Bose metal: Gauge-field fluctuations and scaling for field-tuned quantum phase transitions

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We extend our previous discussion of the Bose metal to the field-tuned case. We point out that the recent observation of the metallic state as an intermediate phase between the superconductor and the insulator in the field-tuned experiments on MoGe films is in perfect consistency with the Bose metal scenario. We establish a connection between general dissipation models and gauge-field fluctuations and apply this to a discussion of scaling across the quantum phase boundaries of the Bose metallic state. Interestingly, we find that the Bose metal scenario implies a possible *two* parameter scaling for resistivity across the Bose metal-insulator transition, which is remarkably consistent with the MoGe data. Scaling at the superconductor-metal transition is also proposed, and a phenomenolgical model for the metallic state is discussed. The effective action of the Bose metal state is described and its low energy excitation spectrum is found to be $\omega \propto k^3$.

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I. INTRODUCTION

In a recent paper,[1] we argued that a system of interacting Cooper pairs may form a gapless nonsuperfluid liquid, i.e., a metallic state, in two dimensions at $T=0$, when no external magnetic field is present. We called this state a Bose metal (BM). Although such an idea might seem rather counter-intuitive offhand since a system of delocalized bosons *typically* forms a superfluid (SF) state at $T=0$, closer thinking suggests that it is not. A Bose metal is possible in the same sense that a spin liquid is possible in quantum antiferromagnets. In fact, we showed that it is just another variety of a spin liquid: an E_2 spin liquid. We found our predictions consistent with the features of the metallic state observed in films of granular superconductors at low temperatures in the absence of any external magnetic field.[27]

In amorphous superconducting films, particularly MoGe, metal-like states analogous to the ones in granular superconductors,[1] have been observed in the presence of a magnetic field. The fact that such a state exists as an intermediate phase between a superconductor (SC) and an insulator (INS), as predicted in Ref. 1, is already a good hint that the Bose metal scenario might be applicable to these systems as well. In this paper, we generalize the arguments which were previously developed for the case of zero applied magnetic field, to the field tuned case. We show that the results of field tuned experiments in superconducting films, where such a metallic state has also been seen,[2–4] also fit in with our concept of a Bose metal.

We first discuss (Sec. II) the fact that gauge field fluctuations present in the vortex system in the JJA model play a key role in the formation of the BM state and show how generalized dissipation models can also similarly lead to an analogous BM state. We then discuss how the concept of the BM state may be generalized to the field tuned case. The observation of three phases in Ref. 5 is consistent with the Bose metal scenario. This leads to the suggestion of new scaling formulas for resistivity across the SC-BM and BM-INS transitions. These are presented in Sec. III and are shown to work very well for the MoGe data. In Sec. IV, we develop a phenomenological model for the metallic state based on the physics described above, which we find is in good agreement with the MoGe data. Finally, in Sec. V, we take this opportunity to discuss some of the general properties of the Bose metal state: e.g., its effective action and the low energy excitation spectrum, etc. Here, we also address the issue of why the BM state has not been observed in the previous analyses of the JJA model.

II. THE JJA MODEL AND EFFECTS OF DISSIPATION

Dynamical gauge field fluctuations: In Ref. 1, we considered a Josephson junction array (JJA) model with onsite and nearest neighbor repulsion for large Cooper pair fillings and zero external gate voltage (i.e., $\Sigma_i \langle \delta\hat{n}_i \rangle = 0$, where $\delta\hat{n}_i$ is the charge fluctuation operator at the site i^1). We showed using the duality transformation relations that this model maps onto a *two*-component *quantum* plasma of vortices (V) and antivortices ($\bar{V}$), i.e., a set of *nonrelativistic* bosons, moving in a dynamically fluctuating gauge field A^μ.

This picture of vortices moving in a fluctuating gauge field is a simple quantum mechanical extension of the results of classical phase fluctuations in a 2D superconductor. In the classical regime, a 2D superconductor (with phase fluctuations) maps onto a two component classical plasma ($V\bar{V}$) undergoing screening by a static electric field ($\vec{E}=-\vec{\nabla}A_0$), as described by Kosterlitz and Thouless (KT).[6] In the quantum regime two effects are important: (a) the Bose statistics of the vortices and antivortices allows for the possibility of their superfluidity; and (b) because of quantum fluctuations, the electric field is no more static but dynamical, i.e., $\vec{E}=-\vec{\nabla}A_0-(1/c_s)\partial\vec{A}/\partial\tau$ and is associated with a corresponding magnetic field $\vec{B}=\vec{\nabla}\times\vec{A}$. Hence, expressed in terms of vortices, in the quantum regime, two processes are important: depairing of $V\bar{V}$ pairs [or, blowing up of vortex loops in $(2+1)$D] and Bose condensation of depaired vortices and antivortices, corresponding to the loss of phase order and growth of charge order for the Cooper pairs, as discussed in Ref. 1. The presence of dynamically fluctuating gauge field $\vec{A}$ causes the vortices and antivortices to pick up random Aharonov-Bohm (AB) phases $\exp(i\int\vec{A}\cdot d\vec{l})$ which makes these two processes distinct from one another. If there were no fluctuating gauge field, then because of the quantum zero point motion, the vortices and antivortices would Bose condense into a phase coherent state as soon as they unbind, i.e., the film becomes insulating. Now because of the gauge field $\vec{A}$, the situation here is different. If the gauge field fluctuations are small, the dephasing induced by random AB phases is weak and superfluidity of V and $\overline{V}$ is retained. On the other hand, if the gauge field fluctuations are very strong, then, because of the random AB phases, the unbound V and $\overline{V}$ fail to produce a phase coherent state, and the system is a nonsuperfluid liquid.

