Ferromagnetism's affect on the Aharonov-Bohm effect

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Aharonov-Bohm (AB) and Altshuler-Aronov-Spivak (AAS) oscillation in a ferromagnetic ring is studied theoretically. Ferromagnetism does not affect the AB effect in an essential way, except that the magnetic field becomes a sum of the external and internal field. AAS oscillation would be suppressed for a minority spin channel in most $3d$ metals because of a strong s-d scattering, as indicated by a large spin dependence of a lifetime. The majority spin channel, in contrast, is expected to survive, due to a small density of states in the d band.

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The coherence of electrons plays an essential role in mesoscopic transport at low temperature. In particular, the new and fascinating field of spin electronics, in which spin polarized electrons are injected in devices, will certainly take advantage of the possibility of electron transport with phase preservation (coherence) in ferromagnetic wires, rings, etc. In general it is believed that ferromagnetism produces strong decoherence and therefore should destroy all effects related to phase preservation of electron. There was even a belief that Aharonov-Bohm (AB) and Altshuler-Aronov-Spivak (AAS) (Ref. 1) oscillations[2] could not be observed if the ring is ferromagnetic. The AB oscillation is due to the interference of the electron wave function going through the two arms of the ring. Considering a magnetic flux of ϕ penetrating though the ring, the electron acquires as it traverses the ring a phase of $e^{\pm i\pi\phi/\phi_0}$, the sign $\pm$ corresponding to the two paths ($\phi_0 \equiv hc/e$ being the flux quantum) (see Fig. 1). The current through the ring is hence proportional to $(e^{i\pi\tilde{\phi}} + e^{-i\pi\tilde{\phi}})^2 = 2[1+\cos(2\pi\tilde{\phi})]$ ($\tilde{\phi} \equiv \phi/\phi_0$). This oscillation decays at the length scale of the elastic mean free path l. For a larger scale an additional path-dependent phase smears out the phase due to the flux. AAS oscillation is due to the interference of the two electron propagator (cooperon), which becomes dominant in disordered cases. The cooperon carries a charge of $2e$ and so the oscillation is $\cos(4\pi\tilde{\phi})$. The effect can be seen if the length of the ring, L, is shorter than the inelastic mean free path, l_φ, which is usually larger than l. These oscillations were observed in gold rings of submicron size.[3] Similar oscillation in the persistent current was observed in Cu rings.[4]

One of the recent interests would be the effect of the perturbation on these oscillations from a quantum dot on the arm.[5,6] Another interesting possibility is to study the case of ferromagnetic rings. The effect of the electronic coherence on the electronic transport in ferromagnetic metals was discussed in the context of magnetoresistance due to a magnetic domain wall in a wire.[7,8] It was shown there that the domain-wall scattering leads to another source of dephasing, and hence reduces the resistivity if the system is weakly localized. Later the resistivity modified by the Berry's phase associated with magnetization was discussed.[9,10] In spite of recent experimental efforts to look at these magnetoresistance arising from the quantum interference, the effect is still to be observed. So far there is no experimental observation of the AB or AAS oscillation in ferromagnets either.

The aim of this paper is to study theoretically the AB and AAS oscillations in a ferromagnetic ring and investigate the possibility of experimental observation. One reason for the difficulty in ferromagnets has been believed to be the existence of the magnetization. The magnetization (M) shifts the field acting on the system $H_{\rm eff} \equiv H+M$, where M corresponds to a field of a few T and hence it can eliminate the coherence.[2,11] This is true in a bulk system, but is not necessarily so in a narrow wire or ring where the magnetic flux inside the sample is less than ϕ_0 (see below).[8] Besides the magnetization, another important feature in ferromagnets is the spin-dependent elastic lifetime.[12] For instance, in Fe, the ratio of the mean free path, l_+/l_- ($+$, $-$ denote majority and minority spin, respectively) is estimated to be ~ 10. It turns out that this feature can suppress the AAS oscillation in minority-spin channel, but not necessarily in the majority channel.

