Brillouin light scattering from periodic multilayers composed of very thin magnetic films

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(Received 17 January 2001; published 4 September 2001)

A simple treatment of the dipole-dipole interaction between the magnetic films in a multilayer has been used to calculate the low-lying spin-wave modes in multilayer structures. It is assumed that in equilibrium the magnetization in each film is parallel to the applied magnetic field. It is also assumed that the magnetic films are thin enough so that the magnetization density remains uniform across the thickness of each film when the magnetization is thermally excited from equilibrium. Surface pinning energies and interlayer exchange coupling are included in the formalism. The calculated spin-wave modes have been used to investigate the scattered light spectrum in a Brillouin light scattering (BLS) experiment. The method has been applied to the cases of a surface mode on an infinite multilayer stack and to multilayers containing 5, 10, and 25 magnetic films having thicknesses up to 50 Å and for nonmagnetic spacer layers having similar thicknesses. The scattered light spectra for such thin-film structures are very simple for multilayers having a total thickness less than, or comparable to, the optical decay length for light propagating through the multilayer structure. For such multilayers only those one or two modes give rise to an appreciable scattered light intensity for which the magnetization precessional amplitude varies very little from one magnetic film to the next. The frequencies corresponding to these strong BLS modes are insensitive to the interlayer exchange coupling because the magnetizations in neighboring films remain nearly parallel as they precess.

DOI: 10.1103/PhysRevB.64.134406 PACS number(s): 76.50.+g, 75.70.−i, 78.35.+c

INTRODUCTION

Brillouin light scattering (BLS) experiments have been extensively used to probe the behavior of magnetic thin films assembled to form a multilayer structure. The experimental and theoretical work done on such structures has been reviewed in the excellent articles by Hillebrands and Güntherodt[1] and by Hillebrands.[2] The theory of BLS for periodic coherent layered structures was initiated by Camley, Rahman, and Mills.[3] These authors calculated the mode frequencies for a semi-infinite stack of identical ferromagnetic films of thickness d_1 separated by nonmagnetic spacer films of thickness d_2. The static magnetization in each layer was taken to be uniform and parallel with the applied magnetic field. The frequencies of thermally excited spin-wave modes supported by the multilayer were calculated using the linearized Landau-Lifshitz equations of motion to describe the spatial variation of the magnetization deviation from equilibrium. The associated magnetic fields were calculated from Maxwell's equations in the magnetostatic approximation, curl $\mathbf{h}=0$. The intensity distribution of the light scattered from such a multilayer stack was calculated using the assumption that each magnetic layer was thick compared with the optical skin depth of the incident light. The BLS calculations were therefore restricted to the case in which the magnetic films were of the order of 200 Å or thicker. However, much of the experimental work on multilayers has been carried out using structures in which the magnetic layers were less than 50 Å thick.[1,2] The theory for such very thin magnetic films has been developed for a periodic, semi-infinite multilayer by Almeida and Mills[4] using an effective-medium approximation in which the magnetization within a thin film remains uniform as it precesses around the equilibrium magnetization direction. The Almeida-Mills effective-medium theory was extended by Nörtemann *et al.*[5] to include perpendicular anisotropies and interlayer exchange interactions. The last two articles were mainly concerned with the calculation of spin-wave spectra when the static film magnetizations were no longer parallel with the applied magnetic field. The case of finite multilayers has been studied by Hillebrands[6] and by Stamps and Hillebrands.[7] These authors used the full Landau-Lifshitz equations of motion including exchange to study the applied-field dependence of the multilayer mode frequencies for multilayers containing a few magnetic films. Perpendicular surface anisotropies and interlayer exchange coupling were included in their calculations: they did not attempt to calculate the BLS spectra to be expected for such multilayer specimens.

In a BLS experiment one does not directly measure the frequencies associated with the various spin-wave modes in the specimen. The BLS experiment measures the frequency dependence of the intensity of the light scattered from those spin-wave modes when the specimen is illuminated by light of a fixed frequency. It turns out that for a multilayer stack composed of magnetic films thinner than 50 Å thick most of the modes do not contribute a BLS signal of appreciable intensity. In this article the frequency distribution of the BLS scattered light intensity will be calculated for a periodic multilayer composed of very thin magnetic films separated by very thin nonmagnetic spacer layers. It will be shown that for such thin films the BLS spectra basically probe only the two modes of the multilayer that have the smallest variation in amplitude from one magnetic film to the next. The BLS spectra for such multilayers are insensitive to the strength of the interlayer exchange coupling because the magnetizations in neighboring films remain almost parallel as they precess around the direction of the equilibrium magnetization.

In the following work an approximate but simple method for treating the dipole-dipole interlayer and intralayer cou-

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pling will be described and will be used to treat three problems.

(1) The normal modes for an infinite multilayer stack will be calculated in the magnetostatic approximation used by Camley, Rahman, and Mills.[3] This calculation will establish the validity of the dipole-dipole coupling approximation mentioned above by showing that the resulting frequency spectrum is in substantial agreement with that calculated by Camley, Rahman, and Mills.[3]

(2) The surface-mode frequency and the BLS intensity will be calculated for an infinite multilayer stack, again using the magnetostatic approximation. It will be shown that the results for the surface-mode frequency are in agreement with the calculation of Camley, Rahman, and Mills.[3] The BLS intensity corresponding to this surface mode will be shown to depend strongly on the strength of the surface anisotropy parameter K_s [see Eq. (1)].

(3) The mode frequencies and BLS intensities will be calculated for multilayer stacks consisting of 5, 10, and 25 identical magnetic films. It will be shown that the BLS spectrum for a multilayer stack whose total thickness is less than a few hundred angstroms is very simple.

MODEL

The model system under consideration is composed of M identical magnetic films, each $2d$ thick, separated by nonmagnetic films D_{nm} thick. The repeat unit consists of one magnetic and one nonmagnetic film and has a thickness $D = D_{nm}+2d$. Here M of these repeat units are deposited on a nonmagnetic substrate that is optically very thick: see Fig. 1. It is assumed that the optical properties of the films and substrate can be characterized by scalar dielectric constants ε_1, ε_2, and ε_s as indicated in Fig. 1. The M repeat units are capped by a nonmagnetic film D_{nm} thick. Each magnetic film is characterized by a uniform magnetization density M_s, and at equilibrium it is assumed that these magnetization vectors are all parallel with the applied magnetic field H and directed along the x axis as indicated in Fig. 1. It is assumed that the magnetic films are isotropic in plane. Real multilayers fabricated by sputtering have been shown to be isotropic, or very nearly isotropic, in plane;[1,2] moreover, this nonessential assumption simplifies the calculations. It is, however, assumed that each magnetic film is subject to an anisotropy whose axis lies along the multilayer normal (the z axis shown in Fig. 1). It is also assumed that the magnetic films are so thin that the magnetization remains uniform across the width of the film as it precesses around the equilibrium direction. This means that the state of each film can be specified by the two magnetization components m_Y^n and m_Z^n where n specifies the film in question. With this assumption the perpendicular anisotropy energy can be specified by a volume density or by a surface density or by a combination of both. In any real system both volume and surface energy density terms are present.[8] In this article the perpendicular anisotropy will be ascribed entirely to a surface energy term for the sake of simplicity. The surface energy term has the form

$$E_s^n = -K_s(m_Z^n/M_s)^2 \quad \text{ergs/cm}^2. \tag{1}$$

K_s is a phenomenological surface energy parameter. Positive K_s corresponds to a torque density that tends to rotate the magnetization into the direction of the specimen normal. Note that the contribution of this term to the total free energy per unit area of a magnetic film will be double Eq. (1) because each film has two interfaces and it is assumed that each interface contributes equally to the total surface energy. This surface energy term gives rise to an effective field acting on the magnetization density in the nth film whose components are[8]

$$h_Y^n = 0, \tag{2a}$$

$$h_Z^n = \left(\frac{2K_s}{dM_s^2}\right) m_Z^n. \tag{2b}$$

It is assumed that each magnetic film interacts with its neighbors through an exchange coupling energy. The form of this interaction between the nth and the $(n+1)$st films is given by

$$E_X^n = -\frac{J}{M_s^2}(m_X^n m_X^{n+1} + m_Y^n m_Y^{n+1} + m_Z^n m_Z^{n+1}) \quad \text{ergs/cm}^2. \tag{3}$$

