Inhomogeneous broadening of polaritons in high-quality microcavities and weak localization

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We theoretically study the elastic scattering of lower cavity polaritons on the static disorder in high quality semiconductor microcavities in the strong coupling regime. We consider the dominant contribution of the resonant scattering on localized exciton levels and calculate for a model density of states the corresponding elastic mean free path. Our analytical results compare well with available linewidth data and substantiate the possibility of weak localization.

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I. INTRODUCTION

Semiconductor microcavities have attracted much attention as they permit to control the interaction between light and quantum well (QW) exciton modes.[1] In the strong coupling regime, when the exciton-photon coupling is larger than the QW exciton and the cavity photon linewidths, the fundamental excitations of the system are a doublet of two-dimensional polaritons separated by the vacuum Rabi splitting. In general, such cavity polaritons are broadened both homogeneously and inhomogeneously, and it can be experimentally determined which kind of broadening prevails. The homogeneous broadening stems mainly from the cavity photon damping due to the finite transmission of the Bragg mirrors as well as from the QW exciton inelastic scattering by phonons. Both contributions may, respectively, be controlled by increasing the quality factor of the cavity and by decreasing the temperature. The inhomogeneous part of the cavity polariton broadening is mainly due to the elastic scattering of QW excitons on the static structural disorder caused by QW width and alloy fluctuations, which can be much reduced in high-quality samples[2–5] and is responsible for Rayleigh scattering.[6] While the theoretical description of the homogeneous broadening of cavity polaritons is well understood,[7,8] the microscopic mechanism responsible for the inhomogeneous broadening has been controversial.

The motional narrowing model proposed in Ref. 9 implies that a cavity polariton, being partly a cavity photon and thus having a very small mass, experiences a much reduced effective disorder potential. However, the corresponding lower polariton (LP) inhomogeneous linewidth, calculated both from a scaling argument[10] and by means of perturbation theory,[11] is found to be

$$\gamma_{\text{mot-narr}}=\frac{W^2 l_c^2 \mu}{2\hbar^2}\,\frac{\Delta^4}{(\omega_0-\omega)^2[\Delta^2+(\omega_0-\omega)^2]}, \qquad (1)$$

where W and l_c are, respectively, the variance and the correlation length of the disorder potential, μ is the cavity photon mass ($\mu\simeq 10^{-5}\, m_*$, m_* being the bare exciton mass), $\hbar\omega_0$ is the bottom of the bare exciton band, $2\hbar\Delta$ is the Rabi splitting (typically several meV), and $\hbar\omega$ is the LP energy. Using typical disorder parameters such as $W\simeq 0.5$ meV and $l_c\simeq 300$ Å,[12] one obtains $\gamma_{\text{mot-narr}}\sim 10^{-5}$ meV, while the experimentally observed typical values are $\gamma_{\text{inh}}\sim 0.1$ meV for state of the art high quality samples and $\gamma_{\text{inh}}\sim 1$ meV for good samples. Thus, the motional narrowing model alone cannot explain the inhomogeneous broadening of the cavity polaritons.

A satisfactory theoretical description of the inhomogeneous broadening of cavity polaritons due to the static disorder scattering has only been given in terms of large scale numerical simulations in one-dimension[14] and later also in the more relevant case of two-dimensions.[15] The absorption linewidths of both polaritons were numerically calculated by examining the photon Green function for normal incidence geometry for various detunings. The LP inhomogeneous linewidth $\gamma_{\text{inh}}\sim 1$ meV was explained in terms of the scattering of the LP to large wave vector bare exciton states. When the disorder strength becomes small compared to the Rabi splitting ($W<\Delta\hbar$) such scattering is suppressed and for very weak disorder ($W\ll\Delta\hbar$) the regime of motional narrowing described by Eq. (1) is reached and the inhomogeneous broadening becomes negligible. The numerical results have also been interpreted[15] in terms of a phenomenological model which simply assumes that the wave vector is conserved,[16–19] but the exciton energy is distributed with an asymmetric line shape[20,21] corresponding to the bare QW exciton density of states. The predictions of this model relating the cavity polariton linewidths to the bare QW exciton absorption line shape are in good agreement with experiments.[22,2] However, from the theoretical point of view, the wave vector conserving approximation has only been jus-

tified by the comparison with the numerical simulations of the actual disorder scattering.[15]

In high-quality samples ($W<\hbar\Delta$), the inhomogeneous broadening of cavity polaritons ($\gamma_{\rm inh}\sim 0.1$ meV) is small compared to their kinetic energy (a fraction of the Rabi splitting), which allows one to examine the problem analytically as done here. The QW disorder potential gives rise to the low-energy tail of bare exciton states which are spatially localized at defects corresponding to potential wells for the exciton center of mass motion.[12] In a recent paper,[23] we have shown that such a defect scatters strongly a LP only when the LP energy is resonant with its bare exciton bound state; otherwise, the defect is nearly ''transparent.'' We therefore consider a microscopic kinetic model (Sec. II) of the disorder induced elastic scattering which describes a LP of a given energy as a wave moving in a statistical distribution of discrete resonant defects. All the other details of the continuously varying QW disorder potential only contribute to the *nonresonant* part of the elastic scattering, which is comparably negligible and determine just the motional narrowing linewidth of Eq. (1). Based on model statistical distributions of bare exciton localized levels (Sec. III), we calculate the *resonant* mean free path of a LP. It allows us to find analytically the dominating contribution to the LP inhomogeneous broadening related to the resonant scattering of a LP by localized exciton levels. Within the limits of validity of the present approach, the calculated broadening (Sec. IV) is in agreement with what is measured in high-quality samples and its functional form provides an alternative theoretical justification of the wave vector conserving model. Furthermore, the analytical expression for the mean free path as a function of the disorder strength enables us to substantiate the possibility of weak localization in high-quality microcavities, as suggested by recent experiments[4,5] and numerical simulations.[13] A few technical details are relegated to the Appendix.

