

## Decoherence in circuits of small Josephson junctions

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We discuss dephasing by the dissipative electromagnetic environment and by measurement in circuits consisting of small Josephson junctions. We present quantitative estimates and determine in which case the circuit might qualify as a quantum bit. Specifically, we analyze a three junction Cooper pair pump and discuss a measurement to determine the decoherence time  $\tau_\varphi$ .

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The rate at which phase coherence is lost is an important factor in deciding whether a system qualifies as a quantum bit. Until now there exist very few experimental tests<sup>1</sup> or theoretical arguments<sup>2,3</sup> to determine this time in proposed quantum bits based on small Josephson junctions (squbits), and moreover only a lower bound of  $\sim 5$  ns has been determined so far. In this Brief Report we estimate quantitatively the dephasing time  $\tau_\varphi$  caused by the electromagnetic environment in single- and multijunction Josephson circuits. This dephasing is closely related to the decoherence in a squbit, thus yielding a direct measure of the decoherence time. Based on our analysis we determine certain limiting factors for the realization of a successful squbit experiment, and as a concrete example we investigate the three junction Cooper pair pump in which the coherent nature of the charge transport induces deviations from the accurate quantized transfer.<sup>4</sup> We demonstrate that the crossover from incoherent to coherent charge transport yields a direct measure of  $\tau_\varphi$ . We also discuss the limitations in implementing the so-called quasiparticle traps often used to suppress single electron effects in Cooper pair transistors and Cooper pair boxes.<sup>5</sup>

We start by relating the dephasing rate of the Josephson junction circuit under consideration to the impedance of the electromagnetic environment that it is imbedded in. By dephasing we mean the deviation of the Josephson phase  $\varphi(t)$  across the junction circuit from its initial value  $\varphi(0)$ . Specifically we are interested in the rms value of the phase deviation  $\sqrt{\langle(\Delta\varphi)^2\rangle} \equiv \sqrt{\langle[\varphi(t) - \varphi(0)]^2\rangle}$ . Voltage fluctuations induced by dissipative circuit elements result in the phase-phase correlation function  $J(t) = \langle[\varphi(t) - \varphi(0)]\varphi(0)\rangle$ ,<sup>6,7</sup> which, based on the fluctuation-dissipation theorem, can be expressed in the form

$$J(t) = 2 \int_0^\infty \frac{d\omega}{\omega} \frac{\text{Re } Z_t(\omega)}{R_K} \times \left\{ \coth\left(\frac{\hbar\omega}{2k_B T}\right) [\cos(\omega t) - 1] - i \sin(\omega t) \right\}, \quad (1)$$

where  $Z_t(\omega)$  is the impedance seen by the circuit/junction whose phase fluctuations we want to determine, and  $R_K = h/e^2 \approx 25.8 \text{ k}\Omega$  is the resistance quantum. It is straightforward to see that  $\langle(\Delta\varphi)^2\rangle$  and  $J(t)$  are related by

$$\langle(\Delta\varphi)^2\rangle = -2 \text{Re } J(t). \quad (2)$$

It can be shown that in the presence of an environment the coherence, i.e., off-diagonal elements in the density matrix of the squbit decays as  $\exp[-2 \text{Re } J(t)]$ .<sup>3,8,9</sup> Thus the decoherence and dephasing are directly related. Other possible sources of decoherence<sup>2</sup> besides the dissipative environment will not be discussed here.

First we consider the case of an array of  $N$  superconducting tunnel junctions. In the following analysis the array is assumed to be homogeneous,  $C_1 = C_2 = \dots = C_N \equiv C$ , and the electromagnetic environment to be purely resistive  $Z(\omega) = R_e$  as shown in Fig. 1(a). [Figure 1(b) refers to the three junction Cooper pair pump to be discussed later.] These simplifications make results more transparent, but it is straightforward to generalize the present method also to other circuits with an arbitrary environment,  $Z(\omega)$ . With the assumptions mentioned we obtain for the total resistance seen by the array<sup>6,7</sup>  $Z_t(\omega) = R_e / (1 + i\omega\tau)$ , where  $\tau = R_e C / N$ . The real part of this can be written in form

$$\text{Re } Z_t(\omega) = \frac{R_e}{1 + \omega^2 \tau^2}. \quad (3)$$

Inserting this into Eq. (1) and using the result for  $J(t)$  when  $\text{Re } Z_t(\omega)$  assumes the Lorentzian form of Eq. (3),<sup>10,11</sup> we