In Eq. (3) the coupling energy has been written for a single pair of films: the total interlayer exchange energy is obtained as the sum over terms like Eq. (3) for $n=1$ to $M-1$. Equation (3) can be rewritten using the approximation

$$m_x = M_s - \frac{1}{2M_s}(m_Y^2 + m_Z^2). \tag{4}$$

The exchange coupling energy between the nth and $(n+1)$st films then takes the form

$$E_X^n = -J + \frac{J}{2M_s^2}[(m_Y^n - m_Y^{n+1})^2 + (m_Z^n - m_Z^{n+1})^2]. \tag{5}$$

This interfilm exchange coupling energy gives rise to effective fields whose components in the nth film are[8]

$$h_Y^n = \frac{J}{2dM_s^2}(-2m_Y^n + m_Y^{n-1} + m_Y^{n+1}). \tag{6a}$$

$$h_Z^n = \frac{J}{2dM_s^2}(-2m_Z^n + m_Z^{n-1} + m_Z^{n+1}). \tag{6b}$$

Notice that film No. 1 has no other film on its left-hand side; therefore, for $n=1$,

$$h_Y^1 = \frac{J}{2dM_s^2}(m_Y^2 - m_Y^1), \tag{7a}$$

$$h_Z^1 = \frac{J}{2dM_s^2}(m_Z^2 - m_Z^1). \tag{7b}$$

Similarly, there is no magnetic film to the right of film No. M, so that the effective-field components for the last film in the stack are given by

$$h_Y^M = \frac{J}{2dM_s^2}(m_Y^{M-1} - m_Y^M), \tag{8a}$$

$$h_Z^M = \frac{J}{2dM_s^2}(m_Z^{M-1} - m_Z^M). \tag{8b}$$

The magnetic films in a multilayer stack interact with each other through dipole-dipole coupling. A deviation of the magnetization density from the direction of the applied magnetic field potentially produces both internal and external magnetic fields. Assuming that any rotation of the magnetization density is uniform across the film thickness, these fields can be calculated in the magnetostatic limit, curl $\mathbf{h}=0$, using the method described by Cochran *et al.*[9] or by using the generalization described by Hurben and Patton.[10] In this article the calculations assume an in-plane spatial dependence of the magnetization components proportional to exp(iqy). The spatial dependence along y (see Fig. 1) comes about because we are concerned with an experimental light scattering geometry in which the plane of incidence of the incident laser light is oriented perpendicular to the applied magnetic field; in terms of the coordinate system shown in Fig. 1 the reflected and incident laser beams lie in the y-z plane. We are also interested in the backscattering configuration in which the scattered light is collected along the direction of the incident light beam. The backscattering configuration is very commonly used for BLS experiments.[11,12] The intensity of the scattered light is the same for both s- and p-polarized incident light for the backscattering geometry. In the present article the incident light is taken to be p-polarized; i.e., the incident optical electric-field vector lies in the plane of incidence. This means that the incident optical magnetic vector is oriented perpendicular to the plane of incidence and therefore has only an x component in the coordinate system of Fig. 1. For an incident optical field having a circular frequency Ω and having a unit amplitude in the cgs system, one has

$$H_X = \exp(-iQy + ik_v z - i\Omega t), \tag{9}$$

where $Q=(\Omega/C)\sin\theta$, $k_v=(\Omega/C)\cos\theta$, and θ is the angle of incidence of the light. This incident light sets up an optical electric field in each magnetic film that gives rise to scattered light whose frequency is shifted up and down by a spin-wave frequency ω. Because of the boundary conditions (continuity of the tangential components of the optical electric and magnetic fields), the optical electric field in a magnetic film must have a spatial variation along y proportional to exp($-iQy$). The outgoing scattered light must have a spatial variation along y proportional to exp($+iQy$) because in the backscattering configuration only that light is collected that is emitted along the direction of the incident beam. It turns out that for the backscattering configuration the light that has been frequency upshifted has been scattered from a spin-wave having a spatial dependence along y proportional to exp(iqy), where $q=2Q$: light that has been frequency downshifted has been scattered by a spin-wave proportional to exp($-iqy$). As a result, one is led to investigate the magnetic fields set up by a precessing magnetization that is uniform across the film, but that has a spatial dependence in the film plane proportional to exp(iqy) where q may be either positive or negative. In a typical experiment using visible laser light incident on the specimen at 45°, the wave vector $q \sim 2\times 10^5\ \mathrm{cm}^{-1}$. The fields set up by the above spatial variation of the magnetization are as follows (suppressing the time dependence exp($-i\omega t$) for convenience)

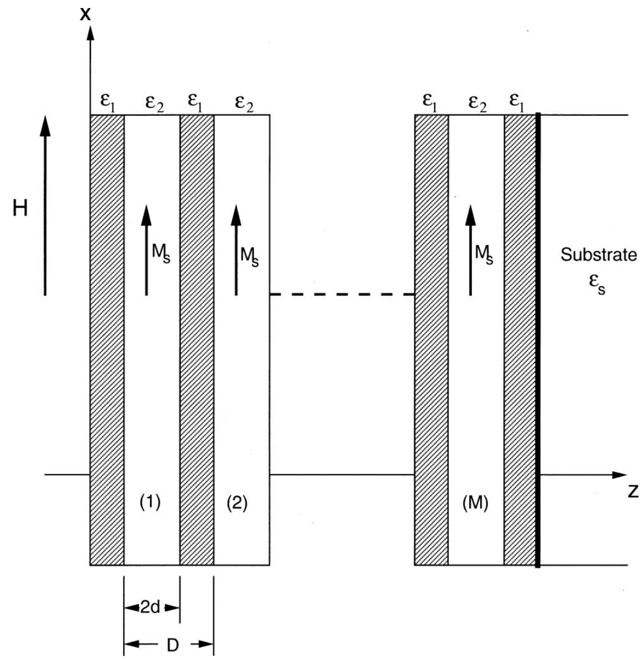

FIG. 1. A schematic diagram of the periodic multilayer geometry used in this article. M thin magnetic films, each $2d$ thick, are separated by nonmagnetic films each D_{nm} thick (shown cross-hatched). $D_{nm}=D-2d$ where D is the periodic repeat distance. Each of the identical magnetic layers has a magnetization density M_s, which in equilibrium is aligned along the applied field H. The optical properties of the films are specified by the dielectric constants ε_1 for the nonmagnetic layers, ε_2 for the magnetic layers, and ε_s for the substrate. The light for the BLS calculations is assumed to be incident in the y-z plane.

(1) Within a film the spatially averaged dipole-dipole field components are

$$h_Y = -\exp(iqy)4\pi m_Y|q|d, \tag{10a}$$

$$h_Z = -\exp(iqy)4\pi m_Z(1-|q|d), \tag{10b}$$

where $2d$ is the thickness of the magnetic film. The expressions (10) neglect terms of order $(qd)^2$ or higher.

(2) The dipole-dipole fields outside the film are given by

$$h_Y = -\exp(iqy)(4\pi m_Y|q|d + 4\pi m_Z iqd)\exp(-|q|z), \tag{11a}$$

$$h_Z = -\exp(iqy)(4\pi m_Y iqd - 4\pi m_Z|q|d)\exp(-|q|z), \tag{11b}$$

for $z>d$, where z is measured from the center of the magnetic film. On the left of the film, $z<-d$, the field components are

$$h_Y = -\exp(iqy)(4\pi m_Y|q|d - 4\pi m_Z iqd)\exp(+|q|z), \tag{11c}$$

$$h_Z = \exp(iqy)(4\pi m_Y iqd + 4\pi m_Z|q|d)\exp(+|q|z). \tag{11d}$$

Equations (11) neglect terms of order $(|q|d)^3$ or higher. The total dipole-dipole contribution to the magnetic field acting on the magnetization in a given film will be the sum of its own internal field, Eq. (10), plus contributions from all of its neighbors calculated using Eqs. (11). This treatment of the dipole fields will be adequate for a multilayer composed of metallic films providing that $|q|d \ll 1$ and providing either that the total thickness of the multilayer stack is much less than the electromagnetic skin depth corresponding to the spin-wave frequency (a microwave frequency) or that the microwave skin depth is much larger than $1/|q|$. The latter conditions are required because conduction currents have been neglected in using curl $\mathbf{h}=0$. In a typical case in which the multilayer is composed of metallic films, the electromagnetic skin depth is of the order of 10^{-4} cm at microwave frequencies. Thus the magnetostatic approximation upon which Eqs. (10) and (11) are based can be expected to be valid for metallic multilayers as thick as 1000 Å in total.

