Josephson current in superconductor-ferromagnet structures with a nonhomogeneous magnetization

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We calculate the dc Josephson current I_J for two types of superconductor-ferromagnet (S/F) Josephson junctions. The junction of the first type is a S/F/S junction. On the basis of the Eilenberger equation, the Josephson current is calculated for an arbitrary impurity concentration. If $h\tau \ll 1$, the expression for the Josephson critical current I_c is reduced to that which can be obtained from the Usadel equation (h is the exchange energy, and τ is the momentum relaxation time). In the opposite limit $h\tau \gg 1$ the superconducting condensate oscillates with period v_F/h and penetrates into the F region over distances of the order of the mean free path l. For this kind of junctions we also calculate I_J in the case when the F layer presents a nonhomogeneous (spiral) magnetic structure with the period $2\pi/Q$. It is shown that for not too low temperatures, the π state which occurs in the case of a homogeneous magnetization ($Q=0$) may disappear even at small values of Q. In this nonhomogeneous case, the superconducting condensate has a nonzero triplet component and can penetrate into the F layer over a long distance of the order of $\xi_T = \sqrt{D/2\pi T}$. The junction of the second type consists of two S/F bilayers separated by a thin insulating film. It is shown that the critical Josephson current I_c depends on the relative orientation of the effective exchange field h of the bilayers. In the case of an antiparallel orientation, I_c increases with increasing h. We establish also that in the F film deposited on a superconductor, the Meissner current created by the internal magnetic field may be both diamagnetic or paramagnetic.

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I. INTRODUCTION

The interplay between ferromagnetism and superconductivity in layered structures has attracted a great interest in the last years. In a rough approximation, these states are antagonistic to each other and the ferromagnetism, being usually much stronger than superconductivity, is supposed to destroy the latter. However, in many cases the coexistence of these two phenomena is possible, even if the superconducting critical temperature T_c is by an order of magnitude lower than the Curie temperature of the ferromagnet. Such is the case when dealing with superconductor-ferromagnet (S/F) hybrid structures. In these systems the mutual interaction of these two states may lead to significant changes of the thermodynamic and transport properties.

In particular for S/F/S systems in equilibrium, one of the most interesting effects is a phase shift by π between weakly coupled superconductors, the so called π state. The possibility of the π state in S/F/S structures was first predicted by Bulaevskii and co-workers[1,2] and studied in later works.[3,4].The transition to the π state manifests itself in a nonmonotonic (and even oscillatory) thickness dependence either of the superconducting critical temperature T_c or of the critical current I_c, and in the change of sign of I_c if the exchange field h exceeds a certain value in the S/F/S junction.[1–7] Although some experiments on the thickness dependence of T_c in S/F structures show that for a certain thickness of the F layer, the ground state of the system may correspond to the π-phase shift between the adjacent superconductors (see, e.g., Ref. 5), this kind of coupling was not observed in other experiments (see, for example, Ref. 6). Only recently, the experiment of Ref. 7 on the measurement of the current $I_c(T)$ in the $Nb/Cu_xNi_{1-x}/Nb$ Josephson junction demonstrated unambiguously the transition from the 0 to the π-phase difference between the superconductors.

In all theoretical works,[1–4] calculations were performed either in the diffusive limit, in which the Usadel equation was applicable, or in the pure ballistic limit, where the elastic scattering by impurities was completely neglected. At the same time, very often the parameters characterizing the samples in experiments, such as the sample size, the mean free path, or the strength of the exchange field, do not correspond to these limits. Therefore, there is a certain need to study the Josephson current in the S/F/S structures not only in extreme limits but also in the intermediate region of the parameters.

In this work, we calculate the critical Josephson currents I_c in a S/F/S junction for arbitrary impurity concentrations. Since the approach based on the Usadel equation[8] (dirty limit) is valid only if the parameter $h\tau$ is small (h is the exchange field of the ferromagnet and τ is the momentum relaxation time), we use in an arbitrary case the more general Eilenberger equation[9,10] in which, generally speaking, the elastic collision integral is not neglected. As mentioned above, in real experiments the parameter $h\tau$ may take different values depending on the sample and therefore our theory can serve as a good description of the experiments.

Moreover, in all theoretical works mentioned previously, it was assumed that the magnetic ordering in the ferromagnetic layers was homogeneous. However, ferromagnetic materials exhibit generally more complex magnetic structures. In strong ferromagnets, like Fe or Ni, the magnetic ground state consists of homogeneously magnetized domains with different relative orientations. Also "weak" ferromagnets, like some ternary compounds with a regular lattice of rare-earth elements, turn out to be superconducting as the crystals undergo a transition into a state with a nonhomogeneous (helicoidal) magnetic order (see Ref. 11 and references therein). A similar nonhomogeneous structure may arise in bilayered S/F structures. For example, an experiment[12] and two theoretical works[13,14] suggested a possible existence of a nonhomogeneous magnetic ordering in the ferromagnetic layer in a S/F system. Also, in experiments on giant magnetoresistance (GMR) in magnetic multilayers employing superconducting contacts, nonhomogeneous magnetic structures can be created artificially (see, e.g., the review Ref. 15 and references therein).

In spite of the importance for the experiments, a theoretical analysis of the influence of a nonhomogeneous magnetization on the properties of S/F junctions is still lacking. Therefore, the second goal of this paper is to investigate the influence of nonhomogeneous magnetic configurations on the supercurrent through different kinds of superconductor-ferromagnet Josephson junctions.

In Sec. III we consider a S/F/S system, with an nonuniform (spiral) magnetic ordering. We derive an expression for the critical current $I_c(Q)$, where Q is the wave vector of the spiral magnetic order. We show that, whereas for $Q=0$ the transition from the 0-phase state to the π-phase state is possible, even small nonzero Q values may restore the 0-phase state. The reason for this is the existence of a triplet component of the superconducting condensate in the ferromagnet due to the proximity effect and the nonhomogeneous magnetic structure. In the limit $h\tau<1$ this component does not decay over the short distance $\sqrt{D/h}$, which corresponds to the length of decay of the usual singlet component, surviving up to a much longer distance $\sim\sqrt{D/2\pi T}$ (D is the diffusion coefficient). The influence of this triplet component on the transport properties of the S/F mesoscopic structures was studied in Ref. 16.

In Sec. IV, we analyze the dc Josephson current in a tunnel junction composed either of two S/F bilayers or of two magnetic superconductors. We derive an expression for the critical current I_c as a function of the relative angle α between the magnetization of both F layers. The most important and surprising result is that for an antiferromagnetic configuration, $\alpha=\pi$, the current I_c increases with increasing exchange field h. The calculated dependence of I_c on various parameters allows us to make some conclusions not only on the magnetic order of the ferromagnetic materials used in S/F structures but also on nonhomogeneous superconducting states predicted by Fulde and Ferrel[17] (FF) Larkin and Ovchinnikov[18] (Lo).

In Sec. V we show that a Meissner current is induced in the F region due to the internal magnetic field of the ferromagnet. The Meissner current density has a different sign at different points, and the total current in the ferromagnet is either diamagnetic or paramagnetic depending on the thickness d of the F film.

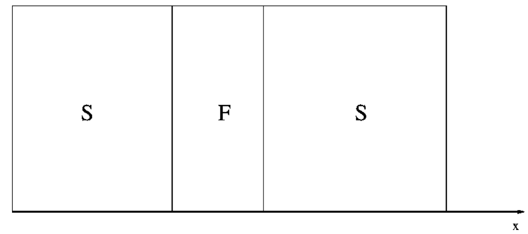

FIG. 1. The S/F/S system.

In the Appendix we present the derivation of the main equations used in this article. All our calculations are based on the Eilenberger[9,10] or on the Usadel[8] equations, generalized to the case of a spin-dependent interaction varying in space. Another approach based mainly on the Bogolyubov–de Gennes equations was widely used for the study of the spin injection from a ferromagnet into unconventional superconductors (see, e.g., Ref. 19 and references therein). In the present work, we restrict ourselves to the case of conventional superconductors with *s*-wave pairing.

