameter. As ξ is diverging when the QCP is approached, it is suggestive to assume that the electrons will adjust their wave functions adiabatically to the local antiferromagnetic background and will therefore show a similar behavior as in the antiferromagnetically ordered phase. If the Fermi surface is sufficiently large, the (staggered) order parameter of the antiferromagnetic phase induces gaps in parts of the Fermi surface with $\epsilon_{\mathbf{k}}\approx\epsilon_{\mathbf{k}\pm\mathbf{Q}}\approx 0$, where $\epsilon_{\mathbf{k}}$ is the dispersion of the quasiparticles measured from the Fermi energy and $\mathbf{Q}$ the ordering wave vector of the antiferromagnet. Will precursors of this effect show up and induce pseudogaps in the paramagnetic phase for sufficiently large ξ? Pseudogaps play an important role in the physics of underdoped cuprates[8–15] and it has been speculated that they are indeed precursors of gaps in either superconducting, antiferromagnetic, flux, or striped phases. In this paper we want to investigate qualitatively on the basis of simple physical arguments under what generic conditions such pseudogaps are expected to occur close to an antiferromagnetic QCP. We will consider only systems where the ordered antiferromagnet is metallic, therefore our discussion might have less relevance for the high-T_c superconductors where the undoped antiferromagnet is a (Mott) insulator.

To define the concept of a pseudogap more precisely, we first analyze the ordered phase where in mean-field theory the Hamilton of the electrons is of the form

$$H_\Delta=\sum_{\sigma,\mathbf{k}}(c^\dagger_{\sigma,\mathbf{k}},c^\dagger_{\sigma,\mathbf{k+Q}})\begin{pmatrix}\epsilon_{\mathbf{k}} & \sigma\Delta\\ \sigma\Delta & \epsilon_{\mathbf{k+Q}}\end{pmatrix}\begin{pmatrix}c_{\sigma,\mathbf{k}}\\ c_{\sigma,\mathbf{k+Q}}\end{pmatrix}. \quad (1)$$

Δ is proportional to the staggered order parameter (assumed to point in z direction) and the $\mathbf{k}$ sum extends over a magnetic Brillouin zone. Close to the "hot lines" on the Fermi surface ("hot points" in two dimensions) with $\epsilon_{\mathbf{k}_h}=\epsilon_{\mathbf{k}_h\pm\mathbf{Q}}=0$, a gap opens (see Fig. 1) and the band structure at $\mathbf{k}=\mathbf{k}_h+\delta\mathbf{k}$ is approximately given by

$$\epsilon^{\pm}_{\delta\mathbf{k}}\approx\tfrac{1}{2}\left((\mathbf{v}_1+\mathbf{v}_2)\,\delta\mathbf{k}\pm\sqrt{[(\mathbf{v}_1-\mathbf{v}_2)\,\delta\mathbf{k}]^2+4\Delta^2}\right), \quad (2)$$

where $\mathbf{v}_1=\mathbf{v}_{\mathbf{k}_h}$ and $\mathbf{v}_2=\mathbf{v}_{\mathbf{k}_h+\mathbf{Q}}$ are the Fermi velocities close to the hot points. The gap is, e.g., visible if one integrates the spectral function $A_{\mathbf{k}}(\omega)$ for $\mathbf{k}$ vectors along a direction $\hat{n}$ in the $(\mathbf{v}_1,\mathbf{v}_2)$ plane perpendicular to $\mathbf{v}_1+\mathbf{v}_2$ (dash-dotted line in Fig. 1) $\tilde{A}(\omega)=\int dk A_{\mathbf{k}_h+k\hat{\mathbf{n}}}(\omega)$. In mean-field theory $\tilde{A}(\omega)$ displays a well-defined gap of size 2Δ. This gap is a consequence of the reduced translational symmetry and is expected to be present in the ordered phase of the antiferromagnet, even in a regime, where the predictions of mean-field theory are quantitatively wrong. Interactions of quasiparticles far away from $\mathbf{k}_h$ with each other and with the spin

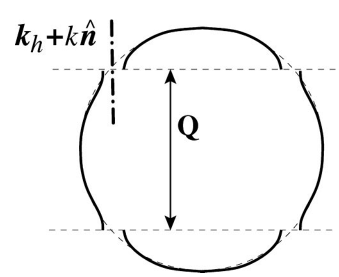

FIG. 1. Schematic plot of the Fermi surface. In the ordered phase of a metallic antiferromagnet gaps open at the boundaries of the magnetic Brillouin zone.

