Stripes, vibrations, and superconductivity

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We propose a model of a spatially modulated collective charge state (CCS) of superconducting cuprates. [For a shorter version of this work, see A. H. Castro Neto, in *Proceedings of the MTSC 2000* [J. Supercond. **13**, 913 (2000)].] The regions of higher carrier density (stripes) are described in terms of one-dimensional (1D) interacting fermions and the regions of lower density as a two-dimensional (2D) interacting bosonic gas of $d_{x^2-y^2}$ hole pairs. The interactions among the elementary excitations are repulsive and the transition to the superconducting state is driven by decay processes. Vibrations of the CCS and the lattice, although not participating directly in the binding mechanism, are fundamental for superconductivity. The superfluid density and the lattice have a strong tendency to modulation with wave vectors $(\pi/a,0)$ and $(0,\pi/a)$ implying a still unobserved dimerized stripe phase in cuprates. The phase diagram of the model has a crossover from 1D to 2D behavior and a pseudogap region where the amplitude of the order parameters are finite but phase coherence is not established. We discuss the nature of the spin fluctuations and the unusual isotope effect within the model.

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I. INTRODUCTION

The experimental evidence for charge and spin inhomogeneities in superconducting cuprates has been accumulating in the past few years.[2,3] It is believed that these inhomogeneities are probably the result of strong competing electron-electron and electron-lattice interactions. The strong interactions can be hinted from the fact that cuprates are related to Mott insulators, not to metals. From the theoretical point of view the major problem has been the description of the doped Mott insulator. It is this state that finally evolves into a superconducting state. The origin of the inhomogeneities can be diverse. Disorder and the charge localization effects that occur close to an insulating Mott state are natural possibilities. Furthermore, there is an emerging point of view that inhomogeneities may be in the heart of the superconducting phase. The difficulty in describing the superconducting state may be related with the fact that an important part of the physics occurs in real space. This should be contrasted with a Fermi liquid description that is dominated by the Fermi surface in momentum space. If the inhomogeneities observed experimentally are essential for the description of the superconducting state then a dual description in terms of real and momentum space is certainly required.[4]

The search for a general principle or a special (maybe hidden) symmetry that facilitates the understanding of these materials is always desirable. It turns out that the intrinsic complexities of these systems are enormous. In fact, the experimental evidence seems to be that all the degrees of freedom, charge, spin, lattice, participate in a fundamental way in their physics. Unfortunately, we may have to sacrifice the simplicity of description that helped us to understand the basic physics of many systems in the past. Besides the strong local interactions that induce large charge fluctuations, long-range interactions associated with the insulating state can suppress the same degrees of freedom. We believe that it is the interplay between these two forces that is fundamental for the understanding of the physics in these systems. In this paper[1] we assume that the physics in cuprates can be divided at least into two length scales. On the one hand there is the short length (high energy) scale that is associated with the behavior of the electrons at atomic distances. This piece of physics is dominated by the strong local interactions that lead to the Mott insulator and it is well described by models of strongly correlated electrons such as the Hubbard or the t-J model. This is a ''highly'' quantum regime where quantum fluctuations at atomic scales determine the physical response. Since the short length scale is usually smaller than the size of systems studied numerically, these studies can provide a good physical insight. Various analytical approaches to these strongly coupled models exist in 2D,[5–7] and although they can also provide insight into the main physical aspects of the problem they are unreliable in dealing with quantum fluctuations. In order to acquire intuition one is forced to look at purely academic models that can be solved beyond mean field. This is the case of the various studies of the Ising t-J model where quantum spin fluctuations are suppressed by a large anisotropy in spin space,[8] the spin-fermion models where the spins are treated classically[9] or models with large number of components.[10] On the other hand, the long wavelength physics of the problem is dominated by long-range interactions such as the Coulomb, Casimir,[11] or entropic interactions that are essentially classical in origin.[12,13] Analytical treatments of these interactions have always been difficult even in statistical mechanical problems in 1D. Therefore, from the theoretical point of view there are technical complications at all length scales. Any theoretical treatment that intends to start from the microscopic picture from the beginning will encounter tremendous difficulties.

In this paper we follow a semiphenomenological approach. At the short length scales we have known for a long time that doped Mott insulators have a big ''aversion'' to charge homogeneity.[14] This happens because there is a large loss of magnetic energy when charge delocalizes. Even before superconductivity was discovered in cuprate oxides the concept of *strings* and quantum confinement of holes was well understood: when a hole moves in an antiferromagnet it produces a string of overturned spins with the energy grow-

ing linearly with the length of the string.[15] When the hopping energy, t, is smaller than the antiferromagnetic exchange, J, strings are the dominant processes in antiferromagnets (although higher energy delocalizing processes called Trugman loops are also possible[16]). When two holes are injected into an antiferromagnet there is a gain in magnetic and kinetic energy if they move together instead of separately. Strings are suppressed because the second hole heals the string generated by the first one. This picture of the diluted Mott insulator is generic. It can be obtained analytically in the Ising t-J model and is also observed in numerical simulations of the SU(2) models as well.[8] Because of the tendency to segregate charge, Mott insulators are at the edge for phase separation into hole-rich and hole-poor regions.[17] Still to this date there is a heated debate about the border to phase separation in the phase diagram of the t-J model.[18] More recently the concept of *stripes* in the t-J model has emerged due to density matrix renormalization group (DMRG) calculations by White and Scalapino (WS).[19] These simulations show clear signs of charge order. In the WS picture when holes are doped into the antiferromagnet they first form pairs that condense into lines of charge with magnetic anti-phase domain walls (ADW). This condensation process, as shown in analytical approaches to the Ising t-J models[20] and in other numerical studies,[21] has to do with the gain in kinetic energy of a *single* hole due to the formation of an ADW (moreover, Trugman loops are suppressed in this case[20]). That is, for a finite linear density of holes (vanishing 2D density) the gain in kinetic energy by making a domain wall is enough to compensate for the loss of magnetic energy due to the formation of magnetic defects. We should stress that we are not concerned with the problem of the phase diagram of the t-J model because this phase diagram most certainly *cannot* be the phase diagram of cuprate oxides. There are many important interactions in cuprates that are not included in the t-J model. We argue below that coupling to the lattice, for instance, is relevant for the experimental phase diagram. We assume that the tendency of the diluted Mott insulator to form bound state pairs of holes is universal.

Long-range interactions and their eventual screening are probably the key to understand the phase diagram of cuprates.[22] At low doping, because of the existence of a large charge gap in the Mott insulator, charge dynamics is suppressed and as a consequence dynamical screening as well. All the physics rests in the spin degrees of freedom. Thus, when a small concentration of hole is introduced into the CuO_2 layers long-range interactions should play a major role. These long-range interactions are effective in suppressing the tendency to phase separation as stressed by Emery and Kivelson (EK) and collaborators[23–25] and lead to the generation of a finite length scale, L_D, associated with domain size.[26] In the absence of a lattice the long-range forces produce blobs or lakes of charge with characteristic size L_D. It turns out, however, that in transition metal oxides the coupling to the lattice is strong and therefore the symmetry of rotations in real space is broken. Charge modulation is therefore the final result of the tendency to phase separation and strong lattice coupling. The formation of this charge modulated phase is *not* the result of a Fermi surface singularity as in the case of 1D charged density waves (CDW). In fact, the CCS state described here has strong similarities with the metallic 2D CDW states observed in Dichalcogenides[27] that have very little resemblance with the 1D CDW states that are insulating.[28] This state is probably better described as coming from the quantum melting of an anisotropic Wigner crystal (or Bragg glass)[22] or from an exotic CDW state.[29] In classical statistical mechanics there are many analogues called liquid crystal phases: nematic, smectic, hexatic, etc.[30] These are phases with long-range orientational order in the absence of translational order. Another more mundane phase where translational symmetry is broken in 2D is an anisotropic *membrane* phase where translational order is broken along the principal vectors of the lattice.[12]

Let us consider a static charge modulated phase induced along the y direction in the 2D system with periodicity Na where a is the lattice spacing. The charge density, $\rho(\mathbf{r})$, can be written as (we use units such that $\hbar = k_B = 1$)

$$\rho(\mathbf{r}) = \rho_0 + \langle \Phi(\mathbf{r}) \rangle \exp\left\{ i \frac{2\pi}{Na} y \right\} + \text{c.c.}, \tag{1.1}$$

where ρ_0 is the background density, and Φ is the complex order parameter that can be rewritten in terms of an amplitude $|\Phi|$ and phase u:

$$\langle \Phi(\mathbf{r}) \rangle = |\Phi| \exp\left\{ -i \frac{2\pi}{Na} u(\mathbf{r}) \right\}. \tag{1.2}$$

The lines of constant phase at wave number $2\pi/(Na)$ are described by

$$\phi = \frac{2\pi}{Na}(y - u) = 2\pi n, \tag{1.3}$$

where $n = 0, \pm 1, \pm 2, \ldots$. In an inert background (such as a Fermi liquid) the local modulations of the density can lead to gaps at the Fermi surface but the system as a whole remains homogeneous. In a Mott insulator this is not necessarily so. In the regions of low charge density the Mott insulator is essentially untouched. In these regions it is energetically advantageous for the system to form bound states of holes with $d_{x^2-y^2}$ symmetry (exactly like in a finite cluster or ladder) with a gain magnetic energy.[21,31,32] In the regions where charge density is large it is energetically more favorable for the system to have a gain of kinetic energy[20] that liberates the holes to move as single particles and create ADW. Thus, the formation of a charge ordered state has to be accompanied by a change in the spin structure of the system with the creation of incommensurate spin fluctuations. The mechanism of gain of energy (kinetic or magnetic) depends strongly on the amount of charge density. Moreover, deformations of amplitude of the order parameter of the CCS are always energetically costly. Therefore, phase fluctuations are the low energy excitations in such systems. For a classical smectic phase, for instance, the free energy is given by

$$F_S=\frac{1}{2}\int d^2r[B(\partial_{||}u_S)^2+K(\partial_\perp^2 u_S)^2]=\frac{1}{2}\sum_{\mathbf{k}}\ \omega_{S,\mathbf{k}}|u_{S,\mathbf{k}}|^2, \tag{1.4}$$

where B is transverse stiffness of the smectic and K is the splay elastic constant. Here $||$ and $\perp$ indicate the gradient of the function in the directions parallel and perpendicular to the ordering direction, respectively. Moreover, $\omega_{S,\mathbf{k}}=Bk_{||}^2+Kk_\perp^4$ is the classical smectic dispersion relation. A membrane phase, on the other hand, would be described by $\omega_{M,\mathbf{k}}=\sqrt{B_{||}^2k_{||}^2+B_\perp^2k_\perp^2}$ where $B_{||}$ ($B_\perp$) are the longitudinal (transverse) compressibilities. In fact, because the charge modulated state is composed of electrons we expect this modulated phase to be quantum in nature, that is, a *quantum liquid crystal*.[33]

Notice that such a smooth charge distribution as proposed in Eq. (1.1) is different from the stripe phases discussed in the context of insulating cuprates and nickelates. In insulating nickelates static charge order is observed in neutron scattering and many other experiments.[34] The key point is that higher *harmonics* of the fundamental Bragg peaks associated with static order are observed in these systems. It implies sharp, well defined and isolated domain wall structures in real space. In superconducting systems where inelastic neutron scattering peaks are observed at equivalent positions there are *no* harmonics observed, even when the fundamental peaks are rather sharp. In $La_{1.48}Nd_{0.4}Sr_{0.12}CuO_4$ (LNSCO) where Tranquada and collaborators observed quasi-static peaks in neutron scattering there are no signs of higher harmonics.[35] This experimental fact signals to a smooth variation of the charge-spin densities in the system. Thus, the idea of well-defined, non-interacting stripes is misleading in the context of superconductivity. Instead one should think of the ''stripes'' as a complex collective state that is driven by competing local interactions and cooperative long-range forces.

The importance of the lattice degrees of freedom has been experimentally verified in essentially all cuprate superconductors and their insulating relatives.[36,37] At this point in time most of the theoretical approaches either focus on electron-electron interactions and overlook the importance of lattice degrees of freedom or mainly electron-phonon interactions[38] disregarding the importance of the strong (short- and long-range) interactions in the problem. Some mean-field approaches, however, have stressed the importance of electron-lattice interaction in the context of nickelates.[6,39] We believe that electron-electron and electron-lattice interactions are equally important because of charge neutrality. Charge neutrality implies that a charge modulated state such as the one defined in Eq. (1.1) has to be strongly coupled to the lattice. One would expect under general considerations that fluctuations of the CCS to appear in the phonon spectrum. Indeed, consider the classical elastic lattice free energy:

$$F_P=\frac{1}{2}\sum_{\mathbf{k}}\ \omega_{P,\mathbf{k}}|u_{P,\mathbf{k}}|^2, \tag{1.5}$$

where $u_{P,\mathbf{k}}$ is the lattice deviation from equilibrium position and $\omega_{P,\mathbf{k}}$ the phonon dispersion relation. In the leading order the coupling between the smectic and lattice is quadratic in the displacements and can be written as

$$F_C=\sum_{\mathbf{k}}\ C_{\mathbf{k}}u_{\mathbf{k}}u_{P,-\mathbf{k}}+\text{c.c.}, \tag{1.6}$$

where $C_{\mathbf{k}}$ are the coupling constants. The problem of the CCS plus phonons can be solved exactly by a simple linear combination of the displacement fields. There are two branches of excitations with frequency given by

$$\Omega_{\pm,\mathbf{k}}=\frac{1}{2}[\omega_{S,\mathbf{k}}+\omega_{P,\mathbf{k}}\pm\sqrt{(\omega_{S,\mathbf{k}}-\omega_{P,\mathbf{k}})^2+4|C_{\mathbf{k}}|^2}] \tag{1.7}$$

leading to a splitting of the vibrational modes. Therefore, the phonon spectrum should be directly affected by the presence of a CCS. Thus, in dealing with the fluctuations of the CCS we have to consider the renormalizations of such fluctuations by the lattice. The full quantum mechanical problem can be quantized exactly like phonons in ordinary crystalline solids and the vibrations of the CCS are described by

$$H_V=\sum_{\mathbf{k},\alpha}\ \Omega_{\alpha,\mathbf{k}}b_{\alpha,\mathbf{k}}^\dagger b_{\alpha,\mathbf{k}}, \tag{1.8}$$

where $b_{\alpha,\mathbf{k}}$ ($b_{\alpha,\mathbf{k}}^\dagger$) is the annihilation (creation) operator for quantum vibrations of the CCS with momentum $\mathbf{k}$ in the branch α and energy $\Omega_{\alpha,\mathbf{k}}$.