II. STATISTICAL KINETIC MODEL AND RESONANT MEAN FREE PATH

An exciton in a nonideal QW can be treated as a particle moving in an effective potential $V(\mathbf{r}_\parallel)$, which represents the microscopic structural disorder averaged by the relative electron-hole motion.[12] The effective potential is characterized by its correlation length l_c of the order of the exciton Bohr radius and its variance $W^2=\langle V(\mathbf{r}_\parallel)V(\mathbf{r}_\parallel)\rangle$ (here, we only consider the case $W<\hbar\Delta$). In a microcavity, the inhomogeneously broadened exciton couples to the cavity photon and two polariton branches appear. The polaritons move in the continuous disorder potential and interact with it through their excitonic component. In addition to the perturbative scattering,[11] the most important feature of disorder is that it introduces localized exciton states. In a previous paper,[23] we have calculated the LP elastic scattering cross section by a single model defect of radius a_0 and depth V_0 which has a bare exciton bound state at a definite energy $\hbar\omega_*$ as given by Eq. (A2) of the Appendix. We found that $\sigma(\omega)$ has a strong and narrow peak at $\omega=\omega_*$, and almost vanishes away from resonance $\sigma(\omega=\omega_*)\sim 100\,a_0$, while $\sigma\sim 10^{-6}\,a_0$ for $\omega\neq\omega_*$, the resonant contribution integrated over the peak width being a factor m_*/μ larger than the nonresonant one. This allows us to single out the resonant scattering of a LP by the localized exciton levels from the nonresonant scattering which can be taken into account perturbatively. In terms of the corresponding mean free paths, we have $l_{\rm res}\ll l_{\rm nonres}$. Thus, we can model the actual smooth and continuous disorder potential acting on the LP center of mass motion by a set of separated scattering centers associated to the negative fluctuations of the potential having a bound state close to the LP energy. The scattering process can be treated as due to a succession of independent collisions only if the corresponding mean free path $l_{\rm res}$ exceeds the size of a typical scatterer $2a_0$: $l_{\rm res}\gg 2a_0$. In addition, $l_{\rm res}$ must exceed the de Broglie wavelength in order to have propagating wave packets: $l_{\rm res}\gg\lambda$; otherwise, strong localization of the LP would occur according to the Ioffe-Regel criterion.[24]

We have then to introduce a model distribution of localized exciton states. Generally speaking, there are families of defects with different depths and extensions which have the same bound level ω_*. However, for a model cylindrical well defect, the LP cross section depends mostly on the position of the resonance, and not on the parameters of the parent well: $\sigma_{\max}(\omega=\omega_*)=4/k_L$, being $k_L=2\pi/\lambda$ the LP wave vector given by Eq. (A1) of the Appendix, and the radius and the depth of the well only weakly influence the width of the resonance. For the sake of simplicity, we therefore model the disorder potential as a set of randomly distributed cylindrical wells of equal radius a_0 and a statistical distribution of depths. It is natural to associate the size of the well with the correlation length of the disorder $a_0=l_c/2$. This approximation is appropriate if the energy of the LP is in the low-energy tail of localized exciton levels, since then the probability that different resonant negative fluctuations may cluster is negligibly small. Consequently, we restrict the LP energy to the range

$$\hbar\omega_0-\hbar\Delta<\hbar\omega<\hbar\omega_0-W. \tag{2}$$

In the following, we measure the energy down from the bottom of the excitonic band in an ideal QW, introducing the notations $\bar{\omega}=\omega_0-\omega$, $\bar{\omega}_*=\omega_0-\omega_*$. The usual formula $l^{-1}=\Sigma_i n_i\sigma_i$ for the mean free path l in a mixture of scatterers with different scattering cross sections σ_i and concentrations n_i can be written in the form

$$l^{-1}(\bar{\omega})=\int d(\hbar\bar{\omega}_*)\rho(\bar{\omega}_*)\sigma(\bar{\omega};\bar{\omega}_*), \tag{3}$$

where $\rho(\bar{\omega}_*)$ is the density of states (DOS) of a bare QW exciton and the detailed expression for the cross section $\sigma(\bar{\omega};\bar{\omega}_*)$ can be obtained from Eq. (A4) of the Appendix.

The important thing is that if the depth of a well is such that $\bar{\omega}_*$ is far from $\bar{\omega}$, then

$$\sigma(\bar{\omega};\bar{\omega}_*)\sim\frac{1}{k_L}\left(\frac{\mu}{m_*}\right)^2. \tag{4}$$

Such nonresonant scattering would contribute as $(\mu/m_*)^2$ to l^{-1}. Even though the approximation of separated defects is unjustified for the nonresonant scattering, it is natural to associate such a contribution to the motional narrowing broadening of Eq. (1). As a matter of fact, we can estimate the nonresonant mean free path as $l_{\text{nonres}} \simeq \hbar v_g / \gamma_{\text{mot-narr}}$, where the group velocity of the LP is given by

$$v_g = \frac{\hbar k_L}{\mu} \frac{\bar{\omega}^2}{\Delta^2 + \bar{\omega}^2}, \tag{5}$$

from which we have

$$l_{\text{nonres}} \simeq 2k_L(\bar{\omega}) \left(\frac{m_*}{\mu}\right)^2 \left(\frac{\hbar^2}{m_* l_c W}\right)^2 \frac{\bar{\omega}^4}{\Delta^4}, \tag{6}$$

exactly of the order of $(m_*/\mu)^2$. If, on the contrary, $\bar{\omega}_* \sim \bar{\omega}$, we have for the cross section

$$\sigma(\bar{\omega}_* \approx \bar{\omega}) = \frac{4\pi\varepsilon}{k_L} \left\{ \frac{\varepsilon/\pi}{\varepsilon^2 + [\bar{\omega}_* - \bar{\omega} + \alpha\varepsilon]^2} \right\}, \tag{7}$$

