Meissner effect in a bosonic ladder

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We investigate the effect of a magnetic field on a bosonic ladder. We show that such a system leads to the one-dimensional equivalent of a vortex lattice in a superconductor. We investigate the physical properties of the vortex phase, such as vortex density and vortex correlation functions, and show that magnetization has plateaus for some commensurate values of the magnetic field. The lowest plateau corresponds to a true Meissner to vortex transition at a critical field H_{c1} that exists although the system has no long-range superconducting order. Implications for experimental realizations such as Josephson-junction arrays are discussed.

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I. INTRODUCTION

The effect of a magnetic field on interacting particles is a long-standing problem. A spectacular case is provided by the type-II superconductor, in which the magnetic field is totally expelled below H_{c_1}, whereas a vortex state exists for $H > H_{c_1}$. This behavior, however, is obtained from the Landau-Ginzburg equation, and it is important to know what happens when interactions and fluctuations have more drastic effects, such as in one-dimensional systems. Indeed, in a one-dimensional conductor at $T=0$, although there is no long-range order, superconductivity in the sense of infinite dc conductivity can nevertheless be present.[1] For a one-dimensional chain, there is no orbital effect, and this question is not relevant. However, a system made of a finite number of coupled chains (a ladder) is still one dimensional (no long-range order can exist) but orbital effects of the magnetic field are present, opening the possibility of such a transition.

Beyond its own theoretical interest the investigation of the effect of a magnetic field on ladder systems is also of direct experimental relevance, due to the various realizations of such ladders.[2–4] Fermionic ladders can be superconducting both from attractive (s-wave) and repulsive (d-wave) interactions. In the attractive case, the system is close to standard superconductors where pairs of fermions can hop from one chain to the other leading to a Josephson-coupling, provided the applied magnetic field is smaller that the spin gap. The system can thus be described as a bosonic ladder. Josephson-junction arrays[5,6] provide also a very direct realization of such a bosonic ladder[7,8] and are thus the prime candidates for observing these effects. This problem of Josephson ladders has been investigated previously in the classical[9] and quantum limit both analytically[10,11] and numerically[12] in the high-field limit of half a flux quantum per plaquette for one-dimensional situations. For the Josephson two-leg ladder, it has been shown[10,11] that a true transition exists between a commensurate and a vortex phase, however the detailed behavior of the vortex phase and the effects of commensurability of the magnetic field remain to be understood.

In this paper we investigate the effect of a magnetic field directly on the bosonic two-leg ladder. We analyze the consequences for transport in such bosonic ladders. The plan of the paper is as follows: in Sec. II the model of the bosonic ladder under magnetic field is introduced and connections with the Josephson ladder are shown. In Sec. III the model is solved and the phase diagram is obtained showing a Meissner phase at low field and phases with vortices above a certain threshold field H_{c1}. These vortex phases are studied in detail in Sec. IV. We show that plateaus in the magnetization for commensurate values of the field exist, in a way similar to the Mott transition in one dimension for commensurate values of the filling. Finally the conclusions of the paper as well as the consequences of our findings for observing the vortex phase in systems such as the Josephson-junction arrays can be found in Sec. V.

II. MODEL

The lattice Hamiltonian of the bosonic two-leg ladder in a magnetic field is

$$\begin{aligned} H= & -t_{\parallel}\sum_{i,p=1,2}\left(b^{\dagger}_{i+1,p}e^{ie^{*}aA_{\parallel,p}(i)}b_{i,p}+b^{\dagger}_{i,p}e^{-ie^{*}aA_{\parallel,p}(i)}b_{i+1,p}\right) \\ & -t_{\perp}\sum_{i}\left(b^{\dagger}_{i,2}e^{ie^{*}A_{\perp}(i)}b_{i,1}+b^{\dagger}_{i,1}e^{-ie^{*}A_{\perp}(i)}b_{i,2}\right) \\ & +U\sum_{i,p}n_{i,p}(n_{i,p}-1)+Vn_{i,1}n_{i,2}, \end{aligned} \qquad (1)$$

where the density $n_{i,p}=b^{\dagger}_{i,p}b_{i,p}$ and the magnetic field is introduced via the Peierls substitution. In addition to their parallel $t_{\parallel}$ and perpendicular $t_{\perp}$ hopping the bosons repel via an on-site U and an interchain V repulsion. An experimental realization of the bosonic two-leg ladder is provided by the Josephson-junction ladder. Its Hamiltonian reads