The effects of these gauge field fluctuations may be seen very clearly in terms of a world line picture for the vortices (Appendix A). We represent this as a bosonic system interacting with a fluctuating gauge field $\vec{A}$. The partition function is[1,7,9]

$$Z_{BA}=\frac{1}{N!}\sum_{P}\int_{\{r_i(\beta)=r_{Pi}(0)\}}\prod_i Dr_i(\tau)D\vec{A}(\vec{r},\tau)\times\exp\left(i\sum_i\int_{r_i(0)}^{r_i(\beta)}\vec{A}\cdot d\vec{r}_i\right)\exp\left(-\int_0^\beta d\tau\left[\sum_i\frac{m}{2}\dot{r}_i^2+\frac{1}{2}\sum_{i\neq j}v(r_i-r_j)\right]\right)\exp(-S_G(\vec{A})),$$

where $S_G(\vec{A})$ denotes the gauge part of the action and P is the permutation of the particles.[25] This type of partition function may be obtained, for example, by substituting $j^0(r,\tau)=\Sigma_i q_i\delta(r-r_i(\tau))$ and $j^\alpha(r,\tau)=\Sigma_i q_i\dot{r}_i^\alpha(\tau)\delta(r-r_i(\tau))$ in Eq. (15) in Ref. 1, where q_i is the vortex charge. The AB phase factor $\exp(i\int\vec{A}\cdot d\vec{r}_i)$ appears explicitly here. When the gauge field fluctuations are weak, this factor is close to 1. So, this case is very similar to the Bose system without any gauge field, and the ground state is an entangled liquid, i.e., a superfluid (Appendix A). Now, when the gauge field fluctuations become very large, the AB phase trapped by the bosons is of order π (modulo 2π), and the phase factor appears in Z_{BA} with fluctuating signs. This would cause cancellation of several terms in Z_{BA} when entangled configurations are present, implying that such configurations enter the partition function with a low weight and are correspondingly high energy states, just like the fermionic case (Appendix A). So, in this case, the ground state is a disentangled liquid, i.e., a nonsuperfluid. Phase separation is not possible because of the long-range interactions present in the (original) vortex Bose system. Hence, as gauge field fluctuations increases, there should be a phase transition from the superfluid to a nonsuperfluid state.

This discussion makes explicit why random AB phases causes disentanglement. The nonsuperfluid state of bosons, thus obtained due to dephasing by random AB phases, is a liquid rather than a solid where the particles are localized by fluctuations, because the gauge field fluctuates with time. Let us say that the vortices get localized by fluctuations. Then, since they see a magnetic field that fluctuates with time, by Faraday's law, they see an induced electric field as well. This electric field will knock the localized particles out of their positions. Thus, the presence of a dynamically fluctuating gauge field in the quantum vortex system leads to the possibility of a non-SF *liquid* phase, and by duality,[10] a consequent possibility of a metallic state for the Cooper pairs, the original players in the system. Because the vortices feel retardation effects due to gauge field fluctuations, they dissipate even in a pure system, thus leading to finite resistivity at $T=0$.

In summary, the effect of AB phases induced by the gauge field fluctuations is to disorder the vortices and to create a *new* variety of quantum liquid, some signatures of which we discuss in this paper.

Effects of dissipation. The following four features about the Bose metal scenario emerge from the description of the JJA model above and in Ref. 1: (i) when the Bose metal is observed, it exists as an intermediate phase between the SC and the INS; (ii) the observation of the BM phase is associated with two phase transitions: one from SC to BM and the other from BM to INS; (iii) the BM phase is dominated by dynamical gauge field (gf) fluctuations, as felt by the vortices; and (iv) there is a competition between gf fluctuations and quantum zero point motion of the vortices at the BM-INS phase boundary: in the INS phase, quantum zero point motion wins and the vortices are in the SF phase.

It has been suggested by Mason and Kapitulnik[2] that dissipation effects will quite generally help the formation of a metallic like state for vortices in the quantum regime. We show here that the effects of a generalized dissipation model on the quantum motion of the vortices also maps onto a set of nonrelativistic bosons moving in a dynamically fluctuating gauge field, just like the JJA model.

Consider a generic dissipation model (no static disorder):

$$S=\int_0^\beta d\tau d^2r\frac{m}{2n}|\vec{j}(r)|^2+\int_0^\beta d\tau d^2rd^2r'\rho(r,\tau)V(r-r')\rho(r',\tau)+\frac{1}{2}\int_0^\beta d\tau d\tau' d^2rj^\alpha(r,\tau)\eta_{\alpha\beta}(r-r',\tau-\tau')j^\beta(r,\tau'), \tag{1}$$

where $j^\alpha(r,\tau)=\Sigma_i q_i\dot{r}_i^\alpha(\tau)\delta(r-r_i(\tau))$, $\rho(r,\tau)=\Sigma_i q_i\delta(r-r_i(\tau))$ and $n=$ average vortex density. $q_i=$vortex charge, and the partition function is $Z=\int Dr_i(\tau)e^{-S}$. The first term is the quantum zero point motion term (also, known as the mass term); the second term is the long-range (logarithmic, in a pure system) interaction among the vortices and the last term is the dissipation term. Summation is implied in the dissipation term and α, β refer to the *space* coordinates. This action can be recast, by virtue of a Hubbard Stratanovich transformation on the associated partition function, as