fluctuations will actually induce some small weight within these (renormalized) gaps but this does not invalidate the mean-field picture: $\tilde{A}(\omega)$ vanishes rapidly in the limit $\omega \to 0$ in the ordered phase as it is obvious from the usual Fermi-liquid phase space arguments. From general scaling arguments one expects in the *paramagnetic* phase close to the QCP, that at $T=0$

$$\tilde{A}(\omega) \sim \omega^{\alpha} f(1/(\omega \xi^{z})) \tag{3}$$

with $f(x \to 0) \approx \text{const}$ and $f(x \to \infty) \sim x^{\alpha}$ where z is a dynamical critical exponent (see below). Within mean-field theory no precursor of the gaps show up and $\alpha = 0$. However, one would expect $\alpha > 0$ if the wave function of the quasiparticles adjusts adiabatically to the local antiferromagnetic order on length scales smaller than ξ.

This paper focuses on the $T=0$ behavior directly at the QCP as we are mainly interested in the question of whether pseudogaps affect the quantum critical behavior and therefore Eq. (3) with $\alpha > 0$ serves as a defintion for a pseudogap. Note that pseudogap physics can be considerably more pronounced in other regimes, e.g., for nearly magnetic metals with Heisenberg or xy symmetry in $d=2$ for low, but finite temperatures in a parameter regime, where the system is deep in the ordered phase at $T=0$. This regime has, for example, been investigated in detail by Vlik and co-workers.[15]

For definiteness, we will consider a model of Fermions $f_{\mathbf{k}\sigma}$ coupled linearly to a collective bosonic field $\Phi_{\mathbf{q}}$ with the following action in imaginary time:[1]

$$S = \int_0^{\beta} d\tau \left[\sum_{\sigma,\mathbf{k}} f^{*}_{\sigma,\mathbf{k}} (\partial_\tau + \epsilon_{\mathbf{k}}) f_{\sigma,\mathbf{k}} + \sum_{\mathbf{q}} \mathbf{\Phi}^{*}_{\mathbf{q}} \frac{1}{J_{\mathbf{q}}} \mathbf{\Phi}_{\mathbf{q}} + \sum_{\mathbf{kq}i\alpha\beta} \Phi^{i}_{\mathbf{q}} f^{\dagger}_{\alpha,\mathbf{k}+\mathbf{q}} \sigma^{i}_{\alpha\beta} f_{\beta,\mathbf{k}} + \text{H.c.} \right], \tag{4}$$

where σ^i are the Pauli matrices and $\beta = 1/T$ the inverse temperature. Integrating out the collective field induces a spin-spin interaction J of the Fermions. For realistic models one should also add charge-charge interactions, which are, however, not expected to change the physics close to a magnetic QCP qualitatively.

Many years ago, Hertz[1] has proposed to describe the QC metallic antiferromagnet in the spirit of a Ginzburg-Landau-Wilson approach in terms of a fluctuating order parameter $\Phi(x,\tau)$ with an effective action

$$S = S_0 + S_{\text{int}}, \tag{5}$$

$$S_0 = \frac{1}{\beta} \sum_{\mathbf{k},\omega_n} \Phi^{*}_{\mathbf{k}\omega_n} [r + (\mathbf{k} \pm \mathbf{Q})^2 + \gamma |\omega_n|] \Phi_{\mathbf{k}\omega_n}, \tag{6}$$

$$S_{\text{int}} = U \int_0^{\beta} d\tau \int d^d\mathbf{r} |\Phi(x,\tau)|^4, \tag{7}$$

where $\omega_n = 2\pi n/\beta$ are bosonic Matsubara frequencies and $\Phi_{\mathbf{k}\omega_n}$ is the Fourier transform of $\Phi(x,\tau)$. The term linear in ω_n is due to the scattering from quasiparticles which induce the Landau damping of the spin fluctuations. As discussed in