II. JOSEPHSON CURRENT IN A S/F/S STRUCTURE

In this section we calculate the dc Josephson current I_J in a S/F/S structure. In order to make the consideration as general as possible we use the Eilenberger equation[9] including the elastic collision term. This allows us to calculate I_J for an arbitrary impurity concentration and to formulate conditions under which the ballistic or diffusive limits can be obtained. In order to find the condensate Green's function $\hat{f}_\omega$ in the F region in an analytical form, we assume that the proximity effect is weak, i.e., $|\hat{f}_\omega|\ll 1$, and linearize the collision term in the Eilenberger equation. This assumption can be reasonable for structures with a big mismatch between the Fermi surfaces in F and S, which leads to a small transmission coefficient T through the S/F interface. If the coefficient T is of the order of unity, we hope that our results are valid at least qualitatively.

We consider the S/F/S structure shown in Fig. 1 and assume that the exchange energy h is homogeneous in the F region (the case of a nonhomogeneous h will be analyzed in the next section). Because of the small interface transparency, one can neglect the suppression of the order parameter Δ in the superconductor due to the proximity of the ferromagnet. We assume also that there are no spin-flip processes in the ferromagnetic region; i.e., the spin-relaxation length is larger than the thickness d of the ferromagnet and there are no spin processes at the S/F interface. The linearized Eilenberger equation in the Matsubara representation has the following form:

$$\mu l\hat{\tau}_3\partial_x\hat{f}+2(\omega_m-ih)\hat{f}=\mathrm{sgn}\,\omega(\langle\hat{f}\rangle-\hat{f}). \qquad (1)$$

Here $\hat{\tau}_3$ is the Pauli matrix, $\mu=\cos\theta$, θ is the angle between the momentum and the x axis, $l=v_F\tau$ is the mean free path, and $\omega_m=\pi T(2m+1)$ is the Matsubara frequency. The angular brackets denote the average over angles: $\langle\cdots\rangle$

$=(1/2)\int_{-1}^{1}d\mu(\cdots)$. Equation (1) is complemented by the boundary conditions at $x=\pm d/2$, which in the case of low transparency take the form[20]

$$\hat{a}=-(\gamma/2)[\operatorname{sgn}\omega\hat{\tau}_3+\hat{s},\hat{g}_s+\hat{f}_s]\cong-\gamma\operatorname{sgn}\omega(\hat{\tau}_3\hat{f}_s)_{x=\pm d/2}, \tag{2}$$

where $\hat{a}$ and $\hat{s}$ are the antisymmetric and symmetric (with respect to μ) parts of $\hat{f}$, $\gamma=T(\mu)/4$ is a parameter describing the transmittance of the interface, $T(\mu)$ is the transmission coefficient, and $\hat{g}_s$ and $\hat{f}_s$ are the quasiclassical normal and anomalous Green's functions of the superconductors. The square brackets denote the commutator. When writing the last equality, we neglect the term proportional to $\hat{f}$, since $|\hat{f}|\sim\gamma$. The condensate function $\hat{f}_s$ in the superconductors can be written as

$$\hat{f}_s(\pm d/2)=[i\hat{\tau}_2\cos(\varphi/2)\pm i\hat{\tau}_1\sin(\varphi/2)]f_s, \tag{3}$$

where $f_s=\Delta/\sqrt{\Delta^2+\omega_m^2}$ and φ is the phase difference between the superconductors.

It is convenient to represent the solution of Eq. (1) as a sum of the symmetric ($\hat{s}$) and the antisymmetric ($\hat{a}$) parts,

$$\hat{f}=\hat{s}+\hat{a}. \tag{4}$$

Substituting this expression into Eq. (1) and separating the symmetric and antisymmetric terms, one obtains two equations which determine the functions $\hat{a}$ and $\hat{s}$:

$$\hat{a}=-\operatorname{sgn}\omega(\mu l/\kappa_\omega)\hat{\tau}_3\partial_x\hat{s}, \tag{5}$$

$$\mu^2l^2\partial_{xx}^2\hat{s}-\kappa_\omega^2\hat{s}=-\kappa_\omega\langle\hat{s}\rangle, \tag{6}$$

where $\kappa_\omega=(1+2|\omega_m|\tau)-\operatorname{sgn}\omega 2ih\tau$. Thus, the problem is reduced to finding the solution for Eq. (6) in the interval $|x|<d/2$ with the boundary conditions given by Eqs. (2) and (5). To this end it is convenient to extend formally the function $\hat{s}$ over the whole x axis and to write Eq. (6) in the following form:

$$\mu^2l^2\partial_{xx}^2\hat{s}-\kappa_\omega^2\hat{s} =-\kappa_\omega\Bigg[\langle\hat{s}\rangle+2\mu l\gamma f_s\sum_{n=-\infty}^{\infty}[i\hat{\tau}_2\cos(\varphi/2) +(-1)^n i\hat{\tau}_1\sin(\varphi/2)]\delta(x-(d/2)(2n+1))\Bigg]. \tag{7}$$

One can prove that the solution of Eq. (7) obeys the boundary conditions.

Performing the Fourier transformation, we find the solution for the Fourier transform of $\hat{s}$,

$$\hat{s}_k=2f_sB\hat{F}, \tag{8}$$

where

$$B=\frac{\kappa_\omega}{(1-\kappa_\omega\langle M^{-1}\rangle)M}[\gamma\mu l-\kappa_\omega l(\mu\gamma\langle M^{-1}\rangle-\langle\mu\gamma/M\rangle)], \tag{9}$$

$$\hat{F}=\sum_{-\infty}^{\infty}\exp[ikd(2n+1)/2][i\hat{\tau}_2\cos(\varphi/2) +(-1)^n i\hat{\tau}_1\sin(\varphi/2)] \tag{10}$$

and

$$M=(\mu lk)^2+\kappa_\omega^2. \tag{11}$$

The function $\hat{s}$ determines the dc Josephson current, as well as the variation of the density of states (DOS) due to the proximity effect. It is given by the inverse Fourier transformation

$$\hat{s}(x)=\int\frac{dk}{2\pi}\exp(-ikx)\hat{s}_k. \tag{12}$$

The current is determined by the expression

$$I_J=\frac{1}{8}G_QN(2\pi i)(2T)\mathrm{Tr}\hat{\tau}_3\sum_{\omega_m}\langle a_\omega\mu\rangle \tag{13}$$

$$=\frac{1}{8}G_QN(2\pi i)(2T)\mathrm{Tr}\hat{\tau}_3\sum_{\omega_m}\langle(\mu\gamma/2)[\hat{s}(d/2),\hat{f}_s]\rangle, \tag{14}$$

where $G_Q=e^2/\hbar$, $N=k_F^2\mathcal{S}/\pi^2$ and $\mathcal{S}$ is the cross-section area of the junction. In writing Eq. (13) we have used the boundary condition, Eq. (2). The summation over Matsubara frequencies is carried out from $m=-\infty$ to $m=+\infty$. Substituting Eqs. (6)–(12) into Eq. (13), we obtain finally the dc Josephson current

$$I_J=I_c\sin\varphi, \tag{15}$$

where

$$I_c=G_QNJ_c \tag{16}$$

and

$$J_c=2\pi T\,\mathrm{Re}\sum_{\omega>0}^{\infty}\sum_{n=0}^{\infty}f_s^2\int\frac{dk}{2\pi}\langle\gamma\mu B\rangle\exp[ikd(2n+1)]. \tag{17}$$

Equation (17) determines the critical current and is valid for any impurity concentration. Its analytical evaluation is rather complicated, since it includes summation over Matsubara frequencies, integration over the momentum k, and the averaging over the angles. Here we will discuss two limiting cases in which Eq. (17) can be simplified.