We consider conduction electrons whose Hamiltonian is given by

$$H = \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k},\sigma} c^\dagger_{\mathbf{k},\sigma} c_{\mathbf{k},\sigma} + H_{\rm imp} + H_s + H_{\rm so}, \tag{1}$$

where $\sigma \equiv \pm$ denotes spin and $\epsilon_{\mathbf{k},\sigma} \equiv \hbar^2\mathbf{k}^2/(2m) - \sigma\Delta$, Δ being the band splitting due to the magnetization. Consider-

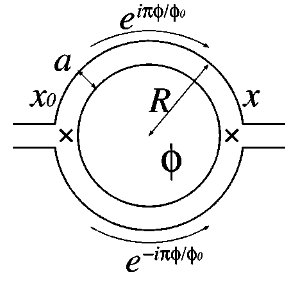

FIG. 1. Ring with a radius of R and thickness of a. Due to a magnetic flux ϕ, the electrons carry a phase of $e^{\pm i\pi\phi/\phi_0}$, depending on the path.

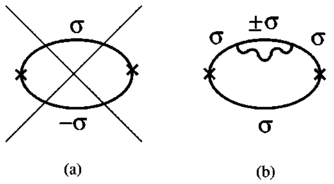

FIG. 2. (a) Direct interference of different spins is not possible, since the current vertex is spin diagonal. (b) Self-energy process due to spin-flip and spin-orbit scattering (denoted by wavy line).

ing a single-domain case in the presence of an external field, Δ is treated as uniform. Considering only s electron, Δ is much smaller than the Fermi energy and we treat the density of states and Fermi wavelength as spin independent. The scattering due to the normal impurities is written as $H_{\rm int}$. We include also a spin-flip (SF) and spin-orbit (SO) interaction due to pointlike impurities:

$$H_{\rm s}=J\sum_{\mathbf{k},\mathbf{k}'}\mathbf{S}\cdot(c^\dagger_{\mathbf{k}'}\boldsymbol{\sigma}c_{\mathbf{k}}),$$

$$H_{\rm so}=i\lambda\sum_{\mathbf{k},\mathbf{k}'}(\mathbf{k}\times\mathbf{k}')\cdot(c^\dagger_{\mathbf{k}'}\boldsymbol{\sigma}c_{\mathbf{k}}),\qquad(2)$$

where $\mathbf{S}$ denotes the spin of the impurity. The lifetime of the conduction electron, τ_σ, is written as

$$\frac{1}{\tau_\sigma}\equiv\frac{1+\gamma_\sigma}{\tau_0}+\frac{1}{\tau^{\rm s}}+\frac{1}{\tau^{\rm so}},\qquad(3)$$

where τ_0 is the elastic lifetime due to normal impurities, i.e., $2\pi nv^2N(0)\tau_0=1$, where $N(0)$ is the density of states, n and v being the concentration and the scattering strength of the impurity, respectively. $\tau^{\rm s}$ and $\tau^{\rm so}$ are lifetime due to SF and SO interaction, $1/\tau^{\rm s}=2\pi(JS)^2N(0)$ and $1/\tau^{\rm so}=(4\pi/3)\lambda^2N(0)k_F^4$, which we assume to be much weaker than impurity scattering. A phenomenological parameter γ_σ is introduced to account for the spin dependence of the lifetime.