where $\alpha \sim 1$ and $\varepsilon \sim (\mu/m_*)\Delta$, as given by Eq. (A11) of the Appendix. We then expand the expression in the brackets in Eq. (7) in powers of the small parameter ε. The leading term is $\delta(\bar{\omega} - \bar{\omega}_*)$. The next term is linear in ε, contains no singularities, contributes as $(\mu/m_*)^2$ to l^{-1}, and again is to be associated with the very small nonresonant scattering. The first term corresponding to resonant scattering gives a contribution to l^{-1} about $m_*/\mu \sim 10^5$ times larger than the nonresonant one, confirming the assumption that the latter may be treated as a perturbative correction. We stress that the width of the resonant cross section is very narrow; precisely this fact allows one to separate out such a dominant contribution to the elastic scattering from the negligible motional narrowing background and, as shown below, prove analytically the validity of the wave vector conserving ''absorption model.''[15] Then, the resonant mean free path reduces to

$$l_{\text{res}}(\bar{\omega}) = \frac{k_L(\bar{\omega})}{(2\pi)^2} \frac{m_*}{\mu} \frac{\hbar\bar{\omega}}{(\hbar\Delta)^2} \frac{V_*(\bar{\omega}) - \hbar\bar{\omega}}{V_*(\bar{\omega})} \frac{1}{\rho(\bar{\omega})}, \tag{8}$$

where we denote by V_* the depth of the well which sustains a bare exciton localized level of energy $\hbar\bar{\omega}_* = \hbar\bar{\omega}$. In order to estimate V_*, we note that the energy $V_* - \hbar\bar{\omega}$ is of the order of the kinetic energy of an exciton confined in a well of width a_0, so that $V_* - \hbar\bar{\omega} \sim \hbar^2/(m_* a_0^2)$. Finally, Eq. (8) takes the form

$$l_{\text{res}}(\bar{\omega}) \simeq \frac{k_L(\bar{\omega})}{(2\pi)^2} \frac{\hbar\bar{\omega}}{(\hbar\Delta)^2 \rho(\bar{\omega})} \frac{(\hbar/\mu a_0^2)}{\bar{\omega} + (\hbar/m_* a_0^2)}. \tag{9}$$

Equation (9) determines the elastic mean free path, dominated by the resonant scattering on localized exciton states, as a function of the statistical characteristics of the disorder potential, such as its variance and correlation length and, most importantly, its bare exciton DOS. In the following section, we discuss the relevant choice of a model bare exciton DOS $\rho(\bar{\omega})$. Then in Sec. IV, we use Eq. (9) to calculate the inhomogeneous broadening of LP induced by the resonant scattering and draw our conclusions.

III. BARE EXCITON DENSITY OF STATES

To make use of Eq. (9) a model for the bare exciton DOS is to be specified. Unfortunately, no analytical computation is possible in the general case, as discussed in the following. We recall[27] that there exist two asymptotics for the DOS $\rho(E)$ of a particle of mass m (in our case, $m = m_*$) in the deep tail of a disorder potential. These asymptotics were found by the method of optimal fluctuations, which consists in the minimization of the functional $\Sigma[V]$, being $\exp(-\Sigma[V])$ the functional of the probability density of the realization V. The value of $\rho(E)$ is mainly determined by the ''optimal fluctuation,'' i.e., the one having a ground state of energy E and minimizing $\Sigma[V]$. In order to solve such a constrained minimization problem, one proceeds for each given energy E through the self-consistent evaluation of the wave function of the ground state $\psi(\mathbf{r})$ in the optimal fluctuation; for Gaussian statistics the wave function ψ is found to obey the following integrodifferential equation:[27,28]

$$-\frac{\hbar^2}{2m}\Delta\psi(\mathbf{r}) - \beta\psi(\mathbf{r}) \int d\mathbf{r}'\, \psi^2(\mathbf{r}') \langle V(\mathbf{r}) V(\mathbf{r}') \rangle = E\psi(\mathbf{r}), \tag{10}$$

where E is known and β is a Lagrange multiplier to be found together with ψ, from which the asymptotic for $\rho(E)$ is eventually obtained. The four relevant length scales in this problem are (i) the correlation length of the potential l_c, (ii) the spatial size $r_\psi(E)$ of the ground state wave function ψ of energy E, (iii) the scale related to the energy of the particle $r_E = \sqrt{(\hbar^2/2m|E|)}$, (iv) the scale related to the strength of the disorder $r_W = \sqrt{(\hbar^2/2mW)}$. In the deep tail $(-E \gg W)$, $\rho(E)$ is expected to be exponentially small; to find the form of such asymptotic behavior is possible in the two limiting cases of disorder potential of either small or large correlation length compared to r_W. To begin with, depending on the relation between r_ψ and l_c, two regimes can be distinguished: (i) the classical limit, which corresponds to $r_\psi \ll l_c$ and (ii) the quantum limit, which corresponds to $r_\psi \gg l_c$ (the latter is the case of white noise potential, when the potential correlation function may be reduced to a δ function). The limitations of validity of these two regimes follow, first, from the relation between l_c and $r_\psi(E)$, and, second, from the requirement to be in the tail of the DOS.

