Microwave absorption in *s*- and *d*-wave disordered superconductors

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We model *s*- and *d*-wave ceramic superconductors with a three-dimensional lattice of randomly distributed 0 and π Josephson junctions with finite self-inductance. The field and temperature dependences of the microwave absoption are obtained by solving the corresponding Langevin dynamical equations. We find that at magnetic field $H=0$ the microwave absoption of the *s*-wave samples, when plotted against the field, has a minimum at any temperature. In the case of *d*-wave superconductors one has a peak at $H=0$ in the temperature region where the paramagnetic Meissner effect is observable. These results agree with experiments. The dependence of the microwave absorption on the screening strength was found to be nontrivial due to the crossover from the weak to the strong screening regime.

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I. INTRODUCTION

A fundamental property of superconductivity is the Meissner effect, i.e., the occurrence of flux expulsion below the superconducting transition temperature and the resulting diamagnetic response to the external magnetic field. Contrary to this behavior a paramagnetic signal was observed in certain ceramic superconductors upon cooling in low enough fields (smaller than 0.1 mT).[1,2] This effect is now referred to as the paramagnetic Meissner effect (PME) or the Wohlleben effect. The nature of the unusual paramagnetic behavior may be related to the appearance of the spontaneous supercurrents (or of orbital moments).[3] The latter occur due to the existence of π junctions characterized by the negative Josephson couplings.[3,4] Furthermore, Sigrist and Rice[5,6] argued that the PME in the high-T_c superconductors is a consequence of the intrinsic unconventional pairing symmetry of the high-temperature superconductors (HTSC's) of $d_{x^2-y^2}$ type.[7] In fact, the PME is succesfully reproduced in a single loop model[5] as well as in a model of interacting junction loops.[8,9] The latter model incorporates a network of Josephson junctions with a random concentration of π junctions. The magnetic screening is taken into account in both of the single and multi-π-junction systems.

The mechanism of the PME based on the *d*-wave symmetry of the order parameter remains ambiguous because it is not clear why this effect could not be observed in many ceramic materials. Furthermore, the paramagnetic response has been seen even in the conventional Nb (Refs. 10–12) and Al (Ref. 13) superconductors and the Nb-AlO_x-Nb tunnel junctions.[14] In order to explain the PME in terms of conventional superconductivity one can employ the idea of the flux compression inside of a sample. Such phenomenon becomes possible in the presence of the inhomogeneities[15] or of the sample boundary.[16] Auletta *et al.*[17] have also observed the PME in the model of special geometry involving only 0 junctions. In our opinion, the PME in this model is of the dynamical nature but not the equillibrium effect as in the *d*-wave model.[9] Thus the intrinsic mechanism leading to the PME is still under debate.[13,18]

One of the most valuable tools to distinguish between the *s*- and *d*-wave symmetry is the study of the microwave absorption (MWA).[18] In fact, Braunish *et al.*[2] have found a nontrivial field dependence of the MWA in samples which display the PME. The MWA has a peak at $H=0$, when plotted against H, whereas for *s*-wave superconductors it has a conventional minimum. Based on a hysteresis in the $M-H$ space (M is a magnetization) Sigrist and Rice have shown that for the one-loop model the peak at $H=0$ would be observed if the dimensionless self-inductance, $\tilde{L}$, exceeds some borderline value $\tilde{L}^*=1$. The relation between $\tilde{L}$ and the inductance, L is as follows:

$$\tilde{L}=\frac{2\pi I_c}{c\Phi_o}L, \tag{1}$$

where Φ_0 and I_c are the flux quantum and the critical current, respectively.

It should be noted that Dominguez *et al.*[19] have qualitatively reproduced the experimental findings for the MWA using the multiloop model. Their results are, however, restricted to the two-dimensional system. More important, the question about the borderline value L^* above which the nontrivial field dependence of the MWA may occur in the *d*-wave interacting loops model was not studied. Also the role of temperature and of the screening have not been explored (the screening, for example, plays a key role in explaining experiments on the aging effect in ceramic superconductors.[20])

Since the underlying mechanism for the PME is still lacking, a careful study of MWA may shed some light on this problem. In this paper we study in detail the MWA in the three-dimensional system which is more relevant to experimental situations than the two-dimensional one. Integrating the corresponding Langevin equations we have made three unusual observations. First, contrary to the one-loop model the MWA in the system with randomly distributed π junctions has a nontrivial field dependence for any value of $\tilde{L}$. In other words, $\tilde{L}^*=0$ for the multiloop model. Second, the peak at $H=0$ is found to disappear for $T>T^*$, where T^* is a borderline temperature below which the PME is observable. T^* was found to grow as the screening is lowered. The third observation is that for both *s*- and *d*-wave ceramics the

MWA decreases with screening not monotonically but it has a minimum at $\tilde{L}\approx 1$. Such a behavior is related to a change in time and length scales when one goes from the weak to the strong screening limit.