First we study the AB oscillation. The calculation itself turns out to be the same as in the nonmagnetic case with Zeeman splitting, since the AB effect is not modified by the ferromagnetism in an essential way. Let us first see the current in the absence of SF and SO interactions. Based on a linear response theory the Boltzmann's contribution to the current though the ring is written as

$$J^0(x)=-\frac{E}{2\pi L}\left(\frac{e}{2m}\right)^2\sum_{x_0}(\partial_x-\partial_{x'})(\partial_{x_0}-\partial_{x_0'})$$
$$\times\sum_\sigma G^{\rm r}_\sigma(x-x_0')G^{\rm a}_\sigma(x_0-x')|_{x'\to x,x_0'\to x_0},\qquad(4)$$

where E is the applied electronic field. $G^{\rm r}$ ($G^{\rm a}$) is retarded (advanced) Green function with self-energy correction due to the impurity. x and x_0 represent the point in the leads. Considering a case where the width of the ring is narrow, we approximate the Green function as in one dimension. The current without the SF and SO interaction is obtained from Eq. (4). By use of $k\equiv 2\pi(n-\phi)/L$ (n is an integer) we obtain

$$\sum_k G^{\rm r}_{k\sigma}=-i\frac{mL}{k_F}[1+2e^{-L/2l_\sigma}e^{ik_FL}\cos(2\pi\tilde{\phi})],\qquad(5)$$

and hence the oscillation part of the current is given as

$$J^0_{\rm AB}=2E\sum_\sigma\sigma_{0\sigma}e^{-L/2l_\sigma}\cos(k_FL)\cos(2\pi\tilde{\phi}),\qquad(6)$$

where $\sigma_{0\sigma}\equiv(e^2/\pi)l_\sigma$ is the Boltzmann conductivity, $l_\sigma=(\hbar k_F/m)\tau_\sigma$ being the elastic mean free path. $L=2\pi R$ is the length of the ring perimeter and k_F is the Fermi wavelength. It is seen that two spin channels independently contribute to the oscillation, and ferromagnetism affects $J^0_{\rm AB}$ only through spin-dependent mean free path l_σ and $\phi\propto(H+M)$. (We consider the case where H and M are perpendicular to the ring.) Note that interference of two different spins shown in Fig. 2(a) is forbidden since the current operator is diagonal in spin.

We now include SF and SO interactions, which mix two spin channels. The lowest-order contribution is a self-energy type [Fig. 2(b)], which is obtained as

$$\delta J_{\rm AB}=\left(\frac{E}{2\pi L}\right)\left(\frac{e}{m}\right)^2\frac{1}{2\pi N(0)}\left(\frac{1}{\tau^{\rm s}}+\frac{1}{\tau^{\rm so}}\right)$$
$$\times\left[\sum_{\mathbf{k},\sigma}\mathbf{k}^2(G^{\rm r}_{\mathbf{k}\sigma})^2G^{\rm a}_{\mathbf{k}\sigma}\sum_{\mathbf{k}'}\frac{1}{3}(G^{\rm r}_{\mathbf{k}'\sigma}+2G^{\rm r}_{\mathbf{k}',-\sigma})+\text{c.c.}\right]$$
$$\simeq-\frac{E}{L}e^2\cos(2\pi\tilde{\phi})N(0)\left(\frac{1}{\tau^{\rm s}}+\frac{1}{\tau^{\rm so}}\right)\sum_\sigma l^2_\sigma(e^{-L/2l_\sigma}$$
$$+e^{-L/2l_{-\sigma}})+O(e^{-2L/l}).\qquad(7)$$

The factors of $e^{-L/2l_{\pm\sigma}}$ corresponds to the electron with spin $\pm\sigma$ traveling around the ring. The total AB current is given by $J^0_{\rm AB}+\delta J_{\rm AB}$.