In the classical limit $r_\psi^2 = 2r_E l_c$ in two dimensions,[27] and the asymptotical form of the DOS is

$$\rho(E) = \frac{2}{(2\pi)^{3/2}} \frac{E^2}{W^3 l_c^2} \exp(-E^2/2W^2), \tag{11}$$

where the prefactor of the exponent for two dimensions can be calculated following the method of Ref. 28 and agrees with the appropriate limit of the recent results of Ref. 29. The restrictions imposed on the energy of the particle are in this case

$$|E| \gg \max\left\{ W; \quad \frac{\hbar^2}{m l_c^2} \right\}. \tag{12}$$

The quantum limit for two dimensions is considered in Ref. 30. Though Eq. (10) greatly simplifies for the case of white-noise potential, it can be solved only numerically. The resulting expression for $\rho(E)$ is

$$\rho(E) \propto |E|^{3/2} \exp(11.8\hbar^2 E/W^2 l_c^2 m), \tag{13}$$

and the restrictions imposed on the energy are then

$$\frac{m W^2 l_c^2}{10\hbar^2} \ll |E| \ll \frac{\hbar^2}{m l_c^2}. \tag{14}$$

On the basis of Eqs. (12) and (14), one can make the following classification for the disorder potentials.[27] The first class represents the potentials with small correlation length ($l_c \ll r_W$). For these potentials the classical asymptotic of Eq. (11) is valid for $|E|$ large enough so that $(2m|E|l_c^2/\hbar^2) \gg 1$. For smaller $|E|$, such that $(l_c/r_W)^4/10 \ll (2m|E|l_c^2/\hbar^2) \ll 1$, the quantum asymptotic of Eq. (13) is valid. On the contrary, for the potentials with large correlation length ($l_c \gg r_W$) only the classical asymptotic of Eq. (11) is valid for energies satisfying $-E \gg W$. In the case of a QW exciton l_c and r_W are of the same order for a typical disorder strength of $W \sim 0.1$–1 meV. Moreover, even the classical asymptotic of Eq. (11) which is always valid for energies very deep in the tail cannot always be reached since the energy of a LP is restricted from below by $\hbar(\omega_0 - \Delta)$. That is why, in general, one must resort to the use of some simple model expressions for the bare exciton DOS, or to large scale numerical simulation.[31] We approximate here the true DOS by (i) Gaussian or (ii) exponential distribution functions with energy independent prefactors

$$\rho_G(\bar{\omega}) = \sqrt{\frac{2}{\pi}} \frac{N}{W} \exp[-(\hbar\bar{\omega})^2/2W^2];$$

$$\rho_{\exp}(\bar{\omega}) = \frac{N}{W} \exp(-\hbar\bar{\omega}/W), \tag{15}$$

where N is the total number of excitonic levels per unit area, typical values given in the literature being $N = 10^{10} - 10^{11}$ states/cm^2.[32] Though such a choice is not analytically justified, the model DOS (15) are able to reproduce both qualitatively and quantitatively the line shape of bare exciton luminescence.[33] In Fig. 1, the two DOS forms of Eq. (15) are compared with the classical asymptotic of Eq. (11) for suitably chosen parameters.

IV. INHOMOGENEOUS BROADENING AND WEAK LOCALIZATION

To understand how realistic are the present results, we calculate the inhomogeneous broadening related to the resonant scattering. First, we compare the mean free path $l_{\rm res}$ given by Eq. (9) with a de Broglie wavelength of a LP $\lambda = 2\pi/k_L$. We notice that due to the smallness of the LP wave vector k_L the inequality $l_{\rm res} \gg 2a_0$ follows from the inequality $l_{\rm res} \gg \lambda$. In Fig. 2, the ratio $l_{\rm res}/\lambda$ is plotted as a function of $\bar{\omega}$ for both exponential and Gaussian DOS for realistic disorder parameters at zero detuning. Our theory is only valid in the frequency intervals where $l_{\rm res}/\lambda > 1$, which are wide enough and correspond to energies of experimental interest. As soon as the inequality $l_{\rm res} > \lambda$ is satisfied, and as long as the inelastic mean free path $l_{\rm inel}$ is also larger than λ, we may apply the kinetic model of Sec. II to a well defined LP wave packet. Then, the inhomogeneous broadening dominated by the resonant scattering can be found as $\gamma_{\rm res} = \hbar v_g / l_{\rm res}$, where v_g is given by Eq. (5). We have

$$\gamma_{\rm res} \simeq (2\pi)^2 \frac{\bar{\omega}^2}{\Delta^2 + \bar{\omega}^2} (\hbar\Delta)^2 \frac{\hbar^2 \rho(\bar{\omega})}{m_*} \frac{V_*}{\hbar\bar{\omega}(V_* - \hbar\bar{\omega})}$$

$$\simeq (2\pi)^2 \frac{\Delta^2 \rho(\bar{\omega}) a_0^2}{\Delta^2 + \bar{\omega}^2} \hbar\bar{\omega} \left(\hbar\bar{\omega} + \frac{\hbar^2}{m_* a_0^2} \right). \tag{16}$$

In Fig. 3, $\gamma_{\rm res}$ is plotted as a function of $\bar{\omega}$ at zero detuning for the same disorder parameters as were used in Fig. 2. Each line is plotted in that frequency interval for which, for the given disorder parameters, the condition $\lambda < l_{\rm res}$ is satisfied. The predicted linewidths of the order of 100 μeV are in reasonable agreement with those reported for high quality samples.[2–4] It is also clear that, for suitable choices of their parameters, both Gaussian and exponential DOS give satisfactory results; in the following, the Gaussian DOS is employed as it is commonly assumed to be the more realistic one.

In Fig. 4, we plot the dependence of $\gamma_{\rm res}$ on detuning for $k_L = 10^4$ cm^{-1} for a Gaussian DOS with various parameters of the disorder for those frequencies for which the inequality $\lambda < l_{\rm res}$ is satisfied. Qualitatively, the dependence is the same as obtained for the case of stronger disorder (see Fig. 2 of Ref. 15). As a matter of fact, the expression for $\gamma_{\rm res}$ given in Eq. (16) can be cast in the same form as the respective term in Eq. (4) of Ref. 15, i.e., the ''absorption model'' broadening given by

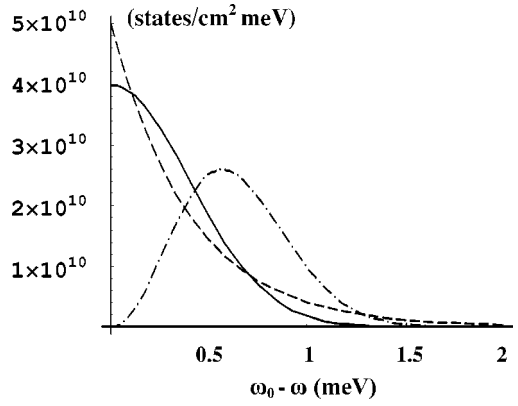

FIG. 1. Comparison of different models for the DOS. Solid line: Gaussian DOS; dashed line: exponential DOS; dot-and-dash line: classical asymptotic of Eq. (11); the chosen parameters are $N = 2 \times 10^{10}$ states/cm^2, $l_c = 2a_0 = 300$ Å, $W = 0.4$ meV.

