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Electron-phonon interaction in impure polycrystalline metals

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We calculate the electron-phonon scattering rate in polycrystalline metals, e.g., $\mathrm{Ti}_{1-x}\mathrm{Al}_x$, in the limit of dilute impurity concentration. We consider the additional contribution due to the Umklapp process of impurity scattering, which has been neglected in all previous nearly free-electron calculations but is important for the present problem. We find that, as a result of including the Umklapp process, the scattering rate in the dirty limit $q_T l \ll 1$ is enhanced by the disorder due to substitutional impurities and random lattice shift of crystallites, where $q_T =$ *thermal phonon wave vector* and $l =$ *electron mean free path*. Specifically, we obtain the scattering rate $1/\tau_{ep} \sim T^2 l^{-1}$, where $T =$ *temperature*, in agreement with previous experiments both in order of magnitude and in functional dependence. This work satisfactorily explains the long-standing discrepancy between theories and experiments regarding the effect of disorder on electron-phonon scattering, for the case of polycrystalline metals with dilute impurity concentration.

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I. INTRODUCTION

The effect of disorder on electron-phonon (*e*-ph) interaction has been a long-standing subject. The primary issue in concern is whether the disorder weakens or enhances the *e*-ph interaction. Clarification of the issue has a direct bearing on the understanding of mesoscopic transport phenomena in impure metals, for which the dephasing of an electron wave caused by various inelastic scattering, including the *e*-ph one, plays an important role.[1] It is also relevant to the question of how the properties of a phonon-mediated superconductor are modified by impurities in the metallic lattice.[2,3]

On the theoretical side, the *e*-ph interaction in disordered systems has been treated for the problem of sound attenuation in impure metals, where the various theories[4–6] agree among themselves as well as with experimental results. On the other hand, in the case of the *e*-ph scattering rate, $1/\tau_{ep}$, theoretical predictions[7–10] in the dirty limit $q_T l \ll 1$ are in conflict with each other, where $q_T = k_B T/\hbar c_s$, with $T =$ *temperature* and $c_s =$ *sound velocity*, and $l =$ *electron mean free path*. For example, Bergman[7] argues for the linear form of Eliashberg function $\alpha^2 F(w \rightarrow 0) \propto w$, which leads, in the dirty limit, to $1/\tau_{ep} \propto T^2$. This is an enhancement over $1/\tau_{ep}^0$, with $1/\tau_{ep}^0 \propto T^3$ being the scattering rate in the clean limit. But the work of Schmid and co-workers,[8,11] based on the transformation to a coordinate system moving with the lattice, gives $1/\tau_{ep} \sim T^4 l$, which is a reduction from $1/\tau_{ep}^0$. On the other hand, the calculation of Takayama[9] predicts an enhancement of $1/\tau_{ep}$ over $1/\tau_{ep}^0$. However, it is later pointed out by Reizer *et al.*[10] that Takayama's calculation does not properly subtract the elastic contribution from the total (impurity and phonon) scattering rate, and hence overestimates $1/\tau_{ep}$. Moreover, Reizer *et al.* have performed an extensive calculation of $1/\tau_{ep}$ in the laboratory coordinate system, which yields the same result as the calculation of Schmid using the moving system approach, and therefore puts more weight on Schmid's prediction of a reduced $1/\tau_{ep} \sim T^4 l$.