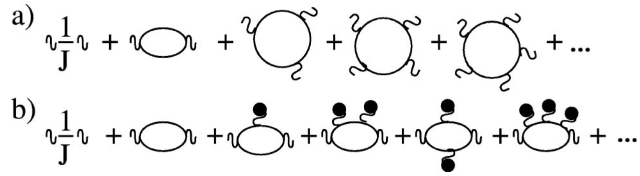

FIG. 2. (a) Effective action $S[\Phi]$ according to Hertz (Ref. 1) for the model defined in Eq. (4) after the electrons have been integrated out. The lines denote free Green's functions $G_0(x-x', \tau-\tau')$ of the electrons, the wiggles are the fields $\Phi(x,\tau)$. (b) Quadratic part of the effective action $\Phi \to \langle \Phi \rangle + \delta\Phi$ in the ordered phase (● denotes the order parameter $\langle \Phi \rangle$). Combinatorial prefactors are omitted both in (a) and (b).

detail in the original paper by Hertz,[1] the action S describes only the leading terms in an expansion which is derived by integrating out the Fermions in Eq. (4). The expansion is shown schematically in Fig. 2(a) (due to time-reversal symmetry, cubic terms vanish in the limit $\omega_n \to 0$). A simple scaling analysis[1] with $k \sim 1/L$, $\omega \sim 1/L^{z_{\text{op}}}$, $\Phi(x,\tau) \sim L^{1-(d+z_{\text{op}})/2}$ with $z_{\text{op}} = 2$ shows that the interaction term S_{int} vanishes $\sim 1/L^{d+z_{\text{op}}-4}$, i.e., U is (dangerously) irrelevant in dimensions $d > 4 - z_{\text{op}} = 2$. Furthermore, higher-order interactions and frequency and momentum dependencies of the effective vertices are even more irrelevant. A pseudogap as it is defined in Eq. (3) would certainly change the critical exponent z_{op}, as it would strongly reduce the damping of the spin fluctuations. As the scaling analysis sketched above gives no indications for such a phenomenon for $d \geqslant 4 - z_{\text{op}}$, it strongly suggests that a strong-coupling effect like a pseudogap should never occur in dimensions $d > 4 - z_{\text{op}}$ at least as long as the (bare) interactions are not too strong.

This line of arguments (which would be completely valid close to a classical phase transitions) is *not* reliable in the case of a quantum phase transition in a metal. This can be seen, for example, by considering the *ordered* phase. An expansion of the Hertz action (5) around the mean field $\Phi = \langle \Phi \rangle + \delta\Phi$ suggests that the transverse spin-fluctuations (assuming Heisenberg or xy symmetry) are damped. However, the Goldstone theorem guarantees that the spin waves are not damped in the limit $\omega, \mathbf{k} \to 0$. The physical origin of this is essentially the same as in the previous discussion of pseudogap formation: the wave function of the electrons adjust to the slowly varying antiferromagnetic background. A simple random-phase approximation (RPA) based upon the mean-field Hamiltonian (1) correctly describes this effect on a qualitative level. It is therefore instructive to investigate how the RPA contribution arise in the effective action $S[\Phi]$. In Fig. 2(b) it is shown that spin-spin interactions Φ^n of arbitrarily high order n are needed to recover the trivial RPA + mean-field result.

Two scenarios seem to be possible to resolve the apparent conflict that contributions which are irrelevant by power counting are important in the ordered phase. The first possibility is that all the higher interactions are indeed *irrelevant* in the sense that the physics of the formation of undamped spin fluctuations does not influence the quantum critical behavior on the paramagnetic side of the phase diagram in any qualitative manner—in technical terms, they are "dangerously irrelevant'' and important only in the ordered phase. The analysis given below suggests that this situation is actually realized in three dimensions. The second possibility is that pseudogap formation is important and that spin-spin interactions of arbitrarily high order have to be kept which implies that Eq. (5) does not describe the physics properly and the ''true'' critical theory cannot be formulated in terms of the order parameter alone but has to include Fermionic modes. For example, the spin-fluctuation theory of the cuprates as it is worked out by Abanov and Chubukov[14] suggests such a scenario in $d=2$. What can go wrong with the simple scaling arguments given above? Belitz and co-workers[16] have recently shown in their analysis of the dirty nearly ferromagnetic metal that scaling is indeed not reliable due to a very simple physical reason: The Hertz action implies that a domain of size ξ fluctuates very slowly on the time scale $\tau_\xi \propto \xi^{z_{\rm op}}$ with $z_{\rm op}=2$. However, in a clean metal there is a much faster and more efficient way to propagate information from one side of a fluctuating domain to the other: ballistic electrons can traverse the domain in the time $\tau_F \propto \xi^{z_F}$ with $z_F=1$. This defines a second dynamical critical exponent z_F (which can be renormalized due to scattering from spin fluctuations; see below). Power counting is not reliable because *two* different dynamical exponents $z_{\rm op}$ and z_F exist simultaneously—while there is only one large length scale ξ, two rather different time scales exist. The question which of these scales is relevant for a given process generally requires a detailed analysis and is not at all obvious. This physics should therefore be investigated in a careful renormalization-group calculation which includes both fermionic and bosonic degrees of freedom. We will not try such an analysis here but instead use a properly modified scaling argument to investigate the possibility of pseudogap formation.