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$$H=\sum_{i,p}\left[-\frac{(e^*)^2}{2C}\frac{\partial^2}{\partial\theta_{i,p}^2}-J_\parallel\cos[\theta_{i+1,p}-\theta_{i,p}-e^*aA_{\parallel,p}(i)]-\frac{J_\perp}{2}\cos[\theta_{i,1}-\theta_{i,2}-e^*A_\perp(i)]\right], \tag{2}$$

where $\theta_{j,p}$ is the Josephson (superconducting) phase and the particle number on the site j,p is given by

$$n_{j,p}=\frac{1}{i}\frac{\partial}{\partial\theta_{j,p}}. \tag{3}$$

The vector potential is given by

$$\oint \mathbf{A}\cdot d\mathbf{l}=A_\perp(i+1)+aA_{\parallel,1}(i)-A_\perp(i)-aA_{\parallel,2}(i)=\Phi, \tag{4}$$

where Φ is the flux of the magnetic field through a plaquette. The low-energy properties of Eq. (1) are more transparent[13] using a "bosonized" representation of the boson operators.[14] One obtains

$$H=\sum_{p=1,2}\int\frac{dx}{2\pi}\left[uK(\pi\Pi_p-e^*A_{\parallel,p})^2+\frac{u}{K}(\partial_x\phi_p)^2\right]-\frac{t_\perp}{\pi a}\int dx\cos[\theta_1-\theta_2+e^*A_\perp(x)]+\frac{Va}{\pi^2}\int dx\,\partial_x\phi_1\partial_x\phi_2+\frac{2Va}{(2\pi a)^2}\int dx\cos(2\phi_1-2\phi_2), \tag{5}$$

where ϕ_i and $\pi\Pi_i=\nabla_x\theta_i$ are conjugate variables and the boson annihilation operator is given by

$$\psi_p(x=na)=\frac{b_n}{\sqrt{a}}=\frac{e^{i\theta_p(x)}}{\sqrt{2\pi a}}, \tag{6}$$

where a is the lattice spacing along the legs of the ladder. u is the sound velocity of the collective density oscillations. The representation makes the analogy between the boson chain and the Josephson ladder transparent. Indeed the Hamiltonian (5) can also be deduced from the Hamiltonian (2) by writing $\cos[\theta_{i,p}-\theta_{i+1,p}-A_{\parallel,p}(i)]\simeq 1-[\theta_{i,p}-\theta_{i+1,p}-A_{\parallel,p}(i)]^2/2$ and taking the continuum limit. In that case, one has

$$u=\sqrt{\frac{(e^*)^2J_\parallel}{C}}a, \tag{7}$$

$$K=\pi\sqrt{\frac{J_\parallel C}{(e^*)^2}}, \tag{8}$$

$$t_\perp=\pi J_\perp, \tag{9}$$

$$V=0. \tag{10}$$

The current operators along and perpendicular to the chains are given by

$$j_p(x)=uKe^*\left(\Pi_p-\frac{e^*}{\pi}A_{\parallel,p}\right), \tag{11}$$

$$j_\perp(x)=\frac{e^*t_\perp}{\pi a}\sin(\theta_1-\theta_2+e^*A_\perp)(x). \tag{12}$$

The Hamiltonian (5) is invariant under the gauge transformations:

$$\theta_p(x)\to\theta_p(x)+e^*f_p(x), \tag{13}$$

$$A_{\parallel,p}(x)\to A_{\parallel,p}(x)+\partial_x f_p(x), \tag{14}$$

$$A_\perp(x)\to A_\perp(x)+f_2(x)-f_1(x). \tag{15}$$

The linear nature of the spectrum is the signature that true superfluidity exists in a single chain even without long-range order.[1] K is the Luttinger parameter, directly related to the compressibility of the system, and incorporating all microscopic interaction effects. For a simple on-site repulsion $1<K<\infty$, with $K=\infty$ being free bosons and $K=1$ hard-core bosons. For more general (longer-range) interactions all values of K are in principle allowed.