However, the enhancement of $1/\tau_{ep}$ is widely observed in experiments,[12,13] in conflict with Schmid's result. Since the result has been independently confirmed by Reizer *et al.* using a different approach, it indicates that Schmid's calculation is basically sound from theoretical point of view, and, to close the discrepancy, the consensus among theorists has recently been that the attention should turn to model assumptions. For example, Belitz *et al.*[14] have introduced strong phonon damping, and demonstrated that it can modify $1/\tau_{ep}$ and lead to agreement with the experiment. But "the physics underlying the strong damping is not known," as they have pointed out. On the other hand, Sergeev *et al.*[15] have invoked static random potentials which do not move with lattice vibrations, and obtained an enhanced $1/\tau_{ep}$. They propose that such static potentials may arise from heavy defects or tough grain boundaries. Without them, e.g., if there is only presence of substitutional disorder, the theory still predicts a reduction in $1/\tau_{ep}$. However, when this theory is compared with the experiment of Wu *et al.* in polycrystalline $\mathrm{Ti}_{1-x}\mathrm{Al}_x$ (Ref. 13) where the enhancement $1/\tau_{ep} \propto T^2 l^{-1}$ has been observed, there are facts that seem hard to reconcile with the theory. First of all, from analyzing their experimental data, one sees that their samples show the property $l^{-1} \propto x$, the Al fraction. When combined with the observed dependence $1/\tau_{ep} \propto T^2 l^{-1}$, it gives $1/\tau_{ep} \propto x$, which strongly suggests that Al-*related disorder* is the reason causing the enhancement. Secondly, Al and Ti are very close in ionic radius, with $r(\mathrm{Al}) = 1.43$ A and $r(\mathrm{Ti}) = 1.46$ A. It favors Al impurities to go to *substitutional sites* in the $\mathrm{Ti}_{1-x}\mathrm{Al}_x$ system. The former statement is also consistent with the x-ray diffraction data[16] which shows that $\mathrm{Ti}_{1-x}\mathrm{Al}_x$ has good crystallinity and has virtually the same lattice constant as pure Ti. Thirdly, Al is actually *lighter in mass* than Ti. These facts, when put together, appear to rule out heavy defects or tough grain boundaries as the reason for the observed enhancement of $1/\tau_{ep}$, in the case of $\mathrm{Ti}_{1-x}\mathrm{Al}_x$ system. Rather, contrary to the theory, this experiment clearly indicates that it is the substitutional disorder that enhances the *e*-ph interaction.

In this work, we shall propose a different model for the explanation of $1/\tau_{ep}$-enhancement. We consider impure metals which are *polycrystals* with *substitutional* disorder, commonly used in the experimental study of disorder physics. The impurity concentration is taken to be in the dilute limit. We shall demonstrate that the substitutional disorder can enhance $1/\tau_{ep}$ in a quite natural way, and lead to the dependence $T^2 l^{-1}$. In Sec. II, we shall discuss our theoretical model. In Sec. III, we shall conclude the study.

II. THEORETICAL MODEL

Our model has two main features that distinguish it from the other models. The first is the inclusion of *Umklapp process* of impurity scattering, which involves the Fourier components of impurity potential at the wave vectors of magnitude$\sim o(2\pi/a)$, where $a=$ *lattice constant*. In contrast the previous models are basically nearly free electron (NFE) calculations and neglect such process. We now give a rough qualitative argument for why such components are important for a full treatment of impurity scattering. We note that, for a host atom in a crystal, the Fourier components of its atomic potential at the reciprocal lattice vectors of magnitude $|G| \sim o(2\pi/a)$ are important for electron dynamics, because they can strongly Bragg reflect the electron and couple a free-electron wave $\exp(ik\cdot r)$ to $\exp[i(k+G)\cdot r]$, which is especially prominent in the case of transition metals, where tight-binding orbitals are better descriptions of electron states than plane waves. Likewise, when an impurity substitutes for a host atom, the impurity potential should be treated with the Fourier components included up to the terms at the wave vectors of magnitude$\sim o(2\pi/a)$, i.e., those involved in the Umklapp process. Below, we shall give an alternative, but more quantitative discussion of the Umklapp impurity scattering.

Let Z be the valence number of a host atom, and Z_{imp} that of an impurity. We write $Z_{\mathrm{imp}}=Z+\Delta Z$, where ΔZ is the valence difference that gives rise to ionic impurity scattering. We neglect any scattering due to the short-range impurity core potential. To estimate the magnitude of Umklapp process, let us consider the scattering $p\rightarrow p+k$ on the Fermi surface, off an impurity at R_i, where p and $p+k$ are wave vectors of an electron. The impurity potential matrix element involved in this scattering is, in previous NFE models,[5–11] taken to be (with crystal volume$=1$)

$$\langle p+k|V_{\mathrm{imp}}(r-R_i)|p\rangle \sim V_{\mathrm{imp}}(k)\exp(-ik\cdot R_i),$$

where $V_{\mathrm{imp}}(k)\equiv -(4\pi e^2 \Delta Z/k^2)$ and the states have been approximated as plane waves. Such an expression includes effectively only the normal process of impurity scattering. In contrast, a full evaluation of the matrix element, with the true *crystal state wave function*, e.g., $|p\rangle=\exp(ip\cdot r)u_p(r)$ where u_p is the cell-periodic Bloch function, gives