For our scaling analysis,[17] we assume that the susceptibility at the QCP is of the form suggested by Eq. (6) ($d \geqslant 2$),

$$\chi_{\mathbf{q}\pm\mathbf{Q}}(\omega) \sim \frac{1}{\mathbf{q}^2+(i\omega)^{2/z_{\rm op}}}. \tag{8}$$

We are mainly interested in the case $z_{\rm op}=2$, smaller values for $z_{\rm op}$ might be relevant if pseudogap formation takes place,[7] larger values have, e.g., been used to fit experiments[6] in $\mathrm{CeCu}_{6-x}\mathrm{Au}_x$ and have been claimed[14,18] to be relevant in $d=2$. It is not difficult to generalize the following arguments for susceptibilities with other $\mathbf{q}$ and ω dependencies.[18]

The strategy of the following scaling analysis is to estimate the effective amplitude of the quasistatic collective field seen by the electrons. Obviously the answer will depend on which time and length scale the electrons probe the background magnetization. The main idea is that a lower bound for the relevant time and length scales can be derived from Heisenberg's uncertainty relation and the effective size of the gap. The main assumptions of the following arguments are discussed in detail in the second half of the paper: we assume that above the upper critical dimension for pseudogap formation, the nature of the electrons is not changed completely by the quantum critical fluctuations. According to the mean-field result (2) a gap of size $\omega^*=\Delta$ opens in a ($d-2$-dimensional) stripe in momentum space of width $k^*=\Delta/v_F$. Below, we will discuss the effect of interactions which can change this relation to $\omega^* \sim (k^*)^{z_F} \sim \Delta^{z_F}$ where $z_F=1$ is the mean-field exponent. Heisenberg's uncertainty relation dictates that the electrons have to see a quasistatic antiferromagnetic background for a time $\tau^* \gtrsim 1/\omega^*$ on a length scale of order $\xi^* \gtrsim 1/k^*$ perpendicular to the direction of the hot lines to develop the pseudogap. What is the effective size of the quasistatic antiferromagnetic order $\langle\Phi\rangle^{\rm eff}_{\xi^*,\tau^*}$ on these length and time scales? The following estimate should at least give an upper bound at the QCP:

$$(\langle\Phi\rangle^{\rm eff}_{\xi^*,\tau^*})^2 \lesssim \int_0^{\omega^*} d\omega \int_{q_\perp<k^*} d^2q_\perp \int_\infty^\infty d^{d-2}q_\parallel \,\mathrm{Im}\,\chi_{\mathbf{q}\pm\mathbf{Q}}(\omega)$$

$$\sim (k^*)^{d+z_{\rm op}-2}+(k^*)^2(w^*)^{(d+z_{\rm op}-4)/z_{\rm op}} \tag{9}$$

$$\sim \Delta^{(d+z_{\rm op}-4)(z_F/z_{\rm op})+2}, \tag{10}$$

where the anisotropic integration of q takes into account that the momentum of the electrons *parallel* to the hot line can vary on the scale k_F. In Eq. (10) we assumed $z_F \leqslant z_{\rm op}$. For our scaling argument, it does not matter whether we use $\mathrm{Im}\,\chi(\omega)$ or, e.g., $\chi(-i\omega)$ in Eq. (9), the version given above is motivated by the estimate of the quasielastic weight obtained in a $T=0$ neutron-scattering experiment with limited resolution ω^* and k^*.