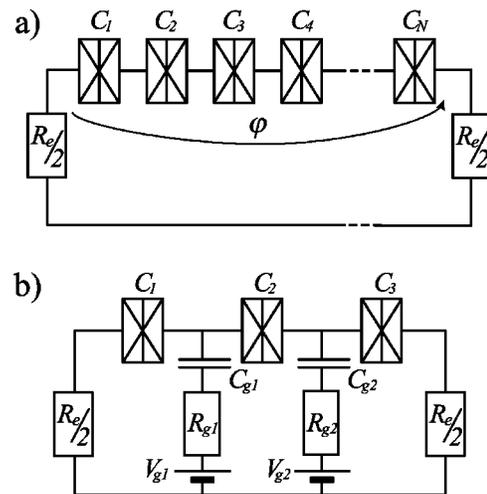


FIG. 1. Schematic views of two circuits studied quantitatively. (a) An  $N$ -junction array with purely resistive electromagnetic environment. (b) A three junction Cooper pair pump with a purely resistive environment in biasing and in gate lines.

obtain an expression for  $\langle(\Delta\varphi)^2\rangle$ . In the limit of zero temperature ( $T\rightarrow 0$ ) we immediately get for  $t\gg\tau$

$$\langle(\Delta\varphi)^2\rangle = 4\frac{R_e}{R_K}[\ln(t/\tau) + \gamma], \quad (4)$$

where  $\gamma\approx 0.57721$  is Euler's constant. In the case of nonzero temperature we consider only the long time ( $\pi k_B T t/\hbar\gg 1$ ) limit which is relevant in most cases, except in the limit of large  $R_e$ . At a realistic measurement temperature, e.g.,  $T=50$  mK, the result is valid in the range  $t\gg 50$  ps, which is the region we are interested in. The long time expansion yields

$$\langle(\Delta\varphi)^2\rangle \approx 4\frac{R_e}{R_K}\left[\frac{\pi k_B T}{\hbar}t - \ln\left(\frac{2\pi k_B T\tau}{\hbar}\right) + \gamma\right], \quad (5)$$

which is valid only at nonzero temperatures and therefore Eq. (4) cannot be recovered from Eq. (5) in the limit of  $T\rightarrow 0$ .

We can also apply the same method to an individual (the  $i$ th) junction to find the phase fluctuations  $\langle(\Delta\varphi_i)^2\rangle$  across it. With the same assumptions as before, we get for the single junction  $\text{Re} Z_{t,i}(\omega) = \text{Re} Z_t(\omega)/N^2$ .<sup>6,7</sup> This immediately yields the relation  $\sum_i \sqrt{\langle(\Delta\varphi_i)^2\rangle} = \sqrt{\langle(\Delta\varphi)^2\rangle}$ , which can be shown to hold also with an arbitrary electromagnetic environment  $Z(\omega)$  in series with the array.

If we define the dephasing time  $\tau_\varphi$  as the value of  $t$  for which  $\langle(\Delta\varphi)^2\rangle = (\pi/2)^2$ , we obtain for zero temperature

$$\tau_\varphi = \tau \exp\left(\frac{\pi^2 R_K}{16 R_e} - \gamma\right) \quad (T=0). \quad (6)$$

Dropping out the small constant terms in Eq. (5) we can write the result at nonzero temperature in the form

$$\tau_\varphi \approx \frac{\pi}{16} \frac{\hbar}{k_B T} \frac{R_K}{R_e} \quad (T>0). \quad (7)$$

For an array with  $N=3$  and  $C=10^{-15}$  F, and for the environment of  $R_e=1$  k $\Omega$  we obtain  $\tau_\varphi\approx 1.5$   $\mu\text{s}$  at zero temperature. A resistance of the environment of the order of the free space impedance,  $R_e=Z_0\approx 377$   $\Omega$ , yields  $\tau_\varphi\approx 1.5\times 10^5$  s. With the same parameters at  $T=50$  mK the decoherence is very fast:  $\tau_\varphi\approx 0.77$  ns and  $\tau_\varphi\approx 2.1$  ns for  $R_e=1$  k $\Omega$  and  $R_e=Z_0$ , respectively. Figure 2 shows the dependence of  $\tau_\varphi$  on the resistance of the environment,  $R_e$ , for a homogeneous three junction array at several different temperatures. Also the zero temperature limit (solid line) corresponding to Eq. (6), is shown. It forms an envelope curve for the finite temperature curves calculated from Eq. (5). It is also seen that only in the limit of low environmental resistance,  $R_e\leq 1$   $\Omega$ , one can obtain long decoherence times  $\tau_\varphi > 1$   $\mu\text{s}$  at realistic measurement temperatures. Such values of  $\tau_\varphi^{-1}$  would possibly allow practical quantum logic operations and measurements to be performed by fast RF-gate lines and by using an RF-SET (radio frequency single electron transistor) as an electrometer.<sup>12</sup>