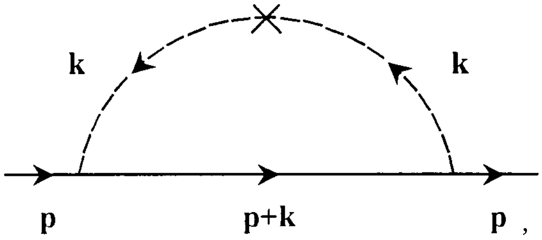
$$\langle p+k|V_{\mathrm{imp}}(r-R_i)|p\rangle=\sum_G V_{\mathrm{imp}}(k-G)C(G;p+k,p) \times\exp[-i(k-G)\cdot R_i], \qquad (1)$$

where

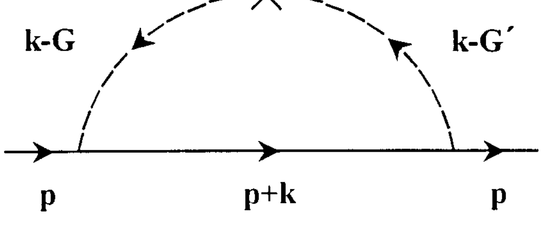
$$C(G;p+k,p)\equiv\int \exp(-iG\cdot r)u^*_{p+k}u_p dr$$

and $G=$reciprocal lattice vectors. In Eq. (1), $V_{\mathrm{imp}}(k-G)$ with $G\neq 0$ describes the Umklapp scattering, while that with $G=0$ the normal scattering. From Eq. (1), we can make quantitative comparison between the normal and the Umklapp contributions, in the following two steps. First k, being the wave vector difference between two Fermi surface states, is of the order of the Fermi wave vector k_f, and so is $k-G$ for small $|G|\leqslant o(2\pi/a)$. This means that $V_{\mathrm{imp}}(k-G)$ and $V_{\mathrm{imp}}(k)$ are of the same order. Secondly, $C(G=0)$ and $C(G\neq 0)$ for $|G|\sim o(2\pi/a)$ are also of the same order. (Other arguments in the function C are omitted whenever it does not cause confusion.) This can be seen as follows. The functions u's in the integral of $C(G)$ are cell periodic and hence have primary Fourier components around $|G| \leqslant o(2\pi/a)$. Expanding u's in Fourier series and evaluating the integral, one can see that, for $|G|\leqslant o(2\pi/a)$, $C(G)$'s are large and have comparable magnitudes [actually, with $|C(G)|\sim o(1)$]. With these two facts put together, it follows that the terms in Eq. (1) with $G=0$ (i.e., the normal process) and with $|G|\sim o(2\pi/a)$ (i.e., the Umklapp process) are about equally important.

Next, we describe how the usual impurity diagrammatic technique[17] is modified, as a result of including the Umklapp process. In the previous NFE models, with only the normal process, the kind of Green's function diagram surviving impurity configuration average is, for example, the following:



where —$=$electron line, ---$=$impurity line, and the vertex denotes the electron-impurity (*e-i*) interaction screened by electron-electron (*e-e*) interaction, i.e., $[V_{\mathrm{imp}}(k)/\varepsilon(k)] \times\exp(-ikR_i)$ (or its complex conjugate), with $\varepsilon(k)=$ *static dielectric function*. The two impurity lines joined to X refer to a double scattering off the same impurity. Throughout the paper, we shall take $V_{\mathrm{imp}}(k)/\varepsilon(k)\sim\text{const}=V_0$, which means isotropic impurity scattering. Now, with the Umklapp process included, the diagram is modified as



(2a)

where the *e-i* vertex now represents $C(G)V_0\exp[-i(k-G)R_i]$ (or its c.c.). Note that, in comparison with the normal scattering case, this diagram is weighted by the following extra factor:

$$C(G)C^*(G')\exp[i(G-G')R_i], \tag{2b}$$

and the diagram is to be summed over G, G', and also the impurity position R_i.