If we assume furthermore that Δ is proportional to $\langle\Phi\rangle^{\rm eff}_{\xi^*,\tau^*}$ as suggested by the mean-field analysis (which should be valid above the upper critical dimension), we obtain the inequality $\Delta^2 \lesssim \mathrm{const}\cdot\Delta^{(d+z_{\rm op}-4)(z_F/z_{\rm op})+2}$. This implies that, at least in a weak-coupling situation, pseudogaps should appear only if

$$d+z_{\rm op} \leqslant 4 \tag{11}$$

which is the central result of this paper. We believe, that it is accidental that Eq. (11) coincides with the condition for the relevance of the Φ^4 interaction (6) in the Hertz model as is evident from the fact that z_F enters the inequality (10). Within the approach of Hertz, $z_{\rm op}=2$ and the critical dimension for pseudogap formation is therefore $d_c=2$. From our scaling arguments we cannot say much about what will happen in $d=d_c=2$ (or for $d<d_c$). Based on the observation, that the ordered phase is not well described by Eq. (5), we suspect that the Hertz description of a quantum critical antiferromagnet is *not* valid in $d=2$—this point of view agrees with the results of Abanov and Chubukov[14] who have analyzed the spin-fermion problem in $d=2$ in a certain large N expansion. In the pseudogap phase we expect by comparison to the ordered phase that $1 \leqslant z_{\rm op} < 2$. Therefore it seems to be possible that the critical dimension is not two but somewhere between 2 and 3 (Abanov and Chubukov claim,[14] however, that $z_{\rm op}$ is larger than 2 in $d=2$ depending on the number of hot spots). In three dimensions, pseudogap formation will probably not invalidate the Hertz approach, at least for weak coupling.

The derivation of Eq. (11) is far from being rigorous and based on a number of assumptions. In the following two of them, which are probably the most important ones, are discussed in more detail. First we consider non-Fermi-liquid effects due to the scattering from singular spin fluctuations, the second aspect concerns strong-coupling effect and the respective role of amplitude and angular fluctuations of the staggered magnetization.

The scattering from spin fluctuations strongly modifies the quasiparticles close to the hot lines. In leading-order perturbation theory, the self-energy of those electrons at $T=0$ is given by

$$\mathrm{Im}\,\Sigma_{\mathbf{k}}(\Omega)\approx g_S^2\sum_{\mathbf{k}'}\int_0^{\Omega}d\omega\,\mathrm{Im}\,\chi_{\mathbf{k}-\mathbf{k}'}(\omega)\mathrm{Im}\,g^0_{\mathbf{k}'}(\omega-\Omega), \tag{12}$$

where g_S is the vertex of the coupling of electrons to spin fluctuations (here, we assume the absence of pseudogap formation and therefore g_S is finite) and $g^0_{\mathbf{k}'}(\omega)\approx 1/(\omega-\epsilon_{\mathbf{k}}+i0^+)$ is the Green's function of the (free) fermions. Using Eqs. (8) and (12) we obtain at the QCP

$$\mathrm{Im}\,\Sigma_{\mathbf{k}_h+\delta\mathbf{k}}(\Omega)\sim\Omega^{1+(d-3)/z_{\mathrm{op}}}\,f\left(\frac{(\delta\kappa)^2}{\Omega^{2/z_{\mathrm{op}}}}\right), \tag{13}$$

where $\delta\kappa\sim\delta\mathbf{k}\cdot\mathbf{v}_{\mathbf{k}_h+\mathbf{Q}}$ is a measure for the distance from the hot line and f is some scaling function with $f(x\to 0)\sim$const and $f(x\to\infty)\sim 1/x^{(5-d)/2}$. For $z_{\mathrm{op}}=2$ and far away from the hot lines, Fermi-liquid behavior is recovered. Our previous arguments suggest that typical frequencies and momenta for the pseudogap formation are $\delta\kappa\sim\Delta$ and $\Omega\sim\Delta^{z_F}$, therefore the typical argument $\Delta^{2(1-z_F/z_{\mathrm{op}})}$ of f is small and the momentum dependence of $\mathrm{Im}\Sigma$ can be neglected for $z_F<z_{\mathrm{op}}$ and will not induce new effects for $z_F=z_{\mathrm{op}}$ (this is the reason that we used $k^*\sim\Delta$ in our scaling analysis). From this we obtain $\Sigma(\Omega_{\mathrm{typical}})\sim\Omega^{1+(d-3)/z_{\mathrm{op}}}$. Below three dimensions, the quasiparticle picture breaks down close to the hot lines and therefore some of our perturbative arguments might fail.[14] Ignoring this possibility, we conclude that typical energies $E_{\mathbf{k}}$ of the (incoherent) Fermionic excitations are determined from $E_{\mathbf{k}}+cE_{\mathbf{k}}^{1+(d-3)/z_{\mathrm{op}}}\sim v_F(k-k_F)$ (because we can neglect the $\mathbf{k}$ dependence of Σ) and therefore