$$S=\int d\tau d^2r\frac{m}{2n}|\vec{j}(r)|^2+\int d\tau d^2rd^2r'\rho(r,\tau)V(r-r')\times\rho(r',\tau)+i\int d^2rd\tau j^{\alpha}(r,\tau)a^{\alpha}(r,\tau)+\int d\tau d\tau' d^2rd^2r' a^{\alpha}(r,\tau)\times K_{\alpha\beta}(r-r',\tau-\tau')a^{\beta}(r',\tau'), \tag{2}$$

with $K_{\alpha\beta}(\omega_n,k)=\eta^{-1}_{\alpha\beta}(\omega_n,k)$ (ω_n=Matsubara frequency). The dissipation kernel $K_{\alpha\beta}$ here is model dependent and the field a^{μ} is introduced through the transformation. The Maxwellian type of coupling $\vec{j}\cdot\vec{a}$ implies that a^{μ} represents a gauge field. For example, for the Caldeira Leggett heat bath $K_{\alpha\beta}(\omega_n,k)=\delta_{\alpha\beta}|\omega_n|/\eta$, which is time-dependent. The dynamical nature of the gauge field follows from the fact that dissipation is associated with velocity dependent forces.

Clearly, this mapping implies two things. First, as the vortices move in the presence of dissipation, they trap a random AB phase $\exp(i\int\vec{a}\cdot d\vec{l})$, and consequently the arguments of dephasing by random AB phases presented earlier in this section imply a disordered liquid phase for a dissipative system as well. For example, if we consider the case of Caldeirra-Leggett heat bath above, we see that: when dissipation η is much weaker than the quantum zero point motion, gauge field fluctuations are weak, the randomness induced by the AB phases is very small and the delocalized vortices are in an entangled state, i.e., a superfluid. However, when dissipation is very strong, gauge field fluctuations are also correspondingly very strong, and the random AB phases can dephase the vortices into a disentangled, i.e., nonsuperfluid, state. The liquidity of this state follows from the dynamical nature of the gauge field. Because of this mapping, we shall quite often use gauge field fluctuations and dissipation interchangeably in this paper. Secondly, the above mapping means that a dissipative quantum vortex system (where vortices have been induced by an external magnetic field) has a very similar phase diagram to that of the JJA model considered in Ref. 1, except that now dislocations are present (see below). There will be an SC phase which consists of dislocation-antidislocation pairs, when quantum fluctuations are weak. As quantum fluctuations are increased (by tuning the field, for example), the pairs will unbind. The state this film would enter depends on the strength of dissipation. If dissipation is weak, the vortices will form a superfluid state and the transition is from SC to INS. On the other hand, when the dissipation is strong, the vortices would first enter an uncondensed liquid like state due to dephasing effect by the AB phases. By duality, this state is a non-SF liquid for the Cooper pairs (see first part of this section), and hence, it is fair to call this a Bose metal. The resistance is induced by the free dislocations in this case. As the field is increased further, quantum zero point motion overcomes dissipation (or, gf fluctuations), the vortices would Bose condense and the film will be insulating. Thus the transition is SC-metal-INS in this case. We explore some of the consequences of this scenario below.

There are two key points which need to be kept track of while generalizing the results for vortices from the zero field case to the field driven case in a dirty film. (a) In the presence of an external field, vortices enter an SC film in the form of an Abrikosov lattice. True long range order is not possible in a 2D system at finite temperatures: dislocation-antidislocation pairs are created in the Abrikosov lattice. As temperature is increased, these pairs unbind (the analog of $V\overline{V}$ unbinding in the zero field case), and (Cooper pair) superconductivity is destroyed. (b) In the presence of static disorder, the Abrikosov lattice is converted into a glass, called a vortex glass (VG). Energy barriers between the various metastable states in this glassy phase are *finite* in 2D.[11–13] One way to understand this is that dislocations are pointlike objects in 2D, and that disorder screens long-range log interaction between the dislocations. Thus, the energy barriers to create dislocation-antidislocation pairs is finite in 2D, and the energy barrier to their motion is very small, particularly in the collective pinning regime,[13] in which we mostly focus on here. This means that as soon as the dislocation-antidislocation pairs are created, they will hop around and induce finite resistance at any finite temperature $\sim\exp(-\epsilon_d/kT)$, where ϵ_d=energy barrier to creation of dislocation pairs. This would imply that true superconductivity should set in at $T=0$. However, at $T=0$, tunneling processes resulting from quantum fluctuations due to quantum zero point motion, etc. can in principle be strong and destroy long-range order in the vortex glass phase, which is discussed in this paper.

III. SCALING AT THE ZERO TEMPERATURE PHASE TRANSITIONS

The presence of a Bose metallic phase in a SC film is associated with two phase transitions: one from SC to BM and another from BM to INS. Correspondingly there will be two scaling behaviors for the resistivity even for the field-tuned case, which we discuss below.