Although the description of the collective state is rather simple because of its Gaussian nature, the description of its internal degrees of freedom (associated with the short length scales) is more complex. In what follows we make an *ad hoc* assumption that the internal charge excitations of this collective state can be divided into two main groups. First, in the regions where the charge density is large [that is, given by the lines of constant phase in Eq. (1.3)] ADW exist due to the local gain in kinetic energy. These regions we call stripes. These high density regions are characterized by single particle excitations (not bound states) and they are essentially confined to 1D lines because of the potential induced by strings.[20] This highly anisotropic electronic fluid should be interacting because of the phase space constraints and in the absence of tunneling between stripes it is described by a Luttinger liquid.[40] In the regions of low density (in the middle of the antiferromagnetic ladders) single particle excitations are suppressed and bound states of holes are energetically more favorable. The simplest of them is a bound state of a pair of holes with $d_{x^2-y^2}$ symmetry.[31] Larger bound state structures like quartets are unlikely to contribute because their quantum dynamics is exponentially suppressed (tunneling matrix elements decay exponentially with the number of particles). Moreover, we assume that this gas of bosons is essentially isotropic and only weakly coupled to the stripe fermions. The main reason for the weak coupling is related with the ''string healing'' process that generates the pairs in first place, that is, the bosons are essentially insensitive to the magnetic structure including the ADW. We argue, however,

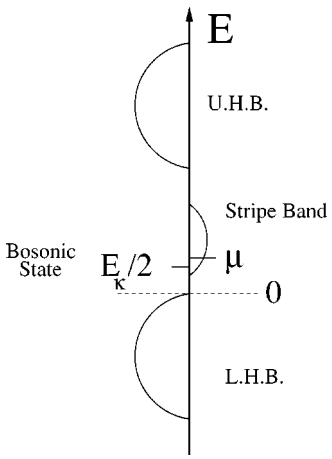

FIG. 1. Schematic plot of the density of states as a function of energy.

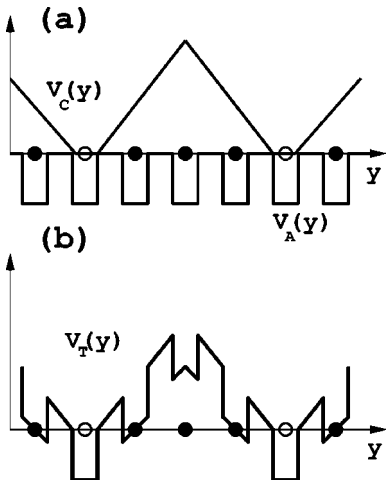

FIG. 2. Effective potential: (a) $V_c(y)$ is the string potential generated by the antiferromagnetic background and $V_A(y)$ is the atomic potential generated by the lattice; (b) resultant potential.

that it is the weak coupling between bosons and stripes that ultimately produces superconductivity in the system.

From the electronic point of view the situation is illustrated in Fig. 1. The Mott insulating state can be described by a filled lower Hubbard band (L.H.B.) and an empty upper Hubbard band (U.H.B.) that are separated by a large energy scale. In between these two bands there is a single electron band associated with the stripe fermions that we call the stripe band. The bound states of holes exist due to the transfer of spectral weight from the L.H.B. to a level above it with energy $E_{\mathbf{k}}/2$ (where $\mathbf{k}$ is the momentum of the boson as we discuss below). The binding energy of the holes is the energy difference between the boson level and the top of the L.H.B. The binding energy can be seen as mediated by the exchange of paramagnons[41] and therefore is of the order of the characteristic magnetic energy in the problem.[42,43] In the undoped system the characteristic energy is simply the exchange constant J but as doping increases the magnetic energy scales are reduced driving the system from the Mott insulator to a more ordinary Fermi liquid state.

The measure of the magnetic energy is the spin stiffness, ρ_s, of the magnetic background. A finite spin stiffness produces the confining potential for the charge carriers of the form[20]

$$V_C(y)=\frac{\rho_s}{a}|y| \tag{1.9}$$

(modulus Na) as shown in Fig. 2(a) $[V_C(y+Na)=V_C(y)]$. The total potential as seen by the holes is a superposition of the atomic potential of the lattice, $V_A(y)[V_A(y+a)=V_A(y)]$, and the magnetic confining potential of the strings. In Fig. 2(b) we show in a simple Kronig-Penney picture the result of the superposition of these two potentials. When the temperature is larger than ρ_s the holes are essentially deconfined. Since the hole pairs only exist at temperatures below ρ_s it is important to estimate this energy scale. In order to do so let us consider the situation in Fig. 3(a) where the exchange within the antiferromagnetic regions is J but across the stripe it is $J'<J$. At long wavelengths the problem maps into a spatially anisotropic Heisenberg model with exchanges J_x and J_y as shown in Fig. 3(b).[44] While J'/J is a short length scale property and depends on the microscopic details[20] the ratio

$$\alpha=\frac{J_x}{J_y} \tag{1.10}$$

depends on the long wavelength properties and determines the region of stability for antiferromagnetic order.[44] Moreover, the effective spin stiffness of the magnetic background depends directly on α. Assuming that $J_y=J$ in Fig. 3(b) (so that the exchange along the direction of the stripes is not modified) it is easy to show that[44]

$$\rho_s(\alpha)=JS^2\sqrt{\alpha}. \tag{1.11}$$

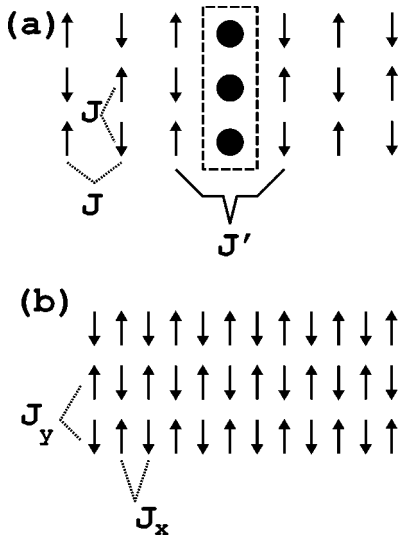

FIG. 3. (a) Magnetic interactions in the presence of stripes: J is the exchange between spins in the antiferromagnetic ladder; J' is the exchange across the stripe. (b) Effective magnetic model with spatially anisotropic couplings.

This choice for the exchange constants was used in Ref. 45 and seems to explain well the data in LNSCO.[46] In the low hole doping regime ($x<0.02$) another choice has to be made since the magnetism is isotropic.[44] It is clear from Eq. (1.11) that a decrease in α reduces the spin stiffness and can lead to a quantum phase transition to a spin liquid state.[47] The main problem is how to relate α of the effective model of Fig. 3(b) to the microscopic model of Fig. 3(a). We use a simple scheme and compare the classical ground state energies of the two problems. In the case of Fig. 3(a) the Hamiltonian is simply

$$H=\sum_{i,j} J_{i,j}\mathbf{S}_i\cdot\mathbf{S}_j, \tag{1.12}$$

where $J_{i,j}=J$ in the antiferromagnetic ladders and $J_{i,j}=J'$ across the ADW. Let us rewrite $J'=\gamma J$ where $0<\gamma<1$ is a microscopic quantity that depends on details of the problem. With this parametrization it is clear that the classical ground state energy can be written as

$$E_0=E_{\mathrm{AF}}(J)+(1-\gamma)JS^2N_{\mathrm{stripes}}N_s, \tag{1.13}$$

where

$$E_{\mathrm{AF}}(J)=-2JS^2N_{\mathrm{spins}} \tag{1.14}$$

is the classical energy of an isotropic antiferromagnet with N_{spins} spins and exchange J. N_s is the number of sites in each direction (N_s^2 is the total number of sites), $N_{\mathrm{stripes}}=N_s/N$ is the number of stripes (N is the separation between stripes in lattice units), and $N_{\mathrm{spins}}=N_s^2-N_s\times N_{\mathrm{stripes}}$ is the number of spins not residing in stripes [$N_{\mathrm{spins}}=N_s^2(1-1/N)$]. Thus, from Eq. (1.13) one gets

$$\frac{E_0}{N_s^2JS^2}=-2+\frac{3-\gamma}{N}. \tag{1.15}$$

On the other hand, for the effective model shown on Fig. 3(b) with the same number of sites we would have

$$E_0=-(J_x+J_y)S^2N_s^2=-J(1+\alpha)S^2N_s^2, \tag{1.16}$$

where we used Eq. (1.10). Comparing Eqs. (1.15) and (1.16) we find

$$\alpha=1-\frac{3-\gamma}{N}. \tag{1.17}$$

This result implies from Eq. (1.11) that

$$\rho_s(N)=JS^2\sqrt{1-\frac{3-\gamma}{N}}. \tag{1.18}$$

Notice that the spin stiffness vanishes at a critical distance N_m between stripes given by

$$N_m=3-\gamma. \tag{1.19}$$

Since $0<\gamma<1$ we see that $2<N_m<3$ [quantum fluctuations increase the value of N_m (Ref. 44)]. The binding energy of the hole pairs is proportional to ρ_s and no bosons can exist at zero temperature when $N<N_m$. Moreover, we identify a temperature scale $T^*(N)\approx\rho_s(N)$ above which no bosons exist [$T^*(N_m)=0$]. So we relate the formation of the hole pairs as the ''pseudogap'' energy scale observed in many different experiments.[48] Our theory, however, breaks down when $T>T^*$.

In the noninteracting picture (with no hybridization between bosons and fermions) we can have two possibilities. In Fig. 1 the boson state is empty of holes (filled with electrons) and the stripes are partially filled up to the chemical potential energy μ. Because bosons and fermions are in thermodynamic equilibrium we work in the grand-canonical ensemble and keep the chemical potential fixed by letting the number of particles fluctuate. Consider the case when

$$\mu>E_{\mathbf{k}}/2.$$

The boson state is unoccupied and there are no bosons in the system. As the chemical potential is reduced (more holes are introduced) the stripes empty and further reduction of the chemical potential pins the energy at the boson level since as more holes are added they can only produce bound states. Thus, there is a continuous transfer of spectral weight from the L.H.B. to the boson level. On the other hand, if the binding energy of the holes increases nothing happens until $\mu=E_{\mathbf{k}}/2$. At this point there would be bosons and stripes coexisting with each other. When the binding energy is increased further the chemical potential follows since bosons become converted into stripe fermions. Once again the chemical potential is pinned at the boson level. Finally when the boson level reaches the top of the stripe band the stripes are completely filled and the ADW disappears. In fact, because of the loss of kinetic energy the domain wall disappears even before this limit is reached.[20] Therefore, we assume that

$$2\mu-E_{\mathbf{k}}\geqslant 0. \tag{1.20}$$

Moreover, it is clear from this picture that the spin degrees of freedom that are responsible for the large magnetic response in these systems leave on the L.H.B. and are separated in energy from the charge degrees of freedom. Therefore, the magnetism can be effectively ''traced out'' of the problem since it only leads to kinematic renormalizations of the various parameters (in other words, the spins follow the charge).

The mechanism for superconductivity discussed in this work requires the coupling of stripe fermions via the exchange of bosons. One can think of this mechanism as the exchange of stripe ''pieces.'' Coherence between the fermions requires the exchange of real bosons. The simplest mechanism for exchange is the decay of the bosons (since they are composite particles) into fermionic degrees of freedom at the stripes. This kind of mechanism can occur when two systems with very different ground states are separated by an interface. In fact, it was proposed long ago that a mechanism of this sort could generate superconductivity at a metal-semiconductor interface (also called exciton superconductivity).[49] Although this kind of proposal has generated controversy in the past[50] there are good indications that they may be good candidates in the case of cuprates.[51] Moreover, the process described here is similar to the ''prox-

imity effect'' mechanism proposed by EK if we neglect retardation effects due to the boson motion.[23–25] One of our main results is: if the stripes static, that is, if we disregard the fluctuations of the CCS, this process is suppressed The ''proximity effect'' mechanism cannot happen with static stripes and real bosons. Therefore, for coherence to be attained, we have to introduce fluctuations of the CCS. This makes our model radically different from the proximity effect proposed by EK since it involves distortions of the CCS and therefore of the lattice as well. Moreover, unlike BCS, the phonons are not part of the binding mechanism that is driven by the exchange of composite fermion pairs. Vibrations, however, are fundamental for the superconductivity. A natural consequence of the mechanism is that fluctuations of the order parameter are necessarily coupled to the fluctuations of the CCS and therefore vortices are coupled to dislocations of the stripe array. Because the superfluid density is low, the interactions between topological defects ultimately determines the phase diagram. Moreover, we claim that static stripes [as the ones obtained in Hartree-Fock solutions of the t-J and Hubbard models[52,53] or DMRG (Ref. 19)] should be insulating due to a 1D CDW instability along the stripe direction. Diagonal stripes do not couple to the bosons because they are oriented along the nodes of the boson wave functions.

Among other things we explain why phonon anomalies that have been observed in neutron scattering[36] occur exactly at the same position in the Brillouin zone where angle resolved photoemission[54] (ARPES) observes the opening of the pseudogap [that is, at $(\pi/a,0)$ and $(0,\pi/a)$]. Moreover, we show that the lattice distortions associated with the phonon anomalies are associated with the modulation of the superfluid density *perpendicular* to the stripes. The theory also predicts an isotope effect and the coexistence of commensurate and incommensurate spin fluctuations. The critical temperature for superconducting order, T_c, is determined by the interplay between topological defects associated with the superfluid bosons (vortices and antivortices) and distortions of the CCS (dislocation loops). The transition at finite temperatures is in the 3D XY universality class. We show that the amplitude of the order parameters is finite at temperatures above T_c up to T^* in the pseudogap region. Moreover, at $T=0$ there is a quantum phase transition as a function of the separation between stripes, N, that is directly related to the hole concentration $x[N\approx 1/(2x)]$. While the order parameters become finite for $N<N_{sp}$ long-range order is only attained at $N=N_c<N_{sp}$. This quantum phase transition is in the 2D XY in a magnetic field universality class. Thus, there is a crossover region at $T=0$, $N_c<N<N_{sp}$, where topological defects prevent long-range order to develop.

The paper is organized as follows: in the next section we introduce the bosonic bound state of holes; in Sec. III we present the Luttinger liquid representation of the stripe fermions; in Sec. IV we discuss the nature of the coupling between bosons and stripe fermions and argue that vibrational degrees of freedom should enter explicitly; in Sec. V we solve the mean-field equations for the problem and determine the mean-field phase diagram; in Sec. VI we discuss the phase fluctuations of the superfluid-superconducting state and how they relate to the phase fluctuations of the CCS; in Sec. VII the nature of the high temperature transition is discussed; Sec. VIII contains the results for the zero temperature transition; Sec. IX contains our conclusions.