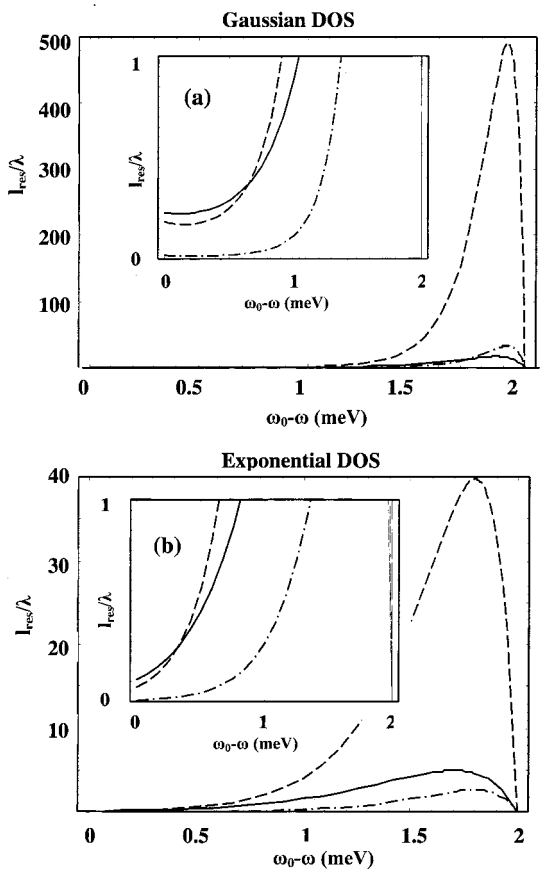

FIG. 2. Relation between λ and $l_{\rm res}$ for model DOS for various parameters of the disorder. Inset shows the frequency intervals at which Ioffe-Regel criterion is violated and the kinetic model is not applicable. (a) Gaussian DOS. Solid line: $a=100$ Å, $W=0.5$ meV, $N=10^{10}$ states/cm^2; dashed line: $a=150$ Å, $W=0.4$ meV, $N=10^{10}$ states/cm^2; dot-and-dash line: $a=200$ Å, $W=0.4$ meV, $N=10^{11}$ states/cm^2. (b) Exponential DOS. Solid line: $a=100$ Å, $W=0.3$ meV, $N=10^{10}$ states/cm^2; dashed line: $a=150$ Å, $W=0.2$ meV, $N=10^{10}$ states/cm^2; dot-and-dash line: $a=200$ Å, $W=0.2$ meV, $N=10^{11}$ states/cm^2.

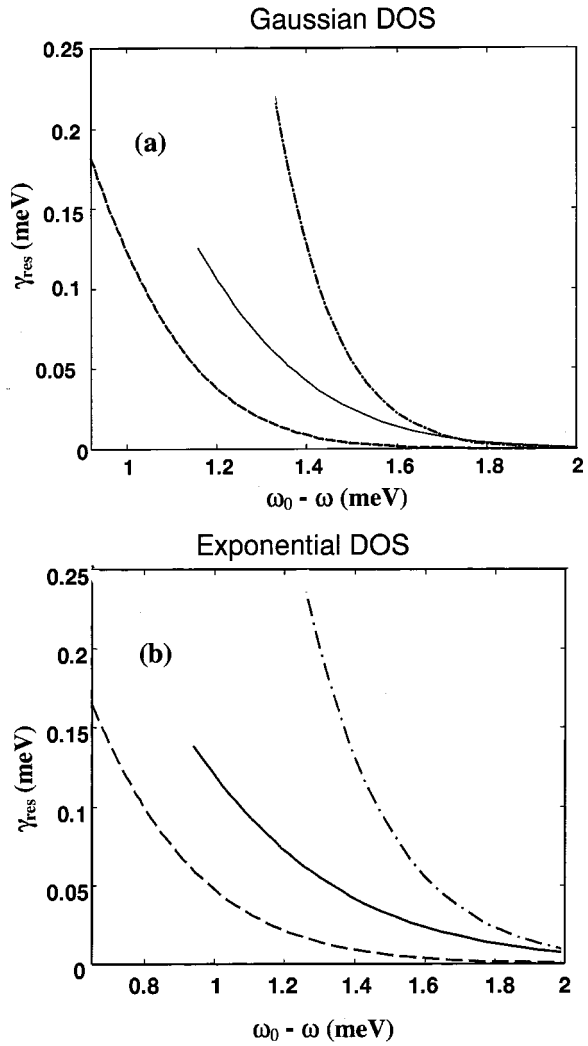

FIG. 3. Inhomogeneous broadening at zero detuning. The parameters of disorder are the same as in Fig. 2.