(1) *SC-BM transition*. The first transition is associated with the unbinding of (quantum) dislocation-antidislocation pairs (or, in a dirty system, when the *free* dislocation-antidislocation pairs come into existence[13]). The film enters a metallic state due to strong gauge field fluctuations. Finite resistance in the film is induced by free dislocations, which is proportional to the free dislocation density n_{df}:[13] $R_{\square}\sim R_Q n_{df}\mu_v$. $R_Q=h/4e^2$ is the quantum of resistance and μ_v=vortex mobility. n_{df} scales as $n_{df}\sim 1/\xi_+^2$. ξ_+=SF correlation length that diverges across the SC-BM phase boundary with an exponent ν_0: $\xi_+\sim(H-H_{c0})^{-\nu_0}$; H_{c0} is the critical field for SC-BM transition. Hence, on the metallic side,

$$R_{\square}\sim(H-H_{c0})^{2\nu_0}. \tag{3}$$

This scaling formula is quite different from traditional quantum SC to non-SC scaling $R_{\square}=f(\delta/T^{1/\nu z})$. Observation of this scaling in Ref. 5 provides good evidence that the metallic phase is a Bose metal. A comment about the value of H_{c0}: since energy barriers to metastable states are finite in 2D in the presence of disorder, one might think that a small amount of quantum fluctuations (zero point motion and dynamical gauge field fluctuations) would destabilize the VG order completely, i.e., $H_{c0}=0$. But, at very low fields, the vortex system moves into the individual pinning regime from the collective pinning regime, and the energy barriers to the vortex motion due to pinning become very large.[13] Hence, it is not inconceivable that, in this very low field regime, at $T=0$, quantum fluctuations and disorder can conspire to produce a pinned vortex state without any free dislocations, and consequently true (Cooper pair) superconductivity. In other words, there is a possibility that $H_{c0}>0$.[5]

(2) *BM-INS transition*. As the field is increased further, quantum zero point motion of the vortices $(\sim\hbar^2 n_v/m_v \sim H)$ increases, and beyond a critical value H_c, the zero point motion overtakes the gauge field fluctuations, the vortices form a SF phase and the film is insulating. Presence of free vortices during this transition motivates a scaling for the resistivity across the BM-INS phase boundary. So far, this phase boundary has been thought to be a SC-INS transition and the predicted scaling[12] is $R_{\square}=f(\delta/T^{1/\nu z})$. Although this scaling formula works at high temperatures, it fails at low temperatures.[2] The *two* parameter scaling formula which we obtain below for the BM-INS transition, however, scales the data across the entire temperature range, both high and low, when the external field is in the critical regime.

In Ref. 1, we mentioned that there is a jump in the vortex superfluid density at the BM-INS phase boundary. Naively, this would imply that this is either a first order or a KT type transition. But, the transition is possibly more subtle than this because it is a phase transition from a (vortex) *superfluid to a gapless nonsuperfluid phase*. In a first order transition, there is no divergent length scale and the length scale is finite through the transition. But, the BM phase is gapless, i.e., characterized by a diverged length scale. The KT transition, which is a close precedent to this, on the other hand, is a transition from a superfluid to a *gapped* nonsuperfluid state. Also, the situation here is fundamentally different from KT because of the involvement of the gauge degrees of freedom. Thus, we suspect that this transition belongs to a very different universality class. In the absence of the knowledge of what exactly this universality class is, it is hard to write down the scaling formula across this BM-INS transition. Below, we suggest a way to scale the resistance across this metal insulator transition based on phenomenological considerations and leave the rest to future research.

Since this is a second order quantum phase transition, there is a diverging correlation length ξ with an exponent ν at this transition, i.e., $\xi\sim|\delta|^{-\nu}$, and a frequency scale Ω, which goes to zero with an exponent z, $\Omega\sim\xi^{-z}$, where $\delta=(H-H_c)$. As the energy dissipated scales as $(V^2/R)t\sim\Omega$ ($V=$ voltage drop, t represents the time), resistance R scales as $R\sim V^2/\Omega^2$. Since the dissipation is due to the vortices, the voltage induced by moving vortices is $V=(h/2e)d\theta/dt$, with $d\theta/dt=2\pi n_f L v$,[14] where v is the vortex velocity, L is the length over which the vortices move and n_f is the vortex density. To obtain the scaling of V, we note that the scaling of L is $L\sim\xi\sim\Omega^{-1/z}$. Since $mv^2\sim\Omega$, the scaling of v is $v\sim\sqrt{\Omega}$. We assume that the scaling of n_f is $n_f\sim|\delta|^{\alpha}$, where α is an exponent to be determined below. n_f is *not* the total vortex density H/Φ_0, but rather the critical fraction of the field induced vortices that participate in the dissipative process. Combining all these factors, we find that the scaling of R is $R\sim\delta^{2\alpha}/\Omega^{1+2/z}$. At any finite temperature, the divergence of ξ is cutoff by temperature T, i.e., $\Omega\sim T$. This also implies that the scaling function is always a function of $\delta/T^{1/\nu z}$, i.e.,

$$RT^{1+2/z}/\delta^{2\alpha}=f(\delta/T^{1/\nu z}),$$

where f represents the scaling function. The right-hand side comes from the fact that $\xi\sim\delta^{-\nu}$, $\Omega\sim\xi^{-z}$ and $\Omega\sim T$, so that $\delta/T^{1/\nu z}$ is the scaling variable. To obtain α, one needs to first note that the resistance saturates to finite values independent of temperature at low temperature; i.e., in the above equation, we must have $f(x)\to x^{-\nu(z+2)}$ in this limit. Plus, since this low temperature resistance of the film is *noncritical* through the metal insulator transition when H is tuned through H_c as can be seen from Eq. (11) below, we should have

$$2\alpha=\nu(z+2). \tag{4}$$

Thus the scaling formula for the resistance is

$$R\left[\frac{T^{1/\nu z}}{\delta}\right]^{\nu(z+2)}=f(\delta/T^{1/\nu z}). \tag{5}$$