The frequencies and amplitudes of the modes supported by the magnetic multilayer can be calculated using the linearized Landau-Lifshitz equations of motion[8] written for a time dependence $\exp(-i\omega t)$:

$$\frac{i\omega}{\gamma}\mathbf{m} = (\mathbf{M}\times\mathbf{H}_{\text{eff}}), \tag{12}$$

where $\gamma=(g/2)(|e|/mC)$ is the gyromagnetic ratio, $\mathbf{M} = \mathbf{M}_s + \mathbf{m}$ where $\mathbf{m}$ is the deviation from equilibrium of the magnetization density in a particular film, and $\mathbf{H}_{\text{eff}}$ is the effective magnetic field acting on that particular film. These effective fields include the applied magnetic field H, the surface anisotropy field, Eq. (2), the interlayer exchange coupling fields, Eqs. (6), (7), and (8), and the dipole-dipole fields, Eqs. (10) and (11). Exchange fields due to the in-plane spatial variation of the magnetization proportional to $\exp(iqy)$ have been neglected for the reasons set forth below in more detail. In essence, if the exchange fields due to the spatial variation of the magnetization across the film (the z dependence) are to be neglected, then it is consistent also to neglect the contribution of the in-plane spatial variation to the exchange fields. It turns out that the small frequency shifts due to in-plane and out-of-plane exchange fields are comparable for films up to 100 Å thick and for $q \sim 10^5\ \text{cm}^{-1}$.

Effective-field components arising from magnetic damping have also been neglected. Damping in magnetic systems is normally very small and results in a resonant line broadening $\Delta\omega$ such that the ratio $\omega_R/\Delta\omega \sim 50$–$100$, where ω_R is the mode frequency and $\Delta\omega$ is the full linewidth at half maximum intensity. The influence of the damping on the frequency of such high-Q resonances can be be safely ignored.

Application of the Landau-Lifshitz equations to a multilayer containing M magnetic films results in a set of $2M$ homogeneous coupled equations for the magnetization components m_Y^n and m_Z^n. These equations can be solved numerically using standard procedures. One obtains solutions corresponding to M positive frequencies and to M negative frequencies for a given value of q greater than zero. The M positive frequencies correspond to spin-wave modes that are proportional to $\exp(+iqy)$ and hence are propagating along the $+y$ direction. These spin waves result in backscattered light that is upshifted in frequency by the spin-wave frequency ω. The M negative frequencies can be associated with backscattered light that is downshifted in frequency by ω: the magnetization amplitudes corresponding to these modes are the complex conjugates of the negative-frequency mode amplitudes. In other words, it is not necessary to solve the equations of motion once for $q>0$ and then a second time for $q<0$. The spin-wave frequencies and mode amplitudes for both directions of propagation (for both $\pm q$) can be obtained from a single calculation.

The effective fields discussed above can be used to calculate the precessional frequency for a single magnetic film. The frequency for a single film having a magnetization that is uniform across the film thickness can be calculated from

$$(\omega/\gamma)^2 = (H + 4\pi M_s|q|d)\left(H + 4\pi M_s - \frac{2K_s}{dM_s} - 4\pi M_s|q|d\right). \tag{13}$$

It is interesting to compare frequencies calculated from the simple expression (13) with frequencies calculated using the coupled Landau-Lifshitz equations including exchange plus Maxwell's equations.[13] The results of such a comparison are shown in Fig. 2 for $q=1.727\times10^5\ \text{cm}^{-1}$ (green argon laser light at 5145 Å incident on the specimen at 45°), for a saturation magnetization $4\pi M_s = 21.0$ kG, and for an exchange parameter $A=2.03\times10^{-6}$ ergs/cm. The difference in frequency between the full calculation[13] and frequencies calculated from Eq. (13) is shown in Fig. 2 for applied magnetic fields of 1 and 6 kOe and for surface pinning parameters $K_s=0$ and 0.5 ergs/cm^2. The frequency difference is less than 0.5 GHz for film thicknesses ranging from 5 to 140 Å. A frequency shift of 0.5 GHz corresponds to an applied magnetic-field change of approximately 0.17 kOe. If the exchange effective fields were to be included in the expression (13), one would have to add the effective field

$$h = \frac{2Aq^2}{M_s} \tag{14}$$

to each of the two factors in Eq. (13). For iron $h=0.07$ kG for $q=1.727\times10^5\ \text{cm}^{-1}$ and $A=2.03\times10^{-6}$ ergs/cm, $M_s = 1.7$ kG. The addition of this effective field to Eq. (13) would result in a graph similar to Fig. 2, but one in which the curves were shifted down so that the frequency shift was approximately zero for a 5-Å-thick film, but correspondingly larger for the thicker films. The neglect of exchange entirely in Eq. (13) results in frequencies that are correct within 0.5 GHz for thicknesses ranging from 5 to 140 Å: this discrepancy is comparable with the line broadening associated with a typical Fabry-Perot interferometer operating with a free spectral range of 50 GHz.

The scale of the mode amplitudes calculated from the $2M$ set of coupled equations described above is fixed by the requirement that each thermally excited mode must contain, on average, an energy of kT greater than the ground-state energy. For convenience, it is explicitly assumed that one is dealing with a multilayer stack having an in-plane area of 1 cm^2. In addition to the surface energy contributions, Eq. (1), and the interlayer exchange contributions, Eq. (3), it is necessary to include the Zeeman energy for each film,[14]

$$E_Z^n = -Hm_X \cong -2dM_sH + \frac{dH}{M_s}[(m_Y^n)^2 + (m_Z^n)^2], \quad (15)$$

plus the dipole-dipole energy for each film,[14]

$$E_d^n = -d(h_Y^n m_Y^n + h_Z^n m_Z^n). \quad (16)$$

(Remember that the magnetic film thickness is $2d$ and that the specimen area is 1 cm^2.) In the work described below, no attempt is made to calculate absolute BLS intensities; only relative intensities are of interest. Therefore the mode amplitudes have been normalized so that each mode contains an excited state energy of 1 erg per cm^2. This normalization procedure gives mode amplitudes that are conveniently of the order of 1 kG.

BLS INTENSITIES

Brillouin scattered light intensities have been calculated following the procedure for thin films described by Cochran and Dutcher[15] and by Cochran.[16] The procedure is divided into two steps. In step (1) the driving electric field amplitude within each magnetic film is calculated. In this step both the nonmagnetic and magnetic films are treated as if they were isotropic and describable by a scalar dielectric constant; i.e., the dependence of the optical dielectric tensor elements on the magnetization is ignored. In step (2) the driving optical electric field amplitude within each magnetic film is combined with the oscillating magnetization components to calculate the optical electric dipole density oscillating at the frequencies $\Omega \pm \omega$, where Ω is the frequency of the incident laser light and ω is the magnetization precessional frequency. The electric dipole densities oscillating at $\Omega \pm \omega$ are then used to calculate the amplitude of the scattered light.

Step (1). It is necessary to calculate the optical electric-field amplitudes in each of the M magnetic layers. For simplicity, it is assumed that the optical properties of each film, as well as those of the substrate, can be specified by complex scalar dielectric constants. This necessarily requires that these materials exhibit cubic symmetry. The off-diagonal dielectric tensor components induced in the magnetic layers by the presence of a magnetization are very small and may be neglected for purposes of calculating the optical electric-field distribution set up in the multilayer structure. It turns out that BLS intensities calculated for the backscattering configuration are the same for both *s*- and *p*-polarized incident light; in this work, the incident laser beam has been taken to be *p* polarized (**E** in the plane of incidence). This means that the light scattered as a result of magnetic excitations in the multilayer structure will be *s* polarized (**E** perpendicular to the plane of incidence). The plane of incidence of the laser light has been assumed to be the *y*-*z* plane (see Fig. 1), and therefore all the incident field amplitudes in the multilayer must be proportional to the factor

$$\exp(-i[Qy + \Omega t]) \quad (17)$$

in order to satisfy the boundary conditions that **E** and **h** must be continuous across any surface of discontinuity. In Eq. (17), $Q = k_0 \sin\theta$ where the angle of incidence is θ and the vacuum wave vector of the incident light is k_0. Within a given film the magnetic field has only an x component, and the spatial variation across the film of that component can be written

$$h_X = a_n \exp(ikz) + b_n \exp(-ikz), \quad (18)$$

where

$$k^2 + Q^2 = \varepsilon k_0^2. \quad (19)$$

The electric-field components can be obtained from the Maxwell equation

$$\mathrm{curl}\,\mathbf{h} = -i\left(\frac{\varepsilon\Omega}{C}\right)\mathbf{E}. \quad (20)$$

The coefficients a_n and b_n are fixed by the boundary conditions at each interface plus the condition that the substrate contain only a wave propagating along the $+z$ direction. Numerical calculation of the field amplitudes in each film can be readily carried out using a computer.