The second main theme of our model is the *random lattice shift* of crystallites in a polycrystal. It has the consequence that only the diagram such as Eq. (2a) *with* $G=G'$ survives the configuration average. To see this, let us consider two neighboring grains in the system, which are typically separated by a low-angle grain boundary created by edge dislocation.[18] If we take our coordinate origin to be at some lattice site of grain 1, then a general lattice site in grain 1 can be expressed in the form $mA+nB+lC$, where m, n, and l are integers, and A, B, and C are lattice basis vectors. On the other hand, because of the edge dislocation between grains, the lattice sites of grain 2 are shifted by a vector s, relative to the lattice of grain 1, and a site in grain 2 is described by the form $mA+nB+lC+s$, where the lattice shift s, being due to the dislocation, is of the order of lattice constant. Therefore, when Eq. (2b) is summed over impurities, a random phase factor difference $\exp[i(G-G')s]$ appears between contributions from different grains. Only the diagram with $G=G'$ survives the configuration average. Note also that the weighting factor in Eq. (2b) then reduces to $|C(G)|^2$. The above discussion still holds if one takes into account that crystal orientation also varies (though only slightly) among grains. Such a variation only induces rotation in G and G' and does not reduce the randomness of the phase factor.

Quite often, the inclusion of Umklapp process does not qualitatively change the physics of disordered systems one is investigating. For example, from the self-energy in diagram (2a), the impurity scattering rate, with the Umklapp process included, becomes

$$1/\tau\sim\sum_k\sum_G|C(G)|^2|V_0|^2\delta(\xi_{p+k}-\xi_p),$$

where ξ is the electronic energy. If we consider only the main contribution from small G, it gives, roughly,

$$1/\tau\sim n_G C_0^2\sum_k|V_0|^2\delta(\xi_{p+k}-\xi_p),$$

where $n_G\sim$number of small G's, e.g., with $|G|\leqslant o(2\pi/a)$, and C_0 is the typical magnitude of $|C(G)|$. Apart from the factor $n_G C_0^2$ [which is actually $o(1)$], the rate $1/\tau$ is basically of the same form as what one would obtain if only the normal process of impurity scattering is considered. In contrast, as we shall show, the inclusion of Umklapp process modifies $1/\tau_{ep}$ in a qualitative way.

We now describe our calculation. Basically, we adopt the same Keldysh formalism of equilibrium Green's functions,[19] as applied to the calculation of energy relaxation rate in disordered metals firstly by Altshuler[20] and later by Reizer *et al.*[10] Great details of this application can be found in their papers and, hence, we shall limit our presentation only to a brief description.

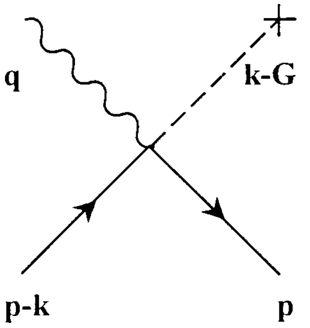
In the Keldysh formalism, four Green's functions are calculated simultaneously. In addition to the usual time-ordered one $G^{--}(r,t;r',t')=-i\langle T[\psi(r,t)\psi^+(r',t')]\rangle$, there are $G^{-+}(r,t;r',t')=-i\langle\psi(r,t)\psi^+(r',t')\rangle$, $G^{+-}(r,t;r',t')=i\langle\psi^+(r',t')\psi(r,t)\rangle$, and $G^{++}(r,t;r',t')=-i\langle\tilde{T}[\psi(r,t)\psi^+(r',t')]\rangle$ with $\tilde{T}$ denoting reverse-time ordering. These functions contain not only dynamic but also distribution information. In particular, the distribution Green's function $G^{+-}(r,t;r',t')$ can be used to write down quantum transport equations, from which expressions of various collision rates can be identified. The advantage of this method is that it properly separates the elastic and inelastic scattering and gives unambiguously the energy relaxation rate, which, in the case of our problem, is exactly the *e*-ph scattering rate $1/\tau_{ep}$.