This is essentially a *two* parameter scaling formula, which is expected of a bosonic system.[15] For MoGe films,[2] $z=1$ and $\nu=4/3$, and hence, $2\alpha=4$. The corresponding plot of Eq. (5) for these films is shown in Fig. 1. The data collapse with this two parameter scaling formula is quite remarkable. We think that this points out that there is a true metal-insulator transition at this critical field. Although resistance usually does not receive scaling under normal circumstances, we found at least one precedent to this in the literature, i.e., when a dangerously irrelevant variable is present.[16] An RG analysis is necessary to cross-check the validity of this scaling and the phase diagram presented earlier.

IV. PHENOMENOLOGY OF FIELD-TUNED EXPERIMENTS

In this part, we discuss the phenomenology of the metallic state in a dirty SC film, close to the metal-insulator transition, by setting up a quantum ''pseudo-temperature'' model.[3] This is based on the idea of competition between dissipation and quantum zero point motion of the vortices present near this transition point, presented so far.

The energy barrier to create of dislocation pairs in a disordered vortex lattice is[13] $\epsilon_d\sim\epsilon_0\ln(H_c/H)$. At high temperatures, dislocations are created by thermal fluctuations, i.e., dislocation density is $n_{dc}\sim\exp(-\epsilon_d/kT)$, and the resistance induced by these is $R_{\square}\sim R_Q n_d\mu_v\sim e^{-\epsilon_d/kT}$. At low temperatures, quantum fluctuations take over, which we model in terms of a quantum (pseudo) temperature T_Q: when T_Q is small, the vortices being bosons are in a SF state, and when high, it is in a disordered/metallic state (real temperature remaining constant and small). Because whether the vortices