In the previous analysis we considered an array of Josephson junctions without gates connected capacitively to the is-

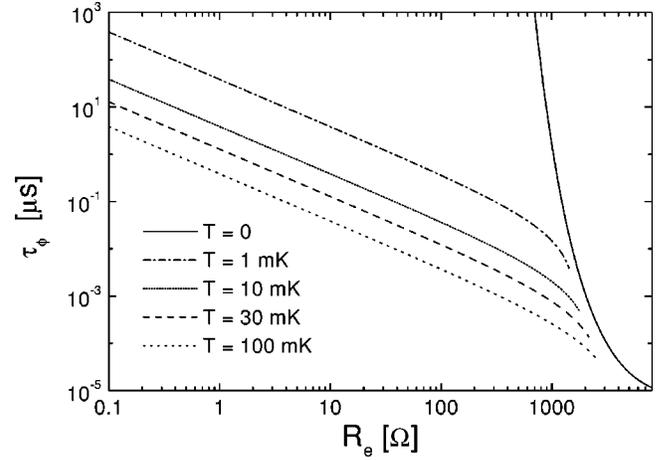


FIG. 2. The dephasing time,  $\tau_\varphi$ , for a three junction array as a function of the series resistance  $R_e$  of the electromagnetic environment. The array is assumed to be homogeneous with junction capacitances of 1.0 fF. The zero temperature curve forms a high resistance envelope of the curves corresponding to finite temperatures. Finite temperature curves are obtained from Eq. (5) and shown only over their range of validity.

lands. These capacitances together with the impedance of the gate lines,  $Z_{g_i}$ , also affect decoherence and should be taken into account. Therefore a quantitative analysis was also applied to the symmetric ( $C_i\equiv C$ ,  $C_{g_i}\equiv C_g$ , and  $Z_{g_i}\equiv R_g$  with all  $i$ ) three junction Cooper pair pump which includes gate lines connected to the islands, as shown in Fig. 1(b). The total impedance seen by the array is

$$Z_t(\omega) = \frac{R_e}{1 + i\omega\tau \frac{6i\omega C + 3g}{6i\omega C + 2g}}, \quad (8)$$

where  $g^{-1}=R_g+1/(i\omega C_g)$  is the impedance of the gate line and  $\tau=R_e C/3$ . The explicit expression for  $\tau_\varphi$  from  $\text{Re} Z_t(\omega)$  does not assume a simple form but can be calculated numerically. Figure 3 shows the influence of the gate lines to the dephasing time as a function of  $R_g$  with several different values of  $R_e$ . It is seen that the effect is positive and very small compared to realistic measurement times. In the case of disconnected gate lines ( $R_g\rightarrow\infty$ )  $\tau_\varphi$  naturally approaches the value of the array without gates, as seen from the figure. In the limit of vanishing  $R_g$  we also recover the result of an array by replacing  $C$  by the effective capacitance  $C_{\text{eff}}=C(C+C_g/2)/(C+C_g/3)$  in Eq. (3).

The influence of dissipation in the gate lines on the dephasing rate is counterintuitive at the first sight: gate lines, even resistive ones, make the dephasing time longer than in an array without gates (Fig. 3). The reason behind this is twofold. First, in our estimates we are interested in the fluctuations of the total phase difference across the array  $\varphi$ , not in those of the individual phases  $\varphi_i$ . Because of the series connection with additive phase differences, each gate line induces an exactly opposite, i.e., a cancelling fluctuation in the neighboring junctions. Thus the noise of the gate resistors does not contribute to the noise in  $\varphi$ . On the other hand, the

