Finite-temperature time-dependent effective theory for the phase field in two-dimensional *d*-wave neutral superconductors

S. G. Sharapov* and H. Beck†
Institut de Physique, Université de Neuchâtel, 2000 Neuchâtel, Switzerland

V. M. Loktev‡
Bogolyubov Institute for Theoretical Physics, Metrologicheskaya Str. 14-b, Kiev, 03143, Ukraine
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We derive finite-temperature time-dependent effective actions for the phase of the pairing field, which are appropriate for a two-dimensional electron system with both nonretarded *d*- and *s*-wave attraction. As for *s*-wave pairing, the *d*-wave effective action contains terms with Landau damping, but their structure appears to be different from the *s*-wave case due to the fact that the Landau damping is determined by the quasiparticle group velocity $\mathbf{v}_g$, which for *d*-wave pairing does not have the same direction as the noninteracting Fermi velocity $\mathbf{v}_F$. we show that for *d*-wave pairing the Landau term has a linear low-temperature dependence and in contrast to the *s*-wave case is important for all finite temperatures. A possible experimental observation of the phase excitations is discussed.

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I. INTRODUCTION

The microscopic derivation of the effective time-dependent Ginzburg-Landau (GL) theory continues to attract attention since an early paper by Abrahams and Tsuneto.[1] Whereas the static GL potential was derived[2] from the microscopic BCS theory soon after its introduction, the time-dependent GL theory is still a subject of interest (see Refs. 1,2,3 and 4 for a review on the problem's history). One of the reasons for this is the presence of Landau damping terms in the effective action. For *s*-wave superconductivity these terms are singular at the origin of energy-momentum space, and consequently they cannot be expanded as a Taylor series about the origin. In other words, these terms do not have a well-defined expansion in terms of space and time derivatives of the ordering field and therefore they cannot be represented as a part of a local Lagrangian. We recall that at $T=0$ and for the static (time-independent) case the Landau damping vanishes, so that either at $T=0$ one still has a local well-defined time-dependent GL theory or for $T\neq 0$ the familiar static GL theory exists. It is known, however, that for *s*-wave superconductivity even though the Landau terms do exist, they appear to be small compared to the main terms of the effective action in the large temperature region $0<T\lesssim 0.6T_c$,[4] where T_c is the superconducting transition temperature. This is evidently related to the fact that only thermally excited quasiparticles contribute to the Landau damping. The number of such quasiparticles at low temperatures appears to be a small fraction of the total charge carriers number in the *s*-wave superconductor due to the nonzero superconducting gap Δ_s which opens over all directions on the Fermi surface.

For a *d*-wave superconductor there are four Dirac points (nodes) where the superconducting gap $\Delta_d(\mathbf{k})$ becomes zero on the Fermi surface. The presence of the nodes increases significantly the number of the thermally excited quasiparticles at given temperature T comparing to the *s*-wave case. Therefore one can expect that the Landau damping should be stronger for superconductors with a *d*-wave gap which is commonly accepted to be the case of high-temperature superconductors (HTSC's).[5] Moreover, it is believed that at temperatures $T\ll T_c$, these quasiparticles are reasonably well described by the Landau quasiparticles, even though such an approach fails in these materials at higher energies.[6] This is the reason why one can hope that a generalization of the BCS-like approach[4] for the 2D *d*-wave superconductivity may be relevant to the description of the low-temperature time-dependent GL theory in HTSC's.

In this work we derive such a theory from a microscopical model with *d*-wave pairing extending the approach of Ref. 1 developed for *s*-wave superconductivity. As known from Ref. 1 the physical origin of the Landau damping is a scattering of the thermally excited quasiparticles ("normal" fluid) with group velocity $\mathbf{v}_g$ from the excitations of phase (or θ) quantums. Such conversion occurs only if the Čerenkov condition, $\Omega=\mathbf{v}_g\mathbf{K}$ for the energy Ω and momentum $\mathbf{K}$ of the θ excitation, is satisfied. This phenomenon in superconductivity is also called Landau damping since its equivalent in the plasma theory was originally obtained by Landau (see, e.g., Ref. 7).

To emphasize the difference between the Landau damping for *s*- and *d*-wave pairing, we also derive for the comparison the corresponding terms for a two-dimensional (2D) *s*-wave superconductor. In addition we compare 2D expressions obtained here with the 3D *s*-wave case studied in Ref. 4. The collective phase oscillations in charged *d*-wave superconductors for clean and dirty cases were recently studied in Ref. 8. Due to the complexity of the corresponding equations they were solved numerically, neglecting damping of the phase excitations. Thus our fully analytical treatment can be very useful for further studies of the phase excitations. We also mention a recent paper[9] where the effective action for the phase mode in the *d*-wave superconductor was obtained using the cumulant expansion. The Landau terms were neglected in Ref. 9 but the effect of Coulomb interaction was taken into account.

Our main results can be summarized as follows:

(i) We find that the main physical difference between the s- and d-wave cases is related to the fact that for d-wave superconductivity the direction of the quasiparticle group velocity $\mathbf{v}_g(\mathbf{k}) \equiv \partial E(\mathbf{k})/\partial \mathbf{k}$ [$E(\mathbf{k})$ is the quasiparticle dispersion law] does not coincide with the Fermi velocity $\mathbf{v}_F$,[6,10] and a gap velocity $\mathbf{v}_\Delta \equiv \partial \Delta_d(\mathbf{k})/\partial \mathbf{k}$ also enters into the Čerenkov condition along with $\mathbf{v}_F$.

(ii) We show that the intensity of the Landau damping has a linear temperature dependence at low T with a coefficient expressed in terms of the anisotropy $\alpha_D \equiv v_F/v_\Delta$ of the Dirac spectrum $E(\mathbf{k}) = \sqrt{v_F^2 k_1^2 + v_\Delta^2 k_2^2}$. Here $k_1(k_2)$ are the projections of the quasiparticle momentum on the directions perpendicular (parallel) to the Fermi surface. The parameters v_F, v_Δ, and α_D proved to be very convenient both in the theory, for example, of the transport phenomena,[6] ultrasonic attenuation[10] in d-wave superconductors, and for the analysis of various experiments.[11]

(iii) We find that the Landau damping is sensitive to the direction of the phason momentum. In particular, for a given node the Landau damping is possible only if the components of the phason momentum $\mathbf{K} = (K_1, K_2)$ (which are defined exactly as the components of the quasiparticle momentum above) satisfy the condition $|\Omega| < \sqrt{v_F^2 K_1^2 + v_\Delta^2 K_2^2}$.

(iv) We derive a simple approximate representation for the Landau damping terms which can be useful for further studies of the d-wave superconductors.

(v) We derive an approximate expression for the propagator of the Bogolyubov-Anderson mode which includes the Landau damping.

(vi) Concerning the mathematical formalism used in the paper, we adapt the bilocal Hubbard-Stratonovich field method of Ref. 12 to the d-wave pairing. Additionally we adjust the technique of the derivative expansion for the "phase only" action (see Ref. 4 and references therein) for the model with the tight-binding spectrum.

The paper is organized as follows: In Sec. II we present our model and write down the partition function using a bilocal Hubbard-Stratonovich field. In Sec. III, we introduce the modulus-phase variables and represent the effective "phase only" action as an infinite series. It appears that the low-energy phase dynamics is contained in the first two terms which are evaluated,respectively, in Secs. IV and V with some details considered in the Appendix. The effective Lagrangians for the d- and s-wave cases without the Landau damping are discussed in Sec. VI. In Sec. VII we derive the damping terms and in detail compare d- and s-wave cases. The approximate forms of the effective action and θ propagator are considered in Sec. VIII. Section IX presents our conclusions and comments on a possible experimental observation of the phase excitations.

II. MODEL

Let us consider the following action:

$$S = -\int_0^\beta d\tau \left[\sum_\sigma \int d^2 r \, \psi_\sigma^\dagger(\tau, \mathbf{r}) \partial_\tau \psi_\sigma(\tau, \mathbf{r}) + H(\tau) \right],$$

$$\mathbf{r} = (x, y), \quad \beta \equiv \frac{1}{T}, \tag{1}$$

where the Hamiltonian $H(\tau)$ is

$$H(\tau) = \sum_\sigma \int d^2 r \, \psi_\sigma^\dagger(\tau, \mathbf{r}) [\varepsilon(-i\nabla) - \mu] \psi_\sigma(\tau, \mathbf{r})$$

$$- \frac{1}{2} \sum_\sigma \int d^2 r_1 \int d^2 r_2 \, \psi_\sigma^\dagger(\tau, \mathbf{r}_2) \psi_{\bar{\sigma}}^\dagger(\tau, \mathbf{r}_1)$$

$$\times V(\mathbf{r}_1; \mathbf{r}_2) \psi_{\bar{\sigma}}(\tau, \mathbf{r}_1) \psi_\sigma(\tau, \mathbf{r}_2). \tag{2}$$

Here $\psi_\sigma(\tau, \mathbf{r})$ is a fermion field with the spin $\sigma = \uparrow, \downarrow, \bar{\sigma} \equiv -\sigma, \tau$ is the imaginary time, and $V(\mathbf{r}_1; \mathbf{r}_2)$ is an attractive potential. For the sake of simplicity we consider the dispersion law $\varepsilon(\mathbf{k}) = -2t(\cos k_x a + \cos k_y a)$ for a model on a square lattice with the constant a including the nearest-neighbor hopping t only. This, however, is not an essential restriction because the final results for the d-wave case will be formulated in terms of the noninteracting Fermi velocity $\mathbf{v}_F \equiv \partial \varepsilon(\mathbf{k})/\partial \mathbf{k}|_{\mathbf{k}=\mathbf{k}_F}$ and the gap velocity $\mathbf{v}_\Delta$ defined in the Introduction. We set $\hbar = k_B = 1$.

The bilocal Hubbard-Stratonovich fields $\Phi(\tau, \mathbf{r}_1; \mathbf{r}_2)$ and $\Phi^\dagger(\tau, \mathbf{r}_1; \mathbf{r}_2)$ (see, e.g., Ref. 12) can be utilized to study the model (1), (2):

$$\exp\left[\int_0^\beta d\tau \int d^2 r_1 \int d^2 r_2 \, \psi_\uparrow^\dagger(\tau, \mathbf{r}_2) \psi_\downarrow^\dagger(\tau, \mathbf{r}_1) V(\mathbf{r}_1; \mathbf{r}_2) \psi_\downarrow(\tau, \mathbf{r}_1) \psi_\uparrow(\tau, \mathbf{r}_2) \right]$$

$$= \int \mathcal{D}\Phi^\dagger(\tau, \mathbf{r}_1; \mathbf{r}_2) \mathcal{D}\Phi(\tau, \mathbf{r}_1; \mathbf{r}_2) \exp\left[-\int_0^\beta d\tau \int d^2 r_1 \int d^2 r_2 \frac{1}{V(\mathbf{r}_1; \mathbf{r}_2)} \left| \Phi(\tau, \mathbf{r}_1; \mathbf{r}_2) \right|^2 \right.$$

$$\left. + \int_0^\beta d\tau \int d^2 r_1 \int d^2 r_2 [\Phi^\dagger(\tau, \mathbf{r}_1; \mathbf{r}_2) \psi_\downarrow(\tau, \mathbf{r}_1) \psi_\uparrow(\tau, \mathbf{r}_2) + \psi_\uparrow^\dagger(\tau, \mathbf{r}_1) \psi_\downarrow^\dagger(\tau, \mathbf{r}_2) \Phi(\tau, \mathbf{r}_1; \mathbf{r}_2)] \right]. \tag{3}$$

On the right-hand side, $1/V(\mathbf{r}_1;\mathbf{r}_2)$ is understood as numeric division, no matrix inversion being implied. The Hermitian conjugate of $\Phi(\tau,\mathbf{r}_1;\mathbf{r}_2)$ includes the transpose in the functional sense, i.e., $\Phi^\dagger(\tau,\mathbf{r}_1;\mathbf{r}_2)\equiv[\Phi(\tau,\mathbf{r}_2;\mathbf{r}_1)]^*$.

Thus in the Nambu variables

$$\Psi(\tau,\mathbf{r})=\begin{pmatrix}\psi_\uparrow(\tau,\mathbf{r})\\ \psi_\downarrow^\dagger(\tau,\mathbf{r})\end{pmatrix},\quad \Psi^\dagger(\tau,\mathbf{r})=(\psi_\uparrow^\dagger(\tau,\mathbf{r})\,\psi_\downarrow(\tau,\mathbf{r})) \tag{4}$$

the partition function can be written as

$$Z=\int \mathcal{D}\Psi^\dagger\mathcal{D}\Psi\mathcal{D}\Phi^\dagger\mathcal{D}\Phi\,\exp\left\{\int_0^\beta d\tau\int d^2r_1\int d^2r_2\left[-\frac{1}{V(\mathbf{r}_1;\mathbf{r}_2)}|\Phi(\tau,\mathbf{r}_1;\mathbf{r}_2)|^2+\Psi^\dagger(\tau,\mathbf{r}_1)[-\partial_\tau-\tau_3\xi(-i\tau_3\nabla)]\right.\right.$$
$$\left.\left.\times\Psi(\tau,\mathbf{r}_2)\delta(\mathbf{r}_1-\mathbf{r}_2)+\Phi^\dagger(\tau,\mathbf{r}_1;\mathbf{r}_2)\Psi^\dagger(\tau,\mathbf{r}_1)\tau_-\Psi(\tau,\mathbf{r}_2)+\Psi^\dagger(\tau,\mathbf{r}_1)\tau_+\Psi(\tau,\mathbf{r}_2)\Phi(\tau,\mathbf{r}_1;\mathbf{r}_2)\right]\right\}, \tag{5}$$

where $\xi(-i\tau_3\nabla)\equiv\varepsilon(-i\tau_3\nabla)-\mu$ and $\tau_3,\tau_\pm=(\tau_1\pm i\tau_2)/2$ are Pauli matrices.