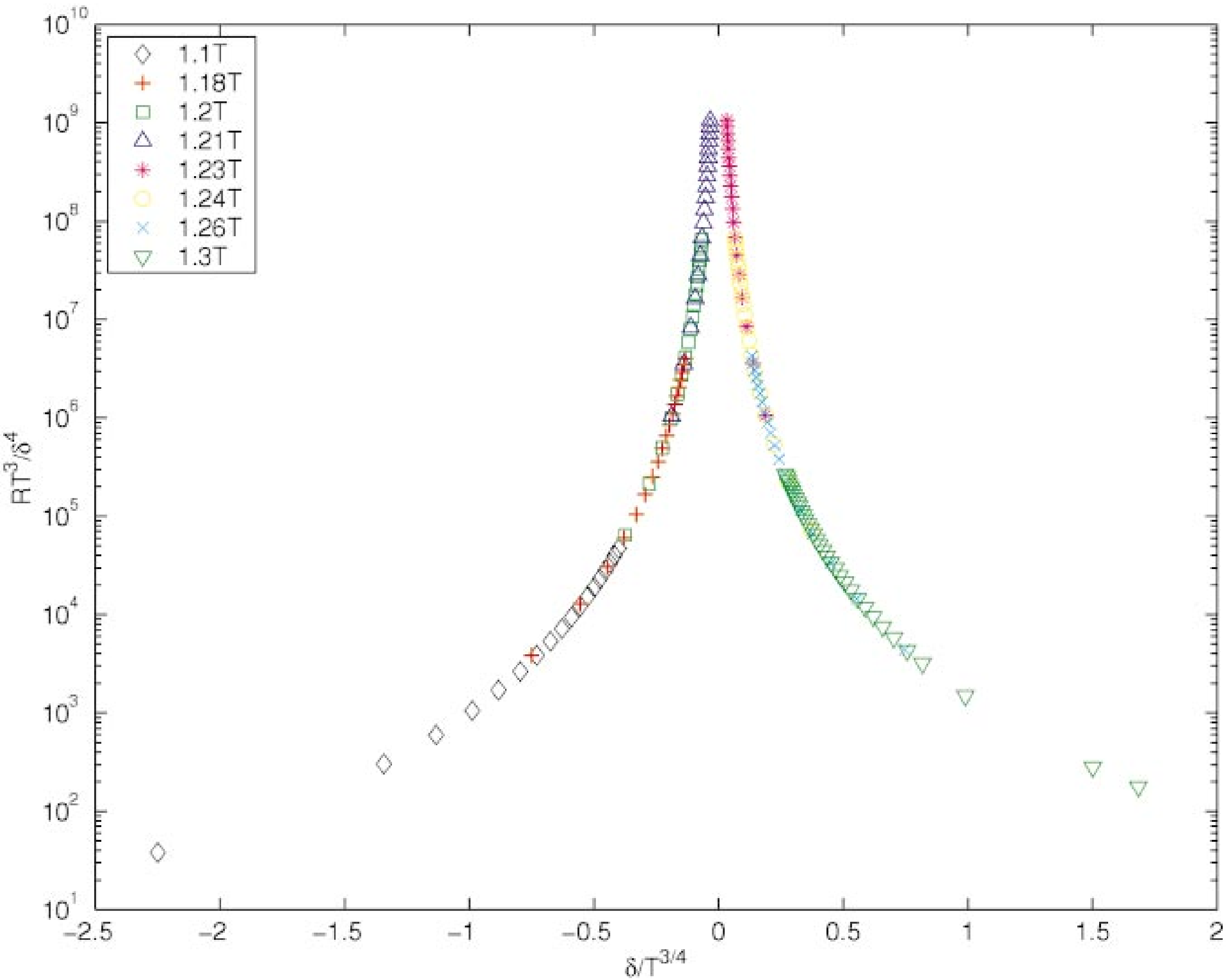

FIG. 1. (Color) Scaling collapse of the MoGe data from Ref. 2 across the metal insulator transition on a log-linear plot using Eq. (5): $\delta = H - H_c$; $H_c = 1.22$ T is the critical field at which the metal insulator transition occurs. The data shown cover the entire low temperature range of measurement: $\sim 0.02 - 0.2$ K.

are in a SF state or non-SF state is determined by the competition between the dissipation η and quantum zero point motion, T_Q should be a function of these two parameters. Since heating a system increases its entropy, and (a) enhanced quantum zero point motion ($\sim n_v/m_v \sim H$) leads to superfluidity in a Bose system, and hence, reduced entropy, while (b) action of dynamical gf fluctuations ($\sim \eta$) increases entropy, we expect T_Q, for not too small fields, to scale as

$$T_Q \sim \frac{\eta^\alpha}{(n_v/m_v)^\beta} \sim \frac{\eta^\alpha}{H^\beta}, \tag{6}$$

where α, β are constants >0. (It can be any monotonic function of η/H in general.) Currently, we do not know how to obtain α and β. In what follows, we shall assume $\alpha = \beta = 1$ and show that this quantum temperature model captures the general phenomenological features of the field-tuned expts in the aforementioned regime very well.

Thus, expressed in terms of the quantum temperature model, as the field increases, the quantum temperature of the system decreases and when it falls below a temperature of the *order* of Kosterlitz Thouless temperature T_{KT} (since vortices here are 2D bosons), the vortices would form a SF, i.e., the film would be insulating. Since $T_{KT} \sim \hbar^2 n_v/m_v \sim H$, the critical field H_c above which the film is insulating is obtained when $T_Q \sim T_{KT}$, i.e., H_c scales as [using Eq. (6)]

$$H_c \sim \sqrt{\eta} \sim 1/\sqrt{R_n}, \tag{7}$$

since η scales inversely as R_n. This makes sense physically, since as η increases, gauge field fluctuations increase and it is harder to create a SF out of the vortices. This inverse dependence of H_c on R_n is broadly consistent with the trend observed in the MoGe films (see Table I in Ref. 17).

When $T_Q > T_{KT}$, the vortices are in a non-SF state and the film displays a metallic response. From Eq. (6), the density of quantum dislocations, by analogy with the thermal case, is then $n_{dQ} \sim \exp(-\epsilon_d/kT_Q)$. Thus, the resistance of this metal at a finite temperature is given by $R_\square \sim R_Q(n_{dC} + n_{dQ})\mu_v$, i.e.,

$$R_{\square} \sim R_n [e^{-\epsilon_d/kT} + e^{-\epsilon_d/kT_Q}]. \tag{8}$$

This simple additive estimate is a good first hand measure when both thermal activation and quantum tunneling processes are going on simultaneously.[18] Here we have used $\mu_v \sim R_n/R_Q$, which is true of the Bardeen-Stephen processes. This implies that when the real temperature T falls below T_Q, the vortices enter the quantum regime. This determines the crossover temperature $T_{\rm cross}$ at which the system moves from the classical to the quantum regime:

$$T_{\rm cross} \sim T_Q \sim \eta/H. \tag{9}$$

Hence, as the field is increased, the crossover temperature decreases, as seen in the Mo-Ge experiments.[3,2] This unusual crossover from the classical to quantum regime, which is very unlike that of a single particle system, results from the strong cooperative effects present in the vortex Bose system.

Thus, from Eq. (8), in the quantum regime, the resistance saturates to

$$R_{\square} \sim R_n e^{-\epsilon_d/kT_Q} \sim R_n \exp\left(-C \frac{H}{H_{c2}} \frac{R_n}{R_Q} \ln\left(\frac{H_c}{H} \right) \right), \tag{10}$$

where we have used dimensionless expressions for η and H by rescaling them with R_Q and H_{c2} respectively. C is a constant of order unity. Equation (10) has the same field dependence as that obtained in Ref. 19 if one expands the logarithm about the critical field H_c, i.e., $\ln(H_c/H) \approx [(H_c - H)/H]$,

$$R_{\square} \sim R_n \exp\left(-\tilde{C} \frac{R_n}{R_Q} \left(\frac{H_c - H}{H_c} \right) \right), \tag{11}$$

It should be noted that the metallic resistance is *noncritical* across the metal-insulator transition.

Thus, despite the simplicity of the quantum temperature model, it captures the basic phenomenology of the experiments very well.

