Kondo effect in a host with fractional statistics: Absence of Kondo logarithms

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By constructing the reflection Dunkl operator we derive several integrable models consisting of a boundary impurity coupled to an electron gas with interactions of the Calogero-Sutherland type. Some of these models were constucted previously using Lax-pair operators. The necessary condition of integrability imposes that the impurity potential has a form similar to that of the bulk interactions. Based on these results we conjecture that a Kondo impurity coupled to the host with long-range interactions of the $1/r^2$ type is also integrable. Using the asymptotic Bethe *Ansatz* we show that there are no Kondo logarithms, and depending on the coupling of the impurity to the host, the impurity spin can either be totally screened, partially screened, or unscreened. On the other hand, for a $1/\sinh^2(r)$ interaction potential a Kondo effect with logarithms is obtained.

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I. INTRODUCTION

In correlated electron metals the screening of the Coulomb potential usually leads to short- or intermediate-range interactions. In low-carrier-density systems, e.g., close to a metal-insulator transition and especially in low dimensions, however, the small number of carriers and their reduced mobility do not provide an effective mechanism for screening, and long-range interactions should be considered. Of special interest is a class of integrable one-dimensional (1D) systems with interactions decreasing with distance r as r^{-2}, $\sin^{-2}(r)$, and $\sinh^{-2}(r)$, frequently referred to as Calogero-Sutherland models[1,2] (CSM's) to honor the pioneering work of these authors. In this paper we present results for the open-boundary CSM and impurities at the boundary. In particular, we explore the implications of long-range interactions on the Kondo effect.

Besides for low carrier-density systems (e.g., underdoped cuprates) the CSM is relevant to fractional statistics and anyons,[3] and spin chain (Haldane-Shastry) models.[4] Numerous common features between the CSM in 1D and the edge states of the fractional quantum Hall effect (FQHE) are known.[3] For instance, in both cases the ground state is a Jastrow-Slater wave function and the excited states are constructed by multiplying polynomials to the ground-state wave function.[2,4,5] The CSM and its generalizations[6–8] have been extensively studied with various methods, in particular, with periodic boundary conditions via the asymptotic Bethe ansatz (ABA).[2,9–11]

The application of the ABA requires an independent proof of integrability. If a model is integrable it suffices to know the asymptotic behavior of the wave functions at long distances, i.e., the phase shifts, which can be obtained without the full knowledge of the many-particle wave functions. The ABA only requires the two-particle phase shifts to classify the states and determine the energy eigenvalues. The exact solution is then valid for any finite density of carriers. An elegant and compact method to construct integrable models is via Dunkl operators.[12,13]

Impurities and boundary effects play a relevant and very similar role in 1D systems. The strong-coupling fixed point of impurity models frequently renormalizes to an equivalent open-boundary impurity problem.[14] Boundary potentials and a Kondo impurity placed at the open end usually lead to similar solutions. This is also to be expected for models with long-range potentials and could be of relevance for the edge states of FQHE. Open boundaries have been studied previously in the context of BC_N-type CSM (Refs. 15–18) and the open Haldane-Shastry spin chain.[19] The quasiparticles in the CSM with r^{-2} and $\sin^{-2}(r)$ potentials obey ideal fractional statistics, so that in this unusual host the Kondo impurity is expected to behave differently from the usual Kondo effect in simple metals[20,21] or in a Luttinger liquid.[22,23] We find that although there is Kondo screening, there are no logarithmic precursors characteristic of asymptotic freedom. On the other hand, for the CSM with $\sinh^{-2}(r)$ potential (which has finite range) we obtain screening with Kondo logarithms.

The structure of the present paper is the following. In Sec. II we rederive the open-boundary CSM with boundary fields for all three cases, namely, the r^{-2}, $\sin^{-2}(r)$, and $\sinh^{-2}(r)$ potentials using Dunkl operators. Conjecturing integrability we present in Sec. III the exact solution of a magnetic-impurity model showing the Kondo effect in a system with an ideal fractional statistics. In Sec. IV we extend our study to a Kondo model with hyperbolic interaction, for which we find that the Kondo effect is similar to that of other short-range interaction models. Finally, concluding remarks follow in Sec. V.