II. ANTIFERROMAGNETIC BOSONS

Consider the problem of two holes in an antiferromagnet. The energy is minimized by the formation a bound state that moves freely through the system.[31,43] One can define an operator that creates such a state at the top of the antiferromagnetic vacuum that is given by

$$P_{\mathbf{k}}^{\dagger}=\frac{1}{\sqrt{S}}\sum_{\mathbf{q}}\Psi_{\mathbf{k}}(\mathbf{q})c_{\uparrow,\mathbf{k}/2-\mathbf{q}}c_{\downarrow,\mathbf{k}/2+\mathbf{q}}, \tag{2.1}$$

where $c_{\sigma,\mathbf{k}}$ destroys an electron (creates a hole) with momentum $\mathbf{k}$ and spin projection σ ($\uparrow$ or $\downarrow$) in the antiferromagnet. $S=(N_s a)^2$ is the area of the system. The wave function of the pair, $\Psi_{\mathbf{k}}(\mathbf{q})$, is normalized $[\Sigma_q|\Psi_{\mathbf{k}}(\mathbf{q})|^2/S=1]$ and in principle depends on the relative and total momentum of the pair. This wave function can be obtained variationally[55] or by solving the Bethe-Salpeter equation for the binding of two holes.[56] The dependence of the wave function on the total momentum is due to the fact that on the lattice the symmetry of the bound state varies with its center of mass momentum and depends strongly on the microscopic details. For the SU(2) t-J model it is known that the $d_{x^2-y^2}$ state has the lowest energy[31] while in the Ising t-J model the p-wave is the ground state.[43,57] The matter of the fact is that the s-wave state is always the state with highest energy and the reason is fairly simple to understand: the strong local repulsion requires the wave function of the pair to be centered at different sites. In other words, s-wave bound states are suppressed by the magnetism. Here we assume the pairs to have $d_{x^2-y^2}$ symmetry.

It is easy to see that the operator defined in Eq. (2.1) is not completely bosonic because of its composite nature.[58] Since the electrons obey anticommutation relations $(\{c_{\sigma,\mathbf{q}},c^{\dagger}_{\sigma',\mathbf{q}'}\}=\delta_{\mathbf{q},\mathbf{q}'}\delta_{\sigma,\sigma'})$ it is easy to show that commutation relations of the bosons is given by

$$[P_{\mathbf{k}},P^{\dagger}_{\mathbf{k}'}]=\delta_{\mathbf{k},\mathbf{k}'}-D_{\mathbf{k},\mathbf{k}'}, \tag{2.2}$$

where

$$D_{\mathbf{k},\mathbf{k}}=\frac{1}{S}\sum_{\mathbf{q}}(|\Psi(\mathbf{k}/2-\mathbf{q})|^2 n_{h,\uparrow,\mathbf{q}}+|\Psi(\mathbf{k}/2+\mathbf{q})|^2 n_{h,\downarrow,\mathbf{q}}). \tag{2.3}$$

Here $n_{h,\sigma}$ is the number operator for holes ($1-n_{c,\sigma}$ where $n_{c,\sigma}$ is the number operator for electrons). Since the magnitude of $D_{\mathbf{k},\mathbf{k}}$ is proportional to density of holes in the antiferromagnet, $D_{\mathbf{k},\mathbf{k}}$ is smaller than x, the total number of holes in the system. Moreover, the largest fraction of holes is actually residing in the stripes. Since Eq. (2.3) is much smaller than one, the violation of the bosonic commutation relations can be disregarded and we can treat the bound states as real bosons. At this point it is convenient to count the number, N_e, of fermions. Assuming that there are N_s/N stripes of

size N_s (in lattice units) the number of antiferromagnetic sites is $(N_s - N_s/N)\times N_s$ and the total number of fermions is

$$N_e = N_s^2(1-1/N) - 2\sum_{\mathbf{k}} P_{\mathbf{k}}^\dagger P_{\mathbf{k}} + \sum_{\mathbf{k},\sigma} \psi_{\mathbf{k},\sigma}^\dagger \psi_{\mathbf{k},\sigma}, \quad (2.4)$$

where the factor of 2 comes from the composite nature of the bosons and the last term is the number of fermions in the stripes (see below).

The Hamiltonian of the composite bosons can be written as

$$H_P = \sum_{\mathbf{k}} (-E_{\mathbf{k}} + 2\mu) P_{\mathbf{k}}^\dagger P_{\mathbf{k}} + \frac{U}{2}\sum_i N_i^2, \quad (2.5)$$

where $E_{\mathbf{k}}$ is the dispersion relation of the bosons and U is the local repulsion between pairs. In Eq. (2.5) μ is the chemical potential for the electrons and it comes with a negative sign because the chemical energy appears as $-\mu N$ with the total number of electrons given in Eq. (2.4).

Suppose there are no other interactions in the problem (that is, stripe fermions and $d_{x^2-y^2}$ bosons do not interact). The Hamiltonian (2.5) is the so-called Bose-Hubbard model. This problem has been studied to a great level of detail and its phase diagram is well known.[59] Let us consider the situation close to the minimum of $E_{\mathbf{k}}$ at, say, $\mathbf{k}=\mathbf{K}$. Expanding close to this point one can rewrite the Hamiltonian for the bosons as

$$H = \sum_i \left[(-E_{\mathbf{K}} + 2\mu) N_i + \frac{U}{2} N_i^2 \right] - t_B \sum_{\langle i,j\rangle} P_i^\dagger P_j + \text{H.c.}, \quad (2.6)$$

where t_B is the hopping energy of the boson. The phase diagram is strongly dependent on the boson chemical potential, μ_B, that is given by

$$\mu_B = -2\mu + E_{\mathbf{K}}. \quad (2.7)$$

In the absence of disorder the superfluid phases are separated from the Mott insulator phases by lines of second order phase transition. At $t_B = 0$ the Mott insulator phases extend in the range $n-1 < \mu_B/U < n$ where n is a positive integer that gives the number of bosons per site. Notice that because of condition (1.20) we must always have $\mu_B \leqslant 0$. Therefore, the only allowed state for the bosons is the Mott insulator with $n=0$, that is, the vacuum. For a finite value of t_B there is a critical value of the hopping above which the system becomes a superfluid. However, even a small amount of disorder suppresses the Mott insulator-superfluid transition by the creation of an insulating state. Disorder is unavoidable in these systems since the charge comes from counter-ions out of the CuO_2 planes. Thus, the conclusion is that if the bosons are decoupled from the stripes the Bose system is in the insulating Bose glass state.

III. STRIPE FERMIONS

As we have argued the single particle excitations at the regions of high density of the CCS is due to the magnetic confinement.[20] In the absence of the CCS the holes can move freely with hopping energy t in both in the x and y directions and the Brillouin zone is defined for $-\pi/a < k_x, k_y \leqslant \pi/a$. In the presence of the CCS the translation symmetry of the lattice perpendicular to the ordering vector is broken. Suppose that the stripes are separated by a distance Na. Then the new Brillouin zone is given by $-\pi/a < k_x \leqslant \pi/a$ and $-\pi/(Na) < k_y \leqslant \pi/(Na)$. Thus the original band has to be folded back into this zone and gaps open in the single particle spectrum generating N new bands.

When $N \gg 1$ the tunneling between stripes is suppressed because of the large distance. A WKB estimate of the transverse tunneling energy using the confining potential (1.9) gives

$$t_\perp(N) \approx \frac{\rho_S(N)^{3/4} N^{3/4}}{m^{1/4} a^{1/2}} e^{-N^{3/2} a \sqrt{2 m \rho_S(N)}}, \quad (3.1)$$

where m is the hole mass. Observe that for $N \gg 1$ Eq. (1.18) shows that $\rho_S \approx J/4$ and therefore the stripe fermions have large gaps in their spectrum. Moreover, $t_\perp$ is exponentially suppressed at large N and interstripe charge fluctuations are suppressed. As N is decreased the confining potential gets weaker and eventually vanishes at N_m given in Eq. (1.19). Close to N_m the system is essentially 2D and the confining potential $V_C(y)$ can be treated as a perturbation of the atomic one and small gaps of order $\rho_S(N)$ open at $\pi/(Na)$. At this point, $N \to N_m^-$, the bandwidth of the lowest band is simply $4t(1-\cos(\pi/N_m)) < 4t$. Exactly at N_m the stiffness vanishes and the system becomes 2D since the holes can move freely, that is $t_\perp(N > N_m) = t$ where t is the hopping energy in the absence of the CCS. A simple and convenient way to parametrize the hopping in the transverse direction in the whole parameter range is

$$t_\perp(N) = t e^{-(N-N_m)/N_0}, \quad (3.2)$$

where $N_0 \approx N_m/\ln[1/(1-\cos(\pi/N_m))]$. The hopping along the stripes, $t_{||}$, is not so sensitive to the distance between stripes. It has been shown[20] that for $N \to \infty$ the hopping energy is reduced from t (as in the uniform system) due to the dressing by strings to a value of the order of $J \approx t$. On the other hand when $N \approx 1$ one has $t_{||} = t$. Thus, $t_{||}$ is a smooth function and its variation with N can be ignored. The anisotropy in the hopping energies leads to anisotropies in the Fermi surface as well. Suppose the stripes are oriented along the x axis. The stripe fermion dispersion relation can be written as

$$\epsilon_{\mathbf{k}}(N) = -2t_{||}\cos(k_x a) - 2t_\perp(N)\cos(N k_y a) \quad (3.3)$$

and it is strongly anisotropic as long as $t_{||} \gg t_\perp$ (this is always true for $N > N_m$). In the limit of $N \gg 1$ the transverse component can be completely disregarded and the Fermi surface is obtained by filling up all the states up to $k_x = \pm k_F$ where k_F is the Fermi momentum [see Fig. 4(a)]. Moreover, the Fermi surface is open along k_y since the system has no dispersion in that direction. If the stripes are oriented along the diagonals on a square lattice then in momentum space the dispersion looks like the one given in Fig. 4(b). Then it is convenient to rotate the orientation of the lattice by $\pi/4$ and to

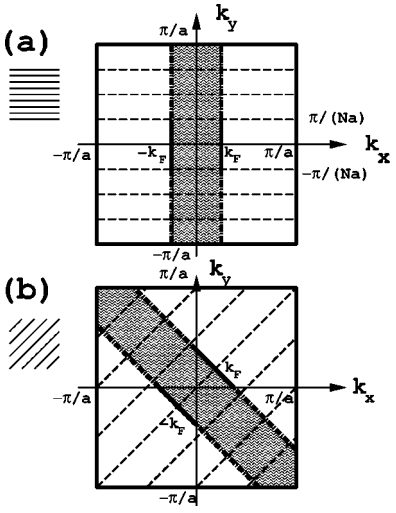

FIG. 4. Momentum space picture for stripes: (a) longitudinal; (b) diagonal.

change the lattice spacing from a to $a/\sqrt{2}$. Introduction of $t_\perp$ leads to a change in the curvature of the Fermi surface by an amount $\delta k_\perp$ close to the Fermi points.

As it has been noted in the context of organic superconductors[60] the strong anisotropy of the hopping produces strong dependence of the physical properties with temperature. At high temperatures, that is, $T>t_\perp(N)$, the uncertainty on the transverse momentum is larger than the size of the Brillouin zone, $\delta k_\perp > \pi/(Na)$, and the curvature of the Fermi surface is blurred by thermal effects. In this case the motion of the electrons perpendicular to the stripes becomes incoherent and electrons become confined by thermal effects to a region of size Na. Thus, the motion of the electrons is essentially 1D and the 2D aspect of the stripe problem can be disregarded. In the opposite limit, $T<t_\perp(N)$, the curvature of the Fermi surface is larger than the thermal effects and 2D physics start to play a role. So there is a 1D to 2D crossover as a function of temperature that occurs at $T_{cr}(N)\approx t_\perp(N)$.

In most of the paper we focus in the region where $N<N_m$ so it makes sense to talk about stripes and the existence of a CCS. In this region the transverse hopping given by (3.2) is exponentially small and the problem can be treated as purely 1D. Consider a single isolated *static* stripe that we describe as a 1D electron gas with dispersion ϵ_k. The noninteracting Hamiltonian is simply

$$H_0=\sum_{k,\sigma}(\epsilon_k-\mu)\psi^\dagger_{k,\sigma}\psi_{k,\sigma}. \tag{3.4}$$

As usual in the case of the 1D system we assume that the physics is dominated by the excitations close to the Fermi points. Expanding the dispersion close to $\pm k_F(\epsilon_{\pm k_F}=\mu)$ we can write the above Hamiltonian as

$$H_0=iv_F\sum_\sigma \int ds(\psi^\dagger_{R,\sigma}(s)\partial_s\psi_{R,\sigma}(s)-\psi^\dagger_{L,\sigma}(s)\partial_s\psi_{L,\sigma}(s)), \tag{3.5}$$

where s is the distance along the stripe (if the stripe is oriented along the x axis then $s=x$, but if the stripe is diagonal then $s=\sqrt{x^2+y^2}$) v_F is the Fermi velocity and $\psi_R(s)$ and $\psi_L(s)$ are right and left moving electrons, respectively. The original fermion operator is written as

$$\psi_\sigma(s)=\psi_{R,\sigma}(s)e^{ik_Fs}+\psi_{L,\sigma}(s)e^{-ik_Fs} \tag{3.6}$$

and these can be bosonized via the transformation[40]

$$\psi_{R,L,\sigma}(s)=\frac{1}{\sqrt{2\pi a}}e^{\pm i\sqrt{\pi}\phi_{R,L,\sigma}(s)}.$$

The fields ϕ can be described in terms of ϕ_σ and θ_σ as $\phi_{R,L,\sigma}(s)=\phi_\sigma(s)\mp\theta_\sigma(s)$. The bosonic fields can be rewritten in terms of charge and spin bosonic modes: $\phi_{c,s}=(1/\sqrt{2})(\phi_\uparrow\pm\phi_\downarrow)$ and $\theta_{c,s}=(1/\sqrt{2})(\theta_\uparrow\pm\theta_\downarrow)$. If the average density on the stripe is ρ_0 then we can write, $\Sigma_{\sigma=\uparrow,\downarrow}\psi^\dagger_\sigma(s)\psi_\sigma(s)=a(\rho_0+\delta\rho)$ and $\delta\rho=\partial_s\phi_\rho/\sqrt{\pi}$ in bosonized form. The Hamiltonian of the problem for *forward* scattering reads[40]

$$H_F=\sum_{\mu=\rho,s}\frac{v_\mu}{2}\int ds\{K_\mu(\partial_s\theta_\mu)^2+K_\mu^{-1}(\partial_s\phi_\mu)^2\}, \tag{3.7}$$

where K_μ are the Luttinger parameters (dependent on the electron-electron interactions) and

$$v_\mu=\frac{v_F}{K_\mu} \tag{3.8}$$

are the charge and spin velocity. For the noninteracting system we have $K_\rho=K_s=1$. By the same token the action that describes the bosonized fermions is given by

$$\begin{aligned}S&=\sum_\mu\frac{K_\mu}{2v_\mu}\int ds\int d\tau[(\partial_\tau\theta_\mu)^2+v_\mu^2(\partial_s\theta_\mu)^2]\\&=\sum_\mu\frac{1}{2K_\mu v_\mu}\int ds\int d\tau[(\partial_\tau\phi_\mu)^2+v_\mu^2(\partial_s\phi_\mu)^2].\end{aligned} \tag{3.9}$$

Besides forward scattering we may also include a backscattering term:

$$\begin{aligned}H_1&\propto\int ds\,\psi^\dagger_{R,\uparrow}(s)\psi^\dagger_{L,\downarrow}(s)\psi_{R,\downarrow}(s)\psi_{L,\uparrow}(s)+\text{H.c.}\\&=g_1\int ds\cos(\sqrt{8\pi}\phi_s(s)),\end{aligned} \tag{3.10}$$

where g_1 is the coupling constant. In principle a Luttinger liquid can also include umklapp terms. The umklapp terms are only important at commensurate fillings and especially at half-filling when they are responsible for the Mott-Hubbard gap. However, half-filling for the stripes implies that there is one electron per site, that is, the stripe is depopulated. This can only happen if the boson level crosses the chemical potential and the stripe state becomes unstable. Throughout the paper we assume that the stripe state is stable and therefore

away from half-filling. Experiments by Tranquada and collaborators in LNSCO indicate that the stripe filling is 1/4.[35] Thus, we disregard the umklapp processes.