$$z_F=\max\left[1,\frac{z_{\mathrm{op}}}{d+z_{\mathrm{op}}-3}\right] \tag{14}$$

which is the value which should be used in our previous arguments for $d+z_{\mathrm{op}}\geqslant 4$, i.e., in the absence of pseudogap formation. An effect which we have not taken into account in our discussion is that generically, close to the antiferromagnetic QCP, a superconducting phase is stabilized,[19] however, at least in $d\geqslant 3$ the ordering temperature of the superconductor T_c is usually much smaller than the typical scale T^* below which the quantum critical behavior of the antiferromagnet dominates. In $d=2$ the situation might be different[19,20] with $T_c\sim cT^*$, where c is a constant of order 1.

It is important to emphasize that our estimate (9) of $\langle\Phi\rangle^{\mathrm{eff}}$ and therefore our main result (11) is based on the assumption that *amplitude* fluctuations of the staggered order parameter are present and can be described by Eq. (8). Electrons adjust their wave functions much better to angular fluctuations of the direction of the staggered magnetization than to fluctuations of its size, because a rotation of the spin-quantization axis does not cost any energy in the long-wavelength limit (assuming weak spin-orbit coupling and/or a sufficient high symmetry of the underlying crystal). This adiabatic adjustment is not included in our estimates. Numerical results of Bartosch and Kopietz,[21] and Millis and Monien[22] show that in $d=1$ amplitude and phase fluctuations have a drastically different effect on pseudogap formation. Nevertheless, our approach to focus on amplitude fluctuations in our previous discussion was valid as within the theory of Hertz [Eq. (5)], the interactions of spin fluctuations are irrelevant and amplitude fluctuations exist for $d>2$. If they are present they should be the dominating mechanism to destroy pseudogap behavior. Below the upper critical dimensions, one expects that amplitude fluctuations are frozen out and only angular fluctuations dominate the critical regime. Even in dimensions larger than 2 such a picture might be appropriate in a strong-coupling regime, e.g., if one considers a Heisenberg model with a large antiferromagnetic coupling J_{AF} coupled to a metal. Unfortunately, the behavior of electrons in such a situation is much less understood. To investigate the pseudogap phase in this case, probably the most obvious theoretical route[24,7] to describe the adiabatic adjustment of the wave function of the electrons is to rotate the quantization axis of the electrons into the *local* direction of the slowly fluctuating order parameter. This approach has been used by a number of authors interested in the pseudogap phase of the cuprates.[9–11] A natural model to discuss this type of physics consists of a nonlinear σ model coupled to the spin $\mathbf{S}(\mathbf{r})=\frac{1}{2}f^\dagger_\alpha(\mathbf{r})\boldsymbol{\sigma}_{\alpha,\beta}f_\beta(\mathbf{r})$ of Fermions *f*. The nonlinear σ model S_σ describes the directional fluctuations of the staggered order parameter $\mathbf{n}$ in the absence of amplitude fluctuations. The action in terms of $\mathbf{n}$ with $\mathbf{n}^2=1$ and the Grassmann fields *f* is given by[23]

$$S=S_f+S_\sigma+S_{f\sigma}, \tag{15}$$

$$S_f=\int_0^\beta d\tau\sum_{\sigma,\mathbf{k}}f^*_{\sigma,\mathbf{k}}(\partial_\tau+\epsilon_{\mathbf{k}})f_{\sigma,\mathbf{k}},$$

$$S_\sigma=\frac{1}{g}\int_0^\beta d\tau\int d^d\mathbf{r}(\partial_\tau\mathbf{n})^2+(v\,\partial_{\mathbf{r}}\mathbf{n})^2,$$

$$S_{f\sigma}=\Delta\int_0^\beta d\tau\int d^d\mathbf{r}\cos(\mathbf{Qr})\mathbf{n}(\mathbf{r},\tau)\mathbf{S}(\mathbf{r},\tau).$$