II. NONMAGNETIC-IMPURITY MODELS

In this section, we study a boundary impurity in the SU(2)-invariant CSM. Since the interaction in the CSM decreases with distance as $1/r^2$, it is necessary for the integrability to consider an electron-impurity interaction proportional to $1/r^2$. Such an impurity potential induces a natural open-boundary confinement without further assumption on a strong-coupling fixed point.[24] We note that the open-boundary CSM with multicomponents has been studied via the Lax-pair operator procedure and transfer-matrix formalism by Yamamoto[18] and Hikami,[17] respectively. Here we rederive the models within a slightly different approach, namely, using Dunkl operators. The Dunkl operator for the trigonometric case has previously been obtained by Hikami by means of a transfer-matrix expansion.

For a 1D system of N electrons and an impurity at the boundary, we define the Dunkl operator[12,13] with boundary reflection,

$$D_j = p_j + i \sum_{l, l\neq j}^{N} (v_{jl} M_{jl} + \overline{v}_{jl} \overline{M}_{jl}) + i u_j M_j, \tag{1}$$

where $v_{jl} = v(x_j - x_l)$, $\overline{v}_{jl} = \overline{v}(x_j + x_l)$, and $u_j = u(x_j)$ are yet undetermined functions, and x_j and p_j are the coordinate and momentum of the jth electron. Here M_{jl} and M_j are the exchange and reflection operators,[13,19]

$$M_{jl} = M_{lj} = M_{jl}^{\dagger}, \quad M_{jl}^2 = 1, \quad M_{jl} A_l = A_j M_{jl},$$
$$M_{jl} A_k = A_k M_{jl} \quad \text{for} \quad k \neq j, l, \tag{2}$$

where A_l is any operator, and

$$M_j x_j = -x_j M_j, \quad M_j p_j = -p_j M_j,$$
$$[M_i, M_j] = 0, \quad \overline{M}_{jl} = M_j M_l M_{jl}. \tag{3}$$

We seek the solutions of $[D_j, D_l]\psi = 0$, where ψ is any antisymmetrized wave function, to define a class of mutually commutative quantities $I_n = \Sigma_{j=1}^{N} (D_j)^n$. If one of the I_n is chosen as the Hamiltonian, then the model is integrable. The commutator of the Dunkl operators is

$$[D_j, D_l] = U_{jl} - \sum_{k, k\neq j,l}^{N} W_{jlk}, \tag{4}$$

where

$$U_{jl} = [(u_j - u_l) v_{jl} + (u_j + u_l) \overline{v}_{jl}](M_{jl} M_j - M_j M_{jl}),$$
$$\begin{aligned} W_{jlk} = {} & (v_{jl} v_{jk} + v_{lk} v_{lj} + v_{kj} v_{kl})(M_{jl} M_{lk} - M_{lk} M_{jl}) \\ & + (v_{jk} \overline{v}_{lj} + \overline{v}_{lj} \overline{v}_{lk} - \overline{v}_{lk} v_{jk})(M_{jk} \overline{M}_{lk} - \overline{M}_{lk} M_{jk}) \\ & + (\overline{v}_{jk} \overline{v}_{lj} - v_{lk} \overline{v}_{jk} + \overline{v}_{lj} v_{lk})(\overline{M}_{jk} M_{lk} - M_{lk} \overline{M}_{jk}) \\ & + (\overline{v}_{jk} v_{lj} - \overline{v}_{lk} \overline{v}_{jk} - v_{lj} \overline{v}_{lk})(M_{jl} \overline{M}_{jk} - \overline{M}_{jk} M_{jl}). \end{aligned}$$

If we choose I_2 as the Hamiltonian we have

$$\begin{aligned} H = {} & \sum_{j=1}^{N} p_j^2 + \sum_{j\neq l} (v_{jl}^2 + \overline{v}_{jl}^2 + v'_{jl} M_{jl} + \overline{v}'_{jl} \overline{M}_{jl}) \\ & + \sum_{j=1}^{N} (u'_j M_j + u_j^2) + \frac{1}{2} \sum_{j\neq l} U_{jl} - \frac{1}{3} \sum_{j\neq l\neq k\neq j} W_{jlk}, \end{aligned} \tag{5}$$

where $u'_j = \partial_j u_j$, $v'_{jl} = \partial_j v_{jl}$, and $\overline{v}'_{jl} = \partial_j \overline{v}_{jl}$. Hence, I_2 is a natural choice for the Hamiltonian, because it leads to the usual parabolic kinetic energy.