$$\gamma_{\rm abs}=2|c_l|^2(\hbar\Delta)^2[-\mathrm{Im}\,G_{\rm ex}(k_L,\overline{\omega})], \tag{17}$$

where the photon fraction of the LP $|c_l|^2$ is $\overline{\omega^2}/(\Delta^2+\overline{\omega^2})$ and the imaginary part of the exciton Green function, as shown in the Appendix, is given within our model by

$$-\mathrm{Im}\,G_{\rm ex}(k_L,\overline{\omega})=(2\pi)^2\frac{\hbar^2}{2m_*}\rho(\overline{\omega})\frac{V_*}{\hbar\overline{\omega}(V_*-\hbar\overline{\omega})}, \tag{18}$$

so that $\gamma_{\rm res}$ (16) is exactly equal to $\gamma_{\rm abs}$ (17). Therefore, within the limits of validity of Eq. (16), i.e., for high quality samples where the resonant scattering is still dominant over the nonresonant one, we can provide analytically a theoretical justification, alternative to the numerical simulations, for the widely used wave vector conserving phenomenological model. We notice that the simplified shape of defect potential we use here does not restrict the validity of our approach as the resonant scattering cross section is insensitive to the detailed values of the potential well parameters ($a_0\sim l_c/2$ and V_*), but only depends on the energy of the localized exciton level. An important merit of the present treatment is that it allows one to relate analytically the inhomogeneous broadening to the phenomenological characteristics of the disorder potential, such as l_c and $\rho(\overline{\omega})$.

Finally, our results also allow us to discuss the conditions under which it is possible to achieve the regime of LP weak localization in a high quality sample.[19] It is clearly seen from Fig. 2 that there exist a rather large energy range where the condition $\lambda<l_{\rm res}$ is satisfied for realistic values of the disorder parameters. However, the homogeneous dephasing due to inelastic processes may wash out the interference effects characteristic of the regime of weak localization (e.g., the coherent backscattering[24–26]). Therefore, for weak localization to occur, the inequality $l_{\rm res}<l_{\rm inel}$ must also be satisfied, where $l_{\rm inel}$ is the LP mean free path related to inelastic scattering processes which can be evaluated as

$$l_{\rm inel}\simeq\frac{\hbar v_g}{|c_{\rm ex}|^2\gamma_{\rm ex}(T)+|c_l|^2\gamma_l}, \tag{19}$$

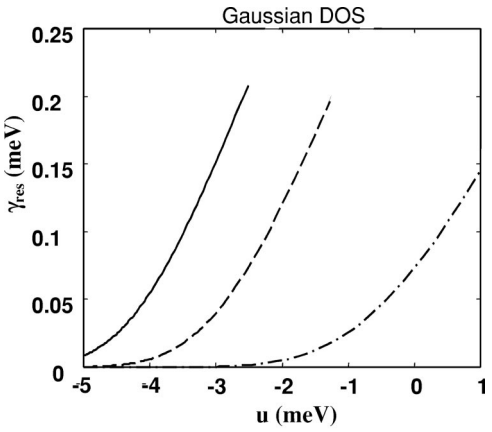

FIG. 4. Inhomogeneous broadening as a function of detuning for Gaussian model DOS. $N=10^{10}$ states/cm^2, $k_L=10000$ cm^{-1}. Solid line: $a=100$ Å, $W=0.4$ meV; dashed line: $a=150$ Å, $W=0.3$ meV; dot-and-dash line: $a=200$ Å; $W=0.2$ meV.

where $|c_{\rm ex}|^2$ and $|c_l|^2$ are, respectively, the bare exciton and the photon fractions of the LP, $\gamma_{\rm ex}(T)$ and γ_l are, respectively, the homogeneous broadenings of the microcavity embedded QW exciton, which is temperature dependent, and of the empty cavity photon. Since in high quality samples the homogeneous broadening is of the same order as (or even prevails over) the inhomogeneous broadening, one has to apply special efforts to increase $l_{\rm inel}$ by reducing $\gamma_{\rm ex}$ and γ_l. The value of γ_l may be reduced by increasing the reflectivity of the DBR mirrors and, thus, the quality factor of the cavity; a value as low as $\gamma_l=120$ μeV has been reported[34] (the sample of Ref. 4 might also have such a low value). In turn, $\gamma_{\rm ex}$ originates mainly from the scattering of the QW exciton by acoustic phonons which, however, can be strongly suppressed at low temperatures in a microcavity with a sufficiently large Rabi splitting.[8,35] In Fig. 5, the three length scales λ, $l_{\rm res}$, and $l_{\rm inel}$ are compared: for $l_{\rm res}$ the Gaussian DOS was used with $a_0=150$ Å, $N=10^{10}$ states/cm^2, $W=0.5$ meV; the inelastic mean free path was calculated with $\gamma_l=120$ μeV and $\gamma_{\rm ex}=20$ μeV. We conclude that, though

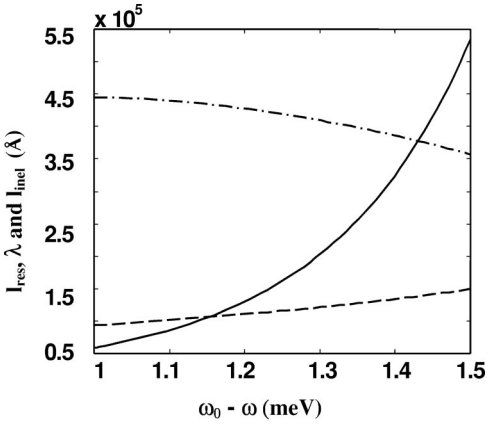

FIG. 5. The three length scales $l_{\rm res}(\bar{\omega})$ (solid line), $\lambda(\bar{\omega})$ (dashed line), and $l_{\rm inel}(\bar{\omega})$ (dot-and-dash line); $a_0=150$ Å, $N=10^{10}$ states/cm^2, $W=0.5$ meV, $\gamma_l=120$ μeV, $\gamma_{\rm ex}=20$ μeV.

the weak localization regime may well be achieved in high quality samples, it puts rather stringent limits to the usable frequency range (to satisfy the inequality $\lambda<l_{\rm res}$) and to the allowed homogeneous broadening (to satisfy the inequality $l_{\rm inel}>l_{\rm res}$). We remark that in Ref. 4 the LP enhanced backscattering in an appropriate frequency range has been observed in a high quality sample with inhomogeneous broadening of the order of 100 μeV. Also, in Ref. 5 the LP in-plane propagation in a high quality sample has been studied, the cavity quality factor, the inhomogeneous broadening and the LP energy being again amenable to the possibility of weak localization. The authors report the detection of two traces, the stronger one in the same direction as the incident beam, and the other one in the opposite direction: the latter might be a manifestation of coherent back scattering.