(a) $h\tau\ll1$ (dirty case). This limit corresponds to a ferromagnet with a weak exchange field h or to an alloy like that used in Ref. 7, for which the condition $h\gg T_c$ is satisfied. In this case the condition $h\tau\ll1$ implies that the quantities $\Delta\tau$ and $T\tau$ are also small. From Eq. (9) one can determine the coefficient B; in this limit $B\cong\langle\mu\gamma\rangle l/[(l^2/3)(k^2+k_+^2)]$,

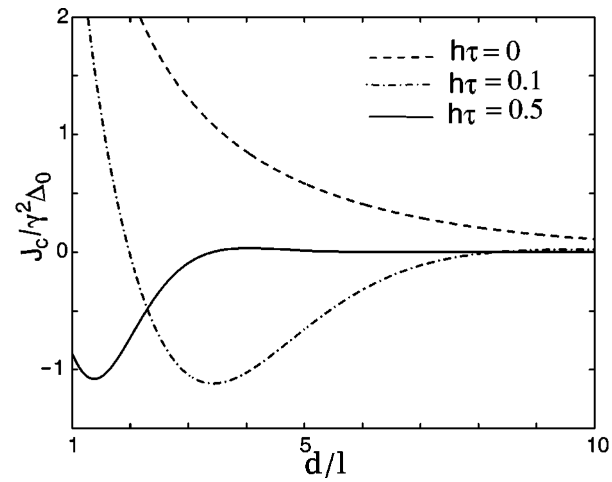

FIG. 2. Dependence of the normalized critical current $J_c/\gamma^2\Delta_0$ on the thickness d of the ferromagnet for $\Delta_0\tau=0.05$ and $T/\Delta_0=0.01$. The curves for $h\tau=0.1$ and $h\tau=0.5$ are multiplied by a factor 10 for clarity.

where $\kappa_+^2=2(\omega-ih)/D$ and $D=v_F l/3$ is the diffusion coefficient. Using Eq. (17) we obtain for the normalized critical current J_c

$$J_c=\frac{3}{4}\langle\mu\gamma\rangle^2\mathrm{Re}(2\pi T)\sum_{\omega>0}\frac{1}{\kappa_+ l\sinh(\kappa_+ d)}\frac{\Delta^2}{\Delta^2+\omega_m^2}. \tag{18}$$

This is the usual expression for the critical current obtained from the Usadel equation in the case of a weak proximity effect (cf. Refs. 4, 7, and 21). In Figs. 2 and 3 we plot J_c as a function of T and d. As has been shown in the previous studies, the function J_c is a rapidly decaying with T and d function which undergoes several oscillations. This oscillatory behavior of the critical current may explain the change from a 0-phase state to a π-phase state observed in Ref. 7.

We consider now another interesting case.

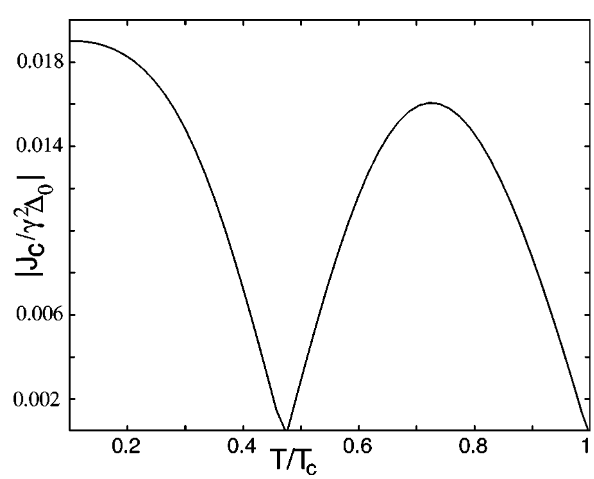

FIG. 3. Dependence of the normalized critical current $J_c/\gamma^2\Delta_0$ on the temperature for $\Delta_0\tau=0.03$, $h\tau=0.06$ and $d/l=\pi$.

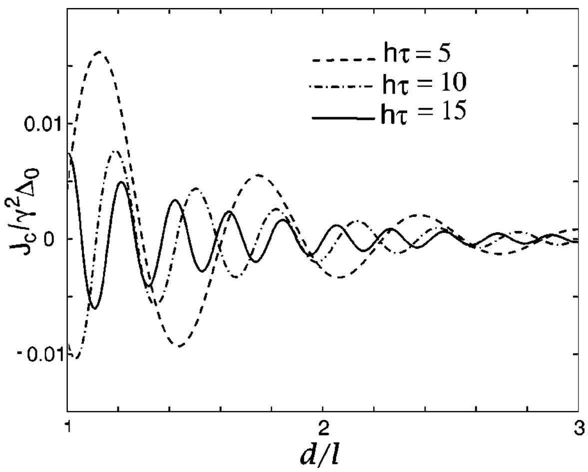

FIG. 4. Dependence of the normalized critical current $J_c/\gamma^2\Delta_0$ on the thickness d of the ferromagnet for $\Delta_0\tau=0.05$ and $T/\Delta_0=0.1$.

(b) $h\tau\gg1$. This condition corresponds to most of the experiments performed on S/F systems, in which F is a "strong" ferromagnet like Ni or Fe. It does not necessarily mean that we are analyzing the clean case (i.e., $T_c\tau>1$), since the value of the exchange field h can be much larger than T_c. For example, if the mean free path l equals ~300 Å, then $\tau^{-1}\sim300$ K$\gg T_c$, whereas $h\tau\gg1$ (we take $v_F=2\times10^7$ cm/s). Therefore in the limit $h\tau\gg1$ we can deal in principle with an arbitrary value of τT_c (although realistic materials and samples correspond to the case $T_c\tau\ll1$). It is worth mentioning that in this case the use of the Usadel equation is not justified.

The condition $h\tau\gg1$ implies that $\kappa_\omega\gg1$, $\kappa_\omega/M\ll1$, and, as one can see from Eq. (9), $B\cong\kappa_\omega(\gamma\mu l)/M$. Performing the integration over k, we find for J_c

$$J_c=\frac{1}{4}(2\pi T)\mathrm{Re}\sum_{\omega>0}f_s^2\left\langle\frac{\mu\gamma^2}{\sinh(\kappa_\omega d/\mu l)}\right\rangle. \tag{19}$$

One can see from Eq. (19) that the critical current oscillates with varying d or h (cf. Ref. 2 where the case $d\ll l$ was considered) and decays with increasing d over the mean free path l. In Figs. 4 and 5 we plot the dependence of J_c on d and T calculated numerically from Eq. (19). We see that in this limit the critical current does not oscillate with the temperature if the exchange field is temperature independent (this assumption is quite reasonable in the case of transition metals as Fe or Ni). It can change sign in a hypothetical case $T_c\tau>1$. In the limit $d/l\gg1$ the critical current is exponentially small and one can perform the angle averaging in Eq. (19). Thus, we find in this limit

$$J_c=(\pi T)\sum_{\omega>0}f_s^2\gamma^2(1)\frac{\sin(2hd/v_F)}{(2hd/v_F)}\exp[-(d/l)(1+2\omega\tau)]. \tag{20}$$

It follows from Eq. (20) that the critical current oscillates as a function of d or h and decays with d exponentially if $d>l$ and as $(hd/v_F)^{-1}$ if $d\cong l$. This power law dependence of

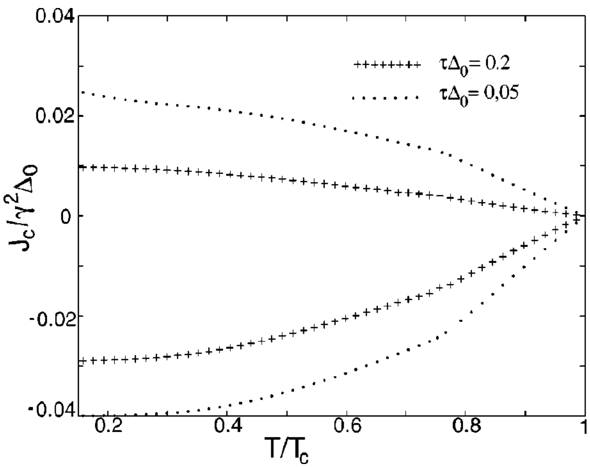

FIG. 5. Dependence of the normalized critical current $J_c/\gamma^2\Delta_0$ on the temperature for $d/l=1$. The two upper curves correspond to the case $h\tau=5$, and the the two lower curves to the case $h\tau=10$. For clarity, in the three lower curves, the values of the critical current have been multiplied by a factor of 5.

I_c on h can be even weaker if γ depends on μ (γ has a maximum at $\mu=1$ and decays with decreasing μ).

III. S/F/S JUNCTION WITH AN NONHOMOGENEOUS MAGNETIZATION

In this section we consider a nonhomogeneous magnetization in the magnetic region. This nonhomogeneity can be due to domain walls or, as in the case of experiments on GMR, due to an artificial layered magnetic structure. In principle, variation of the magnetic moment on the coordinates can be rather complicated, which makes explicit calculations difficult. To simplify the consideration we restrict ourselves with the cases of a magnetic spiral structure in the F region with a wave vector Q and calculate the dependence of the critical Josephson current I_c in a S/F/S junction on the wave vector Q.