II. MODEL

We neglect the charging effects of the grains and consider the following Hamiltonian:[8,9]

$$\mathcal{H}=-\sum_{\langle ij\rangle} J_{ij}\cos(\theta_i-\theta_j-A_{ij})+\frac{1}{2L}\sum_{p}(\Phi_p-\Phi_p^{ext})^2,$$

$$\Phi_p=\frac{\phi_0}{2\pi}\sum_{\langle ij\rangle}^{p}A_{ij}, \quad A_{ij}=\frac{2\pi}{\phi_0}\int_i^j\vec{A}(\vec{r})d\vec{r}, \tag{2}$$

where θ_i is the phase of the condensate of the grain at the ith site of a simple cubic lattice, $\vec{A}$ is the fluctuating gauge potential at each link of the lattice, ϕ_0 denotes the flux quantum, J_{ij} denotes the Josephson coupling between the ith and jth grains, L is the self-inductance of a loop (an elementary plaquette), while the mutual inductance between different loops is neglected. The first sum is taken over all nearest-neighbor pairs and the second sum is taken over all elementary plaquettes on the lattice. Fluctuating variables to be summed over are the phase variables θ_i at each site and the gauge variables A_{ij} at each link. Φ_p is the total magnetic flux threading through the pth plaquette, whereas Φ_p^{ext} is the flux due to an external magnetic field applied along the z direction,

$$\Phi_p^{ext}=\begin{cases} HS & \text{if } p \text{ is on the } \langle xy\rangle \text{ plane} \\ 0 & \text{otherwise,}\end{cases} \tag{3}$$

where S denotes the area of an elementary plaquette. For the d-wave superconductors we assume J_{ij} to be an independent random variable taking the values J or $-J$ with equal probability ($\pm J$ or bimodal distribution), each representing 0 and π junctions. In the case of s-wave ceramics J_{ij} is always positive but distributed uniformly between 0 and $2J$.

It should be noted that model (2) is adequate to describe many dynamical phenomena related to the PME such as the compensation effect,[21] the aging phenomenon,[20] the effect of applied electric fields in the apparent critical current,[22] and the ac resistivity.[23]

In order to study the MWA we have to calculate the linear response to an external electromagnetic field. Using the relation between the MWA and the conductivity and the Kubo formula[24] one can show that this response is proportional to a voltage-voltage correlation function. Integrating over all of frequencies of the electromagnetic field we obtain the following expression for the MWA:

$$\Omega=\frac{4\pi}{cnRT}\sum_i\langle V_i^2\rangle, \tag{4}$$

where $\langle V_i^2\rangle$ is a mean value of the square of the voltage induced by the thermal noise on each junction, n is a light refraction coefficient and R is the normal resistance of the links.

To calculate V_i we model the current flowing between two grains with the resistively shunted junction (RSJ) model,[8,25] which gives the following dynamical equations:

$$\frac{\hbar}{2eR}\frac{d\theta_\mu(\mathbf{n})}{dt}=-\frac{2e}{\hbar}J_\mu(\mathbf{n})\sin\theta_\mu(\mathbf{n}) -\frac{\hbar}{2e\mathcal{L}}\Delta_\nu^-[\Delta_\nu^+\theta_\mu(\mathbf{n})-\Delta_\mu^+\theta_\nu(\mathbf{n})]-\eta_\mu(\mathbf{n},t). \tag{5}$$