III. PHASE DIAGRAM

If one introduces the symmetric and antisymmetric combinations $\phi_{s,a}=(\phi_1\pm\phi_2)/\sqrt{2}$ (and similar combinations for Π and A), Eq. (5) reads[13]

$$H=H_s^0+H_a^0-\frac{t_\perp}{\pi a}\int dx\cos[\sqrt{2}\theta_a+e^*A_\perp(x)]+\frac{2Va}{(2\pi a)^2}\int dx\cos\sqrt{8}\phi_a, \tag{16}$$

where $(\nu=s,a)$

$$H_\nu^0=\int\frac{dx}{2\pi}\left[u_\nu K_\nu(\pi\Pi_\nu-e^*A_\nu)^2+\frac{u_\nu}{K_\nu}(\partial_x\phi_\nu)^2\right] \tag{17}$$

with

$$K_s=K(1+[VKa/\pi u])^{-1/2},$$

$$K_a=K(1-[VKa/\pi u])^{-1/2}.$$

$A_s$describes an Ahronov-Bohm flux threading the system. In the following, we assume that there is no such flux, so $A_s=0$. We also assume that $K_a>1$, so that the term $\cos\sqrt{8}\phi_a$ is irrelevant. This leads to a Hamiltonian similar to the one derived for the Josephson ladder.[10,11] Note that this limit is different from the one considered in Ref. 15, where the term $\cos\sqrt{8}\phi_a$ was relevant and drove the system to a Bose pair superconductor.[13] The total and antisymmetric longitudinal current $j_{s,a}=j_1\pm j_2$ and the current perpendicular to the ladder are given by $(\nu=s,a)$:

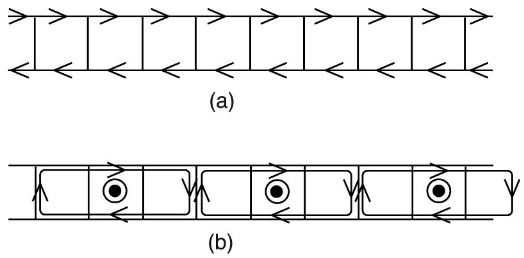

FIG. 1. The phases of the bosonic two-leg ladder in the presence of a magnetic field. (a) The low-field phases. Current only exists along the legs of the ladder leading to screening of the applied magnetic field. (b) The high-field phase. Current exists both on the legs and the rungs of the ladder leading to a vortex lattice (here shown with a vortex at every three sites). At commensurable flux the vortex lattice is perfectly ordered (at $T=0$) whereas away from commensurability the vortex system has only quasi-long-range positional order and no perfect crystalline order.

$$j_\nu = uKe^*\sqrt{2}\left(\Pi_\nu - \frac{e^*}{\pi}A_\nu\right), \tag{18}$$

$$j_\perp = \frac{e^* t_\perp}{\pi a}\sin(\sqrt{2}\theta_a + e^* A_\perp). \tag{19}$$

H_s gives a simple dynamics; the behavior of H_a is richer. In the absence of a magnetic field, H_a is a sine-Gordon Hamiltonian. For $K > \frac{1}{4}$, it develops an order in the field θ_a with $\langle \theta_a \rangle = 0$ and a gap in the excitation spectrum,

$$\Delta_a \sim \frac{u}{a}\left(\frac{t_\perp a}{u}\right)^{1/[2-1/(2K)]}. \tag{20}$$

This describes the locking of the relative phases by Josephson coupling. This implies $\langle j_a \rangle = -uK[(e^*)^2\Phi/\pi a]$, no perpendicular current $\langle j_\perp \rangle = 0$, and

$$\langle j_\perp(x) j_\perp(x) \rangle \sim e^{-|x-x'|/\xi_a} \tag{21}$$

with $\xi_a = u/\Delta_a$. In this low-field phase, there is a current circulating only on the edges of the system (see Fig. 1) and proportional to the applied vector potential. It is just the Meissner current that screens the electromagnetic field. Thus, we can identify the low-field phase to a Meissner phase.[10] Let us now consider the effect of a magnetic field such that the flux per plaquette $\Phi = (2\pi/e^*)p/q$, where p,q are two mutually prime integers. Using the gauge $A_a = 0$, $A_\perp = \Phi x/a$, we can write

$$H_a = \int \frac{dx}{2\pi}\left[u_a K_a (\pi \Pi_a)^2 + \frac{u_a}{K_a}(\partial_x \phi_a)^2\right] - \frac{t_\perp}{\pi a}\int dx \cos\left(\sqrt{2}\theta_a + \frac{2\pi p}{q}\frac{x}{a}\right). \tag{22}$$