Below we consider only the dirty case ($h\tau<1$) and assume again a weak proximity effect. This means that γ must be small enough: $\gamma \ll \sqrt{h\tau}$. In the limit of small $h\tau$ one can use the Usadel equation for finding the condensate function $\hat{f}$. However, in the case of a rotating magnetization (or equivalently a rotating exchange field h) we need to generalize our approach because not only singlet correlators as $\langle\psi_\uparrow\psi_\downarrow\rangle$ are induced in the F region, but also correlators of the type $\langle\psi_\uparrow\psi_\uparrow\rangle$ become nonzero (triplet component). In this case we introduce new 4×4 matrices for the quasiclassical Green's functions (see the Appendix). The 4×4 condensate Green's function $\check{s}$ (to be more exact, its symmetrical part) obeys the generalized Usadel equation in the Matsubara representation

$$-iD\check{g}_0\partial^2_{xx}\check{s}+[\check{M}_h,\check{s}]=0, \tag{21}$$

where $\check{g}_0=\hat{\tau}_3\otimes\hat{\sigma}_0$, $D=v_F l/3$ is the diffusion coefficient, and $\check{M}_h=\hat{\tau}_3\otimes(\hat{\sigma}_0 i|\omega_m|+\hat{\sigma}_3 h\,\mathrm{sgn}\,\omega_m\cos\alpha)-\hat{\tau}_0\otimes\hat{\sigma}_2 h\,\mathrm{sgn}\,\omega_n\sin\alpha$. Here $\alpha=Qx$ is the angle between the z axis and the direction of h. As in the previous section, we neglect corrections to the "normal" Green's function $\check{g}_0$ due to the proximity effect.

Equation (21) is a linear matrix differential equation with space-dependent coefficients. This spatial dependence can be excluded from the consideration by making a rotation in the Nambu spin space and by introducing a new matrix $\check{s}_n$: $\check{s}=\check{U}\check{s}_n\check{U}^\dagger$, where $\check{U}=\hat{\tau}_0\otimes\hat{\sigma}_0\cos(\alpha/2)+i\hat{\tau}_3\otimes\hat{\sigma}_1\sin(\alpha/2)$. After the rotation Eq. (21) acquires the form

$$\partial^2_{xx}\check{s}_n-(Q^2/2)(\check{s}_n-\check{A}\check{s}_n\check{A}^\dagger)+Q(\check{A}\partial_x\check{s}_n+\partial_x\check{s}_n\check{A}^\dagger)$$
$$-[\hat{\tau}_0\otimes(\hat{\sigma}_0|\omega_m|-ih\,\mathrm{sgn}\,\omega_m\hat{\sigma}_3),\check{s}_n]_+=0, \tag{22}$$

where $\check{A}=i\hat{\tau}_3\otimes\hat{\sigma}_1$ and the last term is an anticommutator. If $Q=0$, Eq. (22) coincides with Eq. (21) and can be easily solved. The critical current I_c has been calculated in the previous section, and its dependence on T and d is presented in Figs. 3 and 2. It is seen that the critical current I_c changes the sign at some h of the order of the Thouless energy $\epsilon_d=D/d^2$. The characteristic value of h for the transition to the π state increases with increasing T. This result is well known for both types of Josephson (equilibrium and nonequilibrium) junctions in which the sign-reversal effect of I_c takes place. Earlier than in S/F/S junctions, this effect was observed in four-terminal S/N/S Josephson junctions where a voltage V was applied between the normal reservoirs and therefore an additional dissipative current flows between the N reservoirs[22] were possible.

The critical current changes sign in these junctions due to a shift of the distribution function in the N electrode with respect to the distribution function in the superconductors. In contrast to this case, the sign reversal effect in S/F/S junctions is realized at equilibrium conditions. However, there is a formal analogy between these two cases because the formulas for the critical current can be reduced to each other by shifting the energy scale and by replacing $h \to eV$ (this analogy was noted in Refs. 23 and 21).

In the case of a finite Q, the formulas for $\check{s}$ become more complicated. In order to make them more transparent, we assume that the overlap of the condensate functions $\check{s}$ induced by the different superconductors is weak. This means that max$\{h,T\}$ should be greater than ϵ_d. Then, we represent the solution of Eq. (22) for $\check{s}$ as a sum of two functions

$$\check{s}(x)=\check{U}[\check{S}\check{s}_n(d/2+x)\check{S}^\dagger+\check{S}^\dagger\check{s}_n(d/2-x)\check{S}]\check{U}^\dagger. \tag{23}$$

The matrix $\check{S}$ allows one to take into account the phase difference φ between the superconductors: $\check{S}=[\hat{\tau}_0\cos(\varphi/4)+i\hat{\tau}_3\sin(\varphi/4)]\otimes\hat{\sigma}_0$. The first and second terms in Eq. (23) come from the superconductor at $x=-(d/2)$ and $x=+(d/2)$ respectively. The function $\check{s}_n(x)$ is a solution of Eq. (22) for an infinite S/F system with a vanishing phase $\varphi=0$. The boundary condition for the new matrix $\check{s}_n$ has the form

$$\partial_x\check{s}_n+(Q/2)[\check{A}\check{s}_n+\check{s}_n\check{A}^\dagger]=-(\tilde{\gamma}/l)\check{f}_s. \tag{24}$$

Here $\check{f}_s = i\check{\tau}_2 \otimes \check{\sigma}_3 f_s$ and $\tilde{\gamma} = 3\langle \gamma\mu \rangle$; f_s and γ have been defined in the previous section. It is not difficult to see that the function $\check{s}_n(x)$ has the following form:

$$\check{s}_n(x) = i\hat{\tau}_2 \otimes [\hat{\sigma}_0 S_0(x) + \hat{\sigma}_3 S_3(x)] + i\hat{\tau}_1 \otimes \hat{\sigma}_1 T(x), \tag{25}$$

where the functions $S_0(x)$, $S_3(x)$, and $T(x)$ are the amplitudes of the singlet and triplet components, respectively. All these functions may be represented as a sum of three exponentials corresponding to the eigenvalues of Eq. (22). For example, the expression for $S_3(x)$ is

$$S_3(x) = S_{3+}\exp(-\kappa_+ x) + S_{3-}\exp(-\kappa_- x) + S_{3l}\exp(-\kappa_l x). \tag{26}$$

Identical formulas may be written for the functions $S_0(x)$ and $T(x)$ with the factors in front of the exponentials denoted as $S_{0\pm}$, S_{0l}, and $T_\pm, T_l$ correspondingly. Analytical expressions for the coefficients and eigenvalues can be obtained in the limits of small and large Q, i.e., $DQ^2 \ll h$ or $DQ^2 \gg h$. In the limit of small Q, we find after some algebra

$$\kappa_\pm^2 = 2(|\omega_m| \mp \mathrm{sgn}\,\omega i h)/D, \quad \kappa_l^2 = 2|\omega_m|/D + Q^2 \tag{27}$$

and

$$S_{3\pm} = \mp S_{0\pm} = \tilde{\gamma} f_s / 2(\kappa_\pm l),$$

$$T_l = \frac{1}{2}(\tilde{\gamma} f_s)\frac{(\kappa_- - \kappa_+)Q}{l(\kappa_+\kappa_-)\kappa_l},$$

$$S_{3l} = (\tilde{\gamma} f_s)\frac{(\kappa_+ - \kappa_-)Q^2}{l(\kappa_+\kappa_-)\kappa_h^2}\mathrm{sgn}\omega, \tag{28}$$

where $\kappa_h^2 = -2ih/D$.

We note some new important features that appear at finite Q. If Q is zero, only the first two terms in Eq. (26) are nonzero and the decay is characterized by a short length $\xi_F = \sqrt{D/h}$ (in case of large enough h). If Q is finite, an additional term (the last term) appears in the formula for $S_3(x)$ which, at low temperatures, decays over a much larger length of the order $\sqrt{D/2\pi T}$. Alongside with the last term, the triplet component becomes nonzero. The triplet component contains also a long-range term $T_l \exp(-\kappa_l x)$ which increases with increasing Q. Due to the last term in Eq. (26), the dependence of the critical current on h (or T) is drastically modified even at small Q.