Here we have redefined notation: the site of each grain is at position $\mathbf{n}=(n_x,n_y,n_z)$ (i.e., $i\equiv\mathbf{n}$); the lattice directions are $\mu=\hat{\mathbf{x}},\hat{\mathbf{y}},\hat{\mathbf{z}}$; the link variables are between sites $\mathbf{n}$ and $\mathbf{n}+\mu$ (i.e., link $ij\equiv$ link $\mathbf{n},\mu$); and the plaquettes p are defined by the site $\mathbf{n}$ and the normal direction μ (i.e., plaquette p $\equiv$plaquette $\mathbf{n},\mu$, for example the plaquette $\mathbf{n},\hat{\mathbf{z}}$ is centered at position $\mathbf{n}+(\hat{\mathbf{x}}+\hat{\mathbf{y}})/2$). The forward difference operator $\Delta_\mu^+\theta_\nu(\mathbf{n})=\theta_\nu(\mathbf{n}+\mu)-\theta_\nu(\mathbf{n})$ and the backward operator $\Delta_\mu^-\theta_\nu(\mathbf{n})=\theta_\nu(\mathbf{n})-\theta_\nu(\mathbf{n}-\mu)$. The Langevin noise current $\eta_\mu(\mathbf{n},t)$ has Gaussian correlations

$$\langle\eta_\mu(\mathbf{n},t)\eta_{\mu'}(\mathbf{n}',t')\rangle=\frac{2k_BT}{R}\delta_{\mu,\mu'}\delta_{\mathbf{n},\mathbf{n}'}\delta(t-t'). \tag{6}$$

The local voltage V_i is then given by

$$V_i=\frac{d\theta_i}{dt}. \tag{7}$$

Equation (5) describes the overdamped dynamics. We have tried to include the inertia (capacitive) terms but the results do not change substantially and they are neglected.

In what follows we will consider currents normalized by $I_J=2eJ/\hbar$, time by $\tau=\phi_0/2\pi I_JR$, voltages by RI_J, inductance by $\phi_0/2\pi I_J$ and temperature by J/k_B. Then the dimensionless MWA, $\tilde{\Omega}$ is defined as follows:

$$\tilde{\Omega}=\frac{cnR}{4\pi}\Omega. \tag{8}$$

III. RESULTS

The system of differential Eqs. (5) is integrated numerically by a second-order Runge-Kutta-Helfand-Greenside algorithm for stochastic differential equations.[26] The time step depends on $\tilde{L}$ and is equal to $\Delta t=0.1\tau_J$ and $\Delta t=0.1\tau_J\times\tilde{L}$ for $\tilde{L}>1$ and $\tilde{L}<1$, respectively. We consider the system size $l=8$ (we have made some test runs for $l=12$ and found that the finite-size effects are not substantial). The temporal averages are taken over a time of $10^5\tau_J$ after a transient time of $25\,000\tau_J$. The free boundary conditions are implemented because the magnetization always vanishes for the periodic boundary conditions.[8,9]

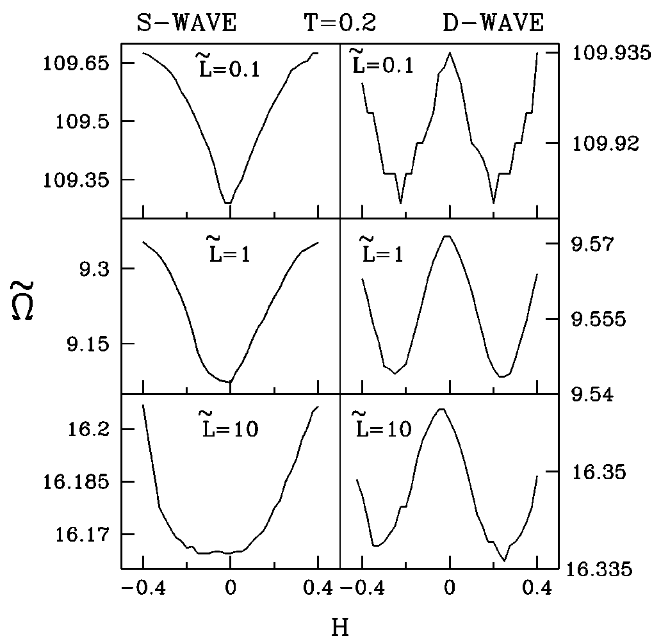

FIG. 1. The field dependence of Ω for *s*- (left panel) and *d*-wave (right panel) ceramic superconductors. We chose $T=0.2$ and $\tilde{L}=0.1$, 1, and 10. The results are averaged over 20 samples.