Step (2). The calculation of the BLS amplitudes. In the presence of an electric field a precessing magnetization induces an electric polarization density $4\pi\mathbf{P}$ that to lowest order in $\mathbf{m}$ is proportional to $(\mathbf{E}\times\mathbf{m})$.[17] In the present case the *p*-polarized optical electric field has only the components E_y and E_z, and these components interact with the magnetization components m_y and m_z to generate a fluctuating polarization density

$$4\pi P_X \propto E_Y m_Z - E_Z m_Y. \quad (21)$$

This optical polarization density oscillates at the frequency $\omega_s = (\Omega + \omega)$ for spin waves propagating along $+y$ and at $\omega_s = (\Omega - \omega)$ for spin waves propagating along $-y$: Ω is the circular frequency of the incident light, and ω is the spin-wave circular frequency. The electric polarization density oscillating at ω_s generates the scattered light, and the scattered light electric vector is polarized along x.[15,16] In the present article we are interested in magnetic films that are thin compared with the optical skin depth. For such films the electric field is very nearly constant over the film thickness and therefore the polarization density $4\pi P_x$ calculated from Eq. (21) can also be taken to be constant over the film thickness. Moreover, for such a thin magnetic film, one can treat the electric dipole density distribution as if it were a δ function having the strength $S = (2d)(4\pi P_x)$ and embedded in the nonmagnetic material. It can be shown[15] that a δ-function

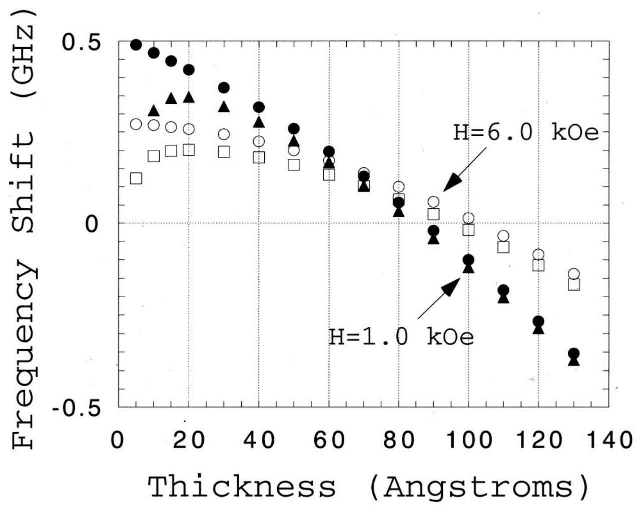

FIG. 2. The difference in frequencies plotted as a function of film thickness for a single magnetic film in which the lowest resonant frequency was calculated in two ways: (1) using the full Landau-Lifshitz formalism plus Maxwell's equations and (2) using the simple theory based on the approximation of a uniform magnetization and no exchange, Eq. (13). Parameters used in the calculation were $4\pi M_s=21.0$ kG, $g=2.09$, exchange parameter $A=2.03\times10^{-6}$ ergs/cm, $q=1.727\times10^{5}$ cm^{-1}, resistivity 10^{-5} Ω cm, and a Gilbert damping parameter $G=10^{7}$ Hz. ● $H=1.0$ kOe, $K_s=0$; ▲ $H=1.0$ kOe, $K_s=0.5$ ergs/cm^2; ○ $H=6.0$ kOe, $K_s=0$; □ $H=6.0$ kOe, $K_s=0.5$ ergs/cm^2.

polarization density having only an x component generates outgoing optical fields such that E_x is continuous at the δ function, but h_y is discontinuous, where $\Delta h_y=i\omega_s/C$. Application of these boundary conditions to the present problem results in

$$E_X^R-E_X^L=0, \tag{22a}$$

$$h_Y^R-h_Y^L=\left(\frac{i\omega_s}{C}\right)S, \tag{22b}$$

where $S=(2d)(4\pi P_x)$ is the strength of the equivalent δ function, E_X^R and h_Y^R are the electric and magnetic field components evaluated at the right-hand interface of the magnetic slab, and E_X^L and h_Y^L are the field components evaluated on the left-hand magnetic film interface. The boundary conditions (22) plus the requirement that there be only an outgoing wave propagating to the left in the vacuum and an outgoing wave propagating to the right in the substrate determine the scattered light amplitude. The vacuum amplitude of the scattered light can be readily calculated using a computer to follow the propagation of the light from its source in a particular magnetic film through the complex multilayer structure to the vacuum on the left and to the substrate on the right.

The scattered light amplitudes generated by the different magnetic films are phase coherent for a given magnetic mode. The amplitudes at the vacuum/multilayer interface must be added together in order to calculate the BLS intensity corresponding to a given mode. The different magnetic modes are uncorrelated in phase; consequently, the total BLS intensity is just the sum of the intensities calculated separately for each mode.

The BLS intensities calculated using the above prescription are integrated intensities: i.e., they correspond to infinitely sharp frequencies. In any real system the magnetic modes will be damped and therefore the scattered light will have a spectral distribution whose width in frequency is proportional to the strength of the damping. Moreover, the response of the Fabry-Perot interferometer used to analyze the scattered light will introduce an additional line broadening. In fact, for most metallic magnetic thin films the observed BLS linewidths are predominantly due to the interferometer resolution. Typically, $\omega/\Delta\omega\sim50$. Both sources of line broadening can be approximately described by a Lorentzian distribution

$$I(f)=\left(\frac{I_0}{\pi}\right)\frac{\Delta}{[(f_s-f)^2+\Delta^2]}, \tag{23}$$

where I_0 is the integrated intensity, f_s is the incident optical frequency shifted up or down by the magnetic mode frequency, and 2Δ is the width of the frequency distribution at half maximum intensity.

APPLICATION TO AN INFINITE MULTILAYER: BULK-MODE FREQUENCIES

Solutions of the Landau-Lifshitz equations for the infinite multilayer problem can be constructed by means of the substitutions

$$m_Y^{n+1}=m_Y^n\exp(i\phi), \tag{24a}$$

$$m_Z^{n+1}=m_Z^n\exp(i\phi). \tag{24b}$$

In other words, one looks for solutions in which the magnetization amplitudes are the same for each film, but there is a constant phase shift ϕ from one film to the next.[3] A band of bulk-mode frequencies is generated as ϕ runs from 0 to π. Using Eqs. (24) and the expressions (10) and (11) for the dipole-dipole fields, it can be shown that the total dipole-dipole field components acting on each magnetic film are the same for each film and are given by

$$h_Y=-4\pi m_Y|q|d\alpha-4\pi m_Z q d\beta, \tag{25a}$$

$$h_Z=-4\pi m_Z+4\pi m_Z|q|d\alpha-4\pi m_Y q d\beta, \tag{25b}$$

where

$$\alpha=\frac{\exp(2|q|D)-1}{\exp(2|q|D)-2\cos(\phi)\exp(|q|D)+1}, \tag{26a}$$

$$\beta=\frac{2\exp(|q|D)\sin(\phi)}{\exp(2|q|D)-2\cos(\phi)\exp(|q|D)+1}. \tag{26b}$$

D is the periodic repeat distance $(D_{nm}+2d)$. The problem has been reduced to that of solving the Landau-Lifshitz equations for a single film because the effective fields acting on each film are the same. The required frequencies can be obtained from

$$\left(\frac{\omega}{\gamma}\right)^2 = H_{\text{eff}}B_{\text{eff}} - (4\pi M_s q d\beta)^2, \tag{27}$$

where

$$H_{\text{eff}} = H + \frac{J}{dM_s}(1-\cos\phi) + 4\pi M_s|q|d\alpha, \tag{28a}$$

$$B_{\text{eff}} = H + 4\pi M_s - \frac{2K_s}{dM_s} + \frac{J}{dM_s}(1-\cos\phi) - 4\pi M_s|q|d\alpha. \tag{28b}$$

Equation (27) can be compared with the exact expression derived by Camley, Rahman, and Mills,[3] their Eq. (2.25), if J and K_s are set equal to zero. It can be seen that the two expressions bear no resemblance to one another: nor does Eq. (27) resemble their Eq. (2.28), the $|q|d \ll 1$, $|q|D \ll 1$ limit of their exact expression (2.25).[18] Nevertheless, Eq. (27) yields frequencies that are in agreement with their exact expression to within 2% for $q = 1.727\times 10^5\ \text{cm}^{-1}$, for $H = 1$ kOe, and for magnetic film thicknesses up to 50 Å. The agreement improves for larger applied magnetic fields. One can conclude that the approximate treatment of the dipole-dipole fields contained in Eqs. (10) and (11) is adequate to describe the dipole-dipole interaction between thin magnetic films.