One possible solution with $U_{jl} = W_{jlk} = 0$ is $v(x) = g x^{-1}$ and $u(x) = \nu x^{-1}$ with g and ν being real parameters. This solution yields the integrable Hamiltonian

$$\begin{aligned} H = {} & \sum_{j=1}^{N} \left[p_j^2 + \frac{\nu(\nu - M_j)}{x_j^2} \right] \\ & + 2 \sum_{l<j} \left[\frac{g(g - M_{jl})}{(x_j - x_l)^2} + \frac{g(g - \overline{M}_{jl})}{(x_j + x_l)^2} \right]. \end{aligned} \tag{6}$$

Since the Hamiltonian acts on antisymmetric wave functions, we can replace $-M_{jl}$ by the spin-exchange operator P_{jl}. In addition $[M_j, H] = 0$, i.e., the Hamiltonian is invariant under reflections. Hence, M_j can be substituted by its eigenvalues ± 1 or by σ_j^z.[19] Here $M_j = \pm 1$ corresponds to a scalar impurity potential, while $M_j = \sigma_j^z$ yields a scalar potential and a boundary magnetic field. In the SU(M) case, we can replace M_j by an $M \times M$-order matrix in spin space with eigenvalues ± 1. If all M_j take the same eigenvalues (± 1) or $M_j = \sigma_j^z$, then $-\overline{M}_{jl}$ in Eq. (6) can be replaced by P_{jl} with the identity $\sigma_j^z \sigma_l^z P_{jl} = P_{jl}$. Model (6) then describes an open Calogero model with boundary impurity (or boundary field). The $(x_j + x_l)$ terms represent a typical feature of the open-boundary system; they describe the interaction between the jth electron and the mirror image of the lth electron or vice versa. The inclusion of the image terms is just equivalent to removing the infinite wall at the boundary.

The Sutherland model and the hyperbolic CSM can be derived similarly. For the Sutherland model, we choose $v_{jl} = g[\cot(x_j - x_l) + i\,\mathrm{sgn}(j-l)]$, $\overline{v}_{jl} = g[\cot(x_j + x_l) - i]$, and $u(x) = (\nu - \delta)(\cot x - i) + 2\delta[\cot(2x) - i]$, which is a solution since M_{jl} and M_j commute with the sign function. The Hamiltonian reads

$$\begin{aligned} H = {} & -\sum_{j=1}^{N} \frac{\partial^2}{\partial x_j^2} + 2 \sum_{l<j} \frac{g(g + P_{jl})}{\sin^2(x_j - x_l)} + 2 \sum_{l<j} \frac{g(g + \overline{P}_{jl})}{\sin^2(x_j + x_l)} \\ & + \sum_{j=1}^{N} \frac{\nu(\nu - M_j)}{\sin^2 x_j} + \sum_{j=1}^{N} \frac{\delta(\delta - M_j)}{\cos^2 x_j}, \end{aligned} \tag{7}$$

where ν and δ are real constants and $\overline{P}_{jl} = M_j M_l P_{jl}$. For the spinless case, the above Hamiltonian corresponds to a CSM of the BC_N type.[15] By rescaling $x_j \to \pi x_j/(2L)$, it is seen that the last two terms describe two impurities (boundary fields) at 0 and L, respectively.

For the hyperbolic CSM, we obtain the following solution, $v_{jl} = g[\coth(x_j - x_l) - \mathrm{sgn}(j-l)]$, $\overline{v}_{jl} = g[\coth(x_j + x_l) + 1]$, $u(x) = (\nu - \delta)(\coth x + 1) + 2\delta[\coth(2x) + 1]$, and the corresponding Hamiltonian is

$$\begin{aligned} H = {} & -\sum_{j=1}^{N} \frac{\partial^2}{\partial x_j^2} + 2 \sum_{l<j} \frac{g(g + P_{jl})}{\sinh^2(x_j - x_l)} + 2 \sum_{l<j} \frac{g(g + \overline{P}_{jl})}{\sinh^2(x_j + x_l)} \\ & + \sum_{j=1}^{N} \frac{\nu(\nu - M_j)}{\sinh^2 x_j} - \sum_{j=1}^{N} \frac{\delta(\delta - M_j)}{\cosh^2 x_j}. \end{aligned} \tag{8}$$

Models (6) and (7) have been derived in Refs. 17 and 18. Note that if we put $M_j=1$ or $M_j=\sigma_j^z$, then $\overline{P}_{jl}=P_{jl}$.