For the interaction part in the Hamiltonian of our system, we write $H_{\rm int}=H_{e\text{-}e}+H_{e\text{-}i}+H_{e\text{-ph}}+H_{e\text{-MI}}$, where $H_{e\text{-}e}$ is the *e-e* interaction, $H_{e\text{-}i}$ the *e-i* interaction, $H_{e\text{-ph}}$ the *e*-ph interaction, and $H_{e\text{-MI}}$ the electron-moving impurity (*e*-MI) interaction. In particular, with the Umklapp process included, we have (with $\hbar=1$)

$$H_{e\text{-}i}=\sum_{p,\sigma}\sum_k{}'\sum_G C(G;p,p-k)V_{\rm imp}(k-G)\times c^+_{p,\sigma}c_{p-k,\sigma}\sum_{R_i}\exp[-i(k-G)R_i],$$

$$H_{e\text{-MI}}=\sum_{p,\sigma}\sum_k{}'\sum_{q,\lambda}{}'\sum_G C(G;p,p-k)\gamma(k-G,q,\lambda)\times c^+_{p,\sigma}c_{p-k,\sigma}(b_{q,\lambda}+b^+_{q,\lambda})\times\sum_{R_i}\exp[-i(k-G-q)R_i],$$

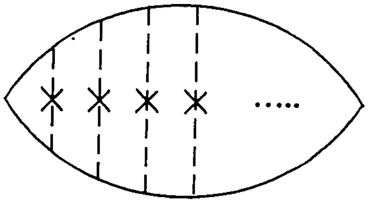
where $\gamma(k-G,q,\lambda)=-iV_{\rm imp}(k-G)(k-G)e_{q\lambda}/(2MNw_{q\lambda})^{1/2}$. In above, c and c^+ are electron operators, b and b^+ phonon operators, $e_{q\lambda}$ a unit vector along the phonon polarization, λ the phonon branch index, $M=$mass of a unit cell, $N=$number density of unit cells, and $w_{q\lambda}=$phonon energy. For later use, we draw the diagram for $H_{e\text{-MI}}$ below



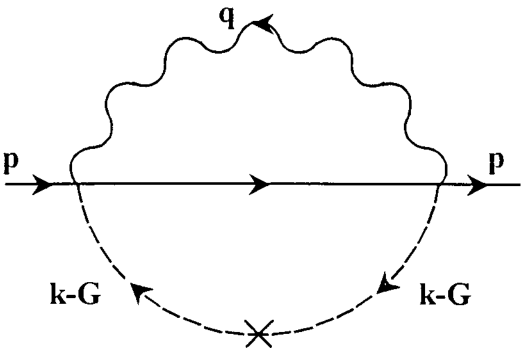
,

where the wavy line is a phonon line, and the vertex is proportional to $i(k-G)e_{q\lambda}V_{\rm imp}(k-G)/\varepsilon(k-G,w)$, with w the frequency of the phonon line. Here, we have included the dynamic screening of the e-MI vertex by the e-e interaction.

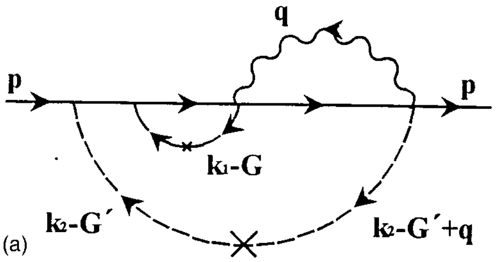
Following previous works, several approximations are made in the calculation. First, because we are considering alloys with dilute impurity concentration and lattice vibrations of long wavelength, we take the phonon dispersion to be just that of the corresponding clean metal. Second, the screening of interactions by e-e interaction is taken into account within the diffusion-modified random phase approximation, where the usual electron-hole bubble is inserted with a ladder of impurity lines as follows:



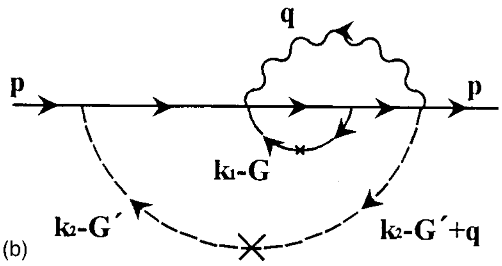
Third, we take the weak e-ph coupling limit and consider only those electron self-energy diagrams which are first-order in the e-ph interaction. The following diagrams are most important and are explicitly shown below:



(3)



(4a)



(4b)

These diagrams and their symmetric counterparts make the largest contributions, I_7 and I_8 (in terms of the symbols of Reizer *et al.*), to $1/\tau_{\rm ep}$. I_7 is given by the diagram in Eq. (3) as follows:

$$I_7(\xi)=\frac{8N_{\rm imp}}{\pi\nu}\int\frac{dpdkdqdw}{(2\pi)^{10}}J_7\frac{R(\xi,w)}{2MNw_{q,\lambda}}\times\mathrm{Im}\,G^A(p,\xi)\mathrm{Im}\,G^A(p+k,\xi+w)\mathrm{Im}\,D^R(q,w).$$

I_8 is given by the diagrams in Eqs. (4a), (4b) and their symmetric counterparts as follows:

$$\begin{aligned}I_8(\xi)=8N_{\rm imp}^2\int&\frac{dpdk_1dk_2dqdw}{(2\pi)^{13}}J_8\frac{R(\xi,w)}{2MNw_{q\lambda}}\\&\times[\mathrm{Im}\,G^A(p+k_1+k_2,\xi+w)]\\&\times\mathrm{Im}\,G^A(p,\xi)][G^R(p+k_2,\xi)\\&\times G^A(p+k_2+q,\xi+w)+\text{c.c.}][\mathrm{Im}\,D^R(q,w)].\end{aligned}$$

In the integrals I_7 and I_8,

$$R(\xi,w)=N_w n_\xi(1-n_{\xi+w})-(1+N_w)(1-n_\xi)n_{\xi+w},$$

$$G^R(p,\xi)=[G^A(p,\xi)]^*=(\xi-\xi_p+i/2\tau)^{-1},$$

$$\begin{aligned}D^R(q,w)&=[D^A(q,w)]^*\\&=(w-w_{q\lambda}+i0)^{-1}-(w+w_{q\lambda}+i0)^{-1}\end{aligned}$$

with N_w the Bose-Einstein distribution and n_ξ the Fermi-Dirac distribution, while J_7 and J_8 are contributions from the e-i and e-MI interaction vertices in the corresponding diagrams, respectively. For the e-MI vertex, we can replace the dynamical screening of impurity potential (at the frequency w of the phonon line) by static screening, and take the screened potential to be V_0, since the contribution to the w integral in I_7 and I_8 mainly comes from the frequency range where $w\sim w_{q\lambda}\sim 0$. This gives

$$J_7=-\sum_G |C(G;p+k,p)|^2[(k-G)e_{q\lambda}]^2|V_0|^2$$

and

$$\begin{aligned}J_8=\sum_{G,G'}&C(G';p+k_2,p)C^*(G';p+k_2+q,p)|V_0|^4\\&\times C(G;p+k_1+k_2,p+k_2)\\&\times C^*(G;p+k_1+k_2,p+k_2+q)\\&\times[(k_1-G)e_{q\lambda}][(k_2-G'+q)e_{q\lambda}].\end{aligned}$$

In the NFE work of Reizer *et al.*, it is found that both I_7 and I_8 have $o(T^2l^{-1})$ as leading-order terms, but they are opposite in sign and exactly cancel out. This leaves them the next-order terms which are $o(T^4l)$. Since other self-energy diagrams not drawn here are also of $o(T^4l)$, it gives, in overall, $1/\tau_{ep}\propto T^4l$. However, we shall show in below that, with the inclusion of Umklapp process, I_7 becomes several times as large as I_8. As a result, the cancellation is not exact in the leading order and it yields $1/\tau_{ep} \propto T^2 l^{-1}$.