The Hamiltonian (22) is similar to the one describing the Mott transition in one dimension,[16] where any commensurability can in principle give a transition depending on K. Indeed, even if the presence of an oscillating phase in Eq. (22) makes $t_\perp$ naively irrelevant, perturbation theory to order q in $t_\perp a/(\pi u_a)$ shows[17,18] that a term

$$H_q = -\frac{g_q}{\pi a}\int dx \cos q\sqrt{2}\theta_a \tag{23}$$

exists with

$$g_q a/(\pi u_a) \sim (t_\perp a/\pi u_a)^q \tag{24}$$

in perturbation. More generally this term is allowed by symmetry and is present in the Hamiltonian even beyond perturbation theory. The term (23) opens a gap,

$$\Delta_{p/q} = \frac{\pi u}{a}\left(\frac{t_\perp a}{\pi u_a}\right)^{(2K_a q)/(4K_a - q^2)} \tag{25}$$

provided $K_a > q^2/4$. The expectation value of θ_a in the gapped phase is $\langle \theta_a \rangle = \sqrt{2}k\pi/q$ where $k = 0, \ldots, q-1$. This corresponds to a q-fold degenerate ground state resulting from the breaking of the discrete translation symmetry, in direct analogy with Mott systems[16] or spin systems.[19,20] The corresponding expectation value of $j_\perp(x)$ is

$$\langle j_\perp(x) \rangle \propto \frac{e^* t_\perp}{\pi a}\sin\left[\frac{2\pi k}{q} + \frac{2\pi p}{a}\frac{x}{a}\right], \tag{26}$$

giving a periodic pattern of the transverse currents of period qa as shown in Fig. 1. This pattern corresponds to a vortex lattice phase pinned to the microscopic lattice with p vortices in a supercell of q sites, leading to an average vortex density $\overline{\rho_V} = p/q$. Let us finally remark that the form of the Hamiltonian (22) implies that the behavior of the system is periodic as a function of the flux with periodicity of one flux quantum.

IV. VORTEX PHASES

Let us now take a flux per plaquette which is an irrational multiple of the quantum of flux. If this value is close to a rational number with a small enough denominator we can decompose $e^*\Phi = 2\pi(p/q) + \delta\Phi$. The most convenient gauge choice is then $A_\perp = (2\pi e^* p)/q$ and $A_a = (\delta\Phi)/(a\sqrt{2})$. Introducing the field P_a conjugate to θ_a, $\pi P_a = \partial_x \phi$, H_a becomes

$$H_a = \int \frac{dx}{2\pi}\left[\frac{u}{K}(\pi P_a)^2 + \frac{u}{K}\left(\partial_x \theta_a + \frac{e^*\Phi}{a\sqrt{2}}\right)^2\right] - \frac{g_q}{\pi a}\int dx \cos q\sqrt{2}\theta_a. \tag{27}$$

The deviation of the flux per plaquette from a rational value in the unit of the quantum of flux causes a commensurate-incommensurate transition.[21–23] At low deviation, the system is in its commensurate phase, $\sqrt{2}\langle \theta_a \rangle = 2\pi k/q$. This implies

$$\langle j_a \rangle = -uK\frac{(e^*)^2\delta\Phi}{\pi a} \tag{28}$$

and no modification of the perpendicular current $\langle \delta j_\perp \rangle = 0$ and

$$\langle \delta j_\perp(x) \delta j_\perp(x') \rangle \sim e^{-|x-x'|/\xi_a} \tag{29}$$

with $\xi_a = u/\Delta_a$.

For a large enough deviation from rationality, $(u/a)\delta\Phi > \Delta_{p/q}$, there is a transition to a phase where the $\cos q\sqrt{2}\theta_a$ operator is irrelevant. In this phase $\langle \partial_x \theta_a \rangle/(\pi\sqrt{2})$ is nonzero and defines an order parameter. In the small applied field case,[10] this order parameter is just the vortex density ρ_V. In the general case, this order parameter measures the deviation $\delta\rho_V = \rho_V - p/q$ of the vortex density from a rational value. The average transverse current is zero in this phase. However, transverse current correlations have an oscillating component that decays with a power law as

$$\langle j_\perp(x) j_\perp(x') \rangle \sim \frac{\cos[2\pi\overline{\rho_V}(x-x')]}{|x-x'|^{1/K_a^*}}, \tag{30}$$

where K_a^* is the renormalized Luttinger parameter. It would be worthwhile to check in a numerical simulation such as Ref. 12 whether such current correlations are present. There is a pattern of transverse currents alternating along the rungs of the ladder. One can identify the solitons in θ_a with vortices surrounded by circulating currents (see Fig. 1). If a vortex is at position x, it causes a jump of 2π in $\theta_1 - \theta_2$ at this point. This enables us to write the density of these one-dimensional objects as[14]

$$\rho_V(x) = \frac{1}{\pi\sqrt{2}} \partial_x \theta_a + \sum_{m=-\infty}^{m=\infty} \frac{C_m}{\pi a} \times \exp(im[2\pi\overline{\rho_V}x + \sqrt{2}(\theta_a - \langle \theta_a \rangle)]). \tag{31}$$