In order to calculate the current I_J, we can use the general expression, Eq. (13). In the dirty limit it can be written as follows:

$$I_J = (\mathcal{S}/16\rho)(2\pi i)(2T)\mathrm{Tr}(\hat{\tau}_3\hat{\sigma}_0)\sum_\omega \check{s}\partial_x\check{s}$$
$$= (\mathcal{S}/16\rho)(2\pi i)(2T)\mathrm{Tr}(\hat{\tau}_3\hat{\sigma}_0)(\tilde{\gamma}/l)\sum_\omega \check{s}\check{f}_s|_{x=d/2}. \tag{29}$$

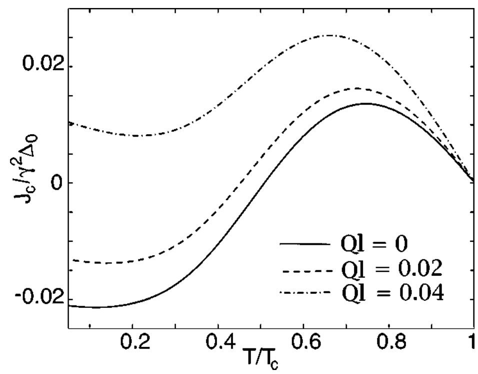

FIG. 6. The dependence of the critical current on T for $h\tau = 0.06$, $\Delta_0\tau = 0.03$, $d/l = \pi$, and different values of Ql.

One can show that this expression does not change its form under the $\check{U}$ transformation and $\check{s}$ in Eq. (29) may be replaced by $\check{s}_n(d/2)$. Performing simple operations, we obtain for the current an expression with the same form as Eq. (15) with

$$I_c = (\mathcal{S}/\rho l)\tilde{\gamma}^2 \sum_{\omega>0} f_s^2 \left[\frac{\exp(-\kappa_+ d)}{\kappa_+ l} + \frac{(Ql)^2}{2(3h\tau)^{3/2}}\exp(-\kappa_l d)\right]. \tag{30}$$

The first term in the brackets corresponds to the term $[2(\kappa_+ l)(\sinh\kappa_+ d)]^{-1}$ in Eq. (17) in the limit of a large exponent. It decays with increasing d over the short characteristic length $\xi_F = \sqrt{D/h}$. The second term in Eq. (30) is caused by the rotation of h along the x axis. It decays with d over the characteristic length κ_l^{-1}, which can be much longer than ξ_F. Therefore this term leads to a drastic change of the critical current. We calculated numerically the critical current I_c and presented its temperature dependence for $h\tau = 0.06$ in Fig. 6. One can see that for this choice of h the critical current is negative at $Q = 0$ (π junction). However, it becomes positive at some finite Ql smaller than $h\tau$. With increasing Q the long-range term in Eq. (26) and the triplet component increase, reach a maximum, and then decrease again to zero at large enough Q.

In the limit of large Q, the coefficients $S_{3\pm}$, $S_{0\pm}$ and the triplet components are small. The coefficient $S_{3l} = \tilde{\gamma} f_s/(kl)$ has the same form as in the absence of h. For the eigenvalues we obtain

$$\kappa_\pm = \pm iQ + \sqrt{2\omega_m/D}, \quad \kappa_l = \sqrt{2(\omega_m/D) + 4h^2/(DQ)^2}.$$

We see that in the limit of large Q the solution for $\check{f}$ has the same form as in a S/N/S structure (no exchange field); i.e., the term $(sf_S/\theta)\exp(-\kappa_l x)$ dominates. The first two terms in the singlet component $S_3(x)$ which contribute to the Josephson current [see Eq. (26)] are small. They oscillate rapidly in space and decay over a large distance of the order of $\xi_T = \sqrt{D/2\pi T}$ (in the limit $DQ^2 < h$). In the main approximation in the parameter (h/DQ^2) the temperature dependence of the critical current I_c is the same as for a S/N/S junction and we do not present this dependence here.

IV. S/F-I-F/S SYSTEM

In this section we consider a layered system consisting of two F/S bilayers separated by an insulating layer (see Fig. 1). In this case the Josephson critical current is determined by the transparency of the insulating layer and depends on the relative orientation of magnetization in the F layers.

We assume that the F and the S layers $d_{F,S}$ are thin enough: $d_{F,S} < \xi_{F,S}$, where $\xi_F = \sqrt{D/h}$ and $\xi_S = \sqrt{D/\Delta}$. First, we analyze the case of a high S/F interface transparency, i.e., $R_{S/F} < \rho_F / \xi_F$. Under these conditions all the Green's functions are nearly constant in space and continuous across the S/F interface.

In order to find the Green's functions $\check{g}^{R(A)}$, we multiply the components (1,1) and (2,2) of the matrix equation (A11) (the Usadel equations) by the density of states $\nu_{F,S}$ in the F and S layers, respectively, and integrate over the thickness of the bilayers. Neglecting the influence of one bilayer on the other [this means that $(\check{g}\partial_x \check{g}) = 0$ at the F/I interface], we obtain the following equation:

$$\text{sgn}\,\omega[\check{M}_h, \check{\mathbf{g}}] + [\hat{\Delta}_S \otimes \hat{\sigma}_3, \check{\mathbf{g}}] = 0, \tag{31}$$

Here the matrix $\check{M}_h$ has the same structure as in Eq. (21), but h has been replaced by $h_F = h(\nu_F d_F)/(\nu_F d_F + \nu_S d_S)$, and $\hat{\Delta}_S = \hat{\Delta}(\nu_S d_S)/(\nu_S d_S + \nu_F d_F)$. We assume that the vector $\mathbf{h}$ in the left layer is oriented along the z axis and has the components $h(0,\sin\alpha,\cos\alpha)$ in the right electrode. One can simplify Eq. (31) in the right bilayer with the help of the transformation (A14). In this case one obtains for the both layers the same equation

$$[\hat{\tau}_3 \otimes (\epsilon\hat{\sigma}_0 + h_F\hat{\sigma}_3), \check{g}] + [\hat{\Delta}_S \otimes \hat{\sigma}_3, \check{g}] = 0. \tag{32}$$

We can solve Eq. (32) by making the ansatz

$$\check{g} = \hat{\tau}_3 \otimes (a_0\hat{\sigma}_0 + a_3\hat{\sigma}_3) + \hat{\Delta}_S \otimes (b_0\hat{\sigma}_0 + b_3\hat{\sigma}_3). \tag{33}$$

From Eq. (32) and the normalization condition (A12) one can obtain the coefficients a's and b's. In the left bilayer $\check{g}$ is given by the expression (33) while in the right bilayer it is given by $\check{g}^{(r)} = \check{U}^{+}\tilde{\check{g}}^{(r)}\check{U}$, i.e.,

$$\check{g}^{(r)} = \hat{\tau}_3 \otimes (a_0\hat{\sigma}_0 + a_3\cos\theta\hat{\sigma}_3) - \hat{\tau}_0 \otimes a_3 \sin\theta\hat{\sigma}_2 + \hat{\Delta}_S \otimes (b_0\cos\theta\hat{\sigma}_0 + b_3\hat{\sigma}_3) - \hat{\tau}_3\hat{\Delta}_S \otimes i b_0 \sin\theta\hat{\sigma}_1.$$

According to Eq. (A16) only the coefficients b_0 and b_3 will enter in the expression for the Josephson current, and they are given by

$$(b_3)_{l,r} = \frac{1}{2}\left(\frac{1}{\xi_+} + \frac{1}{\xi_-}\right)_{l,r}, \quad (b_0)_{l,r} = \frac{1}{2}\left(\frac{1}{\xi_+} - \frac{1}{\xi_-}\right)_{l,r},$$

where $\xi_\pm = \sqrt{\epsilon_\pm^2 - |\Delta_S|^2}$, and $\epsilon_\pm = i\omega_m \pm h$. By writing $\Delta_S = |\Delta_S|\exp(i\varphi)$ on the right side one obtains the following expression for the critical current:

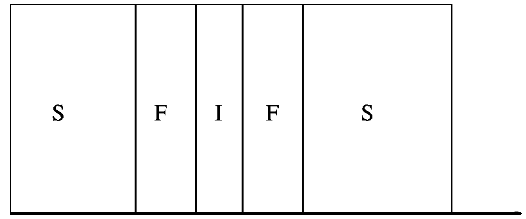

FIG. 7. SF/I/SF system.

$$eV_c(\alpha) \equiv eI_c R_b = 2\pi T \Delta_l \Delta_r \sum_{m>0} \left\{ \text{Re}\left(\frac{1}{\xi_m}\right)_l \text{Re}\left(\frac{1}{\xi_m}\right)_r - \cos\theta\, \text{Im}\left(\frac{1}{\xi_m}\right)_l \text{Im}\left(\frac{1}{\xi_m}\right)_r \right\}, \tag{34}$$

where $\xi_n = \sqrt{(\omega_m + ih_F)^2 + \Delta_S^2}$ and R_b is the tunnel resistance of the I layer. Formula (34) coincides with the formula presented in Ref.[24]. In the latter work the authors considered a Josephson junction consisting of two magnetic superconductors with an oscillating magnetic order. Thus, we have shown that the system of Fig. 7 and that of Ref. 24 are equivalent. However the authors of Ref. 24 did not consider some interesting properties of such structures. We note that the same structure was also analyzed in Ref. 25, where the critical current was calculated for different S/F interface transparencies. The authors have found the conditions under which the system undergoes a transition to the π state; however, they analyzed only the case of parallel magnetization.

Here we consider two limiting cases: (a) a parallel relative orientation of the magnetizations, i.e., $\alpha = 0$, and (b) an antiparallel orientation: $\alpha = \pi$.

In the case $\alpha = 0$ according to Eq. (34), the critical current is given by the expression

$$eV_{c\uparrow\uparrow} \equiv eI_{c\uparrow\uparrow}R_b = 4\pi T \Delta_S^2 \sum_m \frac{\omega_m^2 + \Delta_S^2 - h_F^2}{(\omega_m^2 + \Delta_S^2 - h_F^2)^2 + 4\omega_m^2 h_F^2}. \tag{35}$$

In writing Eq. (35) we assumed that h_F and $|\Delta_S|$ are the same in both bilayers (symmetric structure). The dependence of the critical current on the exchange field h_F was presented in Ref. 26. At $T = 0$ the current I_c is constant up to the value $h_F = \Delta_0$ where it drops to zero; Δ_0 is the effective energy gap Δ_S at zero temperature and zero exchange field. This is a consequence of the fact that the order parameter Δ is also constant. We do not consider here a possible transition to the LOFF phase predicted by Larkin and Ovchinnikov (LO)[18] and Fulde and Ferrell (FF)[17] for the region $0.755\Delta_{S0} < h_F$. We argue that since the homogeneous superconducting state in this region is a metastable state, its realization is possible. Nevertheless, our result is definitely valid for the region of small h_F, and a possible transition to the LOFF phase would manifest itself in a drop of the the critical current.

More interesting is the case when the relative orientation of the magnetizations is antiparallel, i.e., $\alpha = \pi$. Then, the critical current is given by the expression

$$eV_{c\uparrow\downarrow}(\pi)\equiv eI_{c\uparrow\downarrow}R_b = 4\pi T\Delta_S^2\sum_m \frac{1}{\sqrt{(\omega_m^2+\Delta_S^2-h_F^2)^2+4\omega_m^2h_F^2}}. \tag{36}$$

In this case the dependence of I_c on h_F is completely different from that given by Eq. (35). The critical current determined by Eq. (36) increases with increasing h_F (i.e., with increasing either h or d_F) and even diverges at zero temperature when $h_F\rightarrow\Delta_S$. Of course, there is no real divergence of I_c since, for example, finite temperatures smear out this divergency. The dependence of eV_c/Δ_0 on h was presented in Ref. 26. The critical current has a maximum at some value of h_F close to Δ_0. With decreasing T the maximum value of I_c increases and its position is shifted towards Δ_0. For arbitrary relative orientations of magnetizations the expression for $V_c(\alpha)$ can be presented in the form

$$V_c(\alpha)=V_{c\uparrow\uparrow}\cos^2(\alpha/2)+V_{c\uparrow\downarrow}\sin^2(\alpha/2). \tag{37}$$

Therefore, the singular part is always present and its contribution reaches 100% at $\alpha=\pi$. All the conclusions given above remain valid also for two magnetic superconductors with uniformly oriented magnetizations in each layer. We note that in contrast to the case of the spiral structure analyzed in Ref. 24, no π state appears in our model for any effective exchange field $h_F\leqslant\Delta_0$ (at larger h_F the superconductivity is destroyed). As in the previous case of parallel orientations, the state with $h_F=\Delta_0$ might be unreachable for the antiparallel orientation due to the appearance of the inhomogeneous LOFF state. However, the singular behavior of I_c can be realized at smaller values of h in a structure with large enough S/F interface resistance $R_{S/F}$. In this case the bulk properties of the S film are not changed by the proximity of the F film [to be more precise the condition $R_{S/F}>(\nu_F d_F/\nu_S d_S)\rho_F\xi_F$ must be satisfied; ρ_F is the specific resistance of the F film]. Then, as one can readily show,[27] a subgap $\epsilon_{sg}=(D\rho)_F/(R_{S/F}d_F)$ arises in the F layer. The Green's functions in the F layer have the same form as in Eq. (33) with Δ_S replaced by ϵ_{sg}. The singularity in $I_c(h_F)$ occurs at h_F equal to ϵ_{sg}, and the LOFF state does not arise because the subgap ϵ_{sg} is not determined by the self-consistency equation.

For completeness we note that the effect of the relative orientation of magnetization in the F films on the critical temperature T_c of the superconductor was analyzed in Refs. 28 and 29 for a F/S/F structure.

V. ORBITAL EFFECTS

In the preceding sections we have presented the formulas for the condensate function $\hat{s}$ (or $\check{s}$) in the ferromagnetic regions induced by the proximity effect. The amplitude of $\hat{s}$ is determined by the interface transparency, i.e., by the parameter γ, and the penetration length depends essentially on the parameter $h\tau$. The internal magnetic field B of the ferromagnet induces screening currents and leads to some suppression of the condensate function. We have neglected this suppression due to Meissner currents (orbital effects). In order to understand why this approximation can be justified we estimate now the magnitude of these effects.

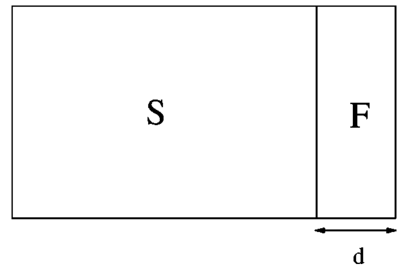

FIG. 8. S/F bilayer.

In the dirty limit the depairing rate due to Meissner current is determined by the energy Dp_s^2, where $p_s=A(d)/\phi_0$ is the condensate momentum, $A(x)=Bx$ is the vector potential, and ϕ_0 is the magnetic flux quantum. The depairing factor can be neglected in the Usadel equation provided the condition

$$Dp_s^2\ll h \tag{38}$$

is satisfied. For example, for $B=1$ kG, $d=100$ Å, $v_F=2.10^7$ cm/s, and $l\sim d$ we obtain $Dp_s^2\sim 50$ mK. If the condition Eq. (38) is met, the condensate function $\hat{s}(x)$ (to be more exact, its Fourier transform) in the ferromagnet is given by Eqs. (8)–(11).

Due to the condensate penetration and the intrinsic magnetic field of the ferromagnet, the Meissner currents arise in the F region. In order to analyze this issue in more detail, we consider the S/F system of Fig. 8.

If we consider the diffusive regime, the condensate function can be found as was done in Sec. II or directly from the Usadel equation. In this limit $(h\tau\ll 1)$ it has the form

$$s_\omega(x)=3\langle\mu\gamma\rangle f_s\frac{\cosh\kappa_+(d-x)}{(\kappa_+l)\sinh(\kappa_+d)}. \tag{39}$$

The current density is expressed in terms of $s(x)$ as

$$j(x)=-\sigma(Bx/\phi_0)(2\pi T)\mathrm{Re}\sum_{\omega>0}s_\omega^2(x). \tag{40}$$

In Fig. 9 we plot the spatial dependence of the current density $j(x)$ which is spontaneously induced in the ferromagnetic film. We can see that $j(x)$ changes sign with varying x. According to the results of Sec. II, in the case $h\tau>1$, the current density changes sign many times on the mean free path.