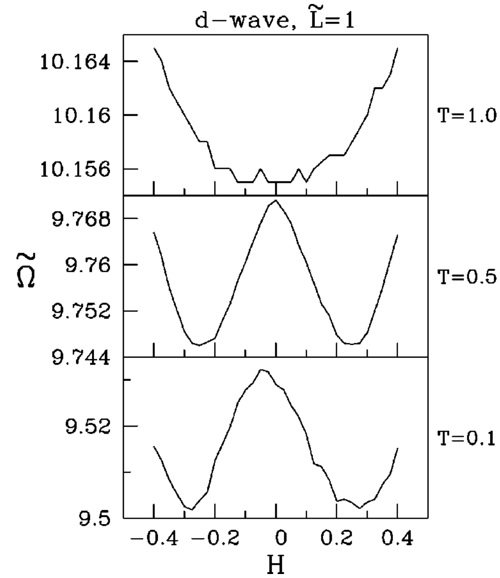

FIG. 2. The field dependence of the MWA for the *d*-wave samples. We took $l=8$, $\tilde{L}=1$ and $T=0.1$, 0.5, and 1. The results are averaged over 10–20 samples.

Figure 1 shows the field dependence of the MWA for $T=0.2$ and for various values of $\tilde{L}$. In the case of *s*-wave superconductors we have the standard minimum at $H=0$ for any value of inductance. It is also true for any *T*. As expected, $\tilde{\Omega}\sim H^2$ at weak fields. For the *d*-wave samples Ω has the unconventional peak at $H=0$. Contrary to the one-loop model[5] such peak is seen not only for $\tilde{L}>1$ but also for $\tilde{L}\leqslant 1$. It should be noted that the height of the peak is very small ($[\tilde{\Omega}(H=0)-\tilde{\Omega}_{min}]/\tilde{\Omega}_{min}$ is of order of 10^{-3}). This is in qualitative agreement with experimental findings[2] that the peak should be low.

In our model (2) the temperature dependence of the critical current is neglected. However, one can show that the dimensionless temperature *T* chosen in Fig. 1 and in all of the figures presented below corresponds to the relevant to experiments real temperature T_R. In fact, the critical current depends not only on temperature but also on conditions under which samples were prepared. The typical value of the critical current density for ceramic superconductors is $\sim 10^6$ A/m^2 (see, for example, Ref. 27). Since the typical size of grains is about 1 μm we have the critical current $I_c\sim 10^{-6}$ A. Using $T_R=JT/k_B=\hbar I_c T/2ek_B$ one obtains $T_R/T\sim 100$ K. Clearly, our dimensionless *T* correctly describes the experimental values of temperature.[2]

It is known that the random π-junction model (2) displays a phase transition to a so called chiral glass.[28] The frustration effect due to the random distribution of π junctions leads to a glass state of quenched-in ''chiralities,'' which are local loop supercurrents circulating over grains and carrying a half quantum of flux. Evidence of this transition has been related to measurements of the nonlinear ac magnetic susceptibility.[29] The question we ask is if there is any correlation between the existence of the chiral glass phase and the anomalous behavior of Ω. As shown in Ref. 28, the chiral glass disappears for $\tilde{L}>\tilde{L}_c$, where $\tilde{L}_c=5-7$. On the other hand, the peak of Ω is obserable for any value of $\tilde{L}$. Therefore there is no one-to-one correlation between the chiral glass and the nontrivial field dependence of the MWA of the ceramic superconductors.

The field dependence of the MWA in the *d*-wave superconductors for $\tilde{L}=1$ and various values of *T* is shown in Fig. 2. At low *T*'s the peak at $H=0$ shows up but *it disappears at high temperatures*. This is our main result. Such an observation was not reported in Ref. 19. Qualitatively, above some borderline temperature, T^* the frustration effect becomes less important and the *d*-wave system should behave like the *s*-wave one.

Figure 3 shows the dependence of T^* on $\tilde{L}$. The question

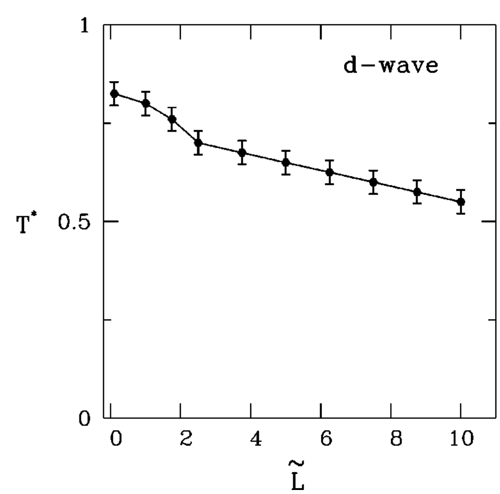

FIG. 3. The inductance dependence of T^* for *d*-wave superconductors.