We were not able to find the Dunkl operator that generates the interaction between a Kondo impurity and the CSM host. However, the similar behavior of boundary potentials and Kondo impurities at an open end in other hosts is a strong plausability argument that the CSM models with boundary impurity are also integrable.

III. KONDO EFFECT IN THE SUTHERLAND MODEL

Of great interest is the problem of an impurity carrying internal degrees of freedom. Unfortunately, we could not derive the condition of integrability for the impurity model via the Dunkl-operator procedure, but this can plausibly be inferred by the reflection Yang-Baxter relation[25] in the sense of the ABA

$$S_{jl}(k_j-k_l)R_j(k_j)S_{jl}(k_j+k_l)R_l(k_l)$$
$$=R_l(k_l)S_{jl}(k_j+k_l)R_j(k_j)S_{jl}(k_j-k_l), \qquad (9)$$

where S_{jl} and R_j are the two-body scattering and the reflection matrices, respectively. Guided by our observations for the scalar impurity we consider the following Kondo model:

$$H=-\sum_{j=1}^{N}\frac{\partial^2}{\partial x_j^2}+\sum_{j=1}^{N}\frac{V+JP_{j0}}{[(2L/\pi)\sin(\pi x_j/2L)]^2}$$
$$+2\sum_{l<j}\frac{g(g+P_{jl})}{\{(2L/\pi)\sin[(\pi/2L)(x_j-x_l)]\}^2}$$
$$+2\sum_{l<j}\frac{g(g+\overline{P}_{jl})}{\{(2L/\pi)\sin[(\pi/2L)(x_j+x_l)]\}^2}, \qquad (10)$$

where P_{j0} is the spin-exchange operator of the jth electron with the boundary impurity of spin 1/2. In fact, such an Hamiltonian, although involving only classical spins, has been derived via symmetry analysis by Polychronakos[16] and has been shown to be integrable for a special value of the boundary-coupling constant. Model (10) presents two main features, namely, (a) the Kondo coupling is long ranged in contrast to usual Kondo models, and (b) the host is rather unusual since the bulk electrons follow an ideal fractional statistics. Although the strong magnetic fields in the FQHE would quench the usual Kondo effect, the above model could be thought of as two degenerate channels of edge states in the FQHE coupled to a two-level system or quantum dot.

The scattering matrix for $g>1/2$ is[8]

$$S_{jl}(k)=-\lim_{\eta\to 0^+}\left[\frac{k-i\eta}{k+i\eta}\right]^g\frac{k-i\eta P_{jl}}{k-i\eta}. \qquad (11)$$

The following two cases satisfy Eq. (9) for model (10), namely, (i) if J and V are parametrized by a real constant $c>2$, $J=2c-1$, and $V=c^2-c+1$, then

$$R_j(k)=\lim_{\eta\to 0^+}\left[\frac{k-i\eta}{k+i\eta}\right]^c\left\{\frac{k-i\eta P_{j0}}{k-i\eta}\right\}^2, \qquad (12)$$

and (ii) if $V=J^2$ provided that $Jg>0$, then

$$R_j(k)=\lim_{\eta\to 0^+}\left[\frac{k-i\eta}{k+i\eta}\right]^J\frac{k-2i\eta P_{j0}}{k-2i\eta}. \qquad (13)$$

Below we study the low-temperature thermodynamics of these two cases we conjecture are integrable.

The periodic motion of particle j consists of its scattering, with each of the particles to the right scattering off the right boundary (impurity site), then scattering with all other electrons while it moves to the left, its reflection at the left boundary with a phase shift π, and its scattering with electrons until it reaches its original position. The transfer matrix then consists of a product of $2(N-1)$ electron-electron scattering matrices and one reflection matrix off the impurity. If the momentum of the electron is k_j and the initial wave function is ψ_0, we have

$$-e^{2ik_jL}S_{j,1}^{+}\cdots S_{j,j-1}^{+}S_{j,j+1}^{+}\cdots S_{j,N}^{+}R_j(k_j)$$
$$\times S_{j,N}^{-}\cdots S_{j,j+1}^{-}S_{j,j-1}^{-}\cdots S_{j,1}^{-}\psi_0=\psi_0, \qquad (14)$$

where $S_{j,l}^{\pm}=S_{jl}(k_j\pm k_l)$. The above $j=1,\dots,N$ eigenvalue equations are simultaneously solved by two nested Bethe *Ansätze* in terms of two sets of rapidities (the charge momenta $\{k_j\}$ and spin rapidities $\{\lambda_\alpha\}$), diagonalizing the Hamiltonian with eigenvalue $E=\Sigma_{j=1}^{N}k_j^2$.