Next we turn to the AAS oscillation. This oscillation is due to a singular behavior of a particle-particle propagator (cooperon) induced by successive scattering by normal impurities. We consider that the case elastic mean free path is governed by the impurity, i.e., $\gamma_\sigma\ll 1$. Similarly to nonmagnetic case,[11] the spin-conserving channel of the cooperon (Fig. 3) is calculated as

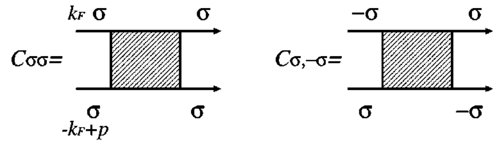

FIG. 3. Cooperon without and with spin flip. Hatched squares denote ladder of impurity scattering.

$$C_{\sigma\sigma}(p)=\frac{1+\gamma_\sigma}{2\pi N(0)\tau_0}\frac{1}{\left[l_\sigma^2p^2+\gamma_\sigma+\kappa^{\rm s}\left(\frac{1}{(1+\gamma_\sigma)^2}-\frac{1}{3}\right)+\kappa^{\rm so}\left(\frac{1}{(1+\gamma_\sigma)^2}+\frac{1}{3}\right)+\frac{1}{(1+\gamma_\sigma)^2}\frac{\tau_0}{\tau_\varphi}\right]}, \tag{8}$$

where p is the momentum of the cooperon. $\kappa^{\rm s}\equiv\tau_0/\tau^{\rm s}$ and $\kappa^{\rm so}\equiv\tau_0/\tau^{\rm so}$ are small parameters. τ_φ is the inelastic lifetime due to the orbital motion by magnetic field, phonons, electron-electron interaction, etc. We consider that the case τ_0/τ_φ is small, i.e., low temperatures and a not very large field (see below). In this case it is seen that $C_{\sigma\sigma}$ glows rapidly at $p\to 0$, which is a result of enhanced backward scattering due to interference,[11] and finite γ_σ results in a cutoff, namely, γ_σ causes dephasing. A spin-flip channel (Fig. 3) is similarly obtained as

$$C_{\sigma,-\sigma}(p)=\frac{(1+\overline{\gamma})^2}{4\pi N\tau_0}\left[\frac{1}{\overline{l}^2p^2+\overline{\gamma}+4(\Delta\overline{\tau})^2+\kappa^{\rm s}\left(\frac{1}{(1+\overline{\gamma})^2}-\frac{1}{3}\right)+\kappa^{\rm so}\left(\frac{1}{(1+\overline{\gamma})^2}+\frac{1}{3}\right)+\frac{1}{(1+\overline{\gamma})^2}\frac{\tau_0}{\tau_\varphi}}\right.$$
$$\left.-\frac{1}{\overline{l}^2p^2+\overline{\gamma}+4(\Delta\overline{\tau})^2+\kappa^{\rm s}\left(\frac{1}{(1+\overline{\gamma})^2}+1\right)+\kappa^{\rm so}\left(\frac{1}{(1+\overline{\gamma})^2}-1\right)+\frac{1}{(1+\overline{\gamma})^2}\frac{\tau_0}{\tau_\varphi}}\right], \tag{9}$$

where $\overline{\gamma}\equiv(\gamma_++\gamma_-)/2$, $\overline{l}=l_0/(1+\overline{\gamma})$, and $\overline{\tau}=\tau_0/(1+\overline{\gamma})$, $l_0\equiv\hbar k_F\tau_0/m$. In this channel Zeeman splitting Δ leads to dephasing. The AAS oscillation arises from the cooperon circulating the ring as described schematically in Fig. 4. If $l_\varphi\equiv l_0\sqrt{\tau_\varphi/\tau_0}$ is shorter than L, the largest contribution comes from circulation of once, which reads

$$J_{\rm AAS}=-\frac{E}{2\pi L}\left(\frac{e}{m}\right)^2\sum_{\mathbf{k}\sigma\sigma'}\mathbf{k}^2G^{\rm r}_{\mathbf{k}\sigma}G^{\rm a}_{\mathbf{k}\sigma}G^{\rm r}_{\mathbf{k}\sigma'}G^{\rm a}_{\mathbf{k}\sigma'}\sum_p C_{\sigma\sigma'}(p)$$
$$\times(e^{i4\pi\tilde{\phi}}e^{ipx}|_{x=L}+e^{-i4\pi\tilde{\phi}}e^{ipx}|_{x=-L}). \tag{10}$$