APPLICATION TO AN INFINITE MULTILAYER: SURFACE MODE

The approximate treatment of the dipole-dipole interaction between magnetic layers in a multilayer can also be used to provide a relatively simple description of the surface mode in a multilayer containing an infinite number of films. At first sight it appears that the dipole fields acting on a surface film, or on a near-surface film, must be quite different from those acting on a film deep within the multilayer stack because the films near the surface are not subject to dipole-dipole fields from the missing films on the left of the surface. However, an examination of Eqs. (11a) and (11b) shows that if, for $q > 0$, one sets

$$m_Z = +im_Y, \tag{29}$$

then this film will generate no external magnetic field on its right-hand side;[19] it will, however, generate an external field on its left-hand side. Thus, if Eq. (29) is satisfied for every magnetic film in the multilayer stack, a film near the surface will experience the same dipole field due to the infinite number of neighbors on its right-hand side as will a film buried deep in the stack. In effect, the surface discontinuity will have been eliminated. Notice that this cancellation of the surface discontinuity does not occur for $q < 0$: this asymmetry gives rise to the well-known fact that the surface mode is nonreciprocal.[3,20] One looks for a surface-mode solution in which the magnetizations in each film precess in phase, but for which the amplitude decays towards the interior by $\exp(-\Delta)$ from one film to the next. The dipole-dipole field components become

$$h_Y = -4\pi m_Y q d S, \tag{30a}$$

$$h_Z = -4\pi m_Z + 4\pi m_Z q d S, \tag{30b}$$

where

$$S = \frac{1}{\tanh([\Delta + |q|D]/2)}. \tag{31}$$

The Landau-Lifshitz equations are the same for every film in the stack if one neglects the fact that the exchange torque on the surface film is different from that on the other films because of the absence of a magnetic film on its left. Trying to take this surface discontinuity into account would introduce a great deal of complication into an otherwise simple problem. In most systems of interest the interlayer exchange interaction is sufficiently weak that the frequency shift caused by the surface exchange discontinuity can be safely ignored. With this approximation the Landau-Lifshitz equations become

$$\left(\frac{i\omega}{\gamma}\right)m_Y = \left[H + 4\pi M_s - \frac{2K_s}{dM_s} + \frac{J}{dM_s}(1-\cosh\Delta) - 4\pi M_s q d S\right]m_Z, \tag{32a}$$

$$\left(\frac{i\omega}{\gamma}\right)m_Z = -\left[H + \frac{J}{dM_s}(1-\cosh\Delta) + 4\pi M_s q d S\right]m_Y. \tag{32b}$$

These two equations must be compatible with the condition $m_z = im_y$. This requirement fixes the allowed value of the parameter S and, therefore, through Eq. (31) determines the parameter Δ which describes the amplitude decay towards the interior of the multilayer stack. From the required compatibility between Eqs. (32a) and (32b), one obtains

$$8\pi M_s q d S = 4\pi M_s - \frac{2K_s}{dM_s} = 4\pi M_{\text{eff}}. \tag{33}$$

Thus, from Eq. (31), the decay parameter Δ is to be determined from

$$\tanh\left(\frac{\Delta + qD}{2}\right) = 2qd\left(\frac{M_s}{M_{\text{eff}}}\right), \tag{34}$$

and the surface-mode frequency is given by

$$\left(\frac{\omega}{\gamma}\right) = H + 2\pi M_{\text{eff}} + \frac{J}{dM_s}(1-\cosh\Delta). \tag{35}$$

This is a remarkably simple result, as has been pointed out by Camley, Rahman, and Mills[3] who derived the same expression for the case $J=0$, $K_s=0$.

Equation (34) can be solved for all $K_s < 0$ for which M_{eff} is greater than M_s. For positive values of K_s one has $M_{\text{eff}} < M_s$, and therefore the ratio of (M_s/M_{eff}) cannot exceed

$1/2qd$ because the tanh function cannot exceed 1. This implies that the assumed ground state of the system in which all static magnetization vectors are parallel cannot be stable as $4\pi M_{\mathrm{eff}} \rightarrow 0$. For $K_s = 0$ and $qd \ll 1$, Eq. (34) becomes

$$\Delta = q(4d - D), \tag{36}$$

and no solution corresponding to $\Delta > 0$ exists if D becomes larger than $4d$. Here D is the periodic repeat distance and $2d$ is the thickness of a magnetic film; therefore, in this limit a surface mode can only exist if the thickness of the nonmagnetic films is less than the thickness of the magnetic films: this feature has been discussed by Camley, Rahman, and Mills.[3] This condition is relaxed for a positive-surface-energy parameter K_s, as has been pointed out by Stamps and Hillebrands.[7]

The intensity of the light scattered from the surface mode has been calculated as a function of nonmagnetic layer thickness for three values of the surface pinning parameter K_s. The results of the calculation are shown in Fig. 3 for a multilayer composed of 10-Å-thick iron films separated by silver films and for an applied field of 1 kOe and for 5145-Å laser light incident at 45° ($q = 1.727 \times 10^5\ \mathrm{cm}^{-1}$). The interlayer exchange coupling parameter J has been set equal to zero. The intensity of the light scattered from the surface mode has been normalized relative to the light intensity scattered from a single 10-Å-thick iron film mounted on a silver substrate and covered by a 10-Å-thick silver layer. It can be seen from Fig. 3 that the scattered light intensity depends strongly on both the nonmagnetic layer thickness and upon the strength of the surface energy parameter. For the case $K_s = 0.5\ \mathrm{ergs/cm}^2$, a surface energy appropriate for the iron/silver system,[8] the BLS intensity in the limit of very thin nonmagnetic layers becomes approximately 14 times larger than that calculated for a single iron film. However, for increasing nonmagnetic layer thickness the intensity falls off and becomes zero at 36 Å. The decrease in intensity with increasing nonmagnetic film thickness, D_{nm}, occurs primarily as a result of the decrease in the value of the decay parameter Δ with increasing D_{nm}. As Δ becomes smaller, the mode energy, which must remain equal to kT, becomes distributed over an ever larger number of magnetic thin films, with the result that the mode amplitudes must decrease with increasing D_{nm}. Eventually, the decay parameter becomes zero, and at that point the surface mode has been transformed into a volume mode and the light scattering amplitude has become almost zero. For example, in the present case in which the magnetic film thickness is 10 Å one finds $\Delta = 0.0614$ for $K_s = 0.5\ \mathrm{ergs/cm}^2$ and for $D_{nm} = 1$ Å. This means that the precessional amplitude in the 16th film has fallen to e^{-1} of the surface film amplitude so that the mode energy is distributed in effect over something like 32 films. On the other hand, for $D_{nm} = 10$ Å the decay factor decreases to $\Delta = 0.0458$, corresponding to an amplitude decrease of e^{-1} at the 22nd film so that the mode energy is then distributed over approximately 44 magnetic films with a corresponding decrease in the mode amplitude and a decrease in BLS intensity of 3.6. Note that the frequency of the surface mode is independent of the nonmagnetic layer thickness so that the effect of increasing D_{nm} is a reduction in BLS scattering intensity with no shift in frequency.