Since the host drives the impurity and is common to both cases, (i) Eq. (12) and (ii) Eq. (13), we first discuss the ABA equations of the host without impurity,

$$e^{2ik_jL}=\prod_{r=\pm}\prod_{l\neq j}^{N}\left(\frac{k_j-rk_l+i\eta}{k_j-rk_l-i\eta}\right)^g\prod_{\alpha=1}^{M}\frac{k_j-r\lambda_\alpha-i\frac{\eta}{2}}{k_j-r\lambda_\alpha+i\frac{\eta}{2}},$$

$$\prod_{r=\pm}\prod_{j=1}^{N}\frac{\lambda_\alpha-rk_j-i\frac{\eta}{2}}{\lambda_\alpha-rk_j+i\frac{\eta}{2}}=\prod_{r=\pm}\prod_{\beta\neq\alpha}^{M}\frac{\lambda_\alpha-r\lambda_\beta-i\eta}{\lambda_\alpha-r\lambda_\beta+i\eta}. \qquad (15)$$

Here the limit $\eta\to 0$ is to be taken before the thermodynamic limit $L\to\infty$. As a consequence no charge-spin bound states can be formed, nor is there the possibility of boundary bound states induced by the impurity. The solutions of the Bethe *Ansatz* equations are then classified into real charge momenta $\{k_j\}$ and squeezed strings of magnons[26] characterized by string rapidities $\{\lambda_{n,\alpha}\}$. In the thermodynamic limit, we denote with $\rho(k)$, $\rho_h(k)$, $\sigma_n(\lambda)$, and $\sigma_{n,h}(\lambda)$ the densities of the rapidities and their holes. It is useful to introduce $\zeta(k)\equiv\rho_h(k)/\rho(k)$ and $\eta_n(\lambda)\equiv\sigma_{n,h}(\lambda)/\sigma_n(\lambda)$. The thermodynamics of the system is determined by

$$\ln\zeta=(k^2-\mu)/T+\left(g-\frac{1}{2}\right)\ln(1+\zeta^{-1})-\frac{1}{2}\ln(1+\eta_1),$$

$$\ln\eta_n^2=\ln(1+\eta_{n-1})+\ln(1+\eta_{n+1}) \qquad (16)$$

with the boundary conditions $1+\eta_0=(1+\zeta^{-1})^{-1}$ and $\lim_{n\to\infty}\ln\eta_n/n=h/T\equiv 2x_0$. Here μ is the chemical potential $[\mu=\pi^2N^2(g+1/2)^2/L^2$ at $T=0]$, h is the magnetic field, and T is the temperature. The free energy of the host is $F=-LT\int\ln[1+\zeta^{-1}(k)]dk/(2\pi)$. The bulk is a Luttinger liquid with ideal fractional statistics.[3,11]

Due to the limit $\eta\to 0$ in the ABA equations the integral kernels are all δ functions, so that Eqs. (16) are algebraic rather than integral equations. When $T\to 0$, $h\to 0$, we have $\zeta(k)\to 0$ for $k^2<\mu$ and $\zeta(k)\to\infty$ for $k^2>\mu$. Therefore, asymptotically the solutions of Eqs. (16) are $1+\eta_n=\sinh^2(nx_0)/\sinh^2(x_0)$ for $\lambda<\sqrt{\mu}$ and $1+\eta_n=\sinh^2[(n+1)x_0]/\sinh^2(x_0)$ for $\lambda>\sqrt{\mu}$.

We now study the low-temperature properties of the impurity for case (i). The free energy of the impurity is

$$
\begin{aligned}
f_{imp}&=-\frac{1}{2}T\ln[1+\eta_1(0)]+\frac{1}{2}(c-1)T\ln[1+\zeta^{-1}(0)]\\
&=\frac{1}{2}(c-1)\bar{\mu}+O(e^{-\bar{\mu}/T}),
\end{aligned}
\tag{17}
$$