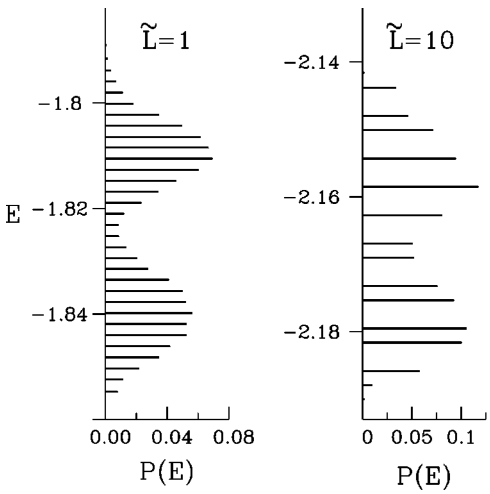

FIG. 4. The energy local minima histogram, $P(E)$, for $\tilde{L}=1$ and 10. We chose the system size $l=6$. The energy bin used for collecting the histogram is equal to 0.002. The local minima are obtained from 10 000 starting configurations by annealing at $T>0$ and quenching at $T=0$.

we ask now is why T^* decreases with $\tilde{L}$. To answer this question we study the dependence of the roughness of the energy landscape on the screening. Since the number of energy local minima should grow with the number of grains (or of spins) exponentially[30] we restrict our calculations to small system sizes. We took $l=6$ and search for the local minima by the annealing procedure at $T>0$ and then by the quenching at $T=0$. The histogram, $P(E)$, collected from local minima which are reached from 10 000 starting configuations is shown in Fig. 4. Obviously, the energy landscape for $\tilde{L}=1$ is more rugged compared to the $\tilde{L}=10$ case.

In order to characterize the roughness of the energy landscape we introduce the parameter δ,

$$\delta=\frac{\sqrt{\langle E_{lm}^2\rangle-\langle E_{lm}\rangle^2}}{\langle E_{lm}\rangle}, \tag{9}$$

where E_{lm} denotes the energy at local minima, $\langle\cdots\rangle$ means averaging over all minima studied. For the results presented in Fig. 4 we have $\delta=0.007$ and 0.012 for $\tilde{L}=10$ and 1, respectively. So the larger is screening the smaller roughness of the energy landscape. The difference between *s*- and *d*-wave ceramics becomes therefore less and less pronounced as the screening increased and T^* should go down with $\tilde{L}$.

In order to understand the nature of T^* we calculate the field-cooled (FC) and zero-field-cooled (ZFC) magnetization. In our model the magnetization is defined as follows:

$$M=\left\langle \frac{1}{l(l-1)^2}\sum_p \frac{\Phi_p^z}{\Phi_0}\right\rangle-\frac{HS}{\Phi_0}, \tag{10}$$

where Φ_p^z is the flux in the *xy* plane, $\langle\cdots\rangle$ denotes the thermal and disorder average. In the FC runs, the temperature is lowered stepwise under a constant field. At each temperature, typically 10^5 time steps are used for thermalization and 4×10^5 steps for averaging. In the ZFC runs, the system is first quenched to a low temperature ($T=0.05$) in zero field and is thermalized during 4×10^5 steps. Then a static field is switched on and the temperature is increased stepwise under the same condition as in the FC regime.

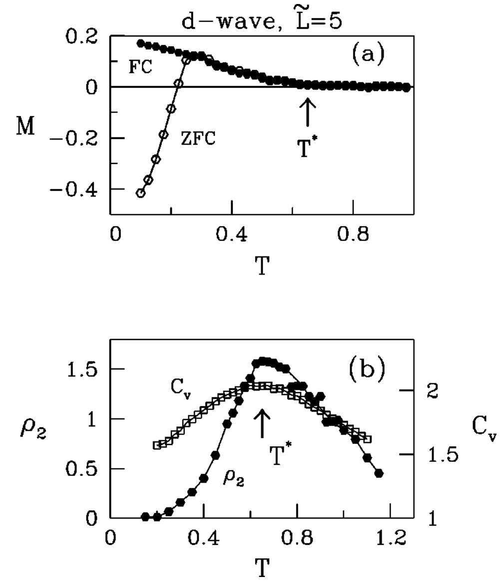

FIG. 5. (a)The temperature dependence of the FC and ZFC magnetization for the *d*-wave superconductors. $\tilde{L}=5$. (b) The same as in (a) but for C_v (right-hand scale) and ρ_2 (left-hand scale). The latter was computed for the frequency of the ac electric field $\omega=0.001$ and its magnitude $I_0=0.1$ (see Ref. 23 for details). The results are averaged over 20–40 samples.