In general an electron-electron attraction on the nearest-neighbor lattice sites can be considered (see, for example, Refs. 13 and 14). The momentum representation for this interaction contains in the pairing channel extended *s*-, *d*-, and even *p*-wave pairing terms:

$$\begin{aligned}V(\mathbf{k}-\mathbf{k}')&=V[\cos(k_x-k_x')a+\cos(k_y-k_y')a]\\&=\frac{V}{2}(\cos k_xa+\cos k_ya)(\cos k_x'a+\cos k_y'a)\\&\quad+\frac{V}{2}(\cos k_xa-\cos k_ya)(\cos k_x'a-\cos k_y'a)\\&\quad+V(\sin k_x\sin k_x'+\sin k_y\sin k_y').\end{aligned} \tag{6}$$

Motivated by HTSC's we consider here *d*-wave pairing only, so that for the Fourier transform of the attracting potential $V(\mathbf{r}_1-\mathbf{r}_2)$ we use

$$V(\mathbf{k}-\mathbf{k}')=V_d(\cos k_xa-\cos k_ya)(\cos k_x'a-\cos k_y'a). \tag{7}$$

As was mentioned in the Introduction, we will compare the main results for the *d*-wave case with the simplest 2D continuum *s*-wave pairing model (see, e.g., Ref. 15 and the review Ref. 16) which has a quadratic dispersion law $\varepsilon(\mathbf{k})=\mathbf{k}^2/2m$ and a local attraction $V(\mathbf{r}_1-\mathbf{r}_2)=V\delta(\mathbf{r}_1-\mathbf{r}_2)$.

III. EFFECTIVE ACTION

While for the model with the local four-fermion attraction[4,15] the modulus-phase variables could be introduced exactly, one should apply an additional approximation for treating the present model.

Let us split the charged Fermi fields $\psi(\tau,\mathbf{r})$ and $\psi^\dagger(\tau,\mathbf{r})$ in Eq. (5) into the neutral Fermi field $\chi(\tau,\mathbf{r})$ (Ref. 17) and charged Bose-field $\exp[i\theta(\tau,\mathbf{r})/2]$,

$$\begin{aligned}\psi_\sigma(\tau,\mathbf{r})&=\chi_\sigma(\tau,\mathbf{r})\exp[i\theta(\tau,\mathbf{r})/2],\\ \psi_\sigma^\dagger&=\chi_\sigma^\dagger(\tau,\mathbf{r})\exp[-i\theta(\tau,\mathbf{r})/2].\end{aligned} \tag{8}$$

It is clear that the terms containing $\delta(\mathbf{r}_1-\mathbf{r}_2)$ in Eq. (5) can be treated similarly to the old model and the problem arises when one deals with the Hubbard-Stratonovich field. To consider this field we introduce the relative $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$ and center-of-mass coordinates $\mathbf{R}=(\mathbf{r}_1+\mathbf{r}_2)/2$. Now we can introduce the modulus-phase representation for the Hubbard-Stratonovich field,

$$\Phi(\tau,\mathbf{r}_1,\mathbf{r}_2)\equiv\Phi(\tau,\mathbf{R},\mathbf{r})=\Delta(\tau,\mathbf{R},\mathbf{r})\exp[i\theta(\tau,\mathbf{R},\mathbf{r})], \tag{9}$$

where $\Delta(\mathbf{R,r})$ is the modulus of the Hubbard-Stratonovich field and $\theta(\mathbf{R,r})$ is its phase.

Assuming that the global phase $\theta(\mathbf{R,r})$ varies slowly over distances on the order of a Cooper pair size and thus is not sensitive to the inner pair structure described by the relative variable $\mathbf{r}$, we can rewrite Eq. (9) as

$$\Phi(\tau,\mathbf{R},\mathbf{r})\approx\Delta(\tau,\mathbf{R},\mathbf{r})\exp[i\theta(\tau,\mathbf{R})]. \tag{10}$$

The approximation we made writing Eq. (10) is in fact equivalent to the Born-Oppenheimer approximation[18] which allows one to separate the dynamics of the Cooper pair formation described by the relative coordinate $\mathbf{r}$ in $\Delta(\tau,\mathbf{R,r})$ from the motion of the superconducting condensate described by the center-of-mass coordinate $\mathbf{R}$ in $\theta(\tau,\mathbf{R})$. If the condensate motion is slow enough this separation becomes possible because the dynamics of the Cooper pair formation can always follow the motion of the condensate. Using the lattice language one can also say about Eq. (10) that the *bond* phase is replaced by the *site* phase.[19]

Applying the transformations (10) and (8) to the terms with the Hubbard-Stratonovich field in Eq. (3) we obtain (the imaginary time τ is omitted)

$$\begin{aligned}&\Phi^\dagger(\mathbf{R};\mathbf{r})\psi_\downarrow(\mathbf{r}_1)\psi_\uparrow(\mathbf{r}_2)\\&\quad=\Phi^\dagger(\mathbf{R};\mathbf{r})\chi_\downarrow(\mathbf{r}_1)\exp\left[\frac{i\theta(\mathbf{R}+\mathbf{r}/2)}{2}\right]\chi_\uparrow(\mathbf{r}_2)\\&\qquad\times\exp\left[\frac{i\theta(\mathbf{R}-\mathbf{r}/2)}{2}\right]\end{aligned}$$

$$\approx \Delta(\mathbf{R};\mathbf{r})\exp[-i\theta(\mathbf{R})]\chi_{\downarrow}(\mathbf{r}_1)\chi_{\uparrow}(\mathbf{r}_2) \times \exp\left[i\theta(\mathbf{R})+\frac{r_\alpha}{2}\frac{r_\beta}{2}\nabla_\alpha\nabla_\beta\theta(\mathbf{R})\right] \approx \Delta(\mathbf{R};\mathbf{r})\chi_{\downarrow}(\mathbf{r}_1)\chi_{\uparrow}(\mathbf{r}_2), \tag{11}$$

where we used the assumption (or hydrodynamical, long-wavelength approximation, see Ref. 20) that $\theta(\mathbf{R})$ varies slowly, $\theta(\mathbf{R}) \gg \xi_0^2[\nabla\theta(\mathbf{R})]^2$. Here ξ_0 is the coherence length which for the BCS theory coincides with an average pair size.

Then the partition function in the modulus-phase variables is

$$Z=\int \Delta\mathcal{D}\Delta\mathcal{D}\theta \exp[-\beta\Omega(\Delta,\partial\theta)], \tag{12}$$

where the effective potential

$$\beta\Omega(\Delta,\partial\theta)=\int_0^\beta d\tau\int d^2r_1\int d^2r_2\frac{\Delta^2(\tau,\mathbf{R},\mathbf{r})}{V(\mathbf{r}_1-\mathbf{r}_2)}-\mathrm{Tr}\ln G^{-1} \tag{13}$$

with the Green's function

$$G^{-1}=\mathcal{G}^{-1}-\Sigma, \tag{14}$$

$$\mathcal{G}^{-1}(\tau_1,\tau_2;\mathbf{r}_1,\mathbf{r}_2)\equiv\langle\tau_1,\mathbf{r}_1|\mathcal{G}^{-1}|\tau_2,\mathbf{r}_2\rangle =[-\hat{I}\partial_{\tau_1}-\tau_3\xi(-i\tau_3\nabla_1)]\delta(\tau_1-\tau_2) \times\delta(\mathbf{r}_1-\mathbf{r}_2)+\tau_1\Delta(\tau_1-\tau_2,\mathbf{R},\mathbf{r}); \tag{15}$$

$$\langle\tau_1,\mathbf{r}_1|\Sigma|\tau_2,\mathbf{r}_2\rangle=\Bigg[\tau_3\Bigg(i\frac{\partial_{\tau_1}\theta}{2}+ta^2\frac{(\nabla_{x_1}\theta)^2}{4}\cos(-ia\nabla_{x_1}) +(x\to y)\Bigg)+\hat{I}\Bigg(-\frac{ita^2\nabla_{x_1}^2\theta}{2}\cos(-ia\nabla_{x_1}) +ta\nabla_{x_1}\theta\sin(-ia\nabla_{x_1})+(x\to y)\Bigg)\Bigg] \times\delta(\tau_1-\tau_2)\delta(\mathbf{r}_1-\mathbf{r}_2). \tag{16}$$

Thus the gauge transformation (8) resulted in the separation of the dependences on Δ and θ, viz. θ is present only in Σ. The similar method of the derivative expansion was used before in Refs. 4 and 21. As pointed out in Ref. 4 the method allows us to maintain explicitly the Galilean invariance (the Landau terms break it) and the continuity equation, while the expansion $\Phi(x)=\Delta+\Phi_1(x)+i\Phi_2(x)$ used recently in Ref. 8 demands the additional enforcement of the conservation laws.[22]

Since the low-energy dynamics in the phase in which $\Delta\neq0$ is determined by the long-wavelength fluctuations of $\theta(x)$, only the lowest-order derivatives of the phase such as $\nabla\theta$, $\partial_\tau\theta$, and $\nabla^2\theta$ need be retained in what follows. However, to take into account the tight-binding electron spectrum the operators $\sin(-ia\nabla)$ and $\cos(-ia\nabla)$ must be kept. Thus in Σ we have omitted higher-order terms in $\nabla\theta$, but in order to keep all relevant terms in the expansion of $\sin(-ia\nabla)$ the necessary resummation was done.[23] One can easily see that for the quadratic dispersion law $2t\to1/(ma^2)$, $\cos(-ia\nabla)\to1$ and $\sin(-ia\nabla)\to-ia\nabla$, so that Eq. (16) reduces to the known expression from Refs. 4 and 21. Thus we arrive at the one-loop effective action

$$\Omega\simeq\Omega_{\mathrm{kin}}(v,\mu,T,\Delta,\partial\theta)+\Omega_{\mathrm{pot}}^{\mathrm{MF}}(v,\mu,T,\Delta), \tag{17}$$

where

$$\Omega_{\mathrm{kin}}(\mu,T,\Delta,\partial\theta)=T\,\mathrm{Tr}\sum_{n=1}^{\infty}\frac{1}{n}(\mathcal{G}\Sigma)^n\Big|_{\partial\Delta/\partial\mathbf{R}=0} \tag{18}$$

and

$$\Omega_{\mathrm{pot}}^{\mathrm{MF}}(\mu,T,\Delta)=\left(\int d^2R\int d^2r\,\frac{\Delta^2(\mathbf{r})}{V(\mathbf{r})} -T\,\mathrm{Tr}\ln\mathcal{G}^{-1}\right)\Bigg|_{\partial\Delta/\partial\mathbf{R}=0}. \tag{19}$$

Deriving the "phase only" action for the s-wave model it was possible to use $\Delta(\mathbf{R,r})=\mathrm{const}$.[16] The d-wave case is more complicated because one should keep the dependence on the relative coordinate, $\Delta(\mathbf{R,r})\approx\Delta(\mathbf{r})$ which is related to the nontrivial pairing. The dependences of the gap Δ on T, μ, and V_d follow from the extremum condition for the mean field ($\partial\Delta/\partial\mathbf{R}=0$) potential $\partial\Omega_{\mathrm{pot}}^{\mathrm{MF}}/\partial\Delta=0$ which results in the usual BCS gap equation. For the d-wave pairing potential (7) one obtains

$$\Delta_d(\mathbf{k})=\frac{\Delta_d}{2}(\cos k_xa-\cos k_ya), \tag{20}$$

where Δ_d is the gap amplitude. In our case there is no need to solve the gap equation and express Δ_d in terms of T, μ, and V_d since in what follows we will use Δ_d, or more precisely the velocity $\mathbf{v}_\Delta$, as the input parameters and will be interested in the low-temperature ($T\ll\Delta_d$) regime.

Thus assuming that $\Delta(\mathbf{R,r})$ does not depend on $\mathbf{R}$ one obtains for the frequency-momentum representation of Eq. (15)

$$\mathcal{G}(i\omega_n,\mathbf{k})=-\frac{i\omega_n\hat{I}+\tau_3\xi(\mathbf{k})-\tau_1\Delta(\mathbf{k})}{\omega_n^2+\xi^2(\mathbf{k})+\Delta^2(\mathbf{k})}, \tag{21}$$

where $\Delta(\mathbf{k})$ is given by Eq. (20) and $\omega_n=\pi(2n+1)T$ is fermionic (odd) Matsubara frequency.

The phase dynamics is contained in the kinetic part Ω_{kin} of the effective action which only involves the single degree of freedom θ. As discussed in Ref. 4, it is enough to restrict ourselves to terms with $n=1,2$ in the infinite series in Eq. (18) since at $T=0$ this would give the right answer for a local time-dependent GL functional which involves the derivatives not higher than $(\nabla\theta)^4$ and $(\partial_t\theta)^2$.