The importance of the backscattering term can be readily understood by a simple perturbative renormalization group (RG) calculation. It is a simple exercise to show that if we shrink the bandwidth of the stripe electrons from E_c to $E_c - dE_c$ the coupling constant g_1 renormalizes according to[40]

$$\frac{\partial g_1}{\partial l} = 2(1-K_s)g_1(l), \tag{3.11}$$

where $l=\ln(1/E_c)$. Thus, g_1 is irrelevant for $K_s<1$ and relevant for $K_s>1$. For repulsive interactions it turns out that $K_s<1$ and therefore the interaction is irrelevant and the system is described by a gapless Luttinger liquid. If interactions are attractive $K_s>1$ and the backscattering is relevant leading to the opening of a *spin gap* signaling tendency to superconducting fluctuations. We have started from a repulsive model and therefore we assume all the interactions to be repulsive throughout the paper. Although the backscattering term is irrelevant it is well known that the slowest decaying correlation function for a repulsively interacting Luttinger liquid is the CDW one (it is equivalent to the case of $K_\rho<1$).

If the CDW is commensurate with the lattice (as it seems to be in the case of LSCO) then the lattice pinning is strong and the system becomes insulating.[28] If the CDW is incommensurate then a Luttinger liquid state is possible but any amount of disorder leads to CDW pinning driving the system again towards an insulating regime. This is certainly undesirable from the phenomenological point of view since these materials are still metallic at finite temperatures. Many approaches that start from a purely 1D description of the stripes are confronted with this serious problem.[61] These approaches usually invoke the introduction of long-range interactions among the stripes. It is hard to reconcile the long range interactions between different stripes with the short range interacting within the stripes. For that to happen the dielectric function of the material should be extremely anisotropic (experimentally it is known that this is not the case[62]). We believe that once the long-range forces have done their work in creating the stripe array the final metallic state screens any long-range forces left, leading the system to purely local interactions. If the interstripe interactions remain long-range the ground state should be a Wigner crystal[22] instead of a CDW and in both cases the stripes should be insulating. We stick with the assumption that after the CCS is formed the interactions between the elementary excitations discussed here are local. The ground state is a CDW state and therefore insulating. In fact this is the picture that arises from mean-field approaches to the stripe problem since in these mean-field studies the stripes are noninteracting.[52,53]

In the case of the CCS state where there are an infinite number of stripes we add a new index to the fermion operator, $\psi_{n,\sigma}(s)$, where n labels the particular stripe. The Fourier transform of the electron operator can be defined as

$$\psi_{n,\sigma}(x) = \sqrt{Na}\int_{-\pi/a}^{\pi/a}\frac{dk_x}{2\pi}\int_{-\pi/(Na)}^{\pi/(Na)}\frac{dk_y}{2\pi}e^{i(k_x x + k_y nNa)}\psi_{\sigma,\mathbf{k}}, \tag{3.12}$$

where the anticommutation relations are preserved as $[\{\psi_{n,\sigma}(s),\psi_{m,\sigma'}(x')\}=\delta(x-x')\delta_{n,m}\delta_{\sigma,\sigma'}$ and $\{\psi_{\sigma,\mathbf{k}},\psi_{\sigma',\mathbf{k}'}\} = (2\pi)^2\delta(\mathbf{k}-\mathbf{k}')]$.

IV. COUPLING BETWEEN STRIPE FERMIONS AND 2D BOSONS

As we have discussed in the past two sections in the absence of coupling between stripe fermions and bosons the ground state is insulating: a Bose-Mott insulator in the antiferromagnetic regions and a CDW state along the stripes. In order to determine the coupling between bosons and fermions we consider simple arguments based on conservation of momentum and energy.

As we have argued, the nature of the elementary excitations in the problem depends strongly on their position in real space. On the one hand, bound pair bosons are not eigenstates of the Luttinger liquid. On the other hand, Luttinger liquid bosons (collective sound waves) are not elementary excitations of the doped Mott insulator. Thus, the simplest process that couples these two types of excitations is a decay process in which bosons are continuously transformed in stripe fermions and vice versa.

Let us consider the case of static stripes. The relevant coupling that conserves linear momentum can be written as

$$V_{\mathbf{k}}(\mathbf{q})P_{-\mathbf{k}}\psi_{\mathbf{q},\uparrow}\psi_{\mathbf{k}-\mathbf{q},\downarrow}, \tag{4.1}$$

where $V_{\mathbf{k}}(\mathbf{q})$ is the boson-stripe coupling that, by conservation of angular momentum, has $d_{x^2-y^2}$ symmetry [that is, $V_{\mathbf{k}}(0)\propto\cos(k_y a)-\cos(k_x a)$]. This is a four-fermion coupling in terms of the original operators [see Eq. (2.1)] and describes a fermionic scattering process where in the final state two electrons always end up in a bound state. This kind of coupling looks unusual because there are only annihilation operators and it corresponds to the vertex shown in Fig. 5(a). It only occurs in this form because we use the electrons as asymptotic states in the scattering process. In fact, Eq. (4.1) is similar to fermion-boson models that have appeared in the literature of superconductivity over the years.[63,64] Notice that since the boson is composed of holes the direction of the current is inverted (holes moving forward are equivalent to electrons moving backward and vice versa). Thus to destroy a boson is equivalent to create two electrons in the antiferromagnetic background. In the process (4.1) a pair of holes is destroyed in the antiferromagnet while a pair of electrons from the stripe hop into the antiferromagnetic media [see Fig. 5(b)].

Let us assume that the coupling between bosons and stripes is weak so that the important degrees of freedom only exist close to the chemical potential. Thus, when a boson hits a stripe there are two main process for decaying. In the first one the two stripe fermions have opposite momentum, that is, $\mathbf{q}\approx k_F\mathbf{x}$ and $\mathbf{k}-\mathbf{q}\approx -k_F\mathbf{x}$ [as shown in Fig. 5(b)]. Thus, linear momentum conservation implies $\mathbf{k}=0$. Moreover, angular momentum cannot be conserved in the scattering pro-

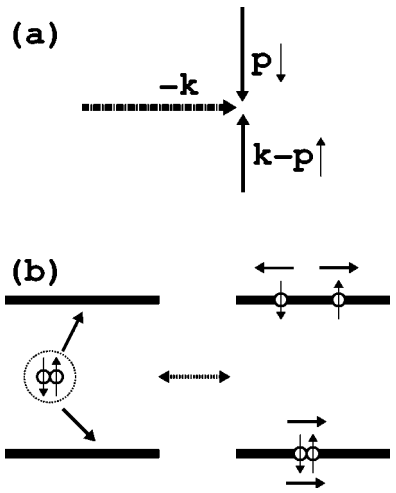

FIG. 5. Basic decaying processes for static stripes: (a) momentum exchange; (b) real space process where a boson decays into stripe fermions.

cess because, relative to the stripe, the two outgoing holes have zero angular momentum while the pair have finite angular momentum. Therefore, we need a sink of momentum. The other process corresponds to boson decay into two fermion moving in the same direction, that is, $\mathbf{q}\approx k_F\mathbf{x}$ and $\mathbf{k}-\mathbf{q}\approx k_F\mathbf{x}$. This process implies $k_x=2k_F$ and $k_y=0$. This is equivalent to a boson moving in the antiferromagnetic ladder parallel to the stripe. Since the boson wave function is restricted to the antiferromagnetic region the amplitude of this coupling is suppressed. The conclusion from this argument is that for a static stripe the decay processes have vanishing phase space.

Consider also the difference between longitudinal and diagonal stripes as shown in Fig. 6. For a longitudinal stripe the coupling between bosons and stripes is possible because the stripes are oriented along the lobes of the boson $d_{x^2-y^2}$ wave function while in the diagonal case the stripes are oriented along the nodes. Thus the coupling between diagonal

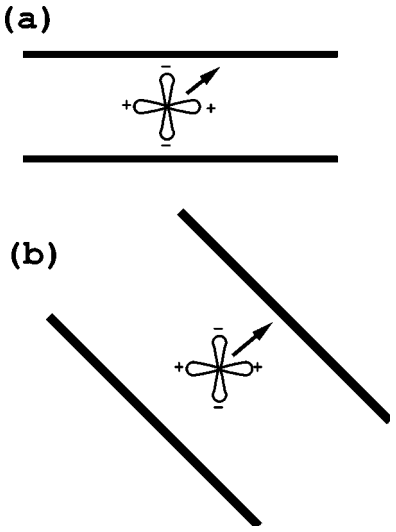

FIG. 6. Coupling of a $d_{x^2-y^2}$ boson wave function to (a) longitudinal and (b) diagonal stripes.

stripes is further reduced as compared to the longitudinal case. Therefore, for diagonal stripes superconductivity is not possible in our model because of the nature of the boson wave function. In fact, recent neutron scattering experiments observed diagonal stripes in the insulating phase of LSCO while vertical and horizontal stripes are observed in the superconducting phase.[65] Moreover, notice that for a given orientation of the boson wave function, horizontal (vertical) stripes couple to the positive (negative) lobe of the wave function.

As we pointed out above we need a sink (a particle or collective mode) for the momentum transferred from the bosons. Moreover, since the bosons are spinless the particle that carries the momentum cannot be a spin-wave or paramagnon. The simplest choice is a vibrational mode: the boson collides with the stripe, breaks into two fermions and produces a vibration. This vibration is due to the fluctuations of the smectic phase of the stripe array. Again, as we stressed previously, stripes are not isolated objects that are independent of each other. Long-range forces, like the Coulomb, the Casimir or simply entropic repulsion keeps them apart and generate a finite stiffness.

Consider the coupling of the bosons with the stripe fermions when the stripe fluctuates a distance u from its equilibrium position. We write the local coupling as

$$P(\mathbf{r})\psi_{\uparrow}(\mathbf{r})\psi_{\downarrow}(\mathbf{r})\delta(y-nNa-u(x,nNa)), \qquad (4.2)$$

where we ensure that the coupling occurs at the position of the stripes. If we assume that CCS fluctuations are small compared to the periodicity in the system, that is, $|u|\ll Na$, the stripes fluctuate in a distance scale much smaller than the interstripe distance. We can expand the Dirac delta function. The first term is just the static stripe problem we discussed above and, as we argued, has no phase space. We therefore keep the next term in the expansion leading to a coupling of the form

$$-V_{\mathbf{k}}(\mathbf{q})\lambda_{\mathbf{p},\alpha}P_{-\mathbf{k}}\psi_{\mathbf{q},\uparrow}\psi_{\mathbf{k}-\mathbf{p}-\mathbf{q},\downarrow}(b_{\mathbf{p},\alpha}+b^{\dagger}_{-\mathbf{p},\alpha}), \qquad (4.3)$$

where $\lambda_{\mathbf{p},\alpha}\propto p_y\rightarrow 0$ when $p\rightarrow 0$ is the stripe-vibration coupling constant. The two processes associated with the interaction in Eq. (4.3) are shown in Fig. 7.

Before we study Eq. (4.3) in detail it is worth investigating the problem via second order time dependent perturbation theory. The transition probability between nonperturbed states is given by

$$W(t)\approx\sum_{\mathbf{k},\mathbf{p},\mathbf{q},\alpha}4|V_{\mathbf{k}}(\mathbf{q})|^2|\lambda_{p,\alpha}|^2(1-\overline{n}_{\mathbf{q}})(1-\overline{n}_{\mathbf{k}+\mathbf{p}-\mathbf{q}})$$
$$\times\frac{\sin^2[(-E_{\mathbf{k}}\pm\Omega_{\mathbf{p},\alpha}+\epsilon_{q_x}+\epsilon_{k_x+p_x-q_x})t/2]}{(-E_{\mathbf{k}}\pm\Omega_{\mathbf{p},\alpha}+\epsilon_{q_x}+\epsilon_{k_x+p_x-q_x})^2}, \qquad (4.4)$$

where $\overline{n}_{\mathbf{q}}$ is the Fermi-Dirac distribution function. Observe that the probability amplitude only grows with time for states such that

$$-E_{\mathbf{k}}\pm\Omega_{\mathbf{p},\alpha}=-\epsilon_{q_x}-\epsilon_{k_x+p_x-q_x}, \qquad (4.5)$$

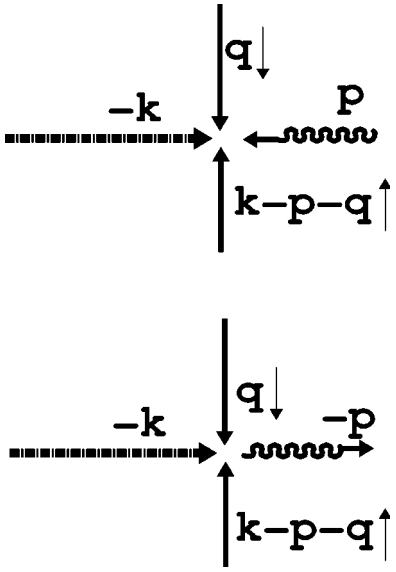

FIG. 7. Scattering processes including vibrations.

where the plus (minus) sign implies absorption (emission) of a vibration. Remembering that $\epsilon_q \approx \mu$ we obtain the condition that $-E_{0,k_y} \pm \Omega_{0,k_y,\alpha} = -2\mu$. Moreover, because λ_{k_y} vanishes at $k_y \to 0$ the largest coupling between bosons and stripes happens at *finite* momentum. Indeed, the coupling increases as one goes towards the Brillouin zone edge. Thus, the largest coupling in the problem occurs at $\mathbf{K}=(0,\pi/a)$, that is, *perpendicular* to the stripes. It may be somewhat surprising that the most important part of the physics occurs in a direction perpendicular to the orientation of the stripes but this effect has been already observed in numerical simulations of spin-fermion models with stripe formation.[9] This result indicates that in the case of the boson condensation we should expect it to occur close to **K** (as photoemission experiments indicate[54]) but not only that: we expect a CCS distortion at the same wave vector so that the momentum

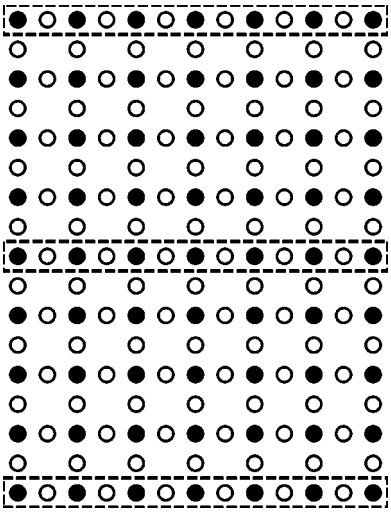

FIG. 8. Real space picture of the normal phase. Open circles: O atoms; filled circles: Cu atoms; dashed line: equilibrium position of the stripes.