To summarize, we find several indications here, viz. (a) occurrence of the metallic state as an intermediate phase between SC and INS phases, (b) scaling behavior at the SC-metal and metal-INS transitions being in accordance with the Bose metal scenario predictions, and (c) the fact that the phenomenology of the metallic state is as would be expected for a Bose metal state, to suspect that the metallic state observed in the magnetic field tuned experiments in the SC films is probably an adiabatic continuation of the Bose metal state presented in Ref. 1.

V. THE EFFECTIVE GAUGE-FIELD ACTION OF THE BOSE METAL

Since the metallic state observed in the field tuned experiments is a similar state of matter as the Bose metal in Ref. 1, we take this opportunity to discuss the general properties of the Bose metal based on the calculations of Ref. 1, and return to some of the unaddressed issues regarding the JJA model in Ref. 1. We first discuss the effective gauge action of the Bose metal based on our analysis of the JJA model in Ref. 1. We obtain certain properties of this metal on the basis of this action, including its low energy excitation spectrum. And then, we revisit the issue of why the Bose metallic phase has not been observed in the previous analyses of the JJA model or the Bose Hubbard (BH) model.

Effective action. It is clear from the discussion in Sec. II that a Bose metal is a liquid of uncondensed bosons (vortices) moving in a transverse gauge field. The effective action of this liquid can be obtained by integrating out the vortices in a one-loop approximation. This calculation has already been done in Appendix E of Ref. 1: the transverse part of the gauge action can be read off directly from Eq. (E3) there. Thus, the effective action of a BM is

$$S([A^\mu]) = \sum_{\omega_n} \int d^2q \Bigg[(q^2 + 1/\xi_+^2) A^0_{\omega_n,q} A^0_{-\omega_n,-q} + \left(\tilde{a} \frac{|\omega_n|}{q} + \tilde{b} q^2 \right) \left(\delta_{\alpha\beta} - \frac{q_\alpha q_\beta}{q^2} \right) A^\alpha_{\omega_n,q} A^\beta_{-\omega_n,-q} \Bigg], \tag{12}$$

where the coefficients $\tilde{a}$ and $\tilde{b}$ are finite and scale with free vortex density and may be read off from Eq. (E3) in Ref. 1 and $\omega_n = 2\pi n T (n = \text{integer})$ are Matsubara frequencies. The spectrum of the longitudinal part of the gauge field follows from the fact that since the vortices and antivortices are free, they screen each other. Since the screening length scales as the density of free vortices n_f and $n_f \sim 1/\xi_+^2$, the above result is obtained. Physically speaking, the above spectrum of the transverse gauge field comes about, because it is dynamically screened[22] and that there is no spontaneous symmetry breaking in this phase. Thus the longitudinal modes are gapped but the transverse modes are gapless.

Since the gauge field A^μ is seen by the vortices, one can use the duality transformation techniques[10,21] on this gauge action to calculate the properties of the *Cooper pairs* (the original bosons) in the Bose metal phase. The superfluid density and the compressibility of the charges, i.e., Cooper pairs, are[10,21]

$$\frac{1}{m} \rho_s^c = \lim_{q \to 0} \lim_{\omega \to 0} \frac{q^2}{q^2 + 1/\xi_+^2} = 0, \tag{13}$$

$$\kappa^c = \lim_{q \to 0} \lim_{\omega \to 0} \frac{q^2}{-i\tilde{a}\frac{\omega}{q} + \tilde{b} q^2} = \frac{1}{\overline{c}^2} = (\text{finite}). \tag{14}$$

Here the superscript c denotes charges and we have used $\tilde{b} = \overline{c}^2$ from Eq. (E3), where $\overline{c}$ is the renormalized plasmon velocity. These results are consistent with the fact that a Bose metal is a nonsuperfluid compressible liquid.

The specific heat of the BM can be calculated from Eq. (12). This has already been done in Ref. 1. The gapless transverse modes make the most important contribution to the specific heat. We found that, in a one-loop approximation,[1] the low temperature specific heat of the BM goes as $C \sim T^{2/3}$. This $T^{2/3}$ power law specific heat, in turn, directly implies that the low energy excitation spectrum of the original Cooper pair system in this liquid state goes as

$$\omega \propto k^3. \tag{15}$$

This unconventional excitation spectrum results from the strong correlation effects present in the system and is consistent with the fact that Landau critical velocity should be zero in a nonsuperfluid Bose system. The density of states is correspondingly divergent: $N(\omega) \sim \omega^{-1/3}$. Obviously, the BM state *cannot* be continued to a Fermi liquid or mapped onto a set of free particles. We shall argue below that this liquid is gapless because it is an E_2 spin liquid, rather than an $SU(2)$ spin liquid.