The result of decreasing the surface energy parameter is to reduce the BLS intensity for fixed nonmagnetic layer thickness and to decrease the value of D_{nm} corresponding to the transition from a surface mode to a bulk mode. This behavior is illustrated in Fig. 3 for $K_s = 0$ and $-0.5\ \mathrm{ergs/cm}^2$.

APPLICATION TO FINITE MULTILAYERS: $K_s = J = 0$

Consider a multilayer having the structure shown in Fig. 1 and composed of M magnetic films. The M modes of such a system can be visualized by plotting the magnetization components m_y and m_z in each film as a function of film number: No. 1 is the magnetic film nearest the vacuum interface and No. M is the film nearest the substrate. Examples of such mode diagrams are shown in Fig. 4 using the example of 25 iron films, each 5 Å thick, separated by 15-Å-thick silver films and subject to an applied magnetic field of 1.0 kOe. The m_z components are $\pi/2$ out of phase with the m_y components so that the magnetization in any given film describes an elliptical precession around the direction of the applied magnetic field. The highest frequency mode, Fig. 4(a), corresponds to the case in which all 25 film magnetizations precess in phase. This is the uniform mode, and the frequency is relatively large because the dipole-dipole contribution to the effective field acting on each film is a maximum. The next-highest frequency corresponds to the configuration shown in Fig. 4(b). As can be seen from the figure, there is a reversal in the phase of the magnetization precession from the first film to the last film. The frequency for this mode is less than the uniform mode frequency because the dipole-dipole field contributions at a particular film tend to cancel when summed over all films in the stack. The number of phase reversals in a given mode increases with mode number until finally the phase changes by π from one film to the next as is illustrated in Fig. 4(c). In this extreme case, the π mode, the magnetizations in neighboring films oscillate 180° out of phase so that the dipole-dipole field contributions from all the neighbors essentially sum to zero. In consequence, the π mode shown in Fig. 4(c) corresponds to the lowest frequency of all the multilayer modes. One feature of the multilayer modes is worthy of particular notice. The lowest-order mode, the uniform mode of Fig. 4(a), displays maximum amplitudes in the outermost films $N = 1$ and $N = 25$. As the mode number increases, the amplitudes of the two outermost films gradually decrease relative to the maximum amplitude of the interior films until for the π mode the outermost two films have almost zero amplitude as shown in Fig. 4(c). If one were dealing with the modes of a single thick uniform slab, one would describe the uniform mode as being unpinned and the π mode as being pinned even though there are no surface pinning energies because $K_s = 0$.

The modes shown in Fig. 4 have been calculated for the particular case $M = 25$, but the patterns illustrated are similar for any number of films. In all instances the high-frequency mode corresponds to the case in which all film magnetizations oscillate in phase and the lowest-frequency mode corresponds to the case in which successive magnetizations os-

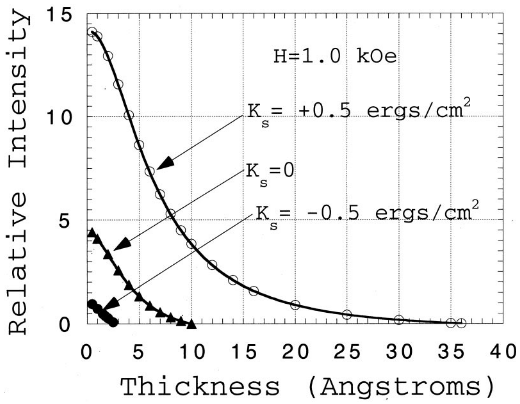

FIG. 3. Relative BLS intensity vs nonmagnetic layer thickness for an infinite stack of iron films 10 Å thick separated by silver layers and for an applied magnetic field of 1.0 kOe. BLS intensities were calculated relative to the BLS intensity calculated for a single 10-Å-thick iron film covered by a 10-Å-thick silver film and mounted on an infinitely thick silver substrate. Parameters used in the calculation were $4\pi M_s = 21.0$ kG, $g = 2.09$, interlayer coupling parameter $J = 0$, $q = 1.727\times 10^5$ cm^{-1}, $\varepsilon_1 = -10.7 + 0.33i$ [silver (Ref. 21)], $\varepsilon_2 = -0.4 + 16.4i$ [iron (Ref. 22)]. ○ $K_s = 0.5$ ergs/cm^2; ▲ $K_s = 0$; ● $K_s = -0.5$ ergs/cm^2.

cillate 180° out of phase. Of course, mode frequencies explicitly depend on the number of magnetic films in the multilayer as well as upon their thicknesses, the thicknesses of the spacer layers, and the strength of the applied magnetic field.

The frequency distribution of the intensity of the light backscattered from a finite multilayer stack depends upon the total stack thickness, *MD*, relative to the characteristic amplitude decay length of the light in the multilayer structure; here, *M* is the number of films in the stack and *D* is the thickness of the repeating unit composed of one magnetic film plus one nonmagnetic film (see Fig. 1). The calculated amplitude of the driving optical electric field in each magnetic film falls off nearly exponentially with distance in a very thick multilayer stack. The characteristic distance *L* corresponding to a decrease of $1/e$ in amplitude is insensitive to the optical properties of the nonmagnetic spacer layers in a multilayer stack composed of iron films and depends only weakly on the ratio of iron thickness $2d$ to the multilayer repeat distance *D* for *d*, *D* both very small compared with the optical skin depths in the iron and spacer materials. For example, if $D = 4d$, the decay length for silver spacers [dielectric constant $\varepsilon = -10.7 + 0.33i$ (Ref. 21)] is $L = 289$ Å, for copper spacers [dielectric constant $\varepsilon = -5.43 + 6.19i$ (Ref. 21)] is $L = 296$ Å, and for vacuum spacers ($\varepsilon = 1$) is $L = 470$ Å, all for 5145-Å incident radiation. For the ratio $D/d = 16$ the decay lengths become $L = 284$ Å for silver, $L = 283$ Å for copper, and $L = 309$ Å for vacuum spacers. Thus, for a typical multilayer based on iron films, the optical decay length is approximately 300 Å. It turns out that for a multilayer stack having a total thickness comparable to *L*, or less than *L*, the calculated BLS spectrum is very simple. For a total stack thickness comparable to *L*, the spectrum consists of two upshifted lines and two downshifted lines. The upshifted and downshifted lines have similar intensities; this is illustrated in Fig. 5 for a stack of 25 iron films, each 5 Å thick, separated by 15-Å-thick silver films. The dielectric constants used for the calculation were the following: silver, $\varepsilon_1 = -10.7 + 0.33i$ (Ref. 21); iron, $\varepsilon_2 = -0.4 + 16.4i$ (Ref. 22); silicon substrate, $\varepsilon_s = 18.05 + 0.55i$ (Ref. 23). The total multilayer thickness in this case is 500 Å. As the total multilayer thickness decreases, the intensities of the downshifted BLS peaks diminish relative to the upshifted peaks and the intensity of the lower-frequency upshifted peak diminishes relative to the intensity of the higher-frequency upshifted peak. Eventually, when the total multilayer stack width has become much smaller than the optical decay length *L*, the BLS intensity pattern consists of a single upshifted peak and a very much weaker downshifted peak. For example, for a multilayer consisting of 25 iron films 1 Å thick interleaved with 1-Å silver films, a total thickness of 50 Å, one obtains a single strong upshifted BLS peak and a downshifted peak having an intensity 44 times weaker. This tendency towards a simplified BLS pattern as the total multilayer width becomes very small compared with the optical decay length *L* can be understood as a consequence of the fact that the driving optical electric-field amplitude tends to become more nearly the same within each magnetic film as the multilayer width becomes smaller. With reference to Fig. 4, it is apparent that the uniform mode of Fig. 4(a) must give a very strong signal if the optical electric field is the same in each film because the scattered light from each film will add in phase. On the other hand, higher-order modes containing phase reversals will tend to give very small BLS signals because the light scattered from a film having a particular phase will be canceled by light scattered from a second film having a phase shifted by 180°. This is the analog of the well-known fact that the BLS signal corresponding to the optical mode (the out-of-phase mode) for a pair of identical, very thin, coupled ferromagnetic films has virtually zero intensity.[16]

The BLS spectrum becomes more complex as the total multilayer thickness exceeds the optical decay length *L*. One obtains spectra containing more and more distinct peaks as the multilayer thickness increases. Moreover, many low-frequency modes are squeezed together to form a broad peak analagous to the spin-wave manifold peak obtained for an infinitely thick bulk magnetic material.[24] This thick-stack limit will not be discussed further.