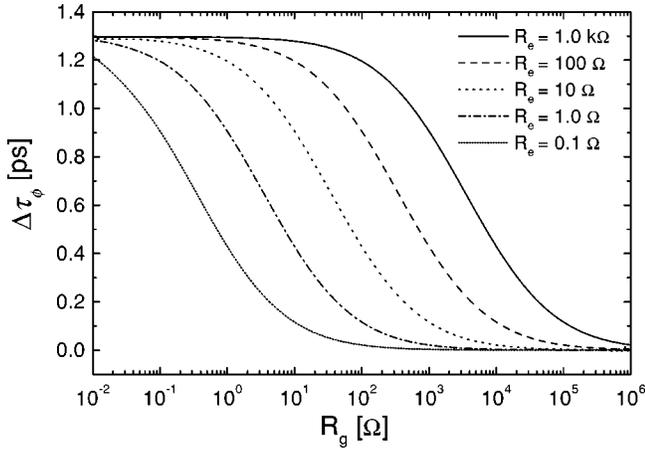


FIG. 3. The difference between the dephasing time,  $\tau_\varphi$ , in the three junction Cooper pair pump [Fig. 1(b)] and the three junction array without gate lines [Fig. 1(a)] as a function of resistance,  $R_g$ , in the gate lines. Capacitances used in the calculations are  $C_1 = C_2 = C_3 = 0.1$  fF and  $C_{g1} = C_{g2} = 0.01$  fF.  $T = 30$  mK.

gate lines decrease the impedance seen by the whole array, and this way the noise in the total phase also decreases. The longest dephasing time is therefore obtained with nonresistive gate lines. Due to the anticorrelated fluctuations in  $\varphi_i$  the sum rule for  $\sqrt{\langle(\Delta\varphi_i)^2\rangle}$ , verified earlier for an  $N$  junction array without gates, does not hold anymore.

Our estimations of the decoherence time also bring up the issue of using so-called quasiparticle traps in single Cooper pair boxes and transistors. The parity effect has been observed in several experiments.<sup>13,14</sup> It is the manifestation of pure Cooper pair effects (not necessarily coherent) in small superconducting islands. Up to now, using aluminum structures, parity effect manifested by a  $2e$ -periodic gate modulation, can be observed reliably only in Josephson junction circuits which are embedded in a resistive environment. Our estimates of  $\tau_\varphi$  now set the limit of how dissipative the quasiparticle traps can be, in order not to destroy the coherent state of the qubits too fast.

A multijunction Josephson pump provides an interesting testground for quantum coherence<sup>4</sup> and it may eventually qualify also as a metrologically accurate current standard when coherence is suppressed by a very dissipative environment.<sup>15</sup> Here we discuss the three junction pump [Fig. 1(b)] whose characteristics are determined by the two energies  $E_J$ , the Josephson coupling energy, and  $E_C$ , the charging energy. In the ideally coherent adiabatic regime, the phase across the array,  $\varphi$ , is constant and no Landau-Zener band crossing occurs,<sup>16</sup> and the optimum charge transfer through the array attains an approximate value (in the lowest order in  $E_J/E_C$ )<sup>4</sup>

$$\frac{I}{2ef} \approx 1 - 9 \frac{E_J}{E_C} \cos(\varphi). \quad (9)$$

Here  $I$  is the current induced by operating the gates and  $f$  is the frequency at which the system makes a wind around a degeneracy node of the charging energy along the closed path on the gate plane ( $V_{g1}, V_{g2}$ ), i.e., the frequency of the

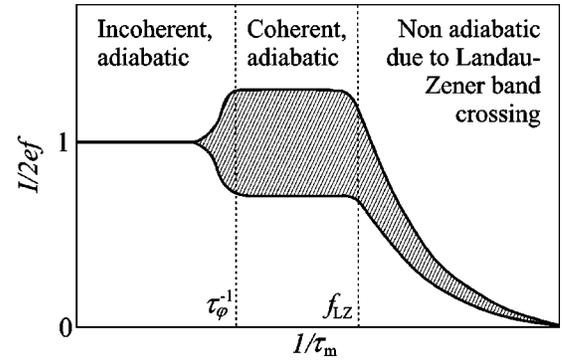


FIG. 4. A schematic presentation of the expected behavior of the pumped current in a three junction Cooper pair pump as a function of inverse of the measuring time  $1/\tau_m$ . The product of pumping frequency  $f$  and the measuring time is supposed to be fixed  $\tau_m f \gg 1$ . The shaded area covers the different values of the phase difference  $\varphi$  across the array.