Integrating the current density $j(x)$ given by Eq. (40), we find the total current $I=\int_0^d dx\, j(x)$:

$$I=-\tfrac{1}{4}(3\langle\mu\gamma\rangle)^2\sigma(Bd^2/\phi_0)\times(2\pi T)\mathrm{Re}\sum_{\omega>0}\frac{f_s^2}{(\kappa_+l)^2\sinh^2(\kappa_x l)}\left[\frac{\sinh^2(\kappa_+d)}{(k_+d)^2}+1\right]. \tag{41}$$

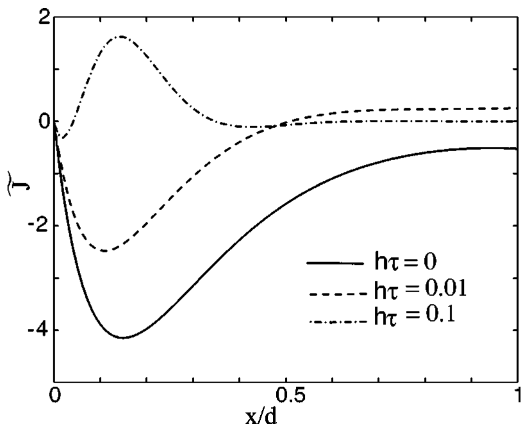

FIG. 9. Spatial dependence of the current density. Here $\tilde{J} = (j/\gamma^2\Delta_0\sigma)\phi_0/Bl$, $d/l=10$, $T/\Delta_0=0.1$, and $\Delta_0\tau=0.05$. For clarity, the values of the current density for $h\tau=0.1$ have been multiplied by a factor of 5.

In Figs. 10 and 11 we presented the total current I as a function of d and $h\tau$, respectively. It is seen that the total current also changes sign; i.e., in the ferromagnetic film either a diamagnetic or paramagnetic current is induced depending on the relation between d and ξ_h.

In the analysis presented here it was assumed that the exchange field h is homogeneous. In a multidomain ferromagnet one expects a more complicated spatial distribution of the Meissner current.

VI. CONCLUSION

We analyzed specific features of a supercurrent in superconductor-ferromagnet structures. In Sec. II we have calculated the Josephson current I_J in a S/F/S junction. It turns out that the product $h\tau$ of the splitting energy h and the momentum relaxation time τ is an important parameter, which determines the approach to be used in the problem. In the dirty limit, i.e., when $h\tau \ll 1$, I_J can be obtained from the

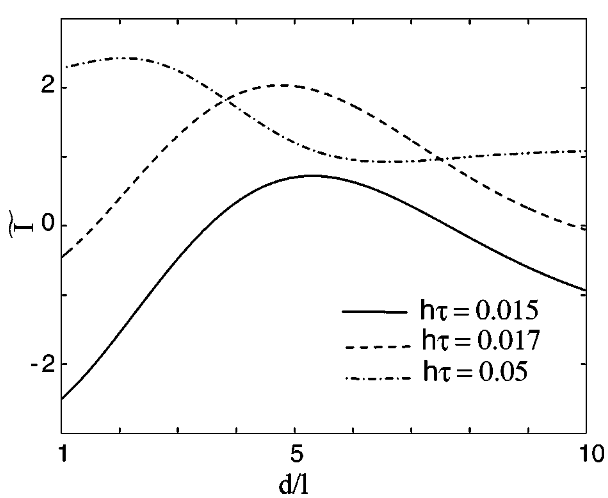

FIG. 10. Dependence of the total current on the thickness d of the ferromagnet film for $\Delta_0\tau=0.05$ and $T/\Delta_0=0.1$. Here $\tilde{I} = (I/\gamma^2\Delta_0\sigma)\phi_0/Bl^2$.

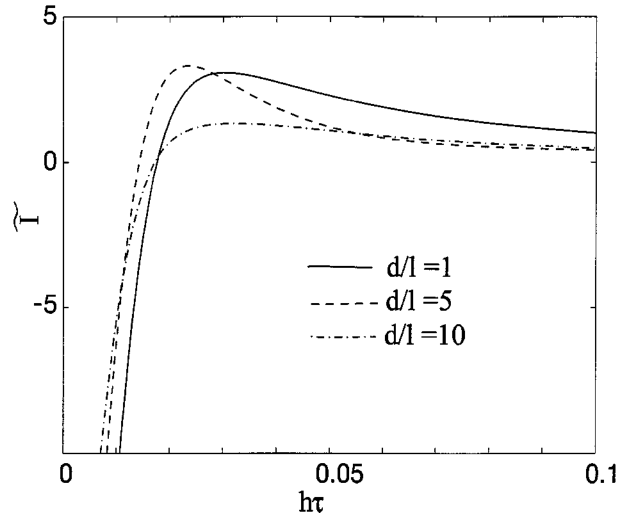

FIG. 11. Dependence of the total current on the parameter $h\tau$ for $\Delta_0\tau=0.05$ and $T/\Delta_0=0.1$. Here $\tilde{I}=(I/\gamma^2\Delta_0\sigma)\phi_0/Bl^2$. At $h\tau=0$, $\tilde{I}(0)=-34,-25,-18$ for $d/l=1,5,10$ respectively.

Usadel equation.[7,30,21] In this limit, the change of sign of the critical current I_c occurs if the thickness of the F layer d is of the order of $\xi_F=\sqrt{D/h}$. The condensate function in the F layer decays exponentially over the length ξ_F and undergoes oscillations with the same period. In the opposite limit ($h\tau \gg 1$) the condensate function oscillates in space with the period v_F/h (as in the pure ballistic case considered in Ref. 2) and decays exponentially on the mean free path l. The critical current I_c decreases with h as a power-law function and is not exponentially small if $d\sim l$.

We have also studied the influence of different inhomogeneous magnetic structures on the critical current through S/F structures. In Sec. III we considered a S/F/S sandwich with an inhomogeneous magnetic order in the F layer described by a vector Q. In the case $Q=0$ we obtained the well-known transition from the 0-phase state to the π-phase state. We have also shown that even for small values of Q and not too low temperatures this transition may not take place (Fig. 6). The reason for a qualitative change of the $I_c(h_F)$ is a long-range term in the singlet component of the condensate function $\check{f}$. This term arises together with the triplet component if Q differs from zero. The long-range part of $\check{f}$ decays in the F film on a length of the order $\sqrt{D/2\pi T}$, which can be much longer than the characteristic length ($\sim\sqrt{D/h}$) of the decay of $\check{f}$ in a homogeneous F layer ($Q=0$).

Our results may be applied to ferromagnets containing domain walls and magnetic multilayers with nonhomogeneous magnetic structures. We used the quasiclassical Green's function approach to describe such structures in a quantitative way. In Sec. IV it was shown that for an antiferromagnetic configuration in the S/F-I-S/F junction, the dependence $I_c(h)$ shows an anomalous behavior: the critical current increases with increasing h or d_F. This means that the Josephson critical current in a junction formed by two ferromagnet-superconductor bilayers may be even larger than the critical current in a similar Josephson junction S/I/S.

In the last section we have considered a S/F bilayer structure. The Meissner currents which are spontaneously excited due to the internal field of the ferromagnetic film have been calculated. The current density oscillates along the x axis and the total Meissner current in the F film may be either diamagnetic or paramagnetic depending on the thickness d and on the exchange field h. All the effects analyzed in our work can be verified experimentally.