IV. FIRST-ORDER TERM OF THE EFFECTIVE ACTION AND THE NODAL APPROXIMATION

In this section we calculate the first ($n=1$) term of the sum appearing in Eq. (18):

$$\Omega^{(1)}_{\rm kin}=T\,{\rm Tr}[\mathcal{G}\Sigma]$$
$$=T\int_0^\beta d\tau\int d^2r\left\{T\sum_{n=-\infty}^{\infty}\int\frac{d^2k}{(2\pi)^2}{\rm Tr}[\mathcal{G}(i\omega_n,\mathbf{k})\tau_3]\right.$$
$$\left.\times\left(i\frac{\partial_\tau\theta}{2}+\frac{ta^2}{4}(\nabla_x\theta)^2\cos k_xa+\frac{ta^2}{4}(\nabla_y)^2\cos k_ya\right)\right\}. \tag{22}$$

Summing over Matsubara frequencies, one obtains

$$\Omega^{(1)}_{\rm kin}=T\int_0^\beta d\tau\int d^2r\left[\int\frac{d^2k}{(2\pi)^2}n(\mathbf{k})\left(i\frac{\partial_\tau\theta}{2}\right.\right.$$
$$\left.\left.+m_{xx}^{-1}(\mathbf{k})\frac{(\nabla_x\theta)^2}{8}+m_{yy}^{-1}(\mathbf{k})\frac{(\nabla_y\theta)^2}{8}\right)\right] \tag{23}$$

with $m_{xx}^{-1}(\mathbf{k})\equiv\partial^2\xi(\mathbf{k})/\partial k_x^2$, $m_{yy}^{-1}(\mathbf{k})\equiv\partial^2\xi(\mathbf{k})/\partial k_y^2$, and

$$n(\mathbf{k})=1-\frac{\xi(\mathbf{k})}{E(\mathbf{k})}\tanh\frac{E(\mathbf{k})}{2T},\quad E(\mathbf{k})=\sqrt{\xi^2(\mathbf{k})+\Delta^2(\mathbf{k})}. \tag{24}$$

For $T\ll\Delta_d$ linearizing the quasiparticle spectrum about the nodes and defining a coordinate system (k_1,k_2) at each node with $\hat{\mathbf{k}}_1(\hat{\mathbf{k}}_2)$ perpendicular (parallel) to the Fermi surface, we can replace the momentum integration in Eq. (23) by an integral over the **k**-space area surrounding each node.[6] If we further define a scaled momentum $\mathbf{p}=(p_1,p_2)=(p,\varphi)$ we can let

$$\int\frac{d^2k}{(2\pi)^2}\to\sum_{j=1}^4\int\frac{dk_1dk_2}{(2\pi)^2}\to\sum_{j=1}^4\int\frac{d^2p}{(2\pi)^2v_Fv_\Delta}$$
$$=\sum_{j=1}^4\int_0^{p_{\rm max}}\frac{pdp}{2\pi v_Fv_\Delta}\int_0^{2\pi}\frac{d\varphi}{2\pi},\quad p_{\rm max}=\sqrt{\pi v_Fv_\Delta}, \tag{25}$$

where $p_1\equiv v_Fk_1=\varepsilon(\mathbf{k})=p\cos\varphi$, $p_2\equiv v_\Delta k_2=\Delta_d(\mathbf{k})=p\sin\varphi$, and $p=\sqrt{p_1^2+p_2^2}=\sqrt{v_F^2k_1^2+v_\Delta^2k_2^2}=E(\mathbf{k})$. Note that for the particular square lattice model used above those velocities are $v_F=2\sqrt{2}ta$ and $v_\Delta=\Delta_da/\sqrt{2}$, respectively.

Using Eq. (25) one can express Eq. (23) in terms of v_F and v_Δ,

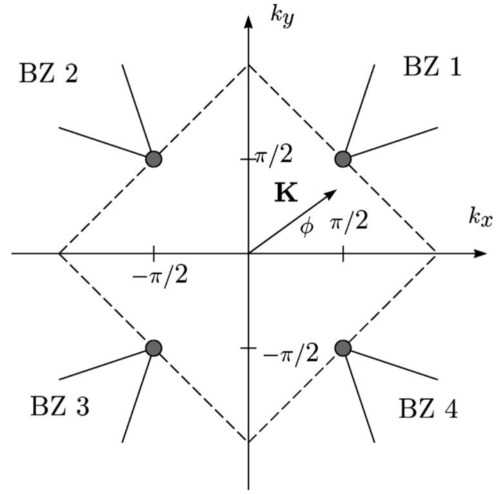

FIG. 1. For $\mu=0$ the Fermi surface (dotted line) represents the points where $\varepsilon(\mathbf{k})=-2t(\cos k_x+\cos k_y)=0$ (in the units where the lattice constant $a=1$). There are four nodes centered at $(\pm\pi/2,\pm\pi/2)$ around which the energy spectrum is linearized. The corresponding nodal subzones [see Eq. (25)] are called BZ j with $j=1,...,4$. For $\alpha_D\ll1$ the Landau damping develops only if the direction of the phason momentum **K** is within one of the ''cones.'' The size of these cones is dependent on the ratio $c=\Omega/v_FK$, so that the cones shrink as $|c|\to1$ [see Eqs. (67)–(69)].

$$\Omega^{(1)}_{\rm kin}=T\int_0^\beta d\tau\int d^2r\left[in_f\frac{\partial_\tau\theta}{2}\right.$$
$$\left.+\int_0^{p_{\rm max}}\frac{pdp}{2\pi v_Fv_\Delta}\int_0^{2\pi}\frac{d\varphi}{2\pi}\tanh\frac{p}{2T}\frac{a^2p\cos^2\varphi}{4}(\nabla\theta)^2\right]$$
$$\approx T\int_0^\beta dr\int d^2r\left[in_f\frac{\partial_r\theta}{2}+\frac{\sqrt{\pi v_Fv_\Delta}}{48a}(\nabla\theta)^2\right], \tag{26}$$

where

$$n_f=\int\frac{d^2k}{(2\pi)^2}n(\mathbf{k}) \tag{27}$$

is the density of carriers. We note that due to the slow convergence of the integrand in Eq. (26) the final expression depends explicitly on the value of the momentum cutoff $p_{\rm max}$ which was defined in Ref. 6 in such a way that the area of the new integration region over four Brillouin subzones (see Fig. 1) is the same as that of the original Brillouin zone. Going from Eq. (23) to Eq. (26) we essentially replace the averaging over the true Fermi surface of the system by the averaging over four nodal subzones. The validity of this approximation can only be justified if the corresponding integrals contain the derivative of the Fermi distribution $n_F(\mathbf{k})$ which is highly peaked in the vicinity of the nodes. This appears to be the case of the temperature-dependent parts of the phase stiffness $J(T)$, compressibility $K(T)$, and the Landau damping terms. For the zero-temperature values $J(T=0)$ and $K(T=0)$ the nodal approximation is not well justified. However, as we show in Sec. VI, this approximation can be justified *a forteriory* for their ratio (see Fig. 2) which determines the velocity of the Bogolyubov-Anderson-Goldstone

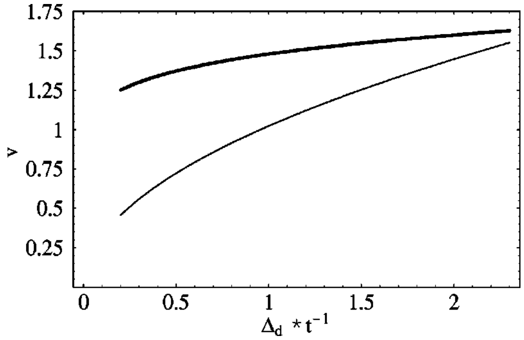

FIG. 2. The dependence of the Goldstone mode velocity $v(T=0)$ on the amplitude of the gap Δ_d. The solid line is the result of numerical calculation with $\xi(\mathbf{k})=-2(\cos k_x+\cos k_y)$, $\Delta(\mathbf{k})=\Delta_d/2(\cos k_x-\cos k_y)$ (we put $t=a=1$, so that Δ_d is expressed in units of t). The thin line is obtained using Eq. (43). The anisotropy of the Dirac spectrum for this case is $\alpha_D=4/\Delta_d$, so that $\Delta_d=0.2$ corresponds to $\alpha_D=20$.

mode. Finally, we stress that after approximation is used, it is impossible to recover the s-wave limit by putting $v_\Delta \to 0$.

V. SECOND-ORDER TERM OF THE EFFECTIVE ACTION

Let us evaluate the trace of the second term in expansion (18):

$$\Omega_{\rm kin}^{(2)}=\frac{T}{2}\,\mathrm{Tr}[\mathcal{G}\Sigma\mathcal{G}\Sigma]. \tag{28}$$

Substituting Eq. (16) into Eq. (28) we obtain that

$$\Omega_{\rm kin}^{(2)}=\Omega_{\rm kin}^{(2)}\{\theta\Omega_n^2\theta\}+\Omega_{\rm kin}^{(2)}\{\theta\mathbf{K}^2\theta\}+\Omega_{\rm kin}^{(2)}\{\theta\mathbf{K}\Omega_n\theta\}, \tag{29}$$

where using $\{\theta\cdots\theta\}$ we denoted symbolically that the corresponding term of Eq. (28) is either diagonal [i.e., its frequency-momentum representation contains $\theta(i\Omega_n,\mathbf{K})\Omega_n^2\,\theta(-i\Omega_n,-\mathbf{K})$ or $\theta(i\Omega_n,\mathbf{K})\mathbf{K}^2\theta(-i\Omega_n,-\mathbf{K})$] as

$$\begin{aligned}\beta\Omega_{\rm kin}^{(2)}\{\theta\Omega_n^2\theta\}=&\frac{T}{2}\sum_{n=-\infty}^{\infty}\int\frac{d^2K}{(2\pi)^2}\,\theta(i\Omega_n,\mathbf{K})\\&\times\left(-\frac{\Omega_n^2}{4}\right)\theta(-i\Omega_n,-\mathbf{K})\\&\times T\sum_{l=-\infty}^{\infty}\int\frac{d^2k}{(2\pi)^2}\,\pi_{33}(i\Omega_n,\mathbf{K};i\omega_l,\mathbf{k});\end{aligned} \tag{30}$$

$$\begin{aligned}\beta\Omega_{\rm kin}^{(2)}\{\theta\mathbf{K}^2\theta\}=&\frac{T}{2}\sum_{n=-\infty}^{\infty}\int\frac{d^2K}{(2\pi)^2}\,\theta(i\Omega_n,\mathbf{K})K_x^2\\&\times\theta(-i\Omega_n,-\mathbf{K})T\sum_{l=-\infty}^{\infty}\int\frac{d^2k}{(2\pi)^2}\\&\times\pi_{00}(i\Omega_n,\mathbf{K};i\omega_l,\mathbf{k})(ta)^2\\&\times\sin(k_x-K_x/2)a\,\sin(k_x+K_x/2)a+(x\to y)\end{aligned} \tag{31}$$

or mixed

$$\begin{aligned}\beta\Omega_{\rm kin}^{(2)}\{\theta\mathbf{K}\Omega_n\theta\}=&-T\sum_{n=-\infty}^{\infty}\int\frac{d^2K}{(2\pi)^2}\,\theta(i\Omega_n,\mathbf{K})\,\frac{K_x\Omega_n}{2}\,\theta(-i\Omega_n,-\mathbf{K})T\sum_{l=-\infty}^{\infty}\int\frac{d^2k}{(2\pi)^2}\bigg[\pi_{03}(i\Omega_n,\mathbf{K};i\omega_l,\mathbf{k})\\&\times\frac{ita}{2}\sin(k_x+K_x/2)a+\pi_{30}(i\Omega_n,\mathbf{K};i\omega_l,\mathbf{k})\,\frac{ita}{2}\sin(k_x-K_x/2)a\bigg]+(x\to y).\end{aligned} \tag{32}$$

In Eqs. (30)–(32) we introduced the following shorthand notations:

$$\begin{aligned}\pi_{ij}(i\Omega_n,\mathbf{K};i\omega_l,\mathbf{k})\equiv&\mathrm{Tr}[\mathcal{G}(i\omega_l+i\Omega_n,\mathbf{k}+\mathbf{K}/2)\tau_i\\&\times\mathcal{G}(\omega_l,\mathbf{k}-\mathbf{K}/2)\tau_j],\quad \tau_i=(\tau_0\equiv\hat{I},\tau_3).\end{aligned} \tag{33}$$

More generally, we can rewrite Eqs. (31) and (32) as follows:

$$\begin{aligned}\beta\Omega_{\rm kin}^{(2)}\{\theta\mathbf{K}^2\theta\}\simeq&\frac{T}{2}\sum_{n=-\infty}^{\infty}\int\frac{d^2K}{(2\pi)^2}\,\theta(i\Omega_n,\mathbf{K})\,\frac{K_\alpha K_\beta}{4}\\&\times\theta(-\Omega_n,-\mathbf{K})\Pi_{00}^{\alpha\beta}(i\Omega_n,\mathbf{K}),\end{aligned} \tag{34}$$

$$\begin{aligned}\beta\Omega_{\rm kin}^{(2)}\{\theta\mathbf{K}\Omega_n\theta\}=&-\frac{T}{2}\sum_{n=-\infty}^{\infty}\int\frac{d^2K}{(2\pi)^2}\,\theta(i\Omega_n,\mathbf{K})\,\frac{i\Omega_nK_\alpha}{4}\\&\times\theta(-i\Omega_n,-\mathbf{K})[\Pi_{03}^{\alpha}(i\Omega_n,\mathbf{K})\\&+\Pi_{30}^{\alpha}(i\Omega_n,\mathbf{K})],\end{aligned} \tag{35}$$

where

$$\begin{aligned}&\Pi_{00}^{\alpha\beta}(i\Omega_n,\mathbf{K})\\&\quad\equiv\sum_{l=-\infty}^{\infty}\int\frac{d^2k}{(2\pi)^2}\,\pi_{00}(i\Omega_n,\mathbf{K};i\omega_l,\mathbf{k})v_{F\alpha}(\mathbf{k})v_{F\beta}(\mathbf{k}),\\&\Pi_{03}^{\alpha}(i\Omega_n,\mathbf{K})\equiv\sum_{l=-\infty}^{\infty}\int\frac{d^2k}{(2\pi)^2}\,\pi_{03}(i\Omega_n,\mathbf{K};i\omega_l,\mathbf{k})v_{F\alpha}(\mathbf{k}),\end{aligned} \tag{36}$$

$v_{F\alpha}(\mathbf{k}) = \partial\xi(\mathbf{k})/\partial k_\alpha$, we used the approximation $v_{F\alpha}(\mathbf{k}) \simeq v_{F\alpha}(\mathbf{k} \pm \mathbf{K}/2)$ and took into account that $(1/2\pi)\int d^2 k v_\alpha v_\beta = \int k dk v^2 \delta_{\alpha\beta}/2$. It is convenient to introduce here

$$\Pi_{33}(i\Omega_n, \mathbf{K}) \equiv \sum_{l=-\infty}^{\infty} \int \frac{d^2k}{(2\pi)^2} \pi_{33}(i\Omega_n, \mathbf{K}; i\omega_l, \mathbf{k}) \tag{37}$$

which would allow us to rewrite Eq. (30) in the same fashion as Eq. (34).