We discuss the functional dependence of $1/\tau_{ep}$ in the following way. First, note that, in the mathematical expressions I_7 and I_8, we can regard the Fermi wave vector k_f as a continuous variable and I_7 and I_8 as functions of k_f. Their domain of k_f consists of an interval $[k_f^-, k_f^+ \sim o(G/2)]$, with k_f small but still large enough to satisfy the condition $k_f l \gg 1$. For example, if $l = 100\,a/\pi$, we take k_f to be $0.1\pi/a$. The enforcement of the foregoing inequality is to make sure that, the same mathematical approximation, e.g., retaining only the lowest-order term in $(k_f l)^{-1}$ in the calculation of I_7 and I_8, can be used throughout the interval of k_f.

We now evaluate $1/\tau_{ep}$ for the case where k_f is small, i.e., near k_f^-. In this case, our argument against the cancellation of $o(T^2 l^{-1})$ terms in $1/\tau_{ep}$ is made in two steps. (A) First, we make the approximation that $u_{kf} \sim u_0$, where u_{kf} and u_0 are Bloch cell-periodic functions at k_f and at the zone center, respectively. The validity of this approximation follows from the *kp* theory of band structures in which one expands u_{kf} in a perturbation series with u_0 being the leading-order term. Moreover, we assume u_0 has the full symmetry of the lattice, e.g., u_0 is *S*-like in the case of a crystal with cubic symmetry. With the approximation that $u_{kf} \sim u_0$, C(G;p_1,p_2) [defined in Eq. (1)] $\sim C(G;0,0)$ when p_1 and p_2 both are restricted to the Fermi surface. Moreover, it is obvious that $|C(G;0,0)| = |C(|G|;0,0)|$, independent of the G direction, which follows from the symmetry of u_0. (B) Next, we note that, in J_7 and J_8, the phonon wave vector $q \sim 0$, and, moreover, the electron wave vectors p, $p+k$, $p+k_2$, and $p+k_1+k_2$ which appear as arguments in various $C(G)$'s there are all restricted to the Fermi surface, both resulting from the fact that contributions to the integrals I_7 and I_8 mainly come from the region with $q \sim 0$ and with the foregoing electron wave vectors $\sim k_f$. So, we can take all C's in J_7 and J_8 as $C(G;k_f,k_f) \sim C(G;0,0)$, according to (A). Then, we have, for the cross term in J_7,

$$2\sum_G |C(G;0,0)|^2 (k e_{q\lambda})(G e_{q\lambda})$$
$$= 2\sum_{|G|} |C(|G|;0,0)|^2 (k e_{q\lambda}) \left(e_{q\lambda} \sum_{\Omega_G} G \right),$$

where $\Sigma_{\Omega_G} G$ denotes the summation of G's over the direction of G. Since $\Sigma_{\Omega_G} G = 0$ with G's distributing symmetrically for a crystal of, say, cubic symmetry, the cross term vanishes. We are then left with only the squared terms

$$J_7 \propto -\sum_G |C(|G|;0,0)|^2 [(k e_{q\lambda})^2 + (G e_{q\lambda})^2] |V_0|^2.$$

Similarly, we have

$$J_8 \propto (1/\tau) \sum_G |C(|G|;0,0)|^2 [k_1 e_{q\lambda} (k_2+q) e_\lambda] |V_0|^2.$$