We have not written down the proper spin Berry phase which is essential to describe the Kondo lattice correctly. For simplicity, we focus in the following on a model with an O(2) symmetry $\mathbf{n}=[0,\sin\phi(\mathbf{r},t),\cos\phi(\mathbf{r},t)]$ and comment below on the more difficult situation with O(3) symmetry. To describe the pseudogap, we define new fields c with a quantization axis rotated in the local direction of the order parameter,[24,7]

$$\begin{pmatrix}c_\uparrow(\mathbf{r},\tau)\\ c_\downarrow(\mathbf{r},\tau)\end{pmatrix}=\exp\left[i\Phi(\mathbf{r},\tau)\frac{\sigma^x}{2}\right]\begin{pmatrix}f_\uparrow(\mathbf{r},\tau)\\ f_\downarrow(\mathbf{r},\tau)\end{pmatrix}. \tag{16}$$

The new fields c, which we call "pseudofermions" in the following, do not transform under a global rotation around the x axis, this implies a separation of spin and charge degrees of freedom[10,11] if the low-energy excitations are well described by c (see below). The advantage of the transformation is that $S_{f\sigma}$ now describes the scattering of the pseudofermions from a *static* order parameter pointing always in the z direction which can be treated nonperturbatively. The pseudofermions are the natural degrees of freedom in a situation, where the single-particle wave function adjusts to the (collective) magnetic background. If one neglects the residual interactions with $\mathbf{n}$, gaps open along the hot lines and the action of the pseudofermions is given by

$$S_c=\int_0^\beta\left(\sum_{\sigma,\mathbf{k}} c^*_{\sigma\mathbf{k}}\partial_\tau c_{\sigma\mathbf{k}}+H_\Delta(c^*,c)\right), \tag{17}$$

where $H_\Delta(c^\dagger,c)$ is the mean-field Hamiltonian (1).

The residual interaction of $\mathbf{n}$ and c arises from the Berry phase $f^*\partial_\tau f$ and kinetic energy of the electrons. The semiclassical contributions $S^{\mathrm{sc}}_{c\sigma}$ is given by the minimal substitution which corresponds to the gauge transformation (16). Using $(\partial_\mu\phi)(\sigma^x)/2)=[(\partial_\mu\mathbf{n})\times\mathbf{n}]\cdot\boldsymbol{\sigma}/2$, we obtain

$$S^{\mathrm{sc}}_{c\sigma}=-i\int_0^\beta d\tau\int_{-\infty}^{\infty}d^d\mathbf{r}\{[\partial_\tau+(\mathbf{v}_F\boldsymbol{\nabla})\mathbf{n}]\times\mathbf{n}\}\cdot\mathbf{S}. \tag{18}$$

In the notation used here, the Fermi velocity $\mathbf{v}_F$ is actually a function of $\mathbf{k}=-i\nabla$ which acts on $c_\sigma(\mathbf{r})$ hidden in $\mathbf{S}(\mathbf{r})$. The action $S^{\mathrm{sc}}_{c\sigma}$ describes an interaction of spin currents.

As the vertex in Eq. (18) vanishes in the limit $\omega,\mathbf{k}\to 0$, Schrieffer[7] has argued that the effect of $S^{\mathrm{sc}}_{c\sigma}$ is small close to the QCP and that therefore pseudogaps and the associated decoupling of spin fluctuations from the fermions are a generic property of an antiferromagnetic QCP. This argument is, however, misleading. One reason is that amplitude fluctuations will destroy the pseudogap in many relevant situations as discussed above. But even in the absence of amplitude fluctuations, the pseudofermions interact strongly with the magnetic fluctuations by a pure quantum effect which is not included in the semiclassical $S^{\mathrm{sc}}_{c\sigma}$. Formally, the origin of the effect is that the rotation of a Fermion by 2π changes its sign! If $\phi(\mathbf{r},\tau)$ in Eq. (16) jumps from 2π to 0, the pseudofermion c abruptly flips its sign, giving rise to a huge contribution to the effective action. There are many possibilities[11,10] to keep track of these sign changes in a path integral, one of them is to rewrite the problem as a local Z_2-gauge theory where the arbitrary sign ± 1 is the origin of the Z_2-symmetry. Here we follow a slightly different route by replacing ϕ in Eq. (16) by $\tilde{\phi}$ with

$$\tilde{\phi}(\tau,\mathbf{r})=\phi(0,0)+\int_{(0,0)}^{(\tau,\mathbf{r})}(\partial_\mu\phi)dr^\mu=\phi(\tau,\mathbf{r})+2\pi n. \tag{19}$$