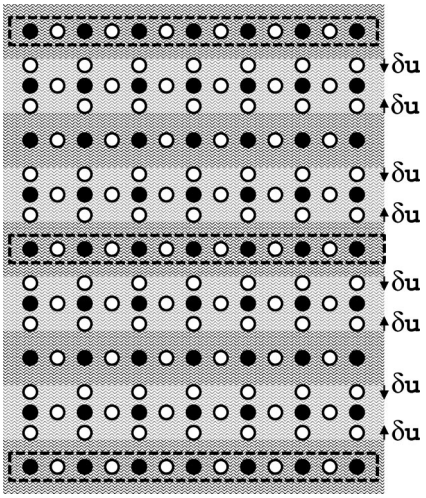

FIG. 9. Real space picture of the ordered phase: δu indicates the possible motion of the O atoms with doubling of the unit cell perpendicular to the stripe orientation. The difference in gray levels represents the modulation of the superfluid density.

carried by the superfluid bosons is transferred to the lattice. Thus, there can be a double condensation: the superfluid density is modulated with finite wave vector **K** and the CCS deforms in order to follow the variation of the superfluid density. Therefore a condensation of the CCS at finite wave vector induces phonon anomalies at the same wave vector. Thus, we expect phonon softening close to $(0,\pi/a)$ together with the condensation of bosons in the same point in momentum space. In the normal state the situation is depicted in Fig. 8: there is no superfluidity and no lattice distortions. In the ordered phase we expect the situation shown in Fig. 9 where the superfluid density is modulated with wavelength a/π in the direction perpendicular to the stripes and the lattice is distorted in the same direction with the same wave vector. Notice that there is a doubling of the unit cell in the direction perpendicular to the stripes and therefore the CCS is dimerized. This effect is similar to the Peierls distortion in 1D systems or the Jahn-Teller effect where there is a lowering of the symmetry with a simultaneous gain in energy.[66] In momentum space the situation is depicted in Fig. 10: phonon anomalies and superfluidity should appear at same point on the edge of the Brillouin zone.

Observe that due to the conservation of energy, $-E_{0,k_y} + 2\mu = \pm \Omega_{0,k_y,\alpha}$, the stability condition (1.20) is satisfied only if emission is allowed. One can generate a boson by creating two electrons at the stripe at the cost of absorption of a vibration. This can be clearly seen in the diagram in Fig. 1 where in order to create a boson two electrons occupying the boson state have to be excited to the Fermi surface by the absorption of a vibration. Conversely, two holes from the stripe hop into the antiferromagnetic ladder and form a boson. Thus, our conclusion is that the most relevant coupling in this problem is given by

$$H_c = -\sum_{\mathbf{k},\mathbf{p},\mathbf{q},\alpha} V_{\mathbf{k}}(\mathbf{q}) \lambda_{\mathbf{p},\alpha} P_{\mathbf{k}} \psi_{\mathbf{q},\uparrow} \psi_{-\mathbf{k}+\mathbf{p}-\mathbf{q},\downarrow} b^{\dagger}_{\mathbf{p},\alpha} + \text{H.c.} \tag{4.6}$$

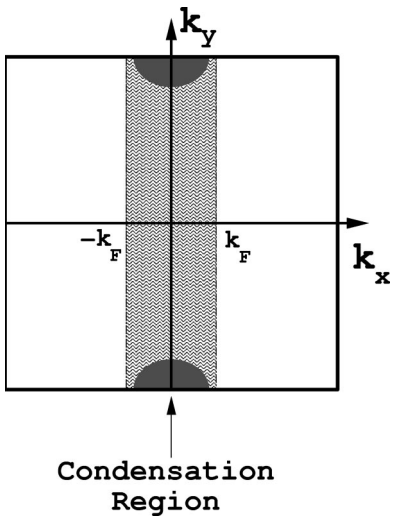

FIG. 10. Momentum space picture of the condensation region for stripes oriented along the X direction.

V. MEAN-FIELD THEORY

In order to understand the phase diagram of this problem it is illuminating to consider the mean-field solution. Let us rewrite the full Hamiltonian by collecting all the terms given in the previous sections (we neglect the interactions between electrons in the stripe for the moment being). The Hamiltonian is written as

$$H=\sum_{\mathbf{k}}\left[(-E_{\mathbf{k}}+2\mu)P_{\mathbf{k}}^{\dagger}P_{\mathbf{k}}+\Omega_{\mathbf{k}}b_{\mathbf{k}}^{\dagger}b_{\mathbf{k}}\right]+\sum_{\mathbf{k},\sigma}(\epsilon_{\mathbf{k}}-\mu)\psi_{\mathbf{k},\sigma}^{\dagger}\psi_{\mathbf{k},\sigma}+\frac{U}{2}\sum_{i}N_i^2-\sum_{\mathbf{k},\mathbf{p},\mathbf{q}}V_{\mathbf{k}}(\mathbf{q})\lambda_{\mathbf{p}}P_{\mathbf{k}}\psi_{\mathbf{q},\uparrow}\psi_{-\mathbf{k}+\mathbf{p}-\mathbf{q},\downarrow}b_{\mathbf{p}}^{\dagger}+\text{H.c.}, \quad (5.1)$$

where we have simplified the problem by considering a single polarization of the vibration field perpendicular to the stripes.

Notice that the Hamiltonian is invariant under the gauge transformation

$$P\rightarrow P\,e^{-i\varphi_B},\qquad b\rightarrow b\,e^{+i\varphi_V},\qquad \psi\rightarrow\psi\,e^{+i(\varphi_B+\varphi_V)/2} \quad (5.2)$$

implying the phase of the bosons and vibrations are coupled through the fermionic degrees of freedom. This symmetry is the one that is broken in the ordered state.

In the mean-field theory we split the coupling term into three different pieces leading to a mean-field Hamiltonian of the form: $H_{\mathrm{MF}}=H_P+H_B+H_S$, where

$$H_P=\sum_{\mathbf{k}}\left[(-E_{\mathbf{k}}+2\mu)P_{\mathbf{k}}^{\dagger}P_{\mathbf{k}}-g_{P,\mathbf{k}}P_{\mathbf{k}}-g_{P,\mathbf{k}}^{*}P_{\mathbf{k}}^{\dagger}\right]+\frac{U}{2}\sum_{i}N_i^2,$$

$$H_B=\sum_{\mathbf{k}}\left[\Omega_{\mathbf{k}}b_{\mathbf{k}}^{\dagger}b_{\mathbf{k}}-g_{B,\mathbf{k}}b_{\mathbf{k}}-g_{B,\mathbf{k}}^{*}b_{\mathbf{k}}^{\dagger}\right], \quad (5.3)$$

$$H_S=\sum_{\mathbf{k},\sigma}(\epsilon_{\mathbf{k}}-\mu)\psi_{\mathbf{k},\sigma}^{\dagger}\psi_{\mathbf{k},\sigma}-\sum_{\mathbf{k},\mathbf{p},\mathbf{q}}g_S(\mathbf{k},\mathbf{p},\mathbf{q})\psi_{\mathbf{q},\uparrow}\psi_{-\mathbf{k}+\mathbf{p}-\mathbf{q},\downarrow}+\text{H.c.},$$

where

$$g_{P,\mathbf{k}}=\sum_{\mathbf{p},\mathbf{q}}V_{\mathbf{k}}(\mathbf{q})\lambda_{\mathbf{p}}\langle\psi_{\mathbf{q},\uparrow}\psi_{-\mathbf{k}+\mathbf{p}-\mathbf{q},\downarrow}\rangle\langle b_{\mathbf{p}}\rangle,$$

$$g_{B,\mathbf{k}}=\lambda_{\mathbf{k}}\sum_{\mathbf{p},\mathbf{q}}V_{\mathbf{p}}(\mathbf{q})\langle P_{\mathbf{p}}\rangle\langle\psi_{\mathbf{q},\uparrow}\psi_{\mathbf{p}+\mathbf{k}-\mathbf{q},\downarrow}\rangle, \quad (5.4)$$

$$g_S(\mathbf{k},\mathbf{p},\mathbf{q})=V_{\mathbf{k}}(\mathbf{q})\lambda_{\mathbf{p}}\langle P_{\mathbf{k}}\rangle\langle b_{\mathbf{p}}\rangle$$

are the mean-field coupling constants. From Eq. (5.3) it becomes obvious the symmetry breaking processes in the ordered phase: spontaneous gauge symmetry breaking leading to superfluidity of the bosons, superconductivity of the fermions and spontaneous symmetry breaking of translational symmetry of the CCS.

Thus, for a transition to a nontrivial conducting state the coupling λV is fundamental. The simplest conducting state is the superfluid state where the charged bosons condense into a macroscopic condensate. In accordance with the discussion in the previous section, conservation rules tell us that the largest coupling occurs at the largest momentum perpendicular to the stripes. Suppose the bosons condense at $\mathbf{k}=\mathbf{K}$, that is, $\langle P_{\mathbf{K}}\rangle=\langle P_{\mathbf{K}}^{\dagger}\rangle=\sqrt{N_B}$. In order to conserve momentum in the interaction in Eq. (5.1) and preserve time reversal symmetry one should also require the vibrations to condense, that is, $\langle b_{\mathbf{K}}^{\dagger}\rangle=\langle b_{\mathbf{K}}\rangle=\sqrt{N_V}$. Notice that the Cooper pairs have zero center of mass momentum. This is equivalent to a static distortion of the lattice with modulation $\mathbf{K}$. Thus, at the mean-field level, a transition to a superfluid state is accompanied by a static lattice distortion. We notice here that the choice of the largest coupling as the one that drives the system into the ordered state is the traditional one in mean-field theories. In fact, the doubling of the unit cell proposed here also occurs in some large N treatments of the t-J models without phonons.[67,68] Thus, this tendency of lowering the energy of the system by breaking the lattice symmetry seems to be quite generic.

The solution of the bosonic problem is straightforward because it only requires a shift of the boson operators by a constant:

$$P_{\mathbf{K}} \rightarrow P_{\mathbf{K}} - \frac{g_{P,\mathbf{K}}}{-E_{\mathbf{K}}+2\mu},$$

$$b_{\mathbf{K}} \rightarrow b_{\mathbf{K}} - \frac{g_{B,\mathbf{K}}}{\Omega_{\mathbf{K}}} \tag{5.5}$$

implying that the actual CCS distortion is given by

$$\delta u(\mathbf{K}) = \frac{\sqrt{2}\mathrm{Re}[g_{B,\mathbf{K}}]}{M_V^{1/2}\Omega_{\mathbf{K}}^{3/2}}, \tag{5.6}$$

where M_V is the mass associated with the vibrations.

The electronic problem can be also readily solved since it is a problem of electrons with a pairing potential. We can write H_S as

$$H_S = \sum_{\mathbf{k}} \left[(\epsilon_{\mathbf{k}} - \mu) \sum_{\sigma} \psi^{\dagger}_{\mathbf{k},\sigma}\psi_{\mathbf{k},\sigma} - \Delta_{\mathbf{k}}\psi_{\mathbf{k},\uparrow}\psi_{-\mathbf{k},\downarrow} + \mathrm{H.c.} \right], \tag{5.7}$$

where the superconducting gap is given by

$$\Delta_{\mathbf{k}} = \lambda_{\mathbf{K}} a^2 \sqrt{\sigma_B^0 \sigma_V^0} V_{\mathbf{k}}(0) \tag{5.8}$$

and therefore has $d_{x^2-y^2}$ symmetry as the pairs that generate it in the first place. Here we have defined

$$\sigma_B^0 = \frac{N_B}{S},$$

$$\sigma_V^0 = \frac{N_V}{S} \tag{5.9}$$

as the superfluid and distortion densities, respectively. The Hamiltonian (5.7) can be diagonalized by a Bogoliubov transformation:

$$\psi_{\mathbf{k},\uparrow} = \cos(\theta_{\mathbf{k}}) d_{\mathbf{k},\uparrow} + \sin(\theta_{\mathbf{k}}) d^{\dagger}_{\mathbf{k},\downarrow},$$

$$\psi^{\dagger}_{-\mathbf{k},\uparrow} = \cos(\theta_{\mathbf{k}}) d^{\dagger}_{\mathbf{k},\downarrow} - \sin(\theta_{\mathbf{k}}) d_{\mathbf{k},\uparrow}, \tag{5.10}$$

where

$$\tan(2\theta_{\mathbf{k}}) = \frac{\Delta_{\mathbf{k}}}{\epsilon_{\mathbf{k}} - \mu} \tag{5.11}$$

and

$$H_S = \sum_{\mathbf{k}} e_{\mathbf{k}} d^{\dagger}_{\sigma,\mathbf{k}} d_{\sigma,\mathbf{k}}, \tag{5.12}$$

where

$$e_{\mathbf{k}} = \sqrt{(\epsilon_{\mathbf{k}} - \mu)^2 + |\Delta_{\mathbf{k}}|^2} \tag{5.13}$$

is the quasiparticle dispersion.

The free energy per unit of area, $f(T)$, can be obtained in a straightforward way:

$$f(T) = (-E_{\mathbf{K}} + 2\mu)\sigma_B^0 + \frac{U}{2}(\sigma_B^0)^2 + \Omega_{\mathbf{K}}\sigma_V^0 + \frac{1}{S}\sum_{\mathbf{k}} [(\epsilon_{\mathbf{k}} - \mu) - e_{\mathbf{k}}(1 - 2n_{\mathbf{k}})], \tag{5.14}$$

where

$$n_{\mathbf{k}} = \frac{1}{e^{\beta e_{\mathbf{k}}} + 1} \tag{5.15}$$

is the quasiparticle occupation. We minimize Eq. (5.14) keeping the number of particles fixed. We obtain two equations:

$$-E_{\mathbf{K}} + 2\mu + U\sigma_B^0 = \frac{\lambda_{\mathbf{K}}^2 a^2 \sigma_V^0}{2S} \sum_{\mathbf{k}} \frac{V_{\mathbf{k}}^2}{e_{\mathbf{k}}} \tanh(\beta e_{\mathbf{k}}/2),$$

$$\Omega_{\mathbf{K}} = \frac{\lambda_{\mathbf{K}}^2 a^2 \sigma_B^0}{2S} \sum_{\mathbf{k}} \frac{V_{\mathbf{k}}^2}{e_{\mathbf{k}}} \tanh(\beta e_{\mathbf{k}}/2) \tag{5.16}$$

and therefore

$$\Omega_{\mathbf{K}}\sigma_V^0 = \sigma_B^0(-E_{\mathbf{K}} + 2\mu + U\sigma_B^0) \approx U(\sigma_B^0)^2 \tag{5.17}$$

that is, the superfluid and vibration density are tied to each other. The last line in Eq. (5.17) comes from the fact that we expect $U \gg -E_{\mathbf{K}} + 2\mu$. Using the above equation we reduce the mean-field problem to a single equation

$$\Omega_{\mathbf{K}} = \frac{\lambda^2 a^2 \sigma_B^0}{2S} \sum_{\mathbf{k}} \frac{V_{\mathbf{k}}^2}{e_{\mathbf{k}}} \tanh(\beta e_{\mathbf{k}}/2). \tag{5.18}$$

Notice that although the coupling constant has *d*-wave symmetry the Fermi surface of an infinite array of 1D stripes has no curvature [as shown in Fig. 4(a)] because the fermions do not propagate in the direction perpendicular to the stripes. Thus, although the order parameter in Eq. (5.8) has $d_{x^2-y^2}$ symmetry, the Fermi surface is fully gapped. The bosons have a tendency to produce a *d*-wave superconductor but the condensate only takes advantage of one of the lobes of the boson wave function. Thus, the final result is an *s*-wave state.