By use of

$$\sum_p\frac{e^{ipL}}{p^2+l_\varphi^{-2}}=\frac{L}{2}l_\varphi e^{-L/l_\varphi}, \tag{11}$$

the AAS current in the dirty case is obtained as[13]

$$J_{\rm AAS}=-\frac{Ee^2}{\pi}\cos(4\pi\tilde{\phi})\sum_\sigma\left[l^{(1)}_{\varphi\sigma}e^{-L/l^{(1)}_{\varphi\sigma}}+\frac{A}{2}(l^{(2)}_\varphi e^{-L/l^{(2)}_\varphi}\right.$$
$$\left.-l^{(3)}_\varphi e^{-L/l^{(3)}_\varphi})\right], \tag{12}$$

The coefficient A is

$$A=\frac{1}{2}\sum_\pm\left[\frac{1}{1+\gamma_\pm}\frac{4(\Delta\tau_0)^2\mp\frac{1}{2}(1+\overline{\gamma})(\gamma_+-\gamma_-)}{\left[4(\Delta\tau_0)^2\mp\frac{1}{2}(1+\overline{\gamma})(\gamma_+-\gamma_-)\right]^2+4(\Delta\tau_0)^2(1+\gamma_\pm)^2}\right]. \tag{14}$$

where the inelastic mean free paths in three channels are

$$l^{(1)}_{\varphi\sigma}\equiv l_\sigma\left[\frac{1}{(1+\gamma_\sigma)^2}\frac{\tau_0}{\tau_\varphi}+\gamma_\sigma+\kappa^{\rm s}\left(\frac{1}{(1+\gamma_\sigma)^2}-\frac{1}{3}\right)+\kappa^{\rm so}\left(\frac{1}{(1+\gamma_\sigma)^2}+\frac{1}{3}\right)\right]^{-1/2},$$

$$l^{(2)}_\varphi\equiv\overline{l}\left[\frac{1}{(1+\overline{\gamma})^2}\frac{\tau_0}{\tau_\varphi}+\overline{\gamma}+4(\Delta\overline{\tau})^2+\kappa^{\rm s}\left(\frac{1}{(1+\overline{\gamma})^2}-\frac{1}{3}\right)+\kappa^{\rm so}\left(\frac{1}{(1+\overline{\gamma})^2}+\frac{1}{3}\right)\right]^{-1/2},$$

$$l^{(3)}_\varphi\equiv\overline{l}\left[\frac{1}{(1+\overline{\gamma})^2}\frac{\tau_0}{\tau_\varphi}+\overline{\gamma}+4(\Delta\overline{\tau})^2+\kappa^{\rm s}\left(\frac{1}{(1+\overline{\gamma})^2}+1\right)+\kappa^{\rm so}\left(\frac{1}{(1+\overline{\gamma})^2}-1\right)\right]^{-1/2}. \tag{13}$$

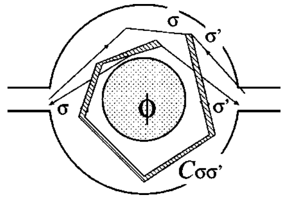

FIG. 4. Schematic diagram of AAS oscillation. The two shaded electron lines are the cooperon depicted in Fig. 3, which circles around the ring and leads to oscillation of $\cos(4\pi\phi/\phi_0)$.