As one could notice the product of the Fermi velocities enters Eq. (34) via Eq. (36). There is nothing surprising in this fact since this piece of the effective action is related to the paramagnetic current correlator $\langle j_\alpha j_\beta \rangle$ (Ref. 9) and in its turn the current operator $\mathbf{j}$ contains the Fermi velocity $\mathbf{v}_F(\mathbf{k})$. We will return to this point considering the Landau term which originates from Eq. (34), so that here we note only that the current correlator term along with the diamagnetic term $\sim (\nabla\theta)^2$ in Eq. (22) from together the mean-field phase stiffness.

The matrix traces π_{ij} and the corresponding expressions for Π_{ij} are calculated in the Appendix. Although the expressions for them are rather lengthy they have a clear physical interpretation which is also discussed in the Appendix.

VI. EFFECTIVE LAGRANGIAN AT $T \neq 0$ WITHOUT THE LANDAU TERMS

The contribution of the first-order term to the effective action is given in Eq. (26). Concerning the second-order term, we note that when the Landau terms are neglected it is enough to set $\Omega_n = 0$ inside Π in Eqs. (30), (34), and (35). To be more precise, the Landau terms arise from the second line of Eqs. (A2) and (A3) which contains ''dangerous'' denominators $1/(E_+ - E_- \pm i\Omega_n)$. One can, however, notice that the second line of Eq. (A2) leads also to the regular terms which are proportional to the derivative $dn_F(E)/dE$. These lead to the second term in the square brackets in Eq. (38) and the whole expression (39) shown below. For the s-wave superconductivity[4,15] in the temperature region $0 < T \lesssim 0.6 T_c$ these terms are very small compared to the main terms. Although the contribution from these terms is still local, their presence breaks the Galilean invariance.[4] This is the reason why it was more natural for Ref. 4 to treat them along with ''true'' Landau terms since they also originate from the same denominators of the second line of Eq. (A2) as was mentioned above. For the d-wave superconductivity this splitting, however, appears to be rather artificial since these terms are not small even for low temperatures due to the presence of the nodal quasiparticles, so here we will consider all regular terms.

For the regular terms from the second-order term we obtain a local effective action, involving time and space derivatives of $\theta(t,\mathbf{r})$:

$$\begin{aligned}
&\beta\Omega_{\mathrm{kin}}^{(2)}\{(\partial_t\theta)^2\} \\
&= \frac{i}{2} \int \frac{d\Omega}{2\pi} \int \frac{d^2K}{(2\pi)^2} \theta(\Omega, \mathbf{K}) \frac{\Omega^2}{4} \\
&\quad \times \theta(-\Omega, -\mathbf{K}) \Pi_{33}(0, \mathbf{K} \to 0) \\
&= i \frac{T}{2} \int dt \int d^2 r \frac{[\partial_t \theta(t,\mathbf{r})]^2}{4} \int \frac{d^2k}{(2\pi)^2} \\
&\quad \times \left[-\frac{\Delta^2(\mathbf{k})}{E^3(\mathbf{k})} \tanh\frac{E(\mathbf{k})}{2T} - \frac{1}{2T} \frac{\xi^2(\mathbf{k})}{E^2(\mathbf{k})} \cosh^{-2}\frac{E(\mathbf{k})}{2T} \right],
\end{aligned} \tag{38}$$

$$\begin{aligned}
&\beta\Omega_{\mathrm{kin}}^{(2)}\{(\nabla\theta)^2\} \\
&= \frac{i}{2} \int \frac{d\Omega}{2\pi} \int \frac{d^2K}{(2\pi)^2} \theta(\Omega, \mathbf{K}) \frac{K_\alpha K_\beta}{4} \\
&\quad \times \theta(-\Omega, -\mathbf{K}) \Pi_{00}^{\alpha\beta}(0, \mathbf{K} \to 0) \\
&= i \frac{T}{2} \int dt \int d^2 r \frac{\nabla_\alpha \theta(t,\mathbf{r}) \nabla_\beta \theta(t,\mathbf{r})}{4} \int \frac{d^2k}{(2\pi)^2} \\
&\quad \times \left[-\frac{1}{2T} \cosh^{-2}\frac{E(\mathbf{k})}{2T} \right] v_{F\alpha}(\mathbf{k}) v_{F\beta}(\mathbf{k}),
\end{aligned} \tag{39}$$

and the mixed term (35) does not contribute to the regular part within the used approximation, since $\Pi_{30}^{\alpha}(0, \mathbf{K} \to 0) = 0$. Evaluating Eqs. (38) and (39) we performed the analytical continuation $i\Omega_n \to \Omega + i0$ back to the real continuous frequencies, so that t is the real time.

Using the nodal expansion (25) to calculate Eqs. (38) and (39), and adding Eq. (26), we finally obtain the regular effective Lagrangian $\mathcal{L}^R$ such that $\beta\Omega_{\mathrm{kin}} = -i \int dt \int d^2 r \mathcal{L}^R(t,\mathbf{r})$ for $T \ll \Delta_d$ and ignoring the Landau terms,

$$\mathcal{L}^R = -\frac{n_f}{2} \partial_t \theta(t,\mathbf{r}) + \frac{K}{2} [\partial_t \theta(t,\mathbf{r})]^2 - \frac{J}{2} [\nabla\theta(t,\mathbf{r})]^2, \tag{40}$$

where the phase stiffness $J \equiv J_{d,s}$ and compressibility $K \equiv K_{d,s}$ are

$$J_d = \frac{\sqrt{\pi v_F v_\Delta}}{24a} - \frac{\ln 2}{2\pi} \frac{v_F}{v_\Delta} T, \quad K_d = \frac{1}{4a\sqrt{\pi v_F v_\Delta}}. \tag{41}$$

The linear time derivative term in Eq. (40) is important for the description of vortex dynamics (see Ref. 9 and references therein), but we omit it in what follows. We stress that the second, temperature-dependent term in J_d follows from Eq. (39) which contains the derivative of the Fermi distribution $dn_F(E)/dE$. Its presence, as we mentioned in Sec. IV, makes the nodal approximation valid.[6] Since we consider only the low-temperature $T \ll \Delta_d$ region, we restrict ourselves by the values Δ_d and v_Δ at $T = 0$, so that all temperature dependences appear to be linear. For higher temperatures it is necessary to take into account that $\Delta_d(T)$ and $v_\Delta(T)$ are, in fact, decreasing functions of T.

It is useful to compare the stiffness and compressibility from Eq. (41) with those parameters derived in Ref. 15 for the continuum 2D model with s-wave pairing:

$$J_s = \frac{n_f}{4m}\left(1 - \int_0^\infty dx \frac{1}{\cosh^2\sqrt{x^2+\Delta_s^2/4T^2}}\right), \quad K_s = \frac{m}{4\pi}. \tag{42}$$

First of all one can see that for low temperatures the superfluid stiffness in Eq. (42) does not contain any term which goes to zero more slowly than $\exp(-\Delta_s/T)$, while Eq. (41) has a term proportional to T. The origin of this difference is well known and related to the presence of the nodal quasiparticles. Second, we can compare the values of the zero-temperature superfluid stiffness which for the continuum translationally invariant system has to be equal to $n_f/4m$, so that all carriers participate in the superfluid ground state.[25] Since the presence of a lattice evidently breaks the continuum translational invariance the superfluid density at $T=0$ in the case of Eq. (41) is less than $n_f/4m$ as can be readily seen from Eq. (23). Finally, since we consider here a neutral system it has the Berezinskii-Kosterlitz-Thouless (BKT) collective mode,[26] $\Omega^2 = v^2K^2$ with $v=\sqrt{J/K}$. One can see that for the continuum s-wave case $v = v_F/\sqrt{2}$ and

$$v = \sqrt{\frac{\pi v_F v_\Delta}{6} - \frac{2\ln 2 a v_F}{\sqrt{\pi}}\sqrt{\frac{v_F}{v_\Delta}}T} \tag{43}$$

for the d-wave model on lattice at $T\sim 0$. Equation (43) gives a rather simple approximate expression for the BKT mode velocity for the lattice model of d-wave superconductor. Comparing in Fig. 2 the results obtained for $v(T=0)$ using Eq. (43) with the numerical computation without the nodal approximation, we can see that even being very simple Eq. (43) predicts the correct behavior of $v(T=0)$.

Following Ref. 9 we estimate the upper values for the frequency Ω in Eq. (38) and the momentum K in Eq. (39). The "phase-only" effective action is appropriate for phase distortions whose energy is smaller than the condensation energy, $E_{\rm cond} \simeq N(0)\Delta^2/2$, where $N(0)$ is the density of states. For the s-wave case (42) this leads to the following restrictions: $\Omega < \Delta_s, v_F K < \Delta_s$, and for the d-wave case (41), $\Omega < \Delta_d \sqrt[4]{1/\alpha_D}$, $v_F K < \Delta_d \sqrt[4]{\alpha_D}$, where α_D is the anisotropy of the Dirac spectrum defined in Introduction.

VII. IMAGINARY PART OF THE LANDAU TERMS FOR 2D *D*-WAVE AND *S*-WAVE CASES

The key values which are necessary for evaluation of the Landau terms are the differences $E_+ - E_-$ and $n_F(E_+) - n_F(E_-)$. Expanding in $\mathbf{K}$,

$$E(\mathbf{k}+\mathbf{K}/2) - (\mathbf{k}-\mathbf{K}/2) = \mathbf{v}_g(\mathbf{k})\mathbf{K}, \tag{44}$$

where the group velocity is given by

$$\mathbf{v}_g(\mathbf{k}) = \nabla_{\mathbf{k}} E(\mathbf{k}) = \frac{1}{E(\mathbf{k})}[\xi(\mathbf{k})\mathbf{v}_F + \Delta(\mathbf{k})\mathbf{v}_\Delta]. \tag{45}$$

It is obvious that due to the gap $\mathbf{k}$ dependence Eq. (44) differs from the s-wave case,[4] where the difference is simply

$$E(\mathbf{k}+\mathbf{K}/2) - E(\mathbf{k}-\mathbf{K}/2) = \frac{\xi(\mathbf{k})}{E(\mathbf{k})}\mathbf{v}_F\mathbf{K} \quad [\Delta(\mathbf{k}) = \Delta_s]. \tag{46}$$

It is convenient to rewrite Eqs. (44) and (45) in terms of the nodal approximation described after Eq. (25). In the vicinity of one of the nodes we have

$$\mathbf{v}_g\mathbf{K} = v_F K_1 \cos\varphi + v_\Delta K_2 \sin\varphi \equiv P\cos(\varphi - \psi),$$

$$E(\mathbf{k}\pm\mathbf{K}/2) \ll \Delta_d, \tag{47}$$

where the momentum $\mathbf{K} = (K_1, K_2)$ of θ particle was also expressed in the nodal coordinate system $\hat{\mathbf{k}}_1, \hat{\mathbf{k}}_2$, so that $P^{(1)} \equiv v_F K_1 = P\cos\psi$, $P^{(2)} \equiv v_\Delta K_2 = P\sin\psi$, and $P = \sqrt{(P^{(1)})^2 + (P^{(2)})^2} = \sqrt{v_F^2 K_1^2 + v_\Delta^2 K_2^2}$. (We denoted the components of $\mathbf{P}$ as $P^{(1)}, P^{(2)}$ to make them different from the node label P_j used in what follows.)

The corresponding substitution in the integrals over $\mathbf{K}$ reads similarly to Eq. (25),

$$\int \frac{d^2K}{(2\pi)^2} \to \sum_{j=1}^{4} \int_0^{P_{\max}} \frac{P_j dP_j}{2\pi v_F v_\Delta} \int_0^{2\pi} \frac{d\psi_j}{2\pi}, \tag{48}$$

where $P_{\max}$ is evidently related to the maximal value of K discussed at the end of Sec. VI. Finally, we can approximate the difference $n_F(E_+) - n_F(E_-)$ as

$$n_F(E_+) - n_F(E_-) \approx \frac{dn_F(E)}{dE}\mathbf{v}_g\mathbf{K} = \frac{dn_F(E)}{dE} P\cos(\varphi - \psi). \tag{49}$$

Having the differences (47) and (49) we can now derive the imaginary part for the Landau terms. In the subsequent subsections we consider all three terms of Eq. (29) for d-wave pairing and compare them with their s-wave counterparts. We would like to note that the Landau terms have also the *real part* which for the 3D s-wave case was considered in detail in Ref. 4. This real part consists of regular and irregular terms. The regular term was already taken into account in Sec. VI and the irregular term is not considered in this paper.