Figure 5(a) shows the FC and ZFC magnetization for $\tilde{L}=5$ at a finite magnetic field $f=HS/\phi_0=0.1$. We can see that T^* is the temperature below which one has an onset of positive magnetization, i.e., the paramagnetic Meissner effect starts to be observed. The irreversibility point occurs at temperatures lower than T^*, and its position is dependent on the heating or cooling rate. We identify T^* to correspond to the onset of chiral short-range order where pair loops begin to appear locally. The main conclusion here is that the anomalous field dependence of the MWA is strongly correlated with the occurrence of the PME. On the other hand, as was shown in Ref. 9, the PME may appear for any value of screening, we conclude that for the multiloop model the borderline value $\tilde{L}^*$ for the anomalous behavior of MWA is equal to 0.

Figure 5(b) shows the temperature dependence of the specific heat C_v and the nonlinear ac resistivity ρ_2 for $\tilde{L}=5$. C_v is defined as porportional to the energy fluctuations, $C_v=\langle(\delta E)^2\rangle/T^2$. The definition of ρ_2 is given in Ref. 23. Clearly, T^* coincides with the peak of C_v and ρ_2.

We now study the dependence of $\tilde{\Omega}$ on the screening. Our results are shown in Fig. 6 for $H=0$ but the qualitative behavior is also valid for $H\neq0$. There is no appreciable difference between *s*- and *d*-wave cases. For a fixed value of $\tilde{L}$, $\tilde{\Omega}(H=0)$ depends on T very weakly. The dependence on screening is more pronounced. From naive arguments the MWA should decrease with $\tilde{L}$ because the screening would

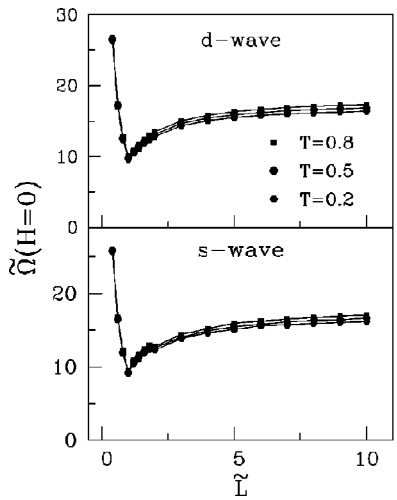

FIG. 6. The inductance dependence of the MWA at $H=0$ for $T=0.2$ (squares), 0.5 (hexagons), and 0.8 (circles). The system size $l=8$. The results are averaged over ten samples.

prevent the absorption in the bulk. Figure 6 shows, however, that $\tilde{\Omega}$ has a minimum at $\tilde{L}=1$. The anomalous dependence of MWA on $\tilde{L}$ may be understood in the following way. The static and dynamic properties of the Josephson arrays are shown[31,25] to change qualitatively if the inductance varies from $\tilde{L}\langle 1$ to $\tilde{L}>1$. The attractive vortex-vortex interaction in the weak screening regime becomes repulsive in the opposite limit. The qualitative change in the dynamic response is related to the change of length and time scales. Since the magnetic screening length goes as $\lambda\sim\tilde{L}^{-1/2}$ (see Ref. 25), for $\tilde{L}<1\lambda$ is larger than the grain size (lattice spacing in the cubic network) while for $\tilde{L}>1\lambda$ becomes smaller than the grain size. For $\tilde{L}<1$ the relaxation time for the field is smaller than the relaxation time for the phases whereas the opposite happens for $\tilde{L}>1$. The decrease of the phase relaxation time compared to the field one should therefore increase the MWA for $\tilde{L}>1$.

IV. CONCLUSION

In conclusion, experimental results of Braunish *et al.*[2] for the MWA can be reproduced by the XY-like model for the *d*-wave superconductor. Although the peak of $\tilde{\Omega}$ is found to be small, its study is useful for elucidating the symmetry of the superconducting order parameter. Within the multiloop model the anomalous behavior should be observable for any value of inductance and if $T<T^*$. At high temperatures there is no qualitative difference between the *s*- and *d*-wave systems. The dependence of the MWA on the screening strength is found to be not monotonic due to the crossover from the weak to the strong screening regime. It would be very interesting to verify this prediction experimentally.

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