ACKNOWLEDGMENT

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APPENDIX:

In this part we present some general formulas for superconductivity in the presence of an exchange field. We consider structures in which the superconducting pairing and the exchange interaction of electrons with ordered, localized magnetic moments take place. The Hamiltonian describing the system under consideration has the form

$$\hat{H}=\sum_{\{p,s\}}(a^{\dagger}_{sp}\{[(\xi_p\delta_{pp'}+eV)+U_{imp}]\delta_{ss'}-(\mathbf{h}\cdot\sigma)\}a_{s'p'}-(\Delta a^{\dagger}_{\overline{sp}}a^{\dagger}_{s'p'}+\text{c.c.})). \tag{A1}$$

Here the summation is carried out over all momenta (p,p') and spins (s,s'), $\xi_p=p^2/2m-\epsilon_F$ is the kinetic energy counted from the Fermi energy ϵ_F, V is a smoothly varying electric potential, $U_{imp}=U(p-p')$ is a potential describing the interaction of electrons with nonmagnetic impurities, and h is an effective ''magnetic field'' caused by the exchange interaction of spins of the free electrons with spins of the localized magnetic moments. The notation $\overline{s}$, $\overline{p}$ means inversion of both spin and momentum. The order parameter Δ must be determined self-consistently. In order to define the Green's function in a customary way we introduce new operators $c^{\dagger}_{ns}$ and c_{ns}, which are related to the creation and anhilation operators $a^{\dagger}_s$ and a_s by the relation (we drop the index p related to the momentum)

$$c_{ns}=\begin{cases} a_s, & n=1,\\ a^{\dagger}_{\overline{s}}, & n=2.\end{cases}$$

The index n operates in the particle-hole (Nambu) space, while the index s operates in the spin space. The operators c_{ns} obey the commutation relations

$$c_{ns}c^{\dagger}_{n's'}+c^{\dagger}_{n's'}c_{ns}=\delta_{nn'}\delta_{ss'}, \tag{A2}$$

$$c_{ns}c_{n's'}+c_{n's'}c_{ns}=\delta_{n\overline{n'}}\delta_{s\overline{s'}}. \tag{A3}$$

In terms of the c_{ns} operators the Hamiltonian can be written in the form

$$\hat{H}=\sum_{\{p,n,s\}}c^{\dagger}_{ns}\mathcal{H}_{(nn')(ss')}c_{n's'}, \tag{A4}$$

where the summation is performed over all momenta, Nambu, and spin indices. The matrix $\check{\mathcal{H}}$ is given by

$$\check{\mathcal{H}}=\tfrac{1}{2}\{[(\xi_p\delta_{pp'}+eV)+U_{imp}]\hat{\tau}_3\otimes\hat{\sigma}_0+\hat{\tilde{\Delta}}\otimes\hat{\sigma}_3-h[(\hat{\tau}_0\otimes\hat{\sigma}_3)\cos\alpha+(\hat{\tau}_3\otimes\hat{\sigma}_2)\sin\alpha]\}. \tag{A5}$$

The matrices $\hat{\tau}_i$ and $\hat{\sigma}_i$ are the Pauli matrices in the Nambu and spin space, respectively; $i=0,1,2,3$, where $\hat{\tau}_0$ and σ_0 are the corresponding unit matrices. We have assumed that the exchange field $\mathbf{h}$ has the components $\mathbf{h}=h(0,\sin\alpha,\cos\alpha)$; this is the case we consider in the next sections. The matrix order parameter equals $\hat{\tilde{\Delta}}=\hat{\tau}_1\mathrm{Re}\,\Delta-\hat{\tau}_2\mathrm{Im}\,\Delta$. Now we can define the matrix Green's functions (in the Nambu$\otimes$spin space) in the Keldysh representation in a standard way:

$$\check{G}(t_i,t'_k)=\frac{1}{i}\langle T_C[c_{ns}(t_i)c^{\dagger}_{n's'}(t'_k)]\rangle, \tag{A6}$$

where the temporal indices take the values 1 and 2, which correspond to the upper and lower branches of the contour C, running from $-\infty$ to $+\infty$ and back to $-\infty$. The quasiclassical Green's functions $\check{g}(t_i,t'_k)$ are defined as usual[9,10]

$$\check{g}(\mathbf{p_F},\mathbf{r})=\frac{i}{\pi}(\hat{\tau}_3\otimes\hat{\sigma}_0)\int d\xi_p\check{G}(t_i,t'_k;\mathbf{p},\mathbf{r}). \tag{A7}$$

We also introduce, as it was done by Larkin and Ovchinnikov,[31] a hypermatrix $\check{\mathbf{g}}$. The matrix elements of $\check{\mathbf{g}}$ are the retarded $\check{g}^R$, advanced $\check{g}^A$, and the Keldysh $\check{g}^K$component. Thus, $\check{\mathbf{g}}$ has the form

$$\check{\mathbf{g}}=\begin{pmatrix}\check{g}^R & \check{g}^K\\ 0 & \check{g}^A\end{pmatrix}. \tag{A8}$$

The functions $\check{g}^{R(A)}$ and $\check{g}^K$ can be expressed in terms of the time-ordered Green's functions $\check{g}(t_i,t_k)$ as follows:

$$\check{g}^{R(A)}=\check{g}(t_1,t'_1)-\check{g}(t_{1(2)},t'_{2(1)}), \tag{A9}$$

$$\check{g}^K=\check{g}(t_1,t_2{}^{\prime})+\check{g}(t_2,t'_1). \tag{A10}$$

The equation for the hypermatrix $\check{\mathbf{g}}$ can be easily derived in the same way as it was done for the case of a superconductor with spin-independent interactions.[31] We are interested in the diffusive limit. The symmetric component of the matrix $\check{\mathbf{g}}$ with respect to the momentum direction $\mathbf{p_F}$ satisfies the equation

$$-iD\nabla(\check{\mathbf{g}}\nabla\check{\mathbf{g}})+i(\hat{\tau}_3\otimes\hat{\sigma}_0.\partial_t\check{\mathbf{g}}+\partial_{t'}\check{g}.\hat{\tau}_3\otimes\hat{\sigma}_0)+eV(t)\check{\mathbf{g}}-\check{\mathbf{g}}eV(t')+[\hat{\Delta}\otimes\hat{\sigma}_3,\check{\mathbf{g}}]+[\check{M}_h,\check{\mathbf{g}}]=0. \tag{A11}$$

Here $D=vl/3$ is the diffusion coefficient, $\check{M}_h=h(\hat{\tau}_3\otimes\hat{\sigma}_3\cos\alpha+\hat{\tau}_0\otimes\hat{\sigma}_2\sin\alpha)$, and

$$\hat{\Delta}=-\hat{\tilde{\Delta}}\hat{\tau}_3=\begin{pmatrix}0 & \Delta\\ -\Delta^* & 0\end{pmatrix}.$$

Equation (A11) is supplemented by the normalization condition

$$\check{\mathbf{g}}\cdot\check{\mathbf{g}}=\check{1} \tag{A12}$$

and by the self-consistency equation

$$\Delta=\frac{\lambda}{16}\mathrm{Tr}(\hat{\tau}_1-i\hat{\tau}_2)\otimes\hat{\sigma}_3\int d\epsilon\check{g}^K. \tag{A13}$$

If the magnetization and, hence, the exchange field h, is constant in the ferromagnetic layers, the angle α in Eq. (A11) can be excluded with the help of the following unitary transformation:

$$\tilde{\check{\mathbf{g}}}=\check{U}^{\dagger}\check{\mathbf{g}}\cdot\check{U}, \tag{A14}$$

where $\check{U}=\hat{\tau}_0\otimes\hat{\sigma}_0\cos(\alpha/2)+i\sin(\alpha/2)\hat{\tau}_3\otimes\hat{\sigma}_1$. We consider F/S structures, therefore we need the boundary conditions for Eq. (A11) at the interface between the conductors "a" and "b" (Ref. 20):

$$\check{\mathbf{g}}_a\partial_x\check{\mathbf{g}}_a=\frac{\rho_a}{2R_{a/b}}[\check{\mathbf{g}}_a,\check{\mathbf{g}}_b], \tag{A15}$$

where ρ_a is the specific resistivity of the conductor a and $R_{a/b}$ is the interface resistance per unit area. We assumed that there are no spin-flip processes at the interface. In the presence of spin processes at the boundary the condition (A15) can be generalized.[32] The current density is determined by the usual expression

$$I_J=\frac{1}{16\rho}\mathrm{Tr}(\hat{\tau}_3\otimes\hat{\sigma}_0)\int \mathrm{d}\,\epsilon(\check{g}^R\partial_x\check{g}^K+\check{g}^K\partial_x\check{g}^A). \tag{A16}$$

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