A. $\Omega_{\rm kin}^{(2)}\{\theta\Omega^2\theta\}$ term

Let us consider first the contribution from $\Omega_{\rm kin}^{(2)}\{\theta\Omega_n^2\theta\}$ [see Eqs. (30) and (A2) for A_-], which after the analytical continuation $i\Omega_n \to \Omega + i0$ takes the form

$$\mathrm{Im}[i\beta\Omega_{kin}^{(2)}\{\theta\Omega^2\theta\}]\approx-\frac{1}{2}\int\frac{d\Omega d^2K}{(2\pi)^3}\theta(\Omega,\mathbf{K})\frac{\Omega^2}{4}\theta(-\Omega,-\mathbf{K})\int\frac{d^2k}{(2\pi)^2}\frac{1}{2}\left(1+\frac{\xi^2-\Delta^2}{E^2}\right)\mathrm{Im}\frac{2}{\mathbf{v}_g\mathbf{K}+\Omega+i0}\frac{dn_F(E)}{dE}\mathbf{v}_g\mathbf{K}$$

$$\approx\sum_{j=1}^{4}\int\frac{d\Omega}{2\pi}\int_0^{P_{\max}}\frac{P_jdP_j}{2\pi v_Fv_\Delta}\int_0^{2\pi}\frac{d\psi_j}{2\pi}\theta(\Omega,\mathbf{K})\frac{\Omega^2}{8}\theta(-\Omega,-\mathbf{K})$$

$$\times\int_0^{p_{\max}}\frac{pdp}{2\pi v_Fv_\Delta}\int_0^{2\pi}d\varphi\cos^2\varphi\,\delta(P_j\cos(\varphi-\psi_j)+\Omega)\frac{dn_F(E)}{dE}P_j\cos(\varphi-\psi_j)$$

$$=\sum_{j=1}^{4}\int\frac{d\Omega}{2\pi}\int_0^{P_{\max}}\frac{P_jdP_j}{2\pi v_Fv_\Delta}\int_0^{2\pi}\frac{d\psi_k}{2\pi}\theta(\Omega,\mathbf{K})\frac{\Omega^2}{8}\theta(-\Omega,-\mathbf{K})\frac{\ln 2}{\pi}\frac{T}{v_Fv_\Delta}$$

$$\times\frac{1}{\sqrt{1-\frac{\Omega^2}{P_j^2}}}\frac{\Omega}{P_j}\left[\frac{\Omega^2}{P_j^2}\cos^2\psi_j+\left(1-\frac{\Omega^2}{P_j^2}\right)\sin^2\psi_j\right]\Theta\left(1-\frac{|\Omega|}{P_j}\right). \tag{50}$$

Here $\Theta(x)$ is the step function and we used the integral

$$\int_0^{\infty}\frac{pdp}{2\pi v_Fv_\Delta}\frac{1}{2T}\frac{1}{\cosh^2\frac{p}{2T}}=\frac{\ln 2}{\pi}\frac{1}{v_Fv_\Delta}T. \tag{51}$$

Integrating over φ we had to take into account that there are two points where the δ function contributes into the integral: $\cos\varphi=-\Omega/P_j$, $\sin\varphi=\sqrt{1-\Omega^2/P_j^2}$ and $\cos\varphi=-\Omega/P_j$, $\sin\varphi=-\sqrt{1-\Omega^2/P_j^2}$.

As we can see from the first equality in Eq. (50) the imaginary part develops when $\Omega=E_+-E_-\approx\mathbf{v}_g\mathbf{K}$. This condition was interpreted in Ref. 1 as the "Čerenkov" irradiation (absorption) condition for the process: "thermally excited θ-fluctuation quantum (phason)+quasiparticle ↔phason+quasiparticle" (or phason being absorbed and scattering thermally excited quasiparticles). As was discussed in Ref. 6 since definite energy is carried by the quasiparticles, this process is defined by their group velocity $\mathbf{v}_g=\partial E(\mathbf{k})/\partial\mathbf{k}$ (Ref. 1) as well as thermal and spin currents.

It is in fact a coincidence that for the s-wave superconductor the directions of $\mathbf{v}_g$ and $\mathbf{v}_F$ are the same, so that for the 2D s-wave superconductor using Eq. (46) instead of Eqs. (44) and (45) one can obtain

$$\mathrm{Im}[i\beta\Omega_{\mathrm{kin}}^{(2)}\{\theta\Omega^2\theta\}]=\frac{1}{2}\int\frac{d\Omega d^2K}{(2\pi)^3}\theta(\Omega,\mathbf{K})\frac{\Omega^2}{4}\theta(-\Omega,-\mathbf{K})\int\frac{d^2k}{(2\pi)^2}\frac{2\xi^2}{E^2}\mathrm{Im}\frac{1}{\frac{\xi}{E}\mathbf{v}_F\mathbf{K}+\Omega+i0}\frac{dn(E)}{dE}\frac{\xi}{E}\mathbf{v}_F\mathbf{K}$$

$$=-\frac{1}{2}\int\frac{d\Omega d^2K}{(2\pi)^3}\theta(\Omega,\mathbf{K})\frac{\Omega^2}{4}\theta(-\Omega,-\mathbf{K})\frac{\Delta_s mc}{2\pi}\int_{-\infty}^{\infty}\frac{dy}{\sqrt{1-c^2\frac{1+y^2}{y^2}}}\frac{dn}{dE}\frac{2|y|}{\sqrt{1+y^2}}\Theta\left(\frac{|y|}{\sqrt{1+y^2}}-|c|\right), \tag{52}$$

where following Ref. 4 we used the notations $y=\xi/\Delta_s$ and $c=\Omega/v_FK$. Comparing Eq. (52) with the 3D case[4] (in the notations of Ref. 4 the term we consider is related to the sum $\overline{B_L}+\overline{C_L}$) one can notice that the only difference between the corresponding expressions is in the square root in the denominator of Eq. (52). It obviously originates from the different measures for the angular integration in 2D and 3D where the extra $\sin\varphi$ is present. Comparing also Eqs. (50) and (52) one can see that their main analytical structure appears to be the same, viz. $\sim\Omega^3/P_j$ for the s-wave case and $\sim\Omega^3/v_FK$ for the s-wave case, so that the momentum variables P_j and v_FK do not appear in the numerator.

We note that the calculation of the ultrasonic attenuation $\alpha_{\text{sound}}(T,\mathbf{K})$ in d-wave superconductors results in the expression

$$\alpha_{\text{sound}}(T,\mathbf{K})\sim\frac{\Omega}{2T}\int\frac{d^2k}{(2\pi)^2}\frac{\xi^2(\mathbf{k})}{E^2(\mathbf{k})}\frac{1}{\cosh^2[E(\mathbf{k})/2T]}\delta(\mathbf{v}_g\mathbf{K}), \tag{53}$$

which has the same structure as Eq. (50). This can be easily seen if one takes into account that for the ultrasound frequency range $\Omega\ll v_FK,\Delta_d$, so that the corresponding terms from Eq. (50) can be simplified as follows:

$$\begin{aligned}\delta(E_+-E_-+\Omega)[n_F(E_-)-n_F(E_+)]\\ &=\delta(E_+-E_-+\Omega)[n_F(\Psi+E_+)-n_F(E_+)]\\ &\simeq\Omega\,\delta(E_+-E_-)\frac{n_F(E)}{dE}\end{aligned} \tag{54}$$

leading to Eq. (53). This is the reason why in the limit $\Omega/v_FK\to 0$ the angular dependence of the Landau damping (50) which we consider in Sec. VIII would appear to be the same as the angular dependence of the ultrasonic attenuation.[10]

B. $\Omega_{\text{kin}}^{(2)}\{\theta\mathbf{K}^2\theta\}$ term

The contribution from $\Omega_{\text{kin}}^{(2)}\{\theta\mathbf{K}^2\theta\}$ [see Eqs. (34) and (A2) for A_-] can be treated in the same manner,

$$\begin{aligned}\mathrm{Im}[i\beta\Omega_{\text{kin}}^{(2)}\{\theta\mathbf{K}^2\theta\}]&\approx-\frac{1}{2}\int\frac{d\Omega\,d^2K}{(2\pi)^3}\theta(\Omega,\mathbf{K})\frac{K_\alpha K_\beta}{4}\theta(-\Omega,\mathbf{K})\int\frac{d^2k}{(2\pi)^2}\frac{1}{2}\left(1+\frac{\xi^2+\Delta^2}{E^2}\right)\\ &\quad\times\mathrm{Im}\frac{2}{\mathbf{v}_g\mathbf{K}+\Omega+i0}v_{F\alpha}(\mathbf{k})v_{F\beta}(\mathbf{k})\frac{dn_F(E)}{dE}\mathbf{v}_g\mathbf{K}\\ &\approx\sum_{j=1}^{4}\int\frac{d\Omega}{2\pi}\int_0^{P_{\max}}\frac{P_j dP_j}{2\pi v_Fv_\Delta}\int_0^{2\pi}d\varphi\,\delta(P_j\cos(\varphi-\psi_j)+\Omega)\frac{dn_F(E)}{dE}P_j\cos(\varphi-\psi_j)\\ &=\sum_{j=1}^{4}\int\frac{d\Omega}{2\pi}\int_0^{P_{\max}}\frac{P_j dP_j}{2\pi v_Fv_\Delta}\int_0^{2\pi}\frac{d\psi_j}{2\pi}\theta(\Omega,\mathbf{K})\frac{P_j^2\cos^2\psi_j}{8}\theta(-\Omega,-\mathbf{K})\frac{\ln 2}{\pi}\frac{T}{v_Fv_\Delta}\\ &\quad\times\frac{1}{\sqrt{1-\dfrac{\Omega^2}{P_j^2}}}\frac{\Omega}{P_j}\Theta\left(1-\frac{|\Omega|}{P_j}\right)\end{aligned} \tag{55}$$

and for the s-wave case

$$\begin{aligned}\mathrm{Im}[i\beta\Omega_{\text{kin}}^{(2)}\{\theta\mathbf{K}^2\theta\}]&=-\frac{1}{2}\int\frac{d\Omega\,d^2K}{(2\pi)^3}\theta(\Omega,\mathbf{K})\frac{K_\alpha K_\beta}{4}\theta(-\Omega,-\mathbf{K})\int\frac{d^2k}{(2\pi)^2}\mathrm{Im}\frac{2}{(\xi/E)\mathbf{v}_F\mathbf{K}+\Omega+i0}v_{F\alpha}v_{F\beta}\frac{dn(E)}{dE}\frac{\xi}{E}\mathbf{v}_F\mathbf{K}\\ &=-\frac{1}{2}\int\frac{d\Omega\,d^2K}{(2\pi)^3}\theta(\Omega,\mathbf{K})\frac{\Omega^2}{4}\theta(-\Omega,-\mathbf{K})\frac{\Delta_s mc}{2\pi}\int_{-\infty}^{\infty}\frac{dy}{\sqrt{1-c^2(1+y^2)/y^2}}\frac{dn}{dE}\frac{2(1+y^2)^{3/2}}{y^2|y|}\\ &\quad\times\Theta\left(\frac{|y|}{\sqrt{1+y^2}}-|c|\right).\end{aligned} \tag{56}$$

Let us now compare the expressions (55) and (56). Evidently their analytical structure is different since for the s-wave case we have again that it is proportional to Ω^3/v_FK, while for d-wave pairing P_j enters the numerator, so that the main structure (excluding the measure of integration over P_j and the square root in the denominator) is $\sim\Omega P_j$. This difference originates from the angular integration in Eqs. (55) and (56) which is performed using the δ function. Since the arguments of the δ function for the d- and s- wave cases are different we have obtained two different answers. Indeed, for the s-wave case the argument is proportional to $\mathbf{v}_F\mathbf{K}$ that coincides with the product $(\mathbf{v}_F\mathbf{K})^2[dn_F(E)/dE]\mathbf{v}_F\mathbf{K}$ outside the δ function [see Sec. V after Eq. (34), where the origin of the term $\sim(\mathbf{v}_F\mathbf{K})^2$ is discussed], so that the angular integration removes K from the numerator. The d-wave case appears to be different since we have $\mathbf{v}_g\mathbf{K}$ inside the δ function and $(\mathbf{v}_F\mathbf{K})^2[dn_F(E)/dE]\mathbf{v}_g\mathbf{K}$ outside, so that the angular integration leaves P_j in the numerator. Despite the fact that ΩP_j

looks substantially less singular than Ω/P_j, the first expression still remains nonanalytical near $P_j \approx 0$ because it is expressed in terms of $\sqrt{\mathbf{K}^2}$ the coordinate representation of which is the nonlocal operator $\sqrt{-\nabla^2}$.

Thus physically the difference between the analytical form of the Landau damping in *d*- and *s*-wave superconductors originates from the $\mathbf{k}$ dependence of the gap $\Delta_d(\mathbf{k})$ which in its turn makes the direction of the quasiparticle group velocity $\mathbf{v}_g$ different from the Fermi velocity $\mathbf{v}_F$. The last velocity, however, still enters the numerator of Eq. (55) since, as was already mentioned in Sec. V, it originates from the current-current correlator. The electrical current is proportional to $\mathbf{v}_F$, not $\mathbf{v}_g$ because quasiparticles carry definite energy and spin, but do not carry definite charge. Therefore the specific form of the Landau damping terms in a *d*-wave superconductor has the same physical origin as the source of the extra terms in the thermal and spin conductivities[6] which are related to the presence in $\mathbf{v}_g$ of both $\mathbf{v}_F$ and $\mathbf{v}_\Delta$ components.

The *s*-wave Landau damping (56) itself can be again related to the 3D case,[4] where it is expressed via the difference $\tilde{B}_L - \tilde{C}_L$.