where $\zeta(0)\sim\exp[-\mu/(g+1/2)T]\equiv\exp(-\bar{\mu}/T)$. The residual entropy of the impurity is zero, $S_{res}=0$, which means that the impurity spin is screened in the ground state, as for the usual single-channel Kondo problem with short-range interactions. Both the susceptibility and the heat capacity of the impurity behave like a local Fermi liquid,[20] but with the leading T dependence arising from $\mu(T)$. Hence, the screening of the spin is not associated with a Kondo energy, indicating that the long-range interactions (both in the bulk and at the boundary) play central roles. It follows from the ABA equations that the impurity contributes to the density of states with a δ-function peak at zero energy, so that the "*impurity level*" is pinned in the core of the Fermi sea, rather than at the Fermi level as for the usual Kondo problem. Therefore, the impurity is screened for $T<\mu$, the only energy scale in the system. The impurity only renormalizes the chemical potential and slightly changes the density of states at the Fermi level (both to the order of L^{-1}), so that the quasiparticles in the system are free. The impurity, after screening, acts just like a foreign particle with zero momentum and is essentially insensitive to thermal activation. Moreover, the Kondo effect occurs for arbitrarily large V/J, in contrast to hosts with short-range correlations, where a large repulsive scalar potential may suppress the Kondo effect completely.[22] On the other hand, this is similar to an impurity in a Fermi liquid,[20] where the scalar potential does not change the fixed point (although T_K is the Fermi energy). For an impurity of arbitrary spin S, P_{j0} is replaced by $(1/2+\vec{\sigma}_j\cdot\vec{S})/(S+1/2)$ in Eq. (10). The same procedure now yields the expected result that the spin is underscreened to an effective spin $S-1/2$. There are, however, no logarithmic corrections as in the traditional Kondo problem with short-range interaction. Hence, the spin $S-1/2$ is *free*, rather than asymptotically free. Screening by multiple electrons does not occur.

In case (ii) the ABA equations (15) are very similar to case (i) and are obtained by using Eq. (13) as the impurity reflection matrix R. The impurity free energy is now

$$
f_{imp}=\frac{1}{2}(J-1)\bar{\mu}+O(e^{-\bar{\mu}/T})-\frac{1}{2}T\ln[\sinh(3x_0)/\sinh(x_0)].
\tag{18}
$$

The first term arises from the charge sector and again yields a low-T specific heat proportional to T with γ of the order of $1/\bar{\mu}$. The second term is more interesting, since it corresponds to half the free energy of a free spin 1. The interpretation of this result is that the impurity spin and its boundary image combine to form an effective spin 1 [see the reflection matrix Equation (13)], which is then partially screened by the Kondo effect. The ground-state residual magnetization and entropy of the impurity are $M_{res}=1/2$ and $S_{res}=\frac{1}{2}\ln 3$. Hence, at $T=0$ the impurity spin acts as if it is completely unscreened, but its entropy is reduced relative to that of a free spin 1/2. Again, there are no logarithmic terms, characteristic of asymptotic freedom, as a consequence of the long-range interactions and the ideal fractional statistics.

IV. KONDO MODEL IN THE HYPERBOLIC CSM

Finally, an integrable Kondo model can also be constructed for the hyperbolic CSM,

$$
\begin{aligned}
H=&-\sum_{j=1}^{N}\frac{\partial^2}{\partial x_j^2}+2\sum_{l<j}\frac{g(g+P_{jl})}{\sinh^2(x_j-x_l)}+2\sum_{l<j}\frac{g(g+\bar{P}_{jl})}{\sinh^2(x_j+x_l)}\\
&+\sum_{j=1}^{N}\frac{(\nu+P_{j0})(\nu-1+P_{j0})}{\sinh^2 x_j}.
\end{aligned}
\tag{19}
$$

The two-electron scattering matrix S_{jl} and the ABA solution for the host with periodic boundary conditions have been obtained.[11] The impurity model is integrable if $\nu=2g+1/2$. In this case, the reflection matrix reads

$$
\begin{aligned}
R_{j0}(k)=&\frac{\Gamma(1+ik)}{\Gamma(1-ik)}\frac{\Gamma(\nu-ik)}{\Gamma(\nu+ik)}\frac{k+i/2+i(\nu-1/2)P_{j0}}{k+i/2+i(\nu-1/2)}\\
&\times\frac{k-i/2+i(\nu-1/2)P_{j0}}{k-i/2+i(\nu-1/2)}.
\end{aligned}
\tag{20}
$$