When the density of vortices is p/q, the potential-energy term (23) can be interpreted as the coupling of the vortex density to a pinning potential of period q lattice spacings. Therefore we recover the interpretation in term of pinning of the vortices by the underlying microscopic lattice and the analogy with the Mott transition[16] in $d=1$. Equation (31) gives the vortex current

$$j_V(x) = -\frac{\partial_t \theta_a}{\pi\sqrt{2}}. \tag{32}$$

In the incommensurate phase, the application of a voltage $V_1 - V_2$ between the chains gives

$$\langle \partial_x \phi_a \rangle = e^* \frac{V_1 - V_2}{u_a K_a \sqrt{2}} \tag{33}$$

and

$$V_1 - V_2 = E_\perp b = -\frac{2\pi}{e^*} j_V, \tag{34}$$

which is just Faraday's law. This completes the identification of the incommensurate phase as an unpinned vortex lattice. Because of the linearization of the spectrum in Eq. (5) there is no Laplace force on vortices. This pathology could be cured by putting back the band curvature.

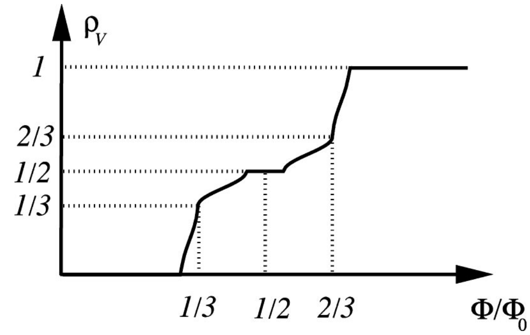

FIG. 2. Sketch of the staircase in the magnetization of the bosonic or Josephson ladders. In the figure, $K_a = 4$, so that only the plateaus obtained for $p/q = 0, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, 1$ survive to the quantum fluctuations. Note that the width of the $\frac{1}{3}$ and $\frac{2}{3}$ plateaus is already extremely reduced compared with the width of the $\frac{1}{2}$ plateau.

In the classical case,[9] a vortex lattice phase is obtained each time the flux per plaquette is a rational multiple of the quantum of flux, leading to a devil's staircase structure in the behavior of the magnetization. Here, the quantum fluctuations wipe out the large fractions for which $q^2 > 4K_a$, so only some plateaus remain as shown in Fig. 2. As quantum fluctuations are reduced, more and more plateaus in the magnetization curve are formed. It is interesting to study how the width of a magnetization plateau increases as a function of K_a or $t_\perp$. Standard renormalization-group calculations of the gap show that the width of a plateau behaves as exp $(-C/\sqrt{t_\perp - t_\perp^c})$ close to the threshold. For $\Phi = \pi/e^*$, a vortex lattice phase has been obtained in numerical simulations.[12] It remains to be seen whether such a phase is a Luttinger liquid or a pinned vortex lattice.

Close to the transition the coefficient K_a^* is universal and $K_a^* = q^2/2$. It is possible in the limit $\delta\Phi \to \delta\Phi_{c1}(p/q)$ to refermionize,[21,23] yielding an average vortex density

$$\langle \rho_V \rangle = \frac{e^*}{2\pi a} \sqrt{\Phi^2 - \Phi_c^2} \tag{35}$$

and a vortex current

$$\langle j_a \rangle = \frac{u(e^*)^2}{2\pi a} (\sqrt{\Phi^2 - \Phi_c^2} - \Phi). \tag{36}$$

The transition from the pinned vortex lattice state to the vortex liquid state is thus continuous. The behavior of the average current as a function of flux is plotted in Fig. 3. This is reminiscent of the behavior of a type-II superconductor, however the growth of the magnetization is here quite different[10] from the standard $B = H + M \sim 1/[\ln^2(H - H_{c1})]$ law. At nonzero temperature, the singularities are rounded off in a way that can be calculated following Ref. 24. In the case of $q=1$, i.e., integer flux per plaquette, close to the transition, one can easily obtain from the refermionization[21,25] the correlation functions of the screening current:

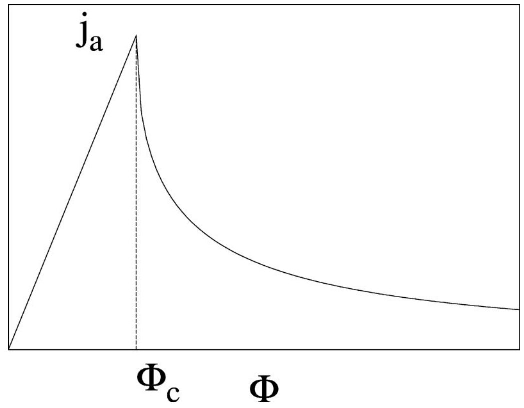

FIG. 3. The behavior of the screening current with Φ. For $\Phi < \Phi_c$, the system is in the Meissner phase and the screening current increases linearly with Φ. For $\Phi > \Phi_c$, the screening current decreases with the applied field.

$$\langle j_a(x) j_a(0)\rangle - \langle j_a\rangle^2 \sim \frac{1}{x^2} + \left(\frac{\Phi_c}{\Phi}\right)^2 \frac{\cos\left[2\pi\bar{\rho}_V \frac{x}{a}\right]}{x^2}, \quad (37)$$

$$\langle j_\perp(x) j_\perp(0)\rangle \sim \left(\frac{\Phi_c}{\Phi}\right)^2 \frac{1}{x^2} + \frac{\cos\left[2\pi\bar{\rho}_V \frac{x}{a}\right]}{x^2} \quad (38)$$

in agreement with Eq. (30) and the universal value $K_a^* = \frac{1}{2}$.

V. CONCLUSIONS

In conclusion we have investigated the effect of a magnetic field on a bosonic two-leg ladder. We have shown that a Meissner state where the magnetic field is expelled exists below a certain critical field H_{c1}, whereas a vortex phase exists above. We have analyzed the physical properties of this vortex phase and have shown that that plateaus in the magnetization for commensurate values of the field exist, in a way similar to the Mott transition in one dimension for commensurate values of the filling. We have determined the magnetization curve and the correlation functions in the vortex phase.

How could the Meissner and the vortex lattice phases be detected in an experiment? First, $H_{c_1} = \Phi_c / ab$ must be smaller than the spin gap. Otherwise, the magnetic field would break the Cooper pairs before vortices are nucleated, a situation reminiscent of type-I superconductivity. Second, in a standard material, $a \simeq 1$ Å so that $H_{c_1} \sim h/(2e^* a^2) \simeq 2.1 \times 10^5$ T making the vortex lattice unobservable. For Josephson-junction ladders, $a \simeq 1$ μm, giving a typical $H_{c1} \simeq 2.1 \times 10^{-3}$ T which is easily achieved. In principle, the formation of the vortex lattice in this system could be observed by magnetization measurements or by resolving individual vortices as was done for other mesoscopic systems.[26] The Meissner transition also affects transport properties at a commensurate filling of an integer number of Cooper pairs per island. In this case, a $\cos 2\phi_s \cos 2\phi_a$ term is present and favors an insulating state.[11] The interchain Josephson coupling $\cos\sqrt{2}\theta_a$ competes with this term making the system conducting when large enough.[27] The application of $H > H_{c1}$ suppresses the interchain Josephson coupling turning the ladder insulating in the vortex phase. The Meissner transition is thus accompanied by a superconductor-insulator transition in this case. At incommensurate filling, the magnetic field still affects transport properties if there is an artificial defect in the ladder, which adds a term

$$V_1 \cos\sqrt{2}\phi_s(x=0)\cos\sqrt{2}\phi_a(x=0) + V_2 \cos\sqrt{8}\phi_s(x=0) \quad (39)$$

to the Hamiltonian. In the Meissner phase, the field $\cos\sqrt{2}\phi_a$ is disordered and the V_2 term dominates transport. If $K>1$ this term is irrelevant and the conductance tends at low temperatures to the quantum of conductance. In the vortex phase on the other hand, the field $\cos\sqrt{2}\phi_a$ has quasi-long-range order and the V_1 term dominates transport, leading (for $K < 2$) to an insulating state at zero temperature. More generally, even when $K<1$ or $K>2$ the conductance has a qualitatively different temperature dependence in the Meissner and in the vortex phases.

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