A series of BLS backscattering calculations have been carried out for iron film thicknesses ranging from 1 to 25 Å and for multilayers containing 5, 10, and 25 magnetic films. The optical properties of silver and copper at 5145 Å have been used for the nonmagnetic films. On the basis of these computer experiments, it can be concluded that the intensity of the upshifted peak corresponding to the uniform mode, the maximum intensity peak, is not very dependent on the magnetic film thickness $2d$, on the repeat period *D*, or on the number of magnetic films in the multilayer, *M*, provided that the total thickness of the multilayer is less than, or comparable to, the optical decay length *L*. For example, for the case of 25 magnetic films and a repeat period $D = 15$ Å a variation of *d* from 1.5 to 7 Å resulted in a twofold increase in the maximum BLS signal strength. For a given thickness $2d$ the

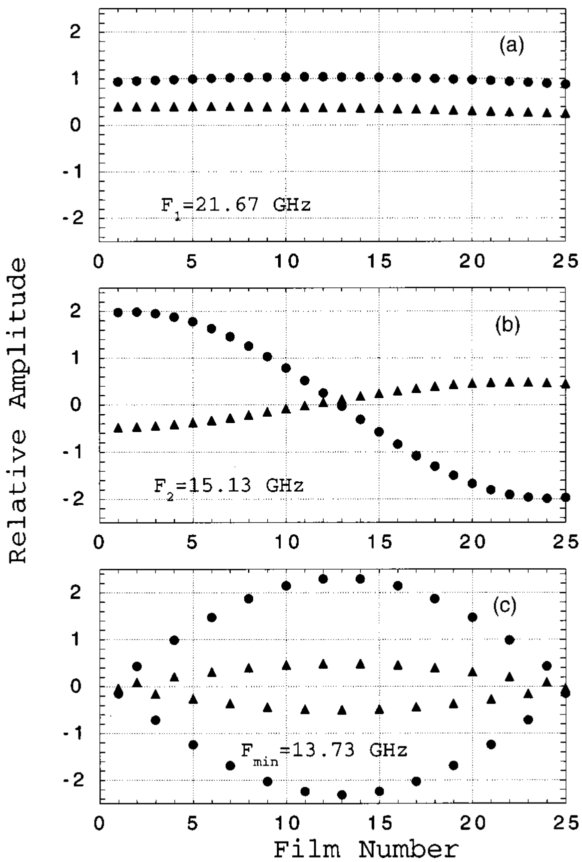

FIG. 4. Magnetization amplitudes vs film number calculated for a multilayer stack consisting of 25 iron films, each 5 Å thick, separated by 15-Å-thick silver films and for an applied magnetic field of 1.0 kOe. ● m_y; ▲ m_z. Note that m_y and m_z are $\pi/2$ out of phase. Mode amplitudes correspond to the same excitation energy in each mode. Magnetic parameters used were $4\pi M_s = 21.0$ kG, $g = 2.09$, $J = K_s = 0$. (a) The mode corresponding to the largest frequency and to the strongest BLS signal. (b) The mode corresponding to the second largest frequency and to the second largest BLS signal. (c) The π mode corresponding to the smallest frequency and to the smallest BLS signal.

maximum BLS signal strength remained the same within 25% for $M = 5$, 10, and 25.

The maximum BLS signal strength calculated for iron/silver and iron/copper multilayers was observed to be within a factor of 2 or 3 the same as that calculated for a single iron film of thickness $2d$ mounted on a thick silver or copper substrate. This is illustrated in Fig. 5 for $d = 2.5$ Å, $D = 20$ Å, and $M = 25$. The BLS intensity has been normalized relative to a single 5-Å iron film mounted on a silver substrate and covered by a 15-Å silver layer. For the extreme cases mentioned in the paragraph above, one finds that (1) for $M = 25$, $d = 1.5$ Å, and $D = 15$ Å the maximum BLS signal for the multilayer was 2.4 times that for the single 3-Å-thick iron film; (2) for $M = 25$, $d = 7$ Å, and $D = 15$ Å the maximum BLS signal was the same for the multilayer and the single 14-Å-thick iron film. Evidently, the complex factors that influence the strength of the light scattering tend to cancel so that the multilayer signal exhibits roughly the same strength as the signal calculated for a single film having the same thickness as each magnetic film in the multilayer.

The pattern of the BLS spectrum shown in Fig. 5 persists to high applied magnetic fields (see Fig. 6) calculated for a stack of 25 iron films 5 Å thick and separated by 15-Å-thick silver films. The variation of the largest frequency with field is very nearly linear. This is reminiscent of the surface-mode behavior expected for an infinite multilayer [see Eq. (35)] or for a bulk magnetic specimen. However, the slope of the straight line shown in Fig. 6 corresponds to a value $g = 2.727$ rather than to the value $g = 2.09$ measured for bulk iron.[25]

EFFECT OF A UNIAXIAL NORMAL ANISOTROPY AND INTERLAYER EXCHANGE COUPLING

The presence of a surface energy, Eq. (1), causes a shift in the mode frequencies because the magnetization enters the Landau-Lifshitz equations through the effective magnetization

$$4\pi M_{\mathrm{eff}} = 4\pi M_s - \left(\frac{2K_s}{dM_s}\right). \tag{37}$$

An increase in the surface energy K_s results in a reduction of the effective magnetization. As the effective magnetization becomes small, the mode frequencies become smaller and the spread between the maximum and minimum mode frequencies becomes smaller. The simple relation between mode number and frequency characteristic of the $K_s = 0$ case becomes more complex. For $K_s = 0$ the uniform mode corresponds to the maximum frequency, and the π mode, having a phase shift of π from one film to the next, corresponds to the minimum frequency. This distribution of mode frequencies is no longer valid for negative effective magnetizations: for $4\pi M_{\mathrm{eff}} < 0$ the uniform mode becomes the minimum frequency and the π mode becomes the maximum frequency. This inversion of the mode frequency distribution has been discussed by Stamps and Hillebrands.[7] However, if the total multilayer thickness is less than the optical decay length L, the upshifted and downshifted BLS spectra simply consists of one or two lines. Typical behavior is illustrated in Fig. 7 for a 25-magnetic-film multilayer composed of 5-Å-thick iron films separated by 15-Å-thick silver films and subjected to an applied field of 6.0 kOe. The interlayer exchange coupling was assumed to be zero. As the surface energy parameter increases at fixed applied field, the frequencies of the two upshifted BLS peaks decrease and converge to the same value for K_s near the value corresponding to zero effective magnetization. (The downshifted peaks exhibit a similar behavior.) As K_s is further increased, the effective magnetization becomes negative, and ultimately the ground state corresponding to all static magnetizations parallel becomes unstable and some, or all, of the static magnetizations tilt away from the direction of the applied field.[7] In the example shown in Fig. 7 in which the applied field is 6 kOe, the parallel ground state becomes unstable for a value of K_s between 0.5 and 0.525 ergs/cm^2.

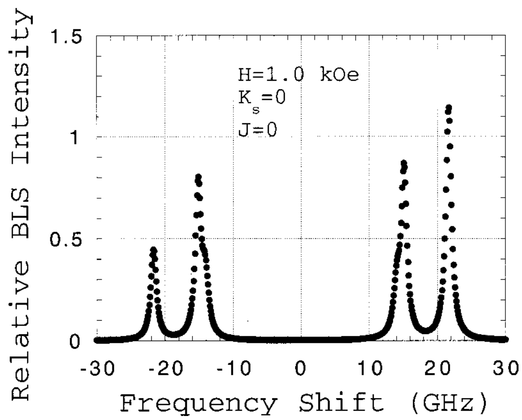

FIG. 5. Relative BLS intensity vs frequency shift calculated for a 25-iron-film multilayer in an applied field of 1.0 kOe. Each iron film was 5 Å thick and was separated from its neighbors by 15-Å-thick silver films. BLS intensities were calculated relative to that for a single 5-Å-thick iron film covered by a 15-Å-thick silver film and mounted on an infinitely thick silver substrate. Parameters used in the calculation were $4\pi M_s = 21.0$ kG, $g = 2.09$, $J = K_s = 0$, $q = 1.727\times 10^5\ \mathrm{cm}^{-1}$, $\varepsilon_1 = -10.7 + 0.33i$ [silver (Ref. 21)], $\varepsilon_2 = -0.4 + 16.4i$ [iron (Ref. 22)], and $\varepsilon_s = 18.05 + 0.55i$ [silicon (Ref. 23)]. An instrumental line broadening of 0.5 GHz has been assumed [see Eq. (23)].