As we discussed in Sec. IX a true *d*-wave superconductor can only develop if the system is made out of domains with horizontal and vertical stripes or if one particle tunneling is included [changing the fermion dispersion relation to Eq. (3.3)]. The case for domains separated by boundaries is almost metallurgically unavoidable in cuprates. Because the domains are usually large the superconducting properties of the domains can be studied as if the domains are macroscopic superconductors. Furthermore, if interstripe tunneling is included the Fermi surface becomes rounded leaving space for a true *d*-wave state to develop (although with a rather anisotropic properties).

Taking the geometry of the Fermi surface into account we can simplify the above equation by using the fact that Eq. (5.15) only depends on k_x and therefore the sum over k_y just gives N_s/N leading to a simpler equation

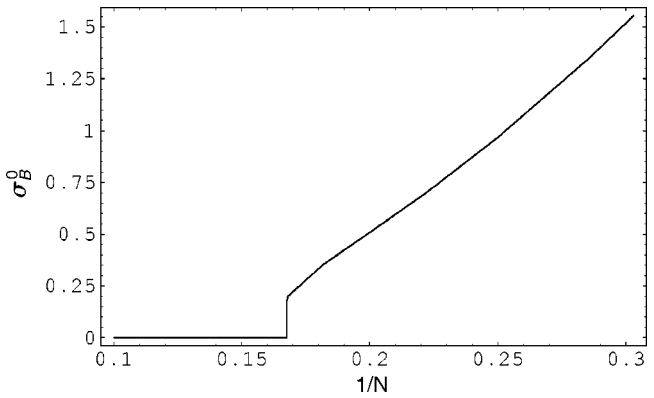

FIG. 11. σ_B^0 as a function of $1/N$.

$$\Omega_{\mathbf{K}}=\frac{\lambda^2 V_0^2 a \sigma_B^0}{N}\int_0^{\pi/a}\frac{dk}{2\pi}\frac{\tanh(\beta e_k/2)}{e_k}, \qquad (5.19)$$

where V_0 is the strength of the coupling $V_{\mathbf{k}}(\mathbf{q})$ at the stripe and

$$e_k=\sqrt{(\epsilon_k-\mu)^2+V_0^2\lambda^2(\sigma_B^0)^2(-E_{\mathbf{K}}+2\mu+U\sigma_B^0)/\Omega_{\mathbf{K}}}.$$

This equation can be solved numerically assuming $\epsilon_{k_x}=-t\cos(k_x a)$.

We can now study the phase diagram as a function of the coupling λV and the distance between stripes N. We expect the characteristic energy scales of the problem to be dependent on the distance between stripes. This is certainly true for the case of antiferromagnetic order where the distance between the stripes determines the strength of the quantum fluctuations.[44] Thus we expect the phonon and boson spectra to change with N, that is, there must be a continuous change in the spectral weight with the interstripe distance. In this paper, for simplicity, we assume that this change is small and the density of states does not change with N (see, however, Ref. 9). In Fig. 11 we plot σ_B^0 as a function of $1/N$ for $E_{\mathbf{K}}=1.9t$, $\mu=-0.6t$, $\Omega_{\mathbf{K}}=1.2\times10^{-3}t$, $U=4t$, and $\lambda V=0.5t$ at $T=0$. Notice that there is a discontinuous jump in σ_B^0 at $N_{sp}\approx 6$ and it grows in a quasilinear way with $1/N$. In Fig. 12 we plot σ_B^0 as a function of T/t for the same parameters given above. Notice that σ_B^0 is weakly dependent on the temperature up to $T=T_{sp}\approx 3.5t$ where it has a discontinuous

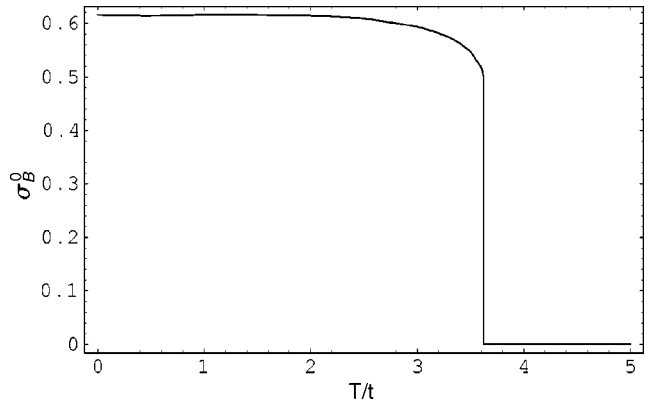

FIG. 12. σ_B^0 as a function of T/t.

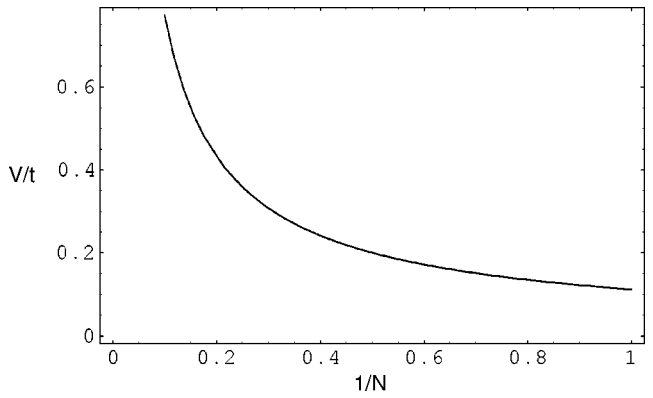

FIG. 13. Critical value of the coupling constant V_c/t as a function of $1/N$.

jump to zero. As one can see from the numerical solution T_{sp} is of the order of the bandwidth of the stripe fermions and therefore very large. As we show below this is not the actual transition temperature because we have not included phase fluctuations into the problem. Thus T_{sp} (like T^* defined in Sec. I) is a crossover temperature scale. We note that $T_{sp}\gg T^*$, that is the mean-field critical temperature is larger than the temperature scale above which makes sense to talk about bosons and confinement. Therefore, T_{sp} is actually not observable since the theory brakes down at T^*.

In order to understand the nature of the phase diagram we simplify the problem by the introduction of a cutoff Λ of the order of the inverse of the lattice spacing and by linearizing the spectrum at the chemical potential. The integral simplifies and at $T=0$ reduces to

$$\Omega_{\mathbf{K}}=\frac{\lambda^2 V_0^2 a\sigma_B^0}{2\pi v_F N}\ln\left(\frac{\Lambda v_F+\sqrt{(\Lambda v_F)^2+\Delta_0^2}}{\Delta_0}\right), \qquad (5.20)$$

where $\Delta_0=V_0\lambda\sigma_B^0\sqrt{(-E_{\mathbf{K}}+2\mu+U\sigma_B^0)/\Omega_{\mathbf{K}}}$ gives the amplitude of the gap. Notice that the transition at the mean-field level is of first order because the right-hand side (RHS) of Eq. (5.20) vanishes as $\sigma_B^0\to 0$ in contrast with the usual BCS equation that contains only the logarithm and is finite in this limit. Some analytical progress can be made in the limit of large U. The critical value of N and the value of the superfluid fraction are given by

$$N_{sp}\approx\frac{\ln(1+\sqrt{2})}{2}\frac{(\lambda V_0)^{4/3}}{(\pi v_F/a)^{1/3}\Omega_{\mathbf{K}}^{2/3}U^{1/3}},$$

$$\sigma_{0,c}\approx\frac{\Omega_{\mathbf{K}}^{1/3}(\pi v_F/a)^{2/3}}{(\lambda V_0)^{2/3}U^{1/3}} \qquad (5.21)$$

that determines the zero temperature mean-field phase diagram. As we vary N there is a critical value of V_0 (that we call V_c) below which order is not possible. In Fig. 13 we plot V_c as a function of $1/N$. We find that $V_c\propto N^{\alpha}$ with $\alpha>0$. This result demonstrates that as the stripes get further apart a larger coupling is required to stabilize long-range order.

As pointed out by EK, the fact that the superfluid density is so low implies large phase fluctuations.[23] If one includes the phase of the order parameters [as in Eq. (5.2)] long-range order requires that

$$\langle P_{\mathbf{K}}\rangle = \langle |P_{\mathbf{K}}|\rangle \langle e^{-i\varphi_B}\rangle \neq 0,$$

$$\langle b_{\mathbf{K}}\rangle = \langle |b_{\mathbf{K}}|\rangle \langle e^{+i\varphi_V}\rangle \neq 0, \tag{5.22}$$

where $\langle |P_{\mathbf{K}}|\rangle \propto \sqrt{\sigma_B^0}$ and $\langle |b_{\mathbf{K}}|\rangle \propto \sqrt{\sigma_V^0}$ are the amplitudes of the order parameters. Thus, ordering also requires $\langle e^{\pm i\varphi_\alpha}\rangle \neq 0$ with $\alpha = B,V$. In our mean-field theory we have fixed arbitrarily $\varphi_\alpha = 0$. This is not correct since fluctuations of φ_α, especially in the 2D system, are fundamental for the development of long-range order. In the next sections we discuss the role of these fluctuations in determining the actual phase diagram. In other words, at high temperatures bosons and superconducting stripe fermions are formed but they are incoherent because of the existence of topological defects: vortex-antivortex pairs in the superfluid and dislocation loops of the distorted CCS. The interesting result of our calculation is that although the amplitudes of the two order parameters appear at the same temperature T^* [as given by Eq. (5.17)] the actual ordering temperatures can be different because they depend on the phase stiffness of the relevant degrees of freedom.

VI. PHASE FLUCTUATIONS

As we discussed at the end of the previous section the problem of phase fluctuations is fundamental for the description of the ordered state. Therefore the topological excitations are fundamental for the determination of the properties of the superconducting state.[69] The simplest way to discuss the importance of phase fluctuations is by studying the partition function of the problem that can be written as

$$Z = \int D\overline{P}DPD\overline{\psi}D\psi D\overline{b}Db\, e^{-\int_0^\beta d\tau L[\overline{P},P,\overline{\psi},\psi,\overline{b},b]}, \tag{6.1}$$

where

$$L = \int d^2r \left\{ \overline{P}(\mathbf{r})[\partial_\tau + E(\nabla) + 2\mu]P(\mathbf{r}) + \frac{U}{2}N^2(\mathbf{r}) + \overline{b}(\mathbf{r})[\partial_\tau + \omega(\nabla)]b(\mathbf{r}) \right\} + \sum_n \left\{ \int dx \sum_\sigma \overline{\psi}_{n,\sigma}(x)[\partial_\tau + \epsilon(\nabla) - \mu]\psi_{n,\sigma}(x) + \int dx \int dx' \sum_{\sigma,\sigma'} \overline{\psi}_{n,\sigma}(x)\psi_{n,\sigma}(x) W_{\sigma,\sigma'}(x-x')\overline{\psi}_{n,\sigma'}(x')\psi_{n,\sigma'}(x') \right\} - \sum_n \int dx V(x,nNa) \times \{P(x,nNa)\psi_{n,\uparrow}(x)\psi_{n,\downarrow}(x)\overline{b}(x,nNa) + \text{c.c.}\} \tag{6.2}$$

is the Lagrangian associated with Hamiltonian (5.1) and β is the inverse temperature. Here we have redefined $\lambda V \to V$ and introduced the electron-electron interactions $W_{\sigma,\sigma'}(x-x')$.

We redefine the fields in order to separate the problem into slow and fast variables:

$$P(\mathbf{r}) = \frac{1}{\sqrt{2}}(P_+(\mathbf{r})e^{-i\mathbf{K}\cdot\mathbf{r}} + P_-(\mathbf{r})e^{i\mathbf{K}\cdot\mathbf{r}}),$$

$$b(\mathbf{r}) = \frac{1}{\sqrt{2}}(b_+(\mathbf{r})e^{-i\mathbf{K}\cdot\mathbf{r}} + b_-(\mathbf{r})e^{i\mathbf{K}\cdot\mathbf{r}}) \tag{6.3}$$

and perform the same expansion as in the case of fermions [see Eq. (3.6)].

Substituting Eqs. (3.6) and (6.3) into the Lagrangian (6.2) we find many terms that oscillate fast. We disregards all the oscillating terms and study slow modes only. The Lagrangian can be written as

$$L = \sum_{\alpha=\pm} L_0[P_\alpha, b_\alpha] + \sum_n L_S[\psi_n] + L_C, \tag{6.4}$$

where

$$L_0[P_\alpha, b_\alpha] = \frac{1}{2}\int d^2r \left\{ \overline{P}_\alpha(\mathbf{r})\left(\partial_\tau + E_0 - \frac{\nabla^2}{2M_B}\right)P_\alpha(\mathbf{r}) + \frac{U}{2}N^2(\mathbf{r}) + \overline{b}_\alpha(\mathbf{r})\left(\partial_\tau + \omega_0 - \frac{\nabla^2}{2M_V}\right)b_\alpha(\mathbf{r}) \right\}, \tag{6.5}$$

where we have assumed that **K** is a *local* minimum in the energy of the bosons and vibrations. This assumption is supported by studies of the t-J model[43] and from the fact that the vibration mode is at the edge of the Brillouin zone. Here, M_B is the effective boson mass. The form of the noninteracting boson-vibration Lagrangian given above is required by Galilean invariance of the problem in the long wavelength limit. Although anisotropies in the dispersion of the modes close to **K** should exist, we disregard them since they are not fundamental in our discussion. Furthermore,

$$L_S[\psi_n]=\int \frac{dx}{a}\Bigg\{\sum_\sigma [\bar{\psi}_{n,R,\sigma}(x)(\partial_\tau+iv_F\partial_x)\psi_{n,R,\sigma}(x)+\bar{\psi}_{n,L,\sigma}(x)(\partial_\tau-iv_F\partial_x)\psi_{n,L,\sigma}(x)]+\sum_{\sigma,\sigma'}[W_{k=0}(\bar{\psi}_{n,R,\sigma}(x)\psi_{n,R,\sigma}(x)$$
$$+\bar{\psi}_{n,L,\sigma}(x)\psi_{n,L,\sigma}(x))(\bar{\psi}_{R,\sigma'}(x,n)\psi_{R,\sigma'}(x,n)+\bar{\psi}_{L,\sigma'}(x,n)\psi_{L,\sigma'}(x,n))]+[W_{k=2k_F}(\bar{\psi}_{n,R,\sigma}(x)\psi_{n,L,\sigma}(x)$$
$$+\bar{\psi}_{n,L,\sigma}(x)\psi_{n,R,\sigma}(x))(\bar{\psi}_{R,\sigma'}(x,n)\psi_{L,\sigma'}(x,n)+\bar{\psi}_{L,\sigma'}(x,n)\psi_{R,\sigma'}(x,n))]\Bigg\} \tag{6.6}$$

is the isolated stripe Lagrangian ($W_{\mathbf{k}}$ is the Fourier transform of the electron-electron interaction) and

$$L_C=-\frac{1}{2}\sum_n \int dx V(x,n)[P_+(x,n)\bar{b}_+(x,n)+P_-(x,n)\bar{b}_-(x,n)][\psi_{n,R,\uparrow}(x)\psi_{n,L,\downarrow}(x)+\psi_{n,L,\downarrow}(x)\psi_{n,R,\uparrow}(x)]+\text{c.c.} \tag{6.7}$$

is the boson-stripe-vibration coupling.