C. $\Omega_{\mathrm{kin}}^{(2)}\{\theta \mathbf{K} \Omega \theta\}$ term

Finally we consider the contribution from $\Omega_{\mathrm{kin}}^{(2)}\{\theta \mathbf{K}\Omega\theta\}$ [see Eqs. (35) and (A3)]:

$$
\begin{aligned}
\mathrm{Im}[i\beta\Omega_{\mathrm{kin}}^{(2)}\{\theta\mathbf{K}\Omega\theta\}] \approx & -\frac{1}{2}\int \frac{d\Omega\, d^2K}{(2\pi)^3}\,\theta(\Omega,\mathbf{K})\,\Omega K_\alpha\,\theta(-\Omega,-\mathbf{K})\int \frac{d^2k}{(2\pi)^2}\,\frac{\xi}{E}\,\mathrm{Im}\frac{1}{\mathbf{v}_g\mathbf{K}+\Omega+i0}\,v_{F_\alpha}(\mathbf{k})\,\frac{dn_F(E)}{dE}\,\mathbf{v}_g\mathbf{K} \\
\approx & \sum_{j=1}^{4}\int \frac{d\Omega}{2\pi}\int_0^{P_{\max}}\frac{P_j\,dP_j}{2\pi v_F v_\Delta}\int_0^{2\pi}\frac{d\psi_j}{2\pi}\,\theta(\Omega,\mathbf{K})\,\frac{\Omega P_j}{4}\cos\psi_j\,\theta(-\Omega,-\mathbf{K}) \\
& \times\int_0^{p_{\max}}\frac{p\,dp}{2\pi v_F v_\Delta}\int_0^{2\pi}d\varphi\,\cos\varphi\,\delta[P_j\cos(\varphi-\psi_j)+\Omega]\,\frac{dn_F(E)}{dE}\,P_j\cos(\varphi-\psi_j) \\
= & -\sum_{j=1}^{4}\int \frac{d\Omega}{2\pi}\int_0^{P_{\max}}\frac{P_j\,dP_j}{2\pi v_F v_\Delta}\int_0^{2\pi}\frac{d\psi_j}{2\pi}\,\theta(\Omega,\mathbf{K})\,\frac{\Omega^2}{8}\,\theta(-\Omega,-\mathbf{K})\,\frac{\ln 2}{\pi}\,\frac{T}{v_F v_\Delta} \\
& \times\frac{2}{\sqrt{1-\dfrac{\Omega^2}{P_j^2}}}\,\frac{\Omega}{P_j}\cos^2\psi_j\,\Theta\!\left(1-\frac{|\Omega|}{P_j}\right)
\end{aligned}
\tag{57}
$$

and for the *s*-wave case

$$
\begin{aligned}
\mathrm{Im}[i\beta\Omega_{\mathrm{kin}}^{(2)}\{\theta\mathbf{K}\Omega\theta\}] = & -\frac{1}{2}\int \frac{d\Omega\, d^2K}{(2\pi)^3}\,\theta(\Omega,\mathbf{K})\,\Omega K_\alpha\,\theta(-\Omega,-\mathbf{K})\int \frac{d^2k}{(2\pi)^2}\,\frac{\xi}{E}\,\mathrm{Im}\frac{1}{\dfrac{\xi}{E}\mathbf{v}_F\mathbf{K}+\Omega+i0}\,v_{F_\alpha}\frac{dn(E)}{dE}\,\frac{\xi}{E}\,\mathbf{v}_F\mathbf{K} \\
= & \frac{1}{2}\int \frac{d\Omega\, d^2K}{(2\pi)^3}\,\theta(\Omega,\mathbf{K})\,\Omega^2\,\theta(-\Omega,-\mathbf{K})\,\frac{\Delta_s mc}{2\pi}\int_{-\infty}^{\infty}\frac{dy}{\sqrt{1-c^2\dfrac{1+y^2}{y^2}}}\,\frac{dn}{dE}\,\frac{(1+y^2)^{1/2}}{|y|}\,\Theta\!\left(\frac{|y|}{\sqrt{1+y^2}}-|c|\right).
\end{aligned}
\tag{58}
$$

Once again the difference between *d*- and *s*-wave cases is related to the different directions of $\mathbf{v}_g$ and $\mathbf{v}_F$. Since this time we had $\cos\phi\,\mathbf{v}_F\mathbf{K}[dn_F(E)/dE]\mathbf{v}_g\mathbf{K}$ outside the δ function, we could not get ΩP_j as in Eq. (55) and obtained again $\sim\Omega^3/P_j$ as in Eq. (50). Furthermore, because we have $\cos\varphi$ instead of $\cos^2\varphi$ as in Eq. (50) we got only $\Omega/P_j\cos\psi_j$ [compare with the square brackets in Eq. (50)] which after multiplying by $\Omega^2 P_j$ gives the final result $\sim\Omega^3/P_j$. It is interesting to note that the mixed term for *d* [see Eq. (57)] and *s* wave [see Eq. (58)] has the opposite sign with respect to other terms. This means that the mixed term describes the energy transfer from the quasiparticles to the phase excitations. It turns out, however, that the whole sum (59) of the Landau damping terms has the sign which corresponds to the phason damping. (We stress that the different sign in front of all final *s*-wave expressions and their *d*-wave counterparts are due to the explicit presence of the derivative of the Fermi distribution, which is negative, in the former expression.)

D. Final expression for the Landau damping term in the *s*-wave case

Since the Landau damping terms for *s*-wave pairing all are $\sim\Omega^3/v_F K$, we can combine Eqs. (52), (56), and (58):

$$\mathrm{Im}[i\beta\Omega_{\mathrm{kin}}^{(2)}] = -\frac{1}{2}\int \frac{d\Omega d^2K}{(2\pi)^3}\,\theta(\Omega,\mathbf{K})\Omega^2\theta(-\Omega,-\mathbf{K})\frac{\Delta_s mc}{2\pi}$$
$$\times\int_{-\infty}^{\infty}\frac{dy}{\sqrt{1-c^2(1+y^2)/y^2}}\,\frac{dn}{dE}\,\frac{1}{2y^2|y|\sqrt{1+y^2}}$$
$$\times\Theta\left(\frac{|y|}{\sqrt{1+y^2}}-|c|\right). \tag{59}$$

This expression coincides with the function $H_i^{(1)}(c)$ introduced in Ref. 4 except for the above-mentioned difference between angular integration in 2D and 3D. Thus the simultaneous derivation of the *s*- and *d*-wave cases allowed us to be sure that all relevant terms were included and to see explicitly why the corresponding *s*- and *d*-wave terms behave so differently.

E. Temperature and energy-momenta dependences of Landau damping

We now compare the conditions on the momenta and energies for the existence of the nonzero Landau terms in *d*- and *s*-wave cases and the temperature dependences of these terms. As one can see from Eqs. (50), (55), and (57) for a given node the imaginary parts develop when $P_j>|\Omega|$ (or $\sqrt{v_F^2K_1^2+v_\Delta^2K_2^2}>|\Omega|$). Thus in contrast to the *s*-wave case when the imaginary part is nonzero for all directions of the phason momentum $\mathbf{K}$ satisfying the condition $-v_F|\mathbf{K}|<\Omega<v_F|\mathbf{K}|$ [or $|c|<1$, where c is defined after Eq. (52)], for the *d*-wave it is sensitive to the direction of the phason momenta as shown in Fig. 3. As we will discuss in Sec. VIII for HTSC's $v_F\gg v_\Delta$, so that the directional anisotropy of the Landau damping becomes very strong. Indeed, as can be seen from Fig. 3 for $|\Omega|/v_F K\lesssim 1$ and $\alpha_D\gg 1$ the Landau damping would exist only in a narrow region of the momenta directions. Furthermore, if the projection K_1 of the phason momentum $\mathbf{K}$ is not exactly zero ($\mathbf{K}\neq\hat{\mathbf{k}}_2$), the condition $|\Omega|<P_j$ can be well replaced by $|\Omega|<v_F|K_1|$. We will discuss the validity of this approximation in Sec. VIII. It is important to stress that the sharp directional dependence discussed here is not related to the nodal approximation and follows only from the gap anisotropy and the fact that the difference of the Fermi distribution functions (49) which is present in Eqs. (50), (55), and (57) has a very sharp $\mathbf{k}$ dependence, so that in principle unbounded [for example, $v_F(\mathbf{k})=\mathbf{k}/m$ for the model with the quadratic dispersion law[4]] functions $v_g(\mathbf{k})$ and $v_F(\mathbf{k})$ in the corresponding integrals were replaced by their values at the Fermi surface.

The final expressions (50), (55), and (57) for the *d*-wave case are in fact simpler than their *s*-wave counterparts, Eqs. (52), (56), and (58) [or the final expression (59)] because the latter expressions still have one integration which depends on the ratio $|c|$ via the Θ function. Thus if $|c|=|\Omega|/v_F K\to 1$, only large $y=\xi/\Delta_s$ contribute into the corresponding integrals. This circumstance and the opening of an isotropic gap Δ_s make the contribution from the Landau terms very small comparing to the main terms. For *d*-wave pairing a relative contribution of the Landau terms does not depend on the ratio Ω/P_j via p integration, so that all frequencies and momenta satisfying the conditions imposed by the Θ functions in Eqs. (50) and (55) have the same temperature-dependent weight.

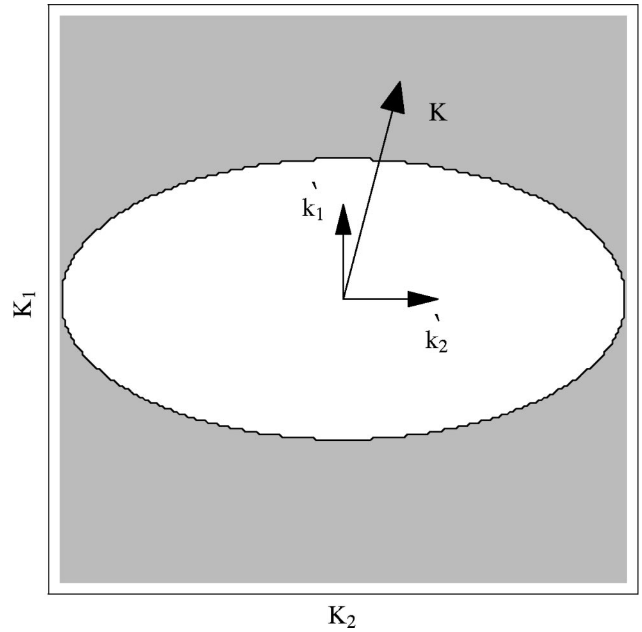

FIG. 3. The directional dependence of the Landau damping for one node imposed by $\Theta(1-|\Omega|/\sqrt{v_F^2K_1^2+v_\Delta^2K_2^2})$ for $\alpha_D=2$. For a given Ω only excitations with the momenta $\mathbf{K}$ which are outside the ellipse (in the shaded region) can contribute into the Landau damping. For a fixed ratio $|\Omega|/v_F|\mathbf{K}|$ the presence of the Landau damping depends not only on $|\mathbf{K}|$, but also on its direction. For example, for a vector having the length of the vector $\mathbf{K}$ shown in the figure the Landau damping is possible only if its direction is sufficiently close to the direction of $\hat{\mathbf{k}}_1$ normal to the Fermi surface.

As in the case with the superfluid density the temperature dependence of the Landau terms is linear in T, but while the superfluid density is nonzero at $T=0$, there is no Landau damping at $T=0$. The linear T dependence and the absence of damping at $T=0$ are obviously related to the fact that for *d*-wave pairing there are only four gapless points on the Fermi surface. It is known, for example, that for a normal (nonsuperconducting) system the Lindhard function (or polarization bubble),

$$L(\Omega,\mathbf{K}) = \int \frac{d^2k}{(2\pi)^2}\,\frac{n_F(\xi_+)-n_F(\xi_-)}{(2\pi)^2\xi_+-\xi_-+\Omega+i0}, \tag{60}$$

has a nonzero imaginary part even at $T=0$ (Ref. 24) if the Fermi surface remains ungapped since the derivative of the

Fermi distribution at $T \to 0$ becomes singular on the entire Fermi surface (or line in 2D).

Finally, we note that there is also an imaginary contribution even from the first, ''superfluid'' term in Eq. (A2) for A_-. As we already mentioned, this term involves pair breaking which for s-wave pairing has the threshold energy $2\Delta_s$. For d-wave pairing this energy is zero for $\mathbf{K}=0$ due to the presence of nodes. Nevertheless, if $\mathbf{K}\neq 0$ a finite energy P_j is necessary to create these excitations and the imaginary contribution is nonzero only for $|\Omega|>P_j$. Besides the analytical structure of this term is regular and it has a higher order than the terms considered here. Thus this term appears to be less important than the Landau terms we just considered.

VIII. APPROXIMATE FORM OF THE EFFECTIVE ACTION AND θ PROPAGATOR

Whereas the local nodal coordinate systems (P_j, ψ_j) are convenient to write down the corresponding contributions from each node, the final result should be presented in the global or laboratory coordinate system (K, ϕ). It is convenient to measure the angle ϕ from the vector $\hat{k}_x$, so that $\phi=0$ corresponds to the corner of the Fermi surface (see Fig. 1) and the first node is at $\phi=\pi/4$. Thus the transformations from the global coordinate system into the local system related to the jth node are

$$P_j = K\sqrt{v_F^2\cos^2\left(\phi-\frac{\pi}{4}+\frac{\pi}{2}(j-1)\right)+v_\Delta^2\sin^2\left(\phi-\frac{\pi}{4}+\frac{\pi}{2}(j-1)\right)},$$

$$\cos\psi_j = \frac{v_F K}{P_j}\cos\left(\phi-\frac{\pi}{4}+\frac{\pi}{2}(j-1)\right),$$

$$\sin\psi_j = \frac{v_\Delta K}{P_j}\sin\left(\phi-\frac{\pi}{4}+\frac{\pi}{2}(j-1)\right), \quad j=1,...,4. \tag{61}$$

The estimates of Ref. 11 show that in HTSC's YBCO $\alpha_D \equiv v_F/v_\Delta = 14$ and in BSCCO $\alpha_D=19$ (see Table I). Thus we can also use the inequality $\alpha_D \gg 1$ in what follows. First of all this inequality implies that for the jth node we may assume that

$$P_j \approx K_{v_F}\left|\cos\left(\phi-\frac{\pi}{4}-\frac{\pi}{2}(j-1)\right)\right|. \tag{62}$$

This approximation is, of course, valid only if $\mathbf{K}$ is not parallel to the Fermi surface ($\mathbf{K}\neq\hat{\mathbf{k}}_2$). We note that this direction which is ''dangerous'' for the jth node is the nodal direction for the neighboring nodes. The size of the dangerous direction where Eq. (62) becomes invalid can be estimated from the condition $Kv_F\sin\Delta\phi \approx Kv_\Delta\cos\Delta\phi$ which gives $\Delta\phi\approx\alpha_D^{-1}$. Since $\alpha_D>15$, Eq. (62) is in fact well justified outside the nodal regions.