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APPENDIX

(1) We consider the problem of the scattering of a LP by a single cylindrical potential well of radius a_0 and of depth V_0, generalizing the results of Ref. 23 to the case of a finite detuning. Let u be the detuning between the cavity mode and the exciton resonance $u=\omega_c(k_\parallel=0)-\omega_0$. Then for the frequency range of interest [see Eq. (2)] the wave vectors of the lower (subscript "L") and the upper (subscript "U") polariton inside (subscript "V") and outside a single cylindrical well are

$$k_L^2=\frac{2\mu}{\hbar}\left[\frac{\Delta^2-\bar{\omega}^2}{\bar{\omega}}-u\right],\quad \kappa_U^2=\frac{2m_*\bar{\omega}}{\hbar}\left[1+\frac{\mu}{m_*}\frac{\Delta^2}{\bar{\omega}^2}\right],$$

$$k_{VL}^2=\frac{2m_*(V_0-\hbar\bar{\omega})}{\hbar^2}\left[1+\frac{\mu}{m_*}\frac{(\hbar^2\Delta^2)}{(V_0-\hbar\bar{\omega})^2}\right],$$

$$\kappa_{VU}^2=\frac{2\mu}{\hbar}\left[\frac{\Delta^2}{V_0/\hbar-\bar{\omega}}+\bar{\omega}+u\right]. \tag{A1}$$

We denote as $\hbar\bar{\omega}_*$ the energy of the bare exciton bound state in the well, measured down from the bottom of the exciton band $\hbar\omega_0$ in an ideal QW. For a given V_0, $\bar{\omega}_*$ can be found as a solution of the equation

$$-\frac{J_0(a_0k_V)}{a_0k_VJ_1(a_0k_V)}\equiv\tilde{g}(\bar{\omega}_*;V_0)$$

$$=\tilde{f}(\bar{\omega}_*;V_0)\equiv-\frac{K_0(a_0\kappa)}{a_0\kappa K_1(a_0\kappa)}, \tag{A2}$$

where J_0, J_1, K_0, and K_1 are Bessel functions, and k_V and κ are bare exciton wave vectors inside and outside the potential well

$$k_V^2=\frac{2m_*(V_0-\bar{\omega}\hbar)}{\hbar^2},\quad \kappa^2=\frac{2m_*\bar{\omega}}{\hbar}. \tag{A3}$$

The scattering cross section of LP[23] is determined by *s*-wave scattering only and is

$$\sigma(\bar{\omega};V_0)=\frac{\pi^2}{k_L}\frac{1+\chi(\bar{\omega};V_0)}{\frac{\pi^2}{4}+R^2(\bar{\omega};V_0)},$$

$$R(\bar{\omega};V_0)=F(\bar{\omega};V_0)-\ln\frac{a_0k_L}{2}, \tag{A4}$$

where $\chi(\bar{\omega};V_0)$ is negligible if $\bar{\omega}\approx\bar{\omega}_*$ and is of the order of unit if $\bar{\omega}$ is far from $\bar{\omega}_*$, and

$$F(\bar{\omega};V_0)=\frac{-\alpha_1 lf+\alpha_2 lg-\alpha_3 gf}{\alpha_1 g-\alpha_2 f+\alpha_3 l}, \tag{A5}$$

$$g(a_0k_{VL})=-\frac{J_0(a_0k_{VL})}{a_0k_{VL}J_1(a_0k_{VL})}\sim 1,$$

$$f(a_0\kappa_U)=-\frac{K_0(a_0\kappa_U)}{a_0\kappa_U K_1(a_0\kappa_U)}\sim 1,$$

$$l(a_0\kappa_{VU})=\frac{I_0(a_0\kappa_{VU})}{a_0\kappa_{VU}I_1(a_0\kappa_{VU})}\approx\frac{\hbar}{\mu a_0^2\Delta}\frac{1}{\beta_4}, \tag{A6}$$

$$\alpha_1(\bar{\omega};V_0)=\alpha_0\left[1+\frac{\mu}{m_*}\beta_1(\bar{\omega})\right],$$

$$\alpha_2(\bar{\omega};V_0)=\alpha_0\left[1+\frac{\mu}{m_*}\beta_2(\bar{\omega})\right],$$

$$\alpha_3(\bar{\omega};V_0)=\alpha_0\frac{\mu}{m_*}\beta_3(\bar{\omega}), \tag{A7}$$

$$\beta_1(\bar{\omega};V_0)=\frac{(\Delta\hbar)^2}{(V_0-\hbar\bar{\omega})^2}+\frac{\Delta^2}{\bar{\omega}^2}-\frac{2(\Delta\hbar)^2}{\bar{\omega}\hbar(V_0-\hbar\bar{\omega})}+\frac{\bar{\omega}\hbar+u\hbar}{V_0-\bar{\omega}\hbar}-1-\frac{u}{\bar{\omega}},$$

$$\beta_2(\bar{\omega};V_0)=\frac{2(\Delta\hbar)^2}{(V_0-\hbar\bar{\omega})^2}+\frac{2\Delta^2}{\bar{\omega}^2}+\frac{\bar{\omega}\hbar+u\hbar}{V_0-\bar{\omega}\hbar}-1-\frac{u}{\bar{\omega}},$$

$$\beta_3(\bar{\omega};V_0)=\left[\frac{\Delta}{\bar{\omega}}\frac{V_0}{V_0-\hbar\bar{\omega}}\right]^2, \tag{A8}$$

$$\beta_4(\bar{\omega};V_0)=\frac{\Delta^2+(\bar{\omega}+u)(V_0/\hbar-\bar{\omega})}{\Delta(V_0/\hbar-\bar{\omega})}.$$