We note that the $(k e_{q\lambda})^2$ term in J_7 and the whole J_8 are of the same forms as what one would obtain if only normal scattering is considered. When we put back in the integrals I_7 and I_8, they both result in $o(T^2 l^{-1})$ contributions to $1/\tau_{ep}$ that cancel with each other just as in the original NFE calculation of Reizer *et al.* This leaves us the $(G e_{q\lambda})^2$ term in J_7 which comes from the Umklapp scattering. It is easy to check that the $(G e_{q\lambda})^2$ term, when integrated, makes an extra $o(T^2 l^{-1})$ contribution to $1/\tau_{ep}$, too. Therefore, after summing I_7 and I_8, we obtain $1/\tau_{ep} \sim o(T^2 l^{-1})$ or, more accurately,

$$\frac{1}{\tau_{ep}} \sim -\frac{\delta(I_7+I_8)}{\delta n_\xi} \sim n_G C_0^2 \frac{3\pi^2}{4} \frac{G_0^2}{k_f^4} \frac{T^2}{I} \left(\frac{\beta_l}{c_l} + 2\frac{\beta_t}{c_t} \right), \quad (5)$$

where

$$\beta_t = \beta_l \left(\frac{c_l}{c_t} \right)^2 = \left(\frac{2\epsilon_f}{3} \right)^2 \frac{\nu}{2MNc_t^2}.$$

In the above, $G_0 \sim o(2\pi/a)$, $\epsilon_f =$ Fermi energy, $c_l =$ sound velocity of longitudinal phonon mode, $c_t =$ that of transverse one, and $\nu =$ electronic density of states. (A) together with (B) is a rigorous argument in the case of small k_f. We stress that this result is significant and distinct from that of Reizer *et al.*, since theirs always gives $o(T^4 l)$ dependence regardless of the value of k_f.

Now, we discuss the functional dependence of $1/\tau_{ep}$ when k_f increases towards $o(G/2)$. Generally speaking, cancellation among terms is less likely to occur than noncancellation, unless there is certain symmetry-related physics behind it. A good example is the optical dipole transition between two states, which is usually finite unless there is inversion symmetry and the two states are of the same parity. In our case, if there were a symmetry-caused cancellation, it would have shown up in the foregoing calculation for a small k_f. The fact that the cancellation does not occur there strongly suggests lack of such symmetry. Moreover, if we write $1/\tau_{ep} = f(k_f) T^2 l^{-1}$ and regard $f(k_f)$ as an analytic function of k_f, with $f(k_f) \geqq 0$, it is hardly likely that the coefficient $f(k_f)$ should go from being nonvanishing in the domain of small k_f, proved earlier, to being identically zero for all large values of k_f. Therefore, we can conclude convincingly against the vanishing of $f(k_f)$ for general values of k_f. To estimate the order of $1/\tau_{ep}$, we could just rely on Eq. (5), or start from the scratch making the simplest assumption $|C(G)| \sim$ constant $= C_0$ in the original expressions of J_7 and J_8 and go through the mathematics once more. In the latter case, we arrive at Eq. (5) again.

Let us apply our theory to the $\mathrm{Ti}_{1-x}\mathrm{Al}_x$ system. We take $n_G = 1$, $C_0 = 1$, $c_l = 6000$ m/s, $c_t = 2950$ m/s, $v_f = 3.2 \times 10^5$ m/s, $M = 7.97 \times 10^{-26}$ kg, $N = 5.66 \times 10^{28}/\mathrm{m}^3$, $k_f = 1.46 \times 10^{10}\,\mathrm{m}^{-1}$, $G_0 \sim k_f$, and obtain the estimate $1/\tau_{ep} \sim 0.1 T^2 l^{-1}$ (in MKS unit), in agreement with the experimental result $1/\tau_{ep} \sim 0.2 T^2 l^{-1}$ of Wu *et al.*[13] in both order of magnitude and functional dependence.

III. CONCLUSION

In summary, we have calculated $1/\tau_{ep}$ in polycrystalline metals in the limit of dilute impurity concentration. We consider the additional contribution due to the Umklapp process of impurity scattering, which has been neglected in all previous NFE calculations but is important for the present problem, and find that $1/\tau_{ep}$ in the dirty limit is enhanced by the disorder due to substitutional impurities and random lattice shift of grains. Specifically, we obtain $1/\tau_{ep} \sim T^2 l^{-1}$, in agreement with previous experiments both in order of magnitude and in functional dependence. This work satisfactorily explains the long-standing discrepancy between theories and experiments regarding the effect of disorder on e-ph scattering, for the case of polycrystalline metals with dilute impurity concentration.

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