Using Eq. (62) we can also rewrite the inequality discussed in Sec. VII E, $|\Omega|<P_j$, imposed by Θ functions which define the region where the Cerenkov condition can be satisfied as

$$|\Omega|<v_F K\left|\cos\left(\phi-\frac{\pi}{4}+\frac{\pi}{2}(j-1)\right)\right|. \tag{63}$$

One can easily see that the condition (63) is in fact equivalent to the condition $|\Omega|<v_F|K_1|$ we already mentioned.

The effective action (18) in the momentum representation in the global coordinate system can be written as

$$\beta\Omega_{\text{kin}} = \frac{i}{2}\int\frac{d\Omega}{2\pi}\int\frac{KdK}{2\pi}\int_0^{2\pi}\frac{d\phi}{2\pi}\theta(\Omega,\mathbf{K})[F^R(\Omega,K) + F^L(\Omega,K,\phi)]\theta(-\Omega,-\mathbf{K}) \tag{64}$$

with the regular [compare with Eqs. (40) and (41)]

$$F^R(\Omega,K) = \frac{K^2}{4}\left(\frac{\sqrt{\pi v_F v_\Delta}}{6a} - \frac{2\ln 2}{\pi}\frac{v_F}{v_\Delta}T\right) - \frac{\Omega^2}{4}\frac{1}{a\sqrt{\pi v_F v_\Delta}} \tag{65}$$

and Landau, $F^L(\Omega,K,\phi)$, parts.

It is possible to write down $F^L(\Omega,K,\phi)$ substituting Eq. (61) directly into Eqs. (50) and (55), but to have a more transparent expression we would like to consider a more simple case, $|\Omega|/v_F K \ll 1$. This condition becomes equivalent to $|\Omega|/P_j \ll 1$ due to the presence of Θ functions in Eqs. (50), (55), and (57) which are cutting out the forbidden domains with $|\Omega|/P_j>1$. Physically, the condition $\Omega/v_F K\ll 1$ is relevant if one, for instance, estimates the Landau damping for the BKT mode (43) because it follows from Eq. (43) that $|\Omega|/v_F K=\sqrt{\pi/(6\alpha_D)}\ll 1$. Thus we obtain

$$F^L(\Omega,K,\phi) = -i\frac{\Omega^3}{K}\frac{\ln 2}{\pi}\frac{T}{v_\Delta v_F^2}\left[f_1\left(\frac{\Omega}{v_F K},\phi\right)+2f_3\left(\frac{\Omega}{v_F K},\phi\right)\right] - i\Omega K\frac{\ln 2}{\pi}\frac{T}{v_\Delta}f_2\left(\frac{\Omega}{v_F K},\phi\right), \tag{66}$$

TABLE I. The Fermi velocities v_F and anisotropies of the Dirac spectrum α_D of $YBa_2Cu_3O_{6.9}$ and $Bi_2Sr_2CaCu_2O_8$ at optimal doping from Ref. 11. The velocities of the phase excitations for the continuum v_{cont} and lattice v_{lat} limits at $T=0$.

	v_F, $\times 10^7$ cm/s	α_D	v_{cont}, $\times 10^7$ sm/s	v_{lat}, $\times 10^6$ sm/c
YBCO	2.5	14	1.77	4.8
BSCCO	2.5	19	1.77	4.2

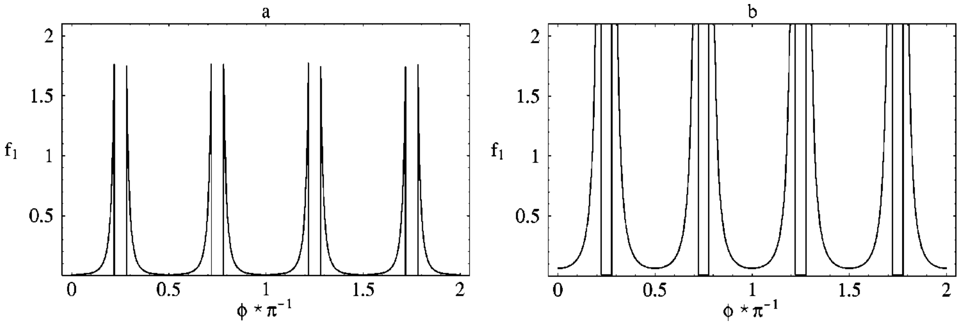

FIG. 4. (a) The angular dependence, $f_1(\Omega/v_F K,\phi)$ given by Eq. (67) of the Landau term $\sim\Omega^3/K$ for $\alpha_D=20$ and $|\Omega|/v_F K=0.1$ (b) The same function calculated directly from Eq. (50).

where

$$f_1\left(\frac{\Omega}{v_F K},\phi\right)=\frac{\alpha_D^{-2}\sin^2(\phi-\pi/4)}{[\cos^2(\phi-\pi/4)+\alpha_D^{-2}\sin^2(\phi-\pi/4)]^{3/2}}\times\Theta\left(|\cos(\phi-\pi/4)|-\frac{|\Omega|}{v_F K}\right)+\frac{\alpha_D^{-2}\cos^2(\phi-\pi/4)}{[\sin^2(\phi-\pi/4)+\alpha_D^{-2}\cos^2(\phi-\pi/4)]^{3/2}}\times\Theta\left(|\sin(\phi-\pi/4)|-\frac{|\Omega|}{v_F K}\right), \tag{67}$$

$$f_2\left(\frac{\Omega}{v_F K},\phi\right)=\frac{\cos^2(\phi-\pi/4)}{[\cos^2(\phi-\pi/4)+\alpha_D^{-2}\sin^2(\phi-\pi/4)]^{1/2}}\times\Theta\left(|\cos(\phi-\pi/4)|-\frac{|\Omega|}{v_F K}\right)+\frac{\sin^2(\phi-\pi/4)}{[\sin^2(\phi-\pi/4)+\alpha_D^{-2}\cos^2(\phi-\pi/4)]^{1/2}}\times\Theta\left(|\sin(\phi-\pi/4)|-\frac{|\Omega|}{v_F K}\right), \tag{68}$$

and

$$f_3\left(\frac{\Omega}{v_F K},\phi\right)=-\frac{\cos^2(\phi-\pi/4)}{[\cos^2(\phi-\pi/4)+\alpha_D^{-2}\sin^2(\phi-\pi/4)]^{3/2}}\times\Theta\left(|\cos(\phi-\pi/4)|-\frac{|\Omega|}{v_F K}\right)-\frac{\sin^2(\phi-\pi/4)}{[\sin^2(\phi-\pi/4)+\alpha_D^{-2}\cos^2(\phi-\pi/4)]^{3/2}}\times\Theta\left(|\sin(\phi-\pi/4)|-\frac{|\Omega|}{v_F K}\right), \tag{69}$$

The functions f_1, f_2, and f_3 are obtained from Eqs. (50), (55), and (57), respectively. Deriving Eqs. (67)–(69) we used the assumption $|\Omega|/P_j \ll 1$ replacing the square roots in Eqs. (50), (55), and (57) by 1. Furthermore, we kept only $\sin^2\psi_j$ from Eq. (50). Nevertheless, we kept the Θ functions which are present in Eqs. (50), (55), and (57) because they are essential in imposing the condition Ω/P_j.

In Figs. 4(a), 5(a), and 6(a) we show the functions f_1, f_2, and f_3 which describe the intensity of Landau damping (f_1 and f_3 for the term $\sim\Omega^3/K$ and f_2 for the term $\sim\Omega K$, respectively) as a function of the direction in the plane. Although the functions f_1 and f_3 describe the directional dependence for the same $\sim\Omega^3/K$ term, we do not combine these functions into the single function because they originate from the different expressions. [We recall that f_1 originates from Eq. (50) and f_3 originates from Eq. (57), respectively.] For the comparison in Figs. 4(b), 5(b), and 6(b) we show the directional dependences calculated by the direct substitution of Eq. (61) into Eqs. (50), (55), and (57) without making the approximations we just described. As one can see, the approximate representation (66) and (67)–(69) despite its relatively simple form (we have used only the terms $\sim\Omega^3/K$ and $\sim\Omega K$) gives reasonably good expression for the Landau damping terms. The biggest discrepancies are seen between Figs. 4(a) and 4(b) because more approximations were made to obtain the f_1 term.

As we mentioned in Sec. VII, the angular dependence described by f_1 [see Fig. 4(a)] coincides with the angular dependence of the ultrasonic attenuation[10] in the limit $\Omega/v_F K\to 0$. Indeed the Θ functions from Eq. (67) disappear when $\Omega/v_F K\to 0$ and Eq. (67) reduces to the corresponding equation from Ref. 10.

We stress also that the analytical structure of the damping terms in Eq. (66) appears to be quite different from the dissipation introduced in Ref. 27 into the "phase only" action basing on the low-frequency conductivity measurements.

Since f_1+2f_3 and f_2 terms are odd functions of the frequency Ω, they integrate to zero in $\Omega_{\rm kin}$. Nevertheless, the damping terms are manifest in the equation of motion and in the propagator of the BKT mode which is considered below.

Comparing Figs. 4, 5, and 6, one can see that the functions f_1 and f_3 have more pronounced angular dependence than f_2. Although as we explained above, the contribution from a given node is zero if for some **K** the condition (63) is not satisfied, this condition can still simultaneously be satisfied for the neighboring nodes, so that the sum over all nodes in f_1-f_3 is not necessarily zero. We note also that by the absolute value both f_1 (or f_3) and f_2 terms are practically the same. This can be easily seen if we rewrite, for example, the f_1 term as

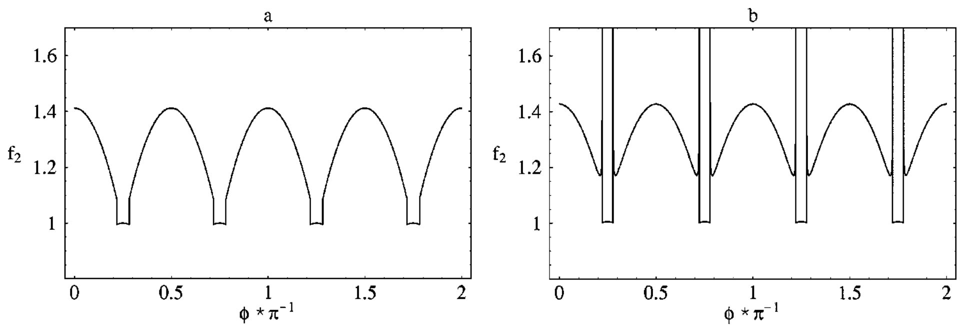

FIG. 5. (a) The angular dependence, $f_2(\Omega/v_F K,\phi)$ given by Eq. (68) of the Landau term $\sim\Omega K$. The parameters α_D and $|\Omega|/v_F K$ are the same as in Fig. 4. (b) The same function calculated directly from Eq. (55).

$$\frac{\Omega^3}{K}\frac{\ln 2}{\pi}\frac{T}{v_\Delta v_F^2}f_1\left(\frac{\Omega}{v_F K},\phi\right)$$
$$=\Omega K\left(\frac{\Omega}{v_F K}\right)^2\frac{\ln 2}{\pi}\frac{T}{v_\Delta}f_1\left(\frac{\Omega}{v_F K},\phi\right)$$
$$\approx\Omega K\frac{\ln 2}{6}\frac{T}{v_F}f_1\left(\sqrt{\frac{\pi}{6\alpha_D}},\phi\right), \tag{70}$$

where writing the last identity we explicitly used the ratio $\Omega/v_F K$ for the BKT mode at $T=0$ given above.

In such a way following Ref. 4 we can write an approximate propagator of the BKT mode near $T=0$ as

$$D_\theta^R(\Omega,\mathbf{K})\approx\left\{\frac{1}{4a\sqrt{\pi v_F v_\Delta}}\left[\Omega^2-K^2\left(\frac{\pi v_F v_\Delta}{6}-\frac{2\ln 2 a v_F}{\pi}\sqrt{\alpha_D}T\right)+2ia\gamma(\phi)T\Omega K\right]\right\}^{-1} \tag{71}$$

with

$$\gamma(\phi)=\ln 2\left\{\frac{1}{6}\sqrt{\frac{\pi}{\alpha_D}}\left[f_1\left(\sqrt{\frac{\pi}{6\alpha_D}},\phi\right)+2f_3\left(\sqrt{\frac{\pi}{6\alpha_D}},\phi\right)\right]+\sqrt{\frac{\alpha_D}{\pi}}f_2\left(\sqrt{\frac{\pi}{6\alpha_D}},\phi\right)\right\}. \tag{72}$$

As discussed in Ref. 4 Eq. (71) has the form of a bosonic propagator with damping and its linewidth has an explicit $\mathbf{K}$ dependence. In contrast to the *s*-wave case, the width depends not only on the absolute value of $|\mathbf{K}|$, but also on the direction of $\mathbf{K}$. The angular dependence for $\gamma(\phi)$ is shown in Fig. 7. Despite a simple form of the propagator (71), the corresponding effective wave operator (the transform of D_θ^{-1} to space and time) has a nonlocal form in coordinate space due to the presence of the damping term which depends[4] on $|\mathbf{K}|$.

Comparing the second term in parentheses of Eq. (71) with the damping term one can see that they have the same order of magnitude showing that even for the low-temperature region the Landau damping becomes important and its magnitude is comparable with the term which describes the linear low-temperature decrease in the superfluid stiffness.