We rewrite $F(\bar{\omega};V_0)$ in the form

$$F(\bar{\omega};V_0)=\frac{\frac{m_*}{\mu}(g-f)+[\beta_2 g-\beta_1 f]}{\beta_3+\frac{m_*a_0^2\Delta}{\hbar}(g-f)}, \tag{A9}$$

and we see that, since the arguments a_0k_V and $a_0\kappa$ of the functions $\tilde{g}$ and $\tilde{f}$ differ from the arguments a_0k_{VL} and $a_0\kappa_U$ of the functions g and f only by terms of the order of $\mu/m_*\sim 10^{-5}$, the magnitude of F has a peculiarity exactly near $\bar{\omega}=\bar{\omega}_*$ [see Eq. (A2)]. If $\bar{\omega}$ is far from $\bar{\omega}_*$, then F is of the order of m_*/μ, and Eq. (4) follows. If, on the contrary, $\bar{\omega}\approx\bar{\omega}_*$, then $F\sim 1$, and it results in a strong and narrow peak in $\sigma(\bar{\omega};V_0)$. Now let us fix the energy of LP $\hbar\bar{\omega}$ and vary the depth of the well V_0 (a_0 is always fixed). The energy of the localized excitonic state also becomes a variable, related to V_0 by Eq. (A2). We denote by V_* the depth of the well which possess the bare exciton level of the energy $\hbar\bar{\omega}_*=\hbar\bar{\omega}$. We expand the function $R(\bar{\omega};V_0)$ near $\bar{\omega}_*=\bar{\omega}$:

$$\begin{aligned}R(\bar{\omega};V_0\approx V_*)&\approx R(\bar{\omega};V_0=V_*)+(\bar{\omega}_*-\bar{\omega})\left[\frac{dR}{dV_0}\frac{dV_0}{d\bar{\omega}_*}\right]_{\bar{\omega}_*=\bar{\omega}}\\&=\frac{m_*}{\mu}\frac{\bar{\omega}_*-\bar{\omega}}{2\Delta}\frac{\bar{\omega}}{\Delta}\frac{V_*(\bar{\omega})-\bar{\omega}\hbar}{V_*(\bar{\omega})}+r(\bar{\omega}),\end{aligned} \tag{A10}$$

where $r(\bar{\omega})=R(\bar{\omega};V_0=V_*)\sim 1$. Then the scattering cross section is given by Eq. (7), where

$$\alpha=\frac{2}{\pi}r(\bar{\omega}),\quad \varepsilon=\pi\frac{\mu}{m_*}\frac{\Delta^2}{\bar{\omega}}\frac{V_*}{V_*-\bar{\omega}\hbar}. \tag{A11}$$

(2) To compare the present results with the ''absorption model'' of Ref. 15, we have to calculate

$$\begin{aligned}-\operatorname{Im}G_{\rm ex}(k_L,\omega)&=\frac{\pi}{S}\left\langle\sum_i|\psi_i(\mathbf{k}_L)|^2\delta(\hbar\bar{\omega}-\hbar\bar{\omega}_i)\right\rangle\\&=\pi|\psi(\mathbf{k}_L)|^2\rho(\bar{\omega}),\end{aligned} \tag{A12}$$

where S is the normalization area, $\psi(\mathbf{k}_L)=\int d\mathbf{r}_\parallel e^{-i\mathbf{k}_L\cdot\mathbf{r}_\parallel}\psi(\mathbf{r}_\parallel)$, and $\psi(\mathbf{r}_\parallel)$ is the wave function of the bare exciton having the energy level $\hbar\bar{\omega}$. The last equality in Eq. (A12) holds since in our model potential all the wells which have the same energy level are identical, and thus $|\psi(\mathbf{k}_L)|^2$ can be taken out from the averaging. The wave function of a bare exciton in a cylindrical well of the radius a_0 is

$$\begin{aligned}&\psi(\mathbf{r}_\parallel)=A_0J_0(k_Vr_\parallel),\quad r_\parallel<a_0,\\&A_0[J_0(a_0k_V)/K_0(a_0\kappa)]K_0(\kappa r_\parallel),\quad r_\parallel>a_0,\end{aligned} \tag{A13}$$

where k_V and κ are determined in Eq. (A3), and A_0 is the normalization coefficient found as

$$|A_0|^2=\frac{1}{\pi a_0^2}\,\frac{K_0^2(a_0\kappa)}{K_0^2(a_0\kappa)J_1^2(a_0k_V)+K_1^2(a_0\kappa)J_0^2(a_0k_V)}. \tag{A14}$$

After the integration, we find

$$\psi(\mathbf{k}_L)=-2\pi A_0 J_0(a_0k_V)\left\{J_0(a_0k_L)\frac{\tilde{f}(\kappa^2-k_L^2)+\tilde{g}(k_V^2-k_L^2)}{\tilde{g}\tilde{f}(k_V^2-k_L^2)(\kappa^2-k_L^2)}\right.$$
$$\left.+a_0k_LJ_1(a_0k_L)\frac{\kappa^2-k_V^2}{(k_V^2-k_L^2)(\kappa^2-k_L^2)}\right\}, \tag{A15}$$

where the functions $\tilde{f}$ and $\tilde{g}$ are defined in Eq. (A2). We expand the right-hand side of (A15) in the powers of the small parameter a_0k_L. We neglect the small term of the order of $(a_0k_L)^2$, and find that $\psi(k_L)$ is k_L independent

$$|\psi(k_L)|^2=\frac{(2\pi)^2}{\pi}\,\frac{\kappa^2+k_V^2}{\kappa^2k_V^2}, \tag{A16}$$

and Eq. (18) follows for all the wave vectors determined in Eq. (2).

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