In Ref. 1, we pointed out that the phase (ϕ_i) and charge fluctuation operators (δn_i) entering the JJA model are the generators of E_2, the Euclidean group in two dimensions; E_2 being a group contraction of $SO(3)$. In this sense, the JJA model is an E_2 "spin" model and the BM phase being the disordered phase of this model should be regarded as an E_2 spin liquid. An extremely important point is that, whereas an $SU(2)$ spin liquid is usually gapped, an E_2 spin liquid is *necessarily gapless*. This follows from the distinction that whereas an $SU(2)$ spin model maps onto bosons in a *constant* magnetic field,[23] an E_2 spin model maps onto bosons in a *fluctuating* magnetic field. Because of the constant magnetic field, the $SU(2)$ spin liquid maps onto a quantum Hall liquid.[23] This means that the effective gauge action has a Chern Simmons term and the excitations are correspondingly gapped. On the other hand, the effective gauge action of the E_2 spin liquid is of Maxwellian type, i.e., *without* any Chern Simmons term:

$$S([\vec{A}]) = \int d\omega d^2 q f(\omega,q) A^{\alpha}_{\omega,q} A^{\alpha}_{-\omega,-q}.$$

Now, if the excitation spectrum is gapped, i.e., $\lim_{\omega,q\to 0} f(\omega,q) = \text{const}$, then this means that there is spontaneous symmetry breaking and the z component of the E_2 spins are ordered, i.e., the system is charge-ordered.[21,1] Hence, the E_2 spin liquid is necessarily gapless. For the JJA model we have been discussing so far, this feature can be seen directly by focusing on the transverse part of the gauge action in Eq. (12).

It is hard to rule out the possibility of excitations with fractional quantum numbers[8] in this liquid. However, since there is no explicit time reversal symmetry breaking in the effective action, we think that such excitations would always exist in pairs being held together strongly by gaugelike forces.

JJA model: what's missing? In this last part of the paper, we revisit the issue of why the Bose metallic phase has not been observed in the previous analyses of the JJA model or the Bose Hubbard (BH) model.

The key issue which distinguishes our analysis from the others is that our calculation has been done in the limit when the average bosonic filling per site is very large. In this limit, the BH model maps onto a JJA model. A lot of current calculations on the JJA model are strongly influenced by the results available for the BH model at low fillings. However, as we stressed in our previous paper, the algebraic properties of the BH model in these two limits are very different: in the limit of low fillings, the model is very close to that of the $SU(2)$ spin model. On the other hand, in the limit of large fillings, it is close to that of the E_2 spin model.

It has been recently pointed out that $SU(2)$ and the E_2 algebras are actually connected by a *singular* transformation[20] so that the results obtained in one limit cannot be continued adiabatically to those in the other limit, i.e., without crossing a phase boundary. This also lends extra support to our argument above that the properties of the spin liquids supported by the two spin models are distinctly different. Hence, it is very important to maintain this distinction between $SU(2)$ and E_2 while performing calculations on the JJA model. The limit of large fillings is good for the SC films, whereas low filling limit is good for helium films.[21]

As a second point, a lot of the calculations on JJA model have been done for the case when an external gate voltage V_g is present. We have worked on the case $V_g = 0$. This situation is close to that of the real granular SC films. And, finally, our calculations have been done in the presence of nearest neighbor interaction V_1. The presence of V_1 brings about nontrivial effects in the JJA model, a quite well explored instance of which is the onset of supersolidity when rescaled gate voltage is close to half integer. More details on these differences may be found in Ref. 21.

There are two previous calculations on the JJA model for $V_g = 0$ and $V_1 \neq 0$: mean field calculations and numerical simulations.[24] The reasons for nonobservation of the BM phase in these are as follows. (a) *Mean field*: A BM phase benefits partly both from the kinetic energy and the potential energy terms. But, in a mean field calculation, as soon as superfluidity is destroyed, the kinetic energy term is completely suppressed. (b) *Numerical simulations*: The problem with this calculation is that the authors investigated only the superfluid phase in the $V_0 - V_1$ plane at $V_g = 0$. Only superfluid density and structure factor $S_{\pi,\pi}$ were measured here, which do not distinguish between a Mott insulator phase and a BM phase: both of these quantities are zero in these states. More simulations addressing this issue might be helpful to crosscheck the existence of the BM phase and its properties.

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APPENDIX A: WORLD LINE PICTURE FOR BOSONS AND FERMIONS

Here we enlist the actions for bosons and fermions in the world line picture, which might help the reader in understanding the world line picture discussion of the gauge field fluctuations in Sec. II.

First, consider a system of N interacting bosons in the world line picture, *without* any gauge field. Partition function of this system is

$$Z_B=\frac{1}{N!}\sum_P \int_{\{r_i(\beta)=r_{Pi}(0)\}}\prod_i Dr_i(\tau) \times \exp\left(-\int_0^\beta d\tau\left[\sum_i \frac{m}{2}\dot{r}_i^2+\frac{1}{2}\sum_{i\neq j} v(r_i-r_j)\right]\right),$$

where P is the permutation of the particles.[25] At low temperatures, the world lines get entangled because of quantum zero point motion effects, which implies a finite superfluid density for the bosons.[26]

Now consider an equivalent system of fermions.[26] Here, the partition function is

$$Z_F=\frac{1}{N!}\sum_P e^{i\pi P}\int_{\{r_i(\beta)=r_{Pi}(0)\}}\prod_i Dr_i(\tau) \times \exp\left(-\int_0^\beta d\tau\left[\sum_i \frac{m}{2}\dot{r}_i^2+\frac{1}{2}\sum_{i\neq j} v(r_i-r_j)\right]\right),$$

the extra phase factor coming from the anticommutativity of the fermions. Because of this phase factor, it is readily clear that entangled configurations enter with random signs and cause cancellation of several terms. Thus, since the entangled configurations enter the partition function with low weight, this means that these configurations are high energy configurations and disentanglement is favored for a fermionic system. And, hence the ground state of a Fermi system is a nonsuperfluid. The case of bosons with gauge field fluctuations discussed in Sec. II is intermediate between these two cases.

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