We stress also the difference between the Landau damping representation in Eqs. (71) and (66). While the propagator (71) relies on the particular dispersion law for the BKT mode with the approximate v given by Eq. (43), our Eq. (66) along with Eqs. (67)–(69) present a rather general representation for the Landau damping terms derivation which did not rely on any particular dispersion law for the phase excitations. Thus it can be, in principle, used to describe the damping of the plasmons in a charged superconductor. The only assumption we made is that $\Omega/v_F K\ll 1$, which was used to derive more simple and transparent representation for the Landau damping. For the more general case one should use the results from Sec. VII.

IX. CONCLUDING REMARKS

It is very important to note that the collective phase excitations described by the propagator similar to Eq. (71) can be and have been studied experimentally. Indeed, the measurements of the order-parameter dynamical structure factor in the dirty Al films allowed us to extract the dispersion relation of the corresponding Carlson-Goldman mode and to investigate its temperature dependence.[28] It is important to stress

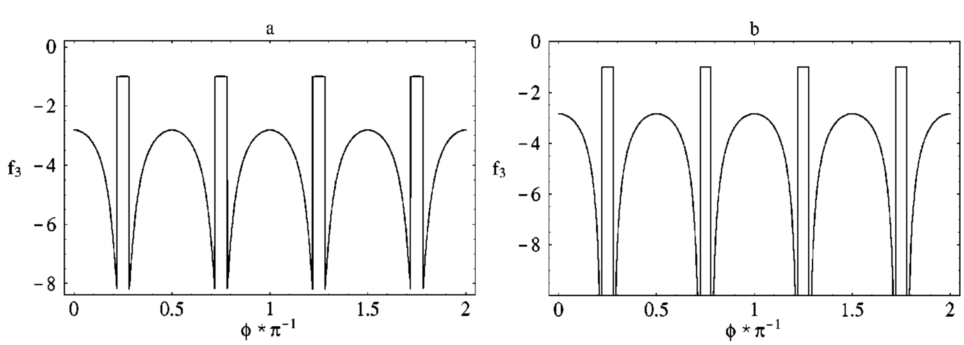

FIG. 6. (a) The angular dependence, $f_3(\Omega/v_F K,\phi)$ given by Eq. (69) of the Landau term $\sim\Omega^3/K$. The parameters α_D and $|\Omega|/v_F K$ are the same as in Fig. 4. (b) The same function calculated directly from Eq. (57).

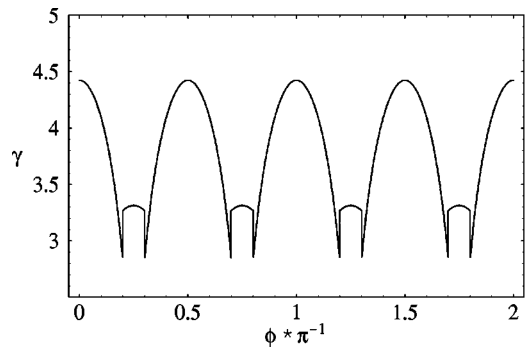

FIG. 7. The angular dependence of the decay constant $\gamma(\Omega v_F K,\phi)$ in θ propagator (71) for $\alpha_D=20$ and $|\Omega|/v_F K = \sqrt{\pi/6\alpha_D}\simeq 0.16$.

that since the real systems are *charged* this mode appears to be different from the soundlike Bogolyubov-Anderson mode which exists in a *neutral* superfluid Fermi liquid. However, as discussed in Ref. 22 (see also Ref. 8 and references therein) the discovery of the Carlson-Goldman collective mode[28] overcame the widespread opinion that the Bogolyubov-Anderson collective oscillations predicted for a neutral superfluid should be forced to the frequencies on the order of the plasma frequency. It appears that if under certain conditions the frequency of the phase excitations is sufficiently low, the normal electron fluid may screen completely the associated electric field, thereby making the theory of the uncharged superconductor applicable.[29] The conditions for screening and occurrence of the Carlson-Goldman mode in *s*-wave superconductors are favorable only if the systems are dirty (to suppress the Landau damping) and $T \lesssim T_c$.[22] As recently claimed in Ref. 8 in *d*-wave superconductors the conditions appear to be much less strict, so that the Carlson-Goldman mode may be observed in the clean systems for T down to $0.2T_c$. Now there are some new questions which would be interesting to address from both experimental and theoretical points of view.

First of all, there is a general question of whether the Carlson-Goldman mode which is the equivalent of the Goldstone mode for the charged system can be observed experimentally. There are also a lot of theoretical questions the study of which would make this kind of experiment possible. Leaving aside the specifics of the Carlson-Goldman mode studied in Ref. 8 we mention some of them which are directly related to the present work.

(i) The Carlson-Goldman experiment[28] measures so-called dynamical structure factor. This factor has a peak associated with the phase excitations the width of which is primarily controlled by the Landau damping. Therefore our result that the intensity of Landau damping has a strong directional dependence should be taken into account in the calculation of the structure factor. In particular, the lowest for the observation of the Carlson-Goldman mode temperature $0.2T_c$ in Ref. 8 is obtained for the nodal direction. As can be seen from, for example, Figs. 4 and 5, the corresponding Landau damping terms become maximal in the vicinity of this direction which should likely result in the widening of the corresponding peak. If this widening is so big that the peak may become unobservable, this would again suggest that the dirty samples should be used for the experiment.

(ii) Taking into account the lattice effects and the dominance of the nodal excitations we have obtained that the velocity of the phase excitations at $T\to 0$ is given by $v_{\mathrm{lat}}=v_F\sqrt{\pi/6\alpha_D}$ [see Eq. (43) and the discussion after Eq. (23)]. This expression appears to be quite different from the well-known continuum result, $v_{\mathrm{cont}}=v_F/\sqrt{2}$. The estimates of the velocities v_{cont} and v_{lat} obtained from the values v_F and α_D (Ref. 11) are presented in Table I. It is seen from these estimates that the difference between the description proceeding from continuum and lattice models can be quite different. However, the estimates of the minimal for the observation of the Carlson-Goldman mode temperature in Ref. 8 are based on the continuum expression $v_{\mathrm{cont}}=v_F/\sqrt{2}$. Thus it would be interesting to reconsider these estimates for the lattice case.

(iii) Finally, as pointed out in Ref. 9 if the plasmon is at finite frequency at $\mathbf{K}\to 0$, the Landau damping does not occur since when Ω is finite and $\mathbf{K}\to 0$ it is impossible to satisfy the Čerenkov condition discussed above. However, a very small value of the plasma frequency in HTSC's suggests that the Landau damping may still be relevant when $\mathbf{K}$ is nonzero and the Čerenkov condition can be satisfied. Thus the corresponding damping terms should be included in the "phase only" actions which are used to describe plasma excitations in *d*-wave superconductors. Predicted anisotropy of the Landau damping would result in the damping anisotropy for the plasma excitations.

To summarize, we have considered the phase only effective action for so-called *neutral* (or *uncharged*) fermionic superfluid with *d*- and *s*-wave pairing in 2D. When the damping terms are included into this action, it turns out nonlocal in coordinate space and its analytical structure for the *d*-wave case is very different from the *s*-wave case. To consider a charged superfluid it is necessary to combine the approaches used in the present paper and in Refs. 8 and 9 to consider the Landau damping in the presence of Coulomb interaction.

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APPENDIX

Substituting Eq. (21) into Eqs. (30) and (34), using the definitions for π_{ij} and Π_{ij} and evaluating the matrix traces for the diagonal terms we obtain

$$A_{\pm}\equiv\begin{bmatrix}\Pi_{00}^{\alpha\beta}(i\Omega_n;\mathbf{K})\\ \Pi_{33}(i\Omega_n,\mathbf{K})\end{bmatrix}=T\sum_{l=-\infty}^{\infty}\int\frac{d^2k}{(2\pi)^2}\frac{2[(i\omega_l+i\Omega_n)i\omega_l+\xi(\mathbf{k}-\mathbf{K}/2)\xi(\mathbf{k}+\mathbf{K}/2)\pm\Delta(\mathbf{k}-\mathbf{K}/2)\Delta(\mathbf{k}+\mathbf{K}/2)]V_{\pm}(\mathbf{k})}{[(i\omega_l+i\Omega_n)^2-\xi^2(\mathbf{k}+\mathbf{K}/2)-\Delta^2(\mathbf{k}+\mathbf{K}/2)][(i\omega_l)^2-\xi^2(\mathbf{k}-\mathbf{K}/2)-\Delta^2(\mathbf{k}-\mathbf{K}/2)]},\tag{A1}$$

$$V_{\pm}(\mathbf{k})\equiv\begin{bmatrix}v_{F\alpha}(\mathbf{k})v_{F\beta}(\mathbf{k}), & \text{``}+\text{''};\\ 1, & \text{``}-\text{''}.\end{bmatrix}$$

Performing in Eq. (A1) the summation over Matsubara frequencies and simplifying the result we arrive at

$$\begin{aligned}A_{\pm}=-\int\frac{d^2k}{(2\pi)^2}\Bigg\{&\frac{1}{2}\left(1-\frac{\xi_-\xi_+\Delta_-\Delta_+}{E_-E_+}\right)\left[\frac{1}{E_++E_-+i\Omega_n}+\frac{1}{E_++E_--i\Omega_n}\right][1-n_F(E_-)-n_F(E_+)]\\&+\frac{1}{2}\left(1+\frac{\xi_-\xi_+\pm\Delta_-\Delta_+}{E_-E_+}\right)\left[\frac{1}{E_+-E_-+i\Omega_n}+\frac{1}{E_+-E_--i\Omega_n}\right][n_F(E_-)-n_F(E_+)]\Bigg\}V_{\pm}(\mathbf{k}),\end{aligned}\tag{A2}$$

where $\xi_{\pm}\equiv\xi(\mathbf{k}\pm\mathbf{K}/2)$, $E_{\pm}\equiv E(\mathbf{k}\pm\mathbf{K}/2)$, and $\Delta_{\pm}\equiv\Delta(\mathbf{k}\pm\mathbf{K}/2)$. The second term in Eq. (A2) can be further simplified if one notices that the term with $1/(E_+-E_--i\Omega_n)$ transforms into the term $(-1)/(E_+-E_-+i\Omega_n)$ when $\mathbf{k}\to-\mathbf{k}$. Note that the $A_{\pm}$ terms do coincide with the corresponding terms in Ref. 4.

For the mixed term (35) one obtains

$$\begin{aligned}\Pi_{03}^{\alpha}(i\Omega_n,\mathbf{K})&=T\sum_{l=-\infty}^{\infty}\int\frac{d_2k}{(2\pi)^2}\frac{2[(i\omega_l+i\Omega_n)\xi(\mathbf{k}-\mathbf{K}/2)+\xi(\mathbf{k}+\mathbf{K}/2)i\omega_l]v_{F\alpha}(\mathbf{k})}{[(i\omega_l+i\Omega_n)^2-\xi^2(\mathbf{k}+\mathbf{K}/2)-\Delta^2(\mathbf{k}+\mathbf{K}/2)][(i\omega_l)^2-\xi^2(\mathbf{k}-\mathbf{K}/2)-\Delta^2(\mathbf{k}-\mathbf{K}/2)]}\\&=\int\frac{d^2k}{(2\pi)^2}\Bigg\{\left(\frac{\xi_+}{2E_+}-\frac{\xi_-}{2E_-}\right)\left[\frac{1}{E_++E_-+i\Omega_n}-\frac{1}{E_++E_--i\Omega_n}\right][1-n_F(E_-)-n_F(E_+)]+\left(\frac{\xi_+}{2E_+}+\frac{\xi_-}{2E_-}\right)\\&\quad\times\left[\frac{1}{E_+-E_-+i\Omega_n}-\frac{1}{E_+-E_--i\Omega_n}\right][n_F(E_-)-n_F(E_+)]\Bigg\}v_{F\alpha}(\mathbf{k})\\&=\int\frac{d^2k}{(2\pi)^2}\Bigg\{\frac{\xi_+}{E_+}\left[\frac{1}{E_++E_-+i\Omega_n}-\frac{1}{E_+-E_--i\Omega_n}\right][1-n_F(E_-)-n_F(E_+)]\\&\quad+\left(\frac{\xi_+}{E_+}+\frac{\xi_-}{E_-}\right)\frac{1}{E_+-E_-+i\Omega_n}[n_F(E_-)-n_F(E_+)]\Bigg\}v_{F\alpha}(\mathbf{k}).\end{aligned}\tag{A3}$$

The get the last identity we again replaced $\mathbf{k}\to-\mathbf{k}$ and took into account that $v_F(\mathbf{k})$ also changes sign under this transformation. One can also check that $\Pi_{30}^{\alpha}(i\Omega_n,\mathbf{K})=\Pi_{03}^{\alpha}(i\Omega_n,\mathbf{K})$. Equation (A3) is also in agreement with the corresponding term in Ref. 4 denoted as D.

The first and second terms in Eqs. (A2) and (A3) have a clear physical interpretation.[24] The first term gives the contribution from ''superfluid'' electrons. The second term give the contribution of the thermally excited quasiparticles (i.e., ''normal''-fluid component). The essential physical difference between the two terms is that the superfluid term involves creation of two quasiparticles, with the minimum excitation energy in the s-wave case being $2\Delta_s$. (For the d-wave case the minimum energy turns out to be zero in the four nodal points, see the discussions at the end of Sec. VII.)

On the other hand, the normal-fluid term involves scattering of the quasiparticles *already present* and the excitation energy in this case can be arbitrary small, as in the normal metal, independently whether we are in the vicinity of the node or not. Of course, the number of the quasiparticles participating in this scattering depends on the value of the gap and drastically increases if the gap is equal to zero in some points. As we shall see, the Landau damping terms originate from the second, normal-fluid term. Note that our Eqs. (A2) and (A3) [as well as Eqs. (30), (34), and (35)] are suitable for studying both s- and d-wave cases.

*On leave of absence from Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine; electronic address: Sergei.Sharapov@unine.ch

†Electronic address: Hans.Beck@unine.ch

‡Electronic address: vloktev@bitp.kiev.ua

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