As the transmission S matrix takes the form

$$
S_{j,l}(k)=-\frac{\Gamma(1+ik_{jl})}{\Gamma(1-ik_{jl})}\frac{\Gamma(g-ik_{jl})}{\Gamma(g+ik_{jl})}\frac{k_{jl}+igP_{jl}}{k_{jl}-ig},
\tag{21}
$$

where $k_{jl}=(k_j-k_l)/2$. The reflection equation (9) is satisfied for $\nu=2g+1/2$. Hence, ν is strongly restricted by the bulk coupling, as it is for impurities in the δ-potential electron gas.[23] In fact, if we rescale $g\to ag$ and let $a\to\infty$, the present model is reduced to the model considered in Ref. 24. The Bethe Ansatz equations (BAE) of the integrable case read

$$e^{2ik_jL}=\frac{\Gamma(1-ik_j)}{\Gamma(1+ik_j)}\frac{\Gamma(2g+1/2+ik_j)}{\Gamma(2g+1/2-ik_j)}\prod_{r=\pm}\prod_{l\neq j}\frac{k_j-rk_l-2ig}{k_j-k_l+2ig}$$
$$\times\prod_{r=\pm}\prod_{j\neq l}\frac{\Gamma(1-i[k_j-rk_l]/2)}{\Gamma(1-i[k_j-rk_l]/2)}\frac{\Gamma(g-i[k_j-rk_l]/2)}{\Gamma(g+i[k_j-rk_l]/2)}$$
$$\times\prod_{\alpha=1}^{M}\frac{k_j-r\lambda_\alpha+ig}{k_j-r\lambda_\alpha-ig},$$
$$\prod_{r=\pm}\prod_{j=1}^{N}\frac{\lambda_\alpha-rk_j+ig}{\lambda_\alpha-rk_j-ig}=\frac{\lambda_\alpha+i(g+1/2)}{\lambda_\alpha-i(g+1/2)}\frac{\lambda_\alpha+i(g-1/2)}{\lambda_\alpha-i(g-1/2)}$$
$$\times\prod_{r=\pm}\prod_{\beta\neq\alpha}\frac{\lambda_\alpha-r\lambda_\beta+2ig}{\lambda_\alpha-r\lambda_\beta-2ig}. \tag{22}$$

Except for the prefactor in the first equation, the structure of the BAE is exactly the same as that of the short-range interaction model.[22,23] A detailed calculation shows that an impurity spin 1/2 is completely screened for $\nu\geqslant 3/2$, partially screened for $3/2\geqslant\nu>1$, and for $0<\nu<1$ the impurity potential is not bounded from below. In view of the similarities of the BAE to those of the short-range interaction model,[22,23] the impurity contributions in this case have the usual logarithmic corrections.

V. CONCLUDING REMARKS

In conclusion, we have constructed several models of magnetic impurities coupled to an electron gas of the CSM type and provided strong plausability arguments for their integrability. The screening of the impurity is influenced by the long-range character of the interactions. From these integrable examples we conclude that the behavior of the impurity is nonuniversal and depends on the coupling between the electrons in the host and that of the electrons to the impurity spin. Screening of the impurity spin by multiple electrons (as in the multichannel Kondo problem) does not occur despite the long-range interactions. The absence of logarithmic terms (usually the trademark of the Kondo effect) is also remarkable. The above models are easily generalized to more internal degrees of freedom [SU(M) invariance], and for SU(2) the extension to an arbitrary spin S is straightforward by replacing P_{j0} by $(1/2+\vec{\sigma}_j\cdot\vec{S})/(S+1/2)$.

The condition of integrability requires that the interaction potential between the electrons and that of the electrons with the boundary impurity have to be of the same form, that is, they must have the same dependence on the coordinates and only the amplitudes may differ. An interesting question is whether similar properties are obtained if the space-dependence of the Kondo coupling is modified. Since this would destroy the integrability and then an exact solution is not available, we can only speculate on this issue. Only if the interaction in the host is long ranged [i.e., of the $1/r^2$ or $1/\sin^2(r)$ type], the particles in the host are free and have fractional statistics. We believe that this statistics is key to the absence of Kondo logarithms and a Kondo scale. We expect that a modified electron-impurity interaction (at the expense of the integrability) does not change this fixed point. Similar conclusions could be inferred for the $1/\sinh^2(r)$ potential, which does not lead to fractional statistics and hence to the ordinary Kondo effect with logarithms. Again, we do not expect that small modifications of the electron-impurity interaction will dramatically affect the Kondo physics.

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