The line integral is along some path in space-time, e.g., $r^\mu(u)=(u\tau,u\mathbf{r})$, where u varies in the interval $[0,1]$ and $\mu=0,1,\ldots,d$ denotes the temporal and spatial directions.[25] The integer n in Eq. (19) is defined in such a way that $\tilde{\phi}$ is continuous along the path $r^\mu(u)$ and therefore the pseudofermions, defined by replacing ϕ by $\tilde{\phi}$ in Eq. (16), will vary smoothly without sudden sign changes along $r^\mu(u)$. But along some other paths, abrupt sign changes are unavoidable. This is obvious by considering the line integral $\int_{(\tau,\mathbf{r})}^{(\tau,\mathbf{r})}(\partial_\mu\phi)dr^\mu=2\pi n$ along some *closed* path in space-time. By definition it has to be a multiple of 2π and n is obviously the number of magnetic vortices of the xy model enclosed in the loop. From this we conclude that the pseudofermions acquire a phase π, i.e., a minus sign, whenever they circle around a magnetic vertex: this is nothing but the well-known Berry phase of a spin forced to move on a circle. A possible interpretation of this result is, that each xy vortex has attached to its core a magnetic flux with half a flux quantum. The pseudoelectrons are strongly interacting with the fluctuating magnetic vortices of the antiferromagnet and it is not obvious whether the gap will survive. A likely possibility is that the interactions are so strong that they are leading to confinement at least in some parameter regime as it has been suggested in the context of the Z_2-gauge theory of fluctuating superconductors.[10] One possible way of confinement is the binding of the pseudofermions to the magnetic excitations in such a way that the resulting degree of freedom is nothing but the original electron $f_{\sigma\mathbf{k}}$. In this case, we do not expect any pseudogaps. As we are not aware of methods which can describe such a confinement transition, it is difficult to give an estimate under which conditions a pseudogaps will occur in the model (5). We can only speculate that the formation of pseudogaps might be controlled by the area density of vortices, i.e., the number of vortices per area n_A piercing through a given area in space time at the QCP, to be compared to $(\Delta/v_F)^2$. Both n_A and Δ are noncritical at the transition. If these are the relevant parameters, then pseudogap behavior is expected only if the density of vortices at the QCP is small.

If the magnet has O(3) instead of xy symmetry, one can follow the same steps which have been discussed before and one faces again the problem that statistical interactions are induced as soon as pseudofermions are introduced. Kübert and Muramatsu[9] have proposed in the context of a theory of a slightly doped t-J model a convenient way to keep track of this statistical interaction with the help of a CP^1 representation of $\mathbf{n}$ using two complex fields z_1 and z_2 with $|z_1|^2+|z_2|^2=1$ and $\mathbf{n}=z^*_\alpha\boldsymbol{\sigma}_{\alpha\beta}z_\beta$. In this language the pseudofermions interact strongly with the CP^1 fields via a local U(1) gauge theory.[9] Again, confinement seems possible.

In this paper, we have investigated the possibility of pseudogap behavior close to the QCP of a nearly antiferromagnetic metal. Based on heuristic scaling arguments we suggest that generically, amplitude fluctuations destroy pseudogaps in dimensions $d>2$. In three dimensions we expect that the Hertz theory is valid at least for not too strong coupling while in $d=2$ it is probably modified due to pseudogap formation and the strong interaction of spin fluctuations and Fermionic modes. These questions should be studied in a renormalization-group treatment of both Fermionic and bosonic modes. We were not able to derive any criteria for pseudogap formation in a situation where amplitude fluctuations are completely frozen out and emphasized that the motion of the Fermions on top of the spin background leads to strong statistical interactions of the Fermionic modes with the excitations of the magnet.

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