Note that oscillation in Eq. (12) is $\cos(4\pi\tilde{\phi})$ since a cooperon carries a charge of $2e$. $l_\varphi^{(i)}$ $(i=1,2,3)$ determines the length scale at which AAS can appear. The main features of the ferromagnetic metal is included in γ, Δ, and τ_φ. For the transport governed by an *s* electron, Zeeman splitting is not strong enough to kill the coherence, i.e., $\Delta\tau_0 \ll 1$. In contrast, a large value of γ expected in pure $3d$ metals may be crucial for AAS. In fact, lifetime asymmetry is estimated to be $\tau_+/\tau_- \simeq O(10)$ in pure Ni, Co, and Fe.[12] This indicates a large value of γ_-, and hence $l_{\varphi-}^{(1)}, l_\varphi^{(2)}, l_\varphi^{(3)} \sim 0$ and cooperons C_{--}, C_{+-}, C_{-+} are killed. This spin asymmetry is believed to arise from *s-d* scattering, which is strong for a minority-spin *s* electron due to a large density of states of the *d* band. (Note that *s-d* scattering is an inelastic process.) This *s-d* interaction is not expected to be strong for a majority spin, since the density of states in the *d* band is not large for a majority spin, and then γ_+ (and hence $l_{\varphi+}^{(1)}$) would remain small. Then C_{++} is not deleted and AAS can arise from majority spin-channel.

The internal magnetic field is, in contrast to what is naively believed, not dangerous in small systems. Its effect is to shift the magnetic field *H*, which enters in $1/\tau_\varphi \equiv 1/\tau_\varphi^{(0)} + 1/\tau_H$, to be $H_{\rm eff} \equiv H + M$. $\tau_\varphi^{(0)}$ is the inelastic lifetime due to phonons, Coulomb interaction, etc.,[11] which we neglect here since we are interested in low temperatures only. The dephasing length due to magnetic field is given by[2]

$$l_H \equiv \sqrt{D\tau_H} = \sqrt{3}\left(\frac{\phi_0}{2\pi a H_{\rm eff}}\right), \tag{15}$$

where *a* is the width of the ring. If *L* is comparable or smaller than l_H, dephasing due to magnetic field is irrelevant. The internal field is $M \sim O(1\ \text{T})$. For this field strength, $l_H \simeq 1200$ Å if $a = 50$ Å. This would be long enough for a ring of $R = 300$ Å ($L/2 = \pi R = 1000$ Å). This estimation may be too severe considering the fact that AB and AAS was observed in a larger ring even under an external field of 8 T in a nonmagnetic case.[3]

In conclusion, we have studied the Aharonov-Bohm (AB) and Altshuler-Aronov-Spivak (AAS) oscillation in a ferromagnetic ring. The AB effect arises if the sample is smaller than the elastic mean free path, in just the same way as in the nonmagnetic case. As for AAS, ferromagnetism can affect it by causing dephasing in three different ways. First is the Zeeman splitting due to the magnetization, which is negligible in the transport of an *s* electron. Second is the dephasing caused by the internal field in addition to the external one. The effect is just to add to the external field, and so becomes irrelevant in a sufficiently small sample. The last is the spin-asymmetric lifetime arising mainly from the *s-d* interaction. *s-d* scattering is an inelastic process and so kills the coherence of the minority spin due to a large density of states in the *d* band in $3d$ metals. The majority spin in contrast is not expected to be affected by this interaction, and thus this channel would remain coherent, leading to AAS oscillation.

We have seen that ferromagnetism does not necessarily kill the coherence in the electronic transport. However, in order to decrease the effect of internal magnetic field, we need a relatively small ferromagnetic sample. Rings, usually made by conventionnal lithography techniques, are too large to give evidence for dephasing effects, such as those predicted in Ref. 7. Extrinsic reasons such as fluctuating spin at the surface might also contribute to the difficulty. The experimental evidence of coherence effects in ferromagnetic material by observing AB and AAS oscillations is undoubtly challenging. Besides ferromagnetic metals such as Fe, Co, Ni, etc., most interesting systems are ferromagnetic semiconductors, where both mean free paths for electrons with or without polarization, and Fermi wavelengths are much larger than in metals. Preliminary experiments are presently done using Mn-doped CeTe rings.

Recently, we found the calculation of cooperon in ferromagnets was recently done also in the context of weak localization.[14]

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