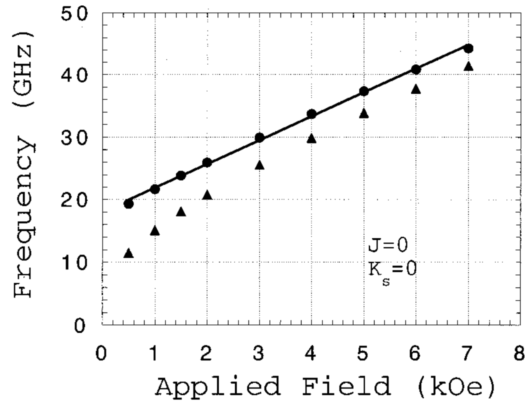

FIG. 6. Frequency shift vs applied magnetic field calculated for a multilayer composed of 25 iron films, each 5 Å thick, separated by 15-Å-thick silver films. The upshifted and downshifted BLS peaks correspond to the same frequency shifts. The magnetic parameters used were $4\pi M_s = 21.0$ kG, $g = 2.09$, $J = K_s = 0$. The solid line is a least-squares fit to the high-frequency points: its equation is $F = 18.03 + 3.817H$.

As the surface energy parameter increases, the BLS peak intensities become stronger, and for $K_s = 0.5\ \mathrm{ergs/cm}^2$ and for an applied field of 6 kOe, they are approximately 10 times as strong as for the case $K_s = 0$. As K_s increases from zero, the mode corresponding to the maximum BLS intensity becomes more like a surface mode in that the magnetization precession in each film becomes more nearly circularly polarized: i.e., $|m_z|$ approaches $|m_y|$, and the amplitudes m_y and m_z decay away from the surface. For K_s near the value where the two BLS peaks converge, $K_s = 0.4\ \mathrm{ergs/cm}^2$ in Fig. 7, all the modes become circularly polarized or very nearly circularly polarized. When K_s is increased to values greater than the crossover value, the mode corresponding to maximum BLS scattering approaches a 1/4 wavelength with a node at the surface, the mode is no longer circularly polarized, and its frequency becomes the minimum frequency. At the same time, the rapidly oscillating mode characterized by a phase shift of π from one film to the next becomes the maximum frequency. This inversion of the mode frequencies for large values of the pinning parameter has been described by Stamps and Hillebrands.[7] These authors have also discussed the effect of interlayer exchange coupling on the mode spectrum. The frequency dependence of the modes becomes very complex in the presence of ferromagnetic interlayer coupling. Despite that complexity, the BLS spectrum remains very simple; one observes either one or two intense upshifted peaks depending upon the value of K_s and the total multilayer thickness. The frequencies and intensities of those BLS peaks are very insensitive to the strength of the exchange coupling. For the example of Fig. 7, for an applied field of 6 kOe and $K_s = 0$ the shift in the high-frequency peak is less than 0.1 GHz as the exchange coupling parameter, J in Eq. (3), is increased from 0 to 0.5 $\mathrm{ergs/cm}^2$. For the same parameters the shift in the lower-frequency peak is approximately 0.3 GHz. These shifts are not sensitive to the values of the applied field or to the value of K_s. The frequencies of the BLS peaks are insensitive to the strength of the exchange coupling because the most intense light scattering is associated with those modes for which the amplitude of precession is very nearly the same for neighboring films, and therefore the effective interlayer exchange fields are small.

The above discussion has been concerned with the particular example of a stack of 25 iron films, each 5 Å thick and separated by 15-Å-thick silver films. Calculations carried out for systems composed of 5 and 10 magnetic films, for film thicknesses up to 50 Å and for multilayer thicknesses less than, or comparable to, the optical decay length L, have given similar results. The quantitative details of the variation of BLS frequencies and intensities with applied magnetic-field strength and with the strength of the surface energy parameter depend upon the magnetic parameters of the films, upon their thicknesses, and upon the thicknesses of the nonmagnetic films. However, in all cases examined the BLS spectra exhibited the following features

(1) Either one or two intense BLS frequencies depending upon the number of magnetic films, the total multilayer thickness, and the strength of the surface energy parameter K_s.

(2) The frequencies of the upshifted and downshifted lines are the same for $K_s = 0$. The intensities of the upshifted BLS peaks may be considerably stronger than the intensities of the downshifted peaks, especially for small applied fields and for a small total multilayer thickness.

(3) The intensities of the BLS peaks are comparable to the intensity of the BLS peaks calculated for a single magnetic film having the same thickness and magnetic properties as the films in the multilayer.

(4) The frequencies calculated for the intense BLS peaks are insensitive to the presence of interlayer exchange coupling.

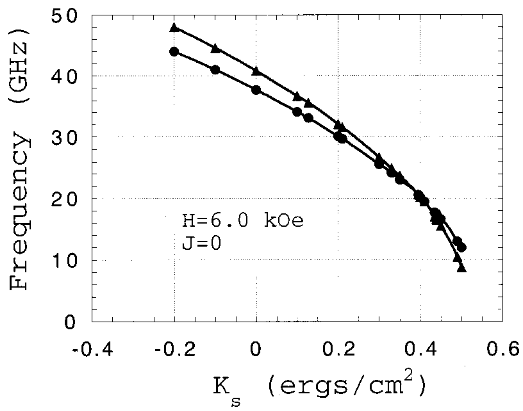

FIG. 7. Calculated BLS upshifted frequencies vs surface energy parameter K_s for a multilayer composed of 25 iron films 5 Å thick and separated by 15-Å-thick silver films in an applied magnetic field of 6.0 kOe. Parameters used were $4\pi M_s = 21.0$ kG, $g = 2.09$, and exchange coupling parameter $J = 0$. The two BLS peaks overlap at $K_s = 0.4$ ergs/cm^2 corresponding to $4\pi M_{\mathrm{eff}} = 1.85$ kG.

CONCLUSIONS

An approximate method has been used to study the Brillouin scattered light spectrum to be expected from multilayers consisting of a periodic repetition of a unit consisting of a thin magnetic layer, 50 Å or less thick, and a comparably thin nonmagnetic layer. In particular, the dipole-dipole interaction between magnetic films has been treated using an extension of the methods described by Cochran *et al.*[9] used to treat pairs of interacting thin films. The most interesting conclusion drawn from the present study is that the scattered light spectrum for multilayers having a total thickness less than the optical decay length L is very simple in that only two of the many modes associated with the periodic multilayer structure give rise to an appreciable BLS signal. The weakness of the BLS signal for the higher-order modes is analogous to the case of a bilayer structure consisting of two identical thin magnetic films. Such films support an acoustic mode in which the magnetizations in the two films precess in phase and an optic mode in which the two magnetizations precess in antiphase. The acoustic mode gives rise to a strong BLS signal, whereas the optic mode yields practically no scattered light because the contributions from each film are in antiphase and cancel. Similarly, the light scattered by the uniform mode in a multilayer composed of identical magnetic thin films is strong and the BLS intensities corresponding to the higher-order modes become progressively weaker due to an increasing number of amplitude phase reversals across the multilayer stack. The frequencies associated with strong BLS signals are insensitive to the exchange coupling between the magnetic films because the magnetization vectors in neighboring films remain very nearly parallel.

It should be noted that the treatment of the dipole-dipole interaction described above and encapsulated in Eqs. (10) and (11) is not restricted to periodic multilayers. It can be easily applied to any planar multilayer system including nonperiodic systems in which the magnetic layers are composed of different materials, have different thicknesses, or are spaced apart by nonmagnetic films having variable thicknesses.

ACKNOWLEDGMENT

The author would like to thank Dr. Milton From for making available the results of his BLS experiments on iron/silver and on iron/copper multilayers. His work stimulated the present study of light scattering by periodic multilayer structures.

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