Due to the symmetry we simplify the problem by changing to amplitude-phase modes via

$$P_+=P_-=\sqrt{\sigma_B}e^{-i\varphi_B},$$
$$b_+=b_-=\sqrt{\sigma_V}e^{+i\varphi_V} \tag{6.8}$$

and use the bosonization technique described previously so that

$$L=\sum_n L_{1D}[\theta_{n,\rho},\phi_{n,s}]+\sum_{\alpha=B,V}L_\alpha[\sigma_\alpha,\varphi_\alpha]+L_C[\Phi,\varphi,\theta_\rho,\phi_s], \tag{6.9}$$

where $L_{1D}[\theta_{n,\rho},\phi_{n,s}]$ is given in Eq. (3.9). Furthermore,

$$L_\alpha=\int d^2r\left[i\sigma_\alpha\partial_\tau\varphi_\alpha+E_\alpha\sigma_\alpha+\frac{U_\alpha}{2}\sigma_\alpha^2+\frac{\sigma_\alpha^{-1}}{8M_\alpha}(\nabla\sigma_\alpha)^2+\frac{\sigma_\alpha}{2M_\alpha}(\nabla\varphi_\alpha)^2\right], \tag{6.10}$$

with $E_B=E_{\mathbf{K}}$, $E_V=\Omega_{\mathbf{K}}$, $U_B=U$, and $U_V=0$ and

$$L_C=-\sum_n \int \frac{dx}{\pi a}V(x,nNa)\sqrt{\sigma_B\sigma_V}e^{-i(\varphi_B(x,nNa)+\varphi_V(x,nNa)-\sqrt{2\pi}\theta_{n,\rho}(x))}\cos(\sqrt{2\pi}\phi_{n,s}(x))+\text{c.c.} \tag{6.11}$$

Observe that the gauge symmetry discussed in the previous section is explicit in the Lagrangian. In order to gauge away the phase fields we define a new bosonic field θ_c:

$$\theta_{n,c}(x)=\theta_{n,\rho}(x)-\frac{\varphi_B(x,nNa)+\varphi_V(x,nNa)}{\sqrt{2\pi}} \tag{6.12}$$

so that L_C simplifies to

$$L_C=-\sum_n \int \frac{dx}{\pi a}V(x,nNa)\sqrt{\sigma_B\sigma_V}\cos(\sqrt{2\pi}\theta_{n,c}(x))\cos(\sqrt{2\pi}\phi_{n,s}(x)) \tag{6.13}$$

and in the final Lagrangian we replace θ_ρ by θ_c and add a new term that reads

$$L_I[\varphi]=\sum_n \int dx\Bigg\{\frac{K_\rho}{4\pi v_\rho}[(\partial_\tau[\varphi_B(x,nNa)+\varphi_V(x,nNa)])^2+v_\rho^2(\partial_x[\varphi_B(x,nNa)+\varphi_V(x,nNa)])^2]$$
$$+\frac{K_\rho}{\sqrt{2\pi}v_\rho}[\partial_\tau[\varphi_B(x,nNa)+\varphi_V(x,nNa)]\partial_\tau\theta_{n,c}(x)+v_\rho^2\partial_x[\varphi_B(x,nNa)+\varphi_V(x,nNa)]\partial_x\theta_{n,c}(x)]\Bigg\}. \tag{6.14}$$

The physical interpretation of Eq. (6.12) is quite interesting. Observe that the fermion current along the stripe is

$$j_n(x)=\frac{1}{\sqrt{\pi}}\partial_x\theta_{n,\rho}(x)$$
$$=\frac{1}{\sqrt{\pi}}\partial_x\theta_{n,c}(x)+\frac{1}{\pi\sqrt{2}}\partial_x\varphi_B(x,nNa)$$
$$+\frac{1}{\pi\sqrt{2}}\partial_x\varphi_V(x,nNa), \tag{6.15}$$

where the first term is just the normal current, the second term is the superfluid *Josephson* current and the last term is a current that is driven by the dislocations in the CCS. Moreover, observe that the bosonic field θ_c is still coupled to the phase fields through Eq. (6.14). Thus, the stripes carry along both superfluid and CCS distortions.

In order to proceed with the calculations we study the phase fluctuations around the saddle point equations for the complete action. At the saddle point we have $\sigma_\alpha=\sigma_\alpha^0$, $\varphi_\alpha=0$ with $\alpha=B,V$. This just gives the mean-field solution given in Sec. V with the difference that at the saddle point the stripe action reads (we drop the subscript n)

$$S=\int_0^\beta d\tau dx\left\{\frac{K_\rho}{2v_\rho}[[\partial_\tau\theta_c(x,\tau)]^2+v_\rho^2[\partial_x\theta_c(x,\tau)]^2]\right.$$
$$\left.+\frac{1}{2K_sv_s}[[\partial_\tau\phi_s(x,\tau)]^2+v_s^2[\partial_x\phi_s(x,\tau)]^2]\right\}$$
$$-\int_0^\beta d\tau\int dx\,\tilde{V}\cos(\sqrt{2\pi}\theta_c(x))\cos(\sqrt{2\pi}\phi_s(x)). \tag{6.16}$$

Equation (6.16) is the interacting version of the Hamiltonian used in the previous section with $\tilde{V}=\sqrt{\sigma_B^0\sigma_V^0}V$. The last term in Eq. (6.16) is the Josephson coupling induced between different stripes by the bosons. A similar coupling was proposed for the case where stripes cross each other.[70] The effect of the pairing term can be easily seen in a renormalization group calculation in first order in V. Repeating the RG calculation done in Sec. III we can easily show that the dimensionless coupling constant $v=V/E_c$ renormalizes as[70]

$$\frac{dv}{dl}=\left[2-\frac{1}{2}\left(K_s+\frac{1}{K_\rho}\right)\right]v(l). \tag{6.17}$$

Notice that the operator associated with V is relevant when $K_s+K_\rho^{-1}<4$ and irrelevant otherwise. For a Hubbard model with local repulsion U_H it requires that $U_H>1.8t$ for the operator to become irrelevant. Thus, even if the stripes are deep inside of a CDW state the coupling V drives the system into a superconductor. When V is large then the last term of Eq. (6.16) acquires a expectation value and the bosonic fields get pinned at their minima:

$$\theta_c=\sqrt{\frac{\pi}{2}}(n+m),$$
$$\phi_s=\sqrt{\frac{\pi}{2}}(n-m), \tag{6.18}$$

where n and m are integers. Thus, from Eq. (6.15) one sees that the only currents circulating along the stripes are the Josephson currents due to the superfluid and CCS fluctuations. Fluctuations around the minima (6.18) are massive (although a superfluid current still flows) and a spin gap opens in the spectrum. Indeed, a simple integration of Eq. (6.17) gives

$$\frac{V_R}{E_R}=\frac{V}{E_c}\left(\frac{E_c}{E_R}\right)^{1-g},$$
$$g=\frac{1}{2}\left(K_s+\frac{1}{K_\rho}\right)-1, \tag{6.19}$$

where V_R and E_R are the renormalized coupling constant and bandwidth. Observe that the RG flow stops when $V_R\approx E_R\approx m$ where

$$m\approx E_c\left(\frac{V}{E_c}\right)^{1/(1-g)}=V\left(\frac{V}{E_c}\right)^{g/(1-g)} \tag{6.20}$$

gives the amplitude of the spin gap (when $g=1$ the operator is marginal and $m\approx E_c\exp\{-E_c/V\}$). Since the stripe modes are massive the coupling terms to the phase modes in Eq. (6.14) are suppressed.

The calculation can proceed in the usual way by expanding the action around the fluctuations of the amplitudes to second order (that is, we write $\sigma_\alpha=\sigma_\alpha^0+\delta\sigma_\alpha$ and integrate over $\delta\sigma$). The final result reads

$$S=\frac{1}{2}\int_0^\beta d\tau\int d^2r\left\{\sum_{\alpha=B,V}\left[i\bar{\sigma}_\alpha\partial_\tau\varphi_\alpha(\mathbf{r},\tau)+\frac{\kappa_\alpha}{4}[\partial_\tau\varphi_\alpha(\mathbf{r},\tau)]^2\right.\right.$$
$$\left.+\frac{\sigma_\alpha}{M_\alpha}[\nabla\varphi_\alpha(\mathbf{r},\tau)]^2\right]+\frac{K_\rho}{2\pi v_\rho Na}\left[\left(\sum_{\alpha=B,V}\partial_\tau\varphi_\alpha(\mathbf{r},\tau)\right)^2\right.$$
$$\left.\left.+v_\rho^2\left(\sum_{\alpha=B,V}\partial_x\varphi_\alpha(\mathbf{r},\tau)\right)^2\right]\right\}, \tag{6.21}$$

where the factor of $1/N$ appears because we have coarse-grained the fields in the direction perpendicular to the stripes.

The parameters that appear in Eq. (6.21) can be obtained directly from the symmetries of the original bosonic action (6.2). The bosons obey periodic boundary conditions $[P(\mathbf{r},\beta)=P(\mathbf{r},0),\ b(\mathbf{r},\beta)=b(\mathbf{r},0)]$ while the fermions obey antiperiodic boundary conditions in the imaginary time direction $[\psi(\mathbf{r},\beta)=-\psi(\mathbf{r},0)]$. Suppose we change the boundary conditions so that $P(\mathbf{r},\beta)=P(\mathbf{r},0)\exp\{-i\delta\varphi\}$. From Eq. (6.2) we can enforce the original boundary conditions if we reinterpret the problem as a shift in the chemical potential of the bosons from μ to $\mu'=\mu-i\delta\varphi/(2\beta)$.[59] Because bosons and fermions are coupled, we are forced to impose new boundary conditions for the fermions, namely

$\psi(\mathbf{r},\beta)=-\psi(\mathbf{r},0)\exp\{i\delta\varphi/2\}$. The whole action is invariant under the change in the boundary conditions if we assume a shift in the chemical potential. If the shift is infinitesimally small we can calculate the change in the free energy due to the change in the boundary conditions:

$$\delta F\approx\frac{\partial F}{\partial\mu}\left(-i\frac{\delta\varphi}{2\beta}\right)+\frac{1}{2}\frac{\partial^2 F}{\partial\mu^2}\left(-i\frac{\delta\varphi}{2\beta}\right)^2 \tag{6.22}$$

and since $\delta\varphi/\beta\approx\partial_\tau\varphi$ we immediately find that

$$N_s^2 a^2\overline{\sigma_B}=\frac{\partial F}{\partial\mu}=\overline{N_e},$$

$$N_s^2 a^2\kappa_B=\frac{\partial^2 F}{\partial\mu^2}, \tag{6.23}$$

where $\overline{N_e}$ is the average number of electrons. Since the number of holes is x we see that $\overline{\sigma_B}=(1-x)/a^2$. Thus, we conclude that $\overline{\sigma_B}$ is the average planar density of electrons.

Notice that so far this is the first equation where the number of holes appears explicitly. In principle the distance between stripes, N, and x should be related but in our theory N is an input. When doping is increased the number of holes in each stripe and/or the distance between stripes can change. In the normal state the stripes are in a CDW state and there is a gap to charge excitations because of commensurability. Since the holes are injected into the system at high temperatures during the annealing of the alloy it seems reasonable to assume that the CDW state is charge rigid (incompressible), that is, instead of increasing the doping of individual stripes one would get more stripes but with the same linear doping, n_s. If this is the case then a simple relation exists between x and N, namely,

$$x=\frac{n_s}{N}. \tag{6.24}$$

The value of n_s is determined by the competition between the gain in the kinetic energy along the stripe versus the loss of energy due to the formation of the ADW.[19,20] This is not a problem we have addressed but DMRG calculations[19] and other approaches estimate that the minimization of the energy occurs at $n_s=1/2$ as proposed by the experiments performed in LNSCO.[35] Notice that Eq. (1.19) implies that the pseudogap temperature scale, T^*, vanishes for $1/6<x=x_{sp}<1/4$. Assuming that $x_{sp}\approx 0.2$ we find that $\gamma\approx 0.5$ and there is a reduction of 50% of the antiferromagnetic exchange across the stripe.

Moreover, κ_B is charge compressibility and the superfluid velocity is given by

$$c_B=\frac{4\sigma_B}{\kappa_B M_B}. \tag{6.25}$$

Since the fluctuations of the CCS have zero chemical potential one concludes immediately that $\overline{\sigma_V}=0$ and κ_V is related to the sound velocity in the CCS that is given by

$$c_V=\frac{4\sigma_V}{\kappa_V M_V}. \tag{6.26}$$

It is convenient to rewrite the action (6.21) in Fourier space:

$$\begin{aligned}S=\frac{1}{2\beta}\sum_{\mathbf{k},n}\Bigg\{\sum_{\alpha=B,V}\Bigg[-\overline{\sigma}_\alpha\omega_n\varphi_\alpha(\mathbf{k},\omega_n)+\Bigg(\kappa_{\alpha,R}\omega_n^2+\frac{\sigma_\alpha}{M_\alpha}\\ \times[(1+\lambda_\alpha)k_x^2+k_y^2]\Bigg)\left|\varphi_\alpha(\mathbf{k},\omega_n)\right|^2\Bigg]+\frac{K_\rho}{2\pi v_\rho Na}\\ \times(\omega_n^2+v_\rho^2 k_x^2)[\varphi_B(\mathbf{k},\omega_n)\varphi_V^*(\mathbf{k},\omega_n)+\varphi_B^*(\mathbf{k},\omega_n)\\ \times\varphi_V(\mathbf{k},\omega_n)]\Bigg\},\end{aligned} \tag{6.27}$$

where $\omega_n=2\pi n/\beta$ is the Matsubara frequency and

$$\kappa_{\alpha,R}=\kappa_\alpha+\frac{K_\rho}{2\pi v_\rho Na},$$

$$\lambda_\alpha=\frac{v_F M_\alpha}{2\pi\sigma_\alpha Na} \tag{6.28}$$

are the renormalized compressibility and anisotropy introduced by the stripes [we used Eq. (3.8)]. Notice that both quantities return to unrenormalized values when the distance between stripes diverges, that is, $N\to\infty$. As a result of the factor λ_α the superconductivity is anisotropic and the correlation length, ξ, is direction dependent. It is easy to see that the ratio of the correlation lengths along the x and y directions is given by

$$\frac{\xi_{x,\alpha}}{\xi_{y,\alpha}}=\frac{1}{1+\lambda_\alpha}. \tag{6.29}$$

The key point of Eq. (6.27) is that the topological excitations of the superfluid state (the vortices) are coupled via the stripe compressibility to the topological excitations of the CCS (the dislocations). In a pictorial way consider first the ordered state depicted on Fig. 9. A defect of the ordered state is shown in Fig. 14 where a dislocation of the lattice distortion leads to a local shift of the superfluid density. In doing so the dislocation can produce vortices. The opposite situation is also possible: vortices shift the superfluid density and drag the lattice with them. This unusual state of affairs is the result of the coupling of the CCS with the lattice and would not happen in ordinary superconductors.