harmonic gate voltages  $V_{gi}$ . (The two gate voltages are phase-shifted by  $\pi/2$ .) The pumped charge per cycle,  $Q = I/f$ , is related but not equal to the geometric phase (Berry's phase) accumulated during one cycle.<sup>4,17,18</sup> Contrary to the pump in the normal state,<sup>19</sup> the coherent adiabatic Josephson pump lacks the ability to pump single charges virtually free of errors, and the relative deviations,  $\approx -9E_J/E_C \cos(\varphi)$ , from the quantized transport are large even for very small values of  $E_J/E_C$ : for example, they can be as large as 9% for  $E_J/E_C = 0.01$ . Yet, if the gates are operated slowly enough, which means  $f \ll 1/\tau_\varphi$ , or the integration time of the measurement is too long,  $\tau_m \gg \tau_\varphi$ , the  $\cos(\varphi)$  term averages to  $\langle \cos(\varphi) \rangle = 0$  during one cycle or during the measurement and the pumping becomes accurate. However, to be able to measure a current and not to destroy the coherent superposition one has to measure the expectation value of the pumped current which means averaging over the large number of pumping cycles. This limits the measuring time,  $\tau_m$ , to be much longer than the time of one cycle,  $\tau_m \gg 1/f$ . Thus the result of Eq. (9) can only be realized if the following inequalities hold:  $1/f \ll \tau_m < \tau_\varphi$ . Another fundamental limit comes from the Landau-Zener band crossing, which sets an upper limit for the operation frequency  $f_{LZ} \sim E_J^2/(\hbar E_C)$ . For typical  $E_J = 0.1$  meV and  $E_C = 1$  meV we obtain  $f_{LZ} \approx 10$  GHz.

Based on these limitations we can summarize our expectations of the dependence of the pump performance at different frequencies (Fig. 4). Here we have chosen the product  $\tau_m f \gg 1$  to be fixed.

(1)  $\tau_m > \tau_\varphi$ .  $I/2ef \approx 1$  because pumping is adiabatic but the phase is undetermined [ $\langle \cos(\varphi) \rangle = 0$ ]. Yet at the lowest frequencies the current becomes very small to measure and the accuracy will be lost in practice.

(2)  $\tau_\varphi^{-1} < \tau_m^{-1} \ll f \leq f_{LZ}$ .  $I/2ef \approx 1 - 9E_J/E_C \cos(\varphi)$ , pumping is adiabatic and coherent. The phase  $\varphi$  will still drift at a time scale longer than  $\tau_\varphi$ , so that measured points will eventually span the whole region between maximum and minimum pumping efficiency (vertical limits of the shaded area in Fig. 4).

(3)  $f \gg \tau_m^{-1} > f_{LZ}$ .  $I/2ef$  decays because the condition for

no band crossing is lost and charge transport does not follow the gating sequence adiabatically.

Alternatively, measurement of the crossover frequency  $\tau_\varphi^{-1}$  could be done by fixing the pumping frequency in the window  $\tau_\varphi^{-1} \ll f < f_{LZ}$  and sweeping the inverse measuring time,  $\tau_m^{-1}$ , from 0 to  $f$ .

Since  $\tau_\varphi$  is presently expected to fall in the range  $\tau_\varphi \gg 5$  ns, in a carefully designed experiment, we would have  $f_C \ll 200$  MHz, yielding a clear separation of the three pumping regimes. In particular, if the decoherence time of a qubit and thus also  $\tau_\varphi$  turns out to be of the order of 1  $\mu$ s, which would allow quantum computation by Josephson qubits in this respect,  $\tau_\varphi^{-1}$  would give an experimentally convenient crossover frequency in the MHz range.

To perform an experiment along the idea sketched in Fig. 4 with distinct regimes one has to have a setup with long enough dephasing time  $\tau_\varphi \gg 10$  ns. This means that the on-chip resistances should be very low, which limits the use of quasiparticle traps. It has already been shown by experiment that with high enough resistances in the biasing circuit the pumped current becomes accurate.<sup>20</sup> Combined with the fact that the parity effect, i.e., the quasiparticle free Cooper pair effect is very difficult to observe without quasiparticle traps, one needs to seek alternative measurement schemes of the pump. One way to avoid decoherence induced by a quasipar-

ticle current is to use a closed superconducting (phase) biasing circuit on the chip with an inductance in series. This way the real part of the impedance of the series environment,  $\text{Re}Z_t(\omega)$ , vanishes. Further, gate lines do not induce any extra decoherence to the system, as shown before, and can therefore be as resistive as needed to filter the feed lines. Thus the major source of decoherence is the resistive impedance of the quasiparticle traps if needed. The pumped current can be measured by a SQUID ammeter inductively connected to the coil in the biasing circuit. This kind of a setup might give a low enough decoherence rate.

In conclusion, we have presented a method to quantitatively estimate the decoherence time due to dissipative electromagnetic environment in circuits consisting of small Josephson junctions. This method allows us, among other things, to discuss the suitability of the system in consideration as a quantum bit. We also suggest a direct measurement of  $\tau_\varphi$  as a crossover between coherent and incoherent pumping in the single Cooper pair pump.

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