Although the theory described by Eq. (6.27) is quadratic, the nature of the elementary topological excitations is not straightforward and we leave their discussion for a later publication. In this paper we consider a simpler problem of the effective theory for each one of the phases. In order to do so we explore the quadratic nature of the action (6.27).

VII. HIGH TEMPERATURES

Let us consider the problem at $T_c<T<T^*$ so that the amplitude of the order parameters is well developed but

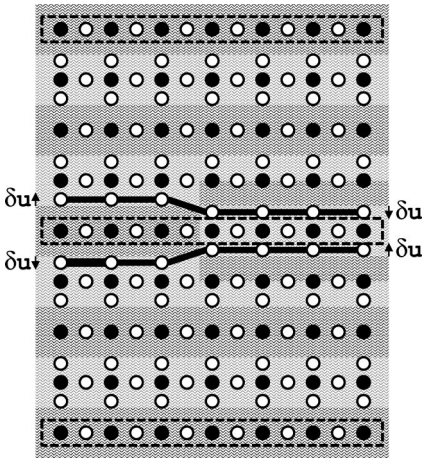

FIG. 14. Topological defect of the CCS.

phase coherence has not been established. It is obvious from Eq. (6.21) that this effect is possible due to presence of singular solutions of the field equations, that is, due to the presence of vortex-antivortex pairs and CCS dislocations loops. At high temperatures we can disregard the time derivatives in Eq. (6.21) since the imaginary time direction shrinks to zero and the phase fields become independent of τ. The effective action reads

$$S_H=\frac{\beta}{2}\int d^2r\left\{\sum_{\alpha=B,V}\frac{\sigma_\alpha}{M_\alpha}[\nabla\varphi_\alpha(\mathbf{r})]^2 + \frac{v_F}{2\pi Na}\left(\sum_{\alpha=B,V}\partial_x\varphi_\alpha(\mathbf{r})\right)^2\right\}. \tag{7.1}$$

Equation (7.1) describes two 2D XY models coupled along the x direction.

Let us look first at the effective field theory for φ_B by tracing out the φ_V modes explicitly. This is equivalent to calculate the renormalization of one of the modes by the Gaussian fluctuations of the other. Thus, in what follows the topological excitations of these fields are not directly coupled to each other but only to their ''spin waves.'' This approach is valid in the weak coupling limit of $N\gg 1$. It is a simple exercise to show that the effective action becomes

$$S_B=\frac{\beta}{2}\sum_{\mathbf{k}} g_B(\mathbf{k})|\varphi_B(\mathbf{k})|^2, \tag{7.2}$$

where

$$g_B(\mathbf{k})=\mathbf{k}^2\frac{E_BE_V\mathbf{k}^2+E_C(E_B+E_V)k_x^2}{E_V\mathbf{k}^2+E_Ck_x^2} \tag{7.3}$$

is the effective phase propagator. We have defined

$$E_\alpha=\frac{\sigma_\alpha}{M_\alpha},$$

$$E_C=\frac{v_F}{2\pi Na}. \tag{7.4}$$

The superfluid order parameter correlation function can be calculated directly from the knowledge of Eq. (7.2). Indeed,

$$\langle P^\dagger(\mathbf{r})P(0)\rangle=\sigma_B\langle e^{i\varphi_B(\mathbf{r})}e^{-i\varphi_B(0)}\rangle=\sigma_B\exp\left\{-\frac{1}{2}G_B(\mathbf{r})\right\}, \tag{7.5}$$

where

$$G_B(\mathbf{r})=\mathrm{Re}\left\{\frac{1}{\beta}\sum_{\mathbf{k}}\frac{1-e^{i\mathbf{k}\cdot\mathbf{r}}}{g_B(\mathbf{k})}\right\} \tag{7.6}$$

is the relevant correlation function. The problem is simplified when we realize that it is possible to write

$$\frac{1}{g_B(\mathbf{k})}=\frac{1}{E_B+E_V}\frac{1}{\mathbf{k}^2} + \frac{E_V/E_B}{E_V+E_B}\frac{1}{[1+E_C(E_B^{-1}+E_V^{-1})]k_x^2+k_y^2} \tag{7.7}$$

that is the sum of the correlation function for an isotropic 2D XY model plus the one for an anisotropic 2D XY model. The integrals can be easily done in Eq. (7.6) and for $x\Lambda, y\Lambda\gg 1$ where Λ is a ultraviolet cutoff (of the order of the inverse of the lattice spacing) we find

$$G_B(x,y)\approx\frac{E_V/E_B}{2\pi\beta(E_V+E_B)\sqrt{1+E_C(E_B^{-1}+E_V^{-1})}} \times\ln\left[\Lambda\sqrt{\frac{x^2}{1+E_C(E_B^{-1}+E_V^{-1})}+y^2}\right] + \frac{1}{2\pi\beta(E_V+E_B)}\ln[\Lambda\sqrt{x^2+y^2}], \tag{7.8}$$

which, for each direction separately, can be written as

$$G_B(s)\approx\frac{1}{2\pi\beta\rho_B}\ln(\Lambda|s|), \tag{7.9}$$

where s can be either x or y and

$$\rho_B=(E_B+E_V)\left[1+\frac{E_V/E_B}{\sqrt{1+E_C(E_B^{-1}+E_V^{-1})}}\right]^{-1}. \tag{7.10}$$

This result can be interpreted as the *effective superfluid stiffness* of an isotropic 2D XY model. The error in making this approximation is equivalent to an anisotropic change in the order parameter (that is irrelevant to the problem of phase coherence discussed here). An analogous calculation can be done for the phase field φ_V and it is obvious that one has only to exchange E_V by E_B in the expressions above in order to get

$$\rho_V=(E_B+E_V)\left[1+\frac{E_B/E_V}{\sqrt{1+E_C(E_B^{-1}+E_V^{-1})}}\right]^{-1} \quad (7.11)$$

that can be thought of as the effective stiffness of the CCS fluctuations. An interesting consequence of our calculations is that if $M_V\to\infty$ then $E_V\to 0$ and according to Eq. (7.11) $\rho_V\to 0$. The transition to the static CCS deformed phase is driven to zero temperature. Thus, while the system becomes a superfluid-superconductor at $T_{c,B}$, at any finite temperature dislocations of the CCS do not allow for long-range order.

We conclude that the transition from the ordered to the disordered phase of the superfluid or/and the CCS is due to the unbinding of the topological excitations (Kosterlitz-Thouless):[71] vortex-antivortex pairs in the case of the superfluid and dislocation loops of the modulated CCS. The transition temperature to the ordered phase can be estimated directly from Eqs. (7.10) and (7.11) by[71]

$$T_{\mathrm{KT},\alpha}\approx\frac{\pi}{2}\rho_\alpha \quad (7.12)$$

and the superconducting correlation length diverges as

$$\xi_\alpha(T)\approx a\exp\left\{\frac{b}{\sqrt{T/T_{\mathrm{KT},\alpha}-1}}\right\}, \quad (7.13)$$

where b is a number of order of unit.

Although our calculation is completely 2D the transition described here only produces quasi-long-range order. True long-range order occurs via the coupling between planes. Thus, the real transition into the ordered phase at finite temperatures is of the 3D-XY type. In order to estimate the actual transition temperature, T_c, we assume that the coupling energy per unit of length, U_c, is small compared to T_{KT}, and therefore the 2D correlation length is well-developed when the system undergoes the phase transition. The transition temperature is defined by the amount of energy required to destroy phase coherence between two regions of size ξ^2 in different planes separated by a distance c,

$$T_{c,\alpha}\approx cU_c[\xi_\alpha(T_{c,\alpha})/a]^2, \quad (7.14)$$

that is a transcendental equation for $T_{c,\alpha}$. Because of the exponential dependence of ξ with the temperature in Eq. (7.13) we can solve this equation to logarithmic accuracy:

$$T_{c,\alpha}\approx T_{\mathrm{KT},\alpha}\left(1+\frac{4b^2}{\ln^2(T_{\mathrm{KT},\alpha}/cU_c)}\right). \quad (7.15)$$

This result indicates that T_c depends only weakly on c and it is very close to T_{KT}.

VIII. ZERO TEMPERATURE

When $T\to 0$ we can replace the sum over the Matsubara frequencies in Eq. (6.27) by an integral over frequency. Observe that at small frequencies the dominant term in Eq. (6.27) is the one that is linear in the frequency. Thus, as in the case of interacting bosons in 2D the universality of the transition is not 2D XY but 2D XY in a magnetic field.[59] We neglect all the quadratic terms in the frequency and retain only the linear one. Notice that this is equivalent to assume that $|\omega|\ll\bar{\sigma}_B/\kappa_{B,R}<2\pi v_\rho Na\bar{\sigma}_B/K_\rho$. Thus, the effect of the linear term is to introduce a high frequency cutoff given by

$$\omega_c=\frac{\bar{\sigma}_B}{\kappa_{B,R}}. \quad (8.1)$$

The equal time correlation function is

$$\langle P^\dagger(\mathbf{r},0)P(0,0)\rangle=\exp\left\{-\frac{1}{2}\sum_{\mathbf{k},\omega}\frac{1}{g_B(\mathbf{k})}\left|1-e^{i\mathbf{k}\cdot\mathbf{r}}-i\bar{\sigma}_B\omega\right|^2\right\}$$
$$\approx\exp\left\{-\mathrm{Re}\left[\omega_c\sum_{\mathbf{k}}\frac{1-e^{i\mathbf{k}\cdot\mathbf{r}}}{g_B(\mathbf{k})}\right]\right\}, \quad (8.2)$$

where the leading order term in ω vanishes because the integral is symmetric. Observe that this is the same result of the previous section with the temperature T replaced by $2\omega_c$. Thus, at zero temperature the system also has an unbinding transition of topological excitations when $N=N_c$ so that

$$\omega_c(N_c)\approx\frac{\pi}{4}\rho_\alpha(N_c). \quad (8.3)$$

Using Eqs. (6.24), (6.28), and (8.1) we see that

$$\omega_c(N)=\frac{1-\frac{1}{2N}}{\kappa_B+\frac{K_\rho}{2\pi v_\rho Na}} \quad (8.4)$$

is a monotonically decreasing function of $1/N$. On the other hand, from Eq. (7.10),

$$\rho_B(N)=\frac{\sigma_B(N)}{M_B}\frac{1+\frac{\sigma_V(N)M_B}{\sigma_B(N)/M_V}}{1+\frac{\frac{\sigma_V(N)M_B}{\sigma_B(N)/M_V}}{\sqrt{1+\frac{v_F}{2\pi Na}\left(\frac{M_B}{\sigma_B^0(N)}+\frac{M_V}{\sigma_V^0(N)}\right)}}}, \quad (8.5)$$

that is a rather complicated expression in terms of N. We have seen from Eq. (5.17) that

$$\frac{\sigma_V(N)}{\sigma_B(N)}\gg 1 \quad (8.6)$$

since the electronic energies are much larger than the vibrational ones. Assuming that this is the case we can write

$$\rho_B(N)\approx\frac{\sigma_B(N)}{M_B}\sqrt{1+\frac{v_F}{2\pi Na}\frac{M_B}{\sigma_B^0(N)}}, \quad (8.7)$$

that is a monotonically increasing function of $1/N$. Thus Eq. (8.3) has a solution at $1/N_c$ provided that

$$\omega_c(1/N_{sp}) > \frac{\pi}{4}\rho_B(1/N_{sp}), \tag{8.8}$$

otherwise there would be no solution and the system would be insulating. Notice that $N_c < N_{sp}$ implying that at zero temperature there is a region $N_c < N < N_{sp}$ where superfluidity exists even at zero temperature but without phase coherence. Observe that according to this theory (even at zero temperature) there must be a region where bosons and stripes coexist but no long-range order is possible.

Moreover, borrowing the results from Ref. 59 we find that at $N \approx N_c$ we must have the superconducting correlation length diverging as

$$\xi(N) \propto \left(\frac{1}{N} - \frac{1}{N_c}\right)^{-\nu}, \tag{8.9}$$

the superfluid density behaving as

$$\sigma_B(N) \propto \left(\frac{1}{N} - \frac{1}{N_c}\right)^{\nu z}, \tag{8.10}$$

and the boson compressibility given by

$$\kappa_B(N) \propto \left(\frac{1}{N} - \frac{1}{N_c}\right)^{\nu(2-z)}, \tag{8.11}$$

where ν and z are the critical exponents that at the mean-field level are $z = 1/\nu = 2$. Moreover, close to N_c the transition temperature scales linearly with the superfluid density

$$T_c(N) \propto \sigma_B(N) \tag{8.12}$$

in complete agreement with the phenomenological Uemura relation observed in all superconducting cuprates.[72]

IX. CONCLUSIONS

In this paper we have proposed a model of a spatially modulated collective charge state of the cuprates where the elementary excitations change character in real space depending on the local charge density. The problem is simplified by assuming that there are two main kind of excitations, namely, Luttinger liquid degrees of freedom in the regions of high density (stripes) and $d_{x^2-y^2}$ bound state of fermions in the regions of lower density (antiferromagnetic ladders), that continuously transform into each other. We have shown that as a consequence of momentum conservation vibrations of the collective state should be present in order to produce sufficient phase space for condensation. In the presence of static or diagonal stripes superconductivity is not possible because of the phase constraints of the former and the $d_{x^2-y^2}$ symmetry of the boson wave function in the latter (in agreement with the experimental observations[65]). Therefore, unlike the BCS theory, phonons are not the basic mechanism of pairing but without vibrations superconductivity would not be possible.

Our main result is the phase diagram in Fig. 15. There $T^* \approx \rho_s(N)$, given in Eq. (1.18), is the temperature scale below which $d_{x^2-y^2}$ bound states can form due to the confining potential generated by the antiferromagnetic background. This energy scale is set by the magnetic forces and is of order of the magnetic stiffness. Because of this constraint our theory is only valid below the crossover line. Observe that the point where T^* vanishes at $N = N_m$ is not a quantum critical point as some theories assume[6] but it sets the scale for a crossover from 1D to 2D behavior [see Eq. (3.2)]. At temperatures below T^* bosons and lattice deformations start to appear in the spectrum at $(\pm\pi/a, 0)$ and $(0, \pm\pi/a)$ but these deformations are dynamic in nature since long range order can only be attained at low temperatures due to phase fluctuations. Thus, the actual phase transition is driven by topological defects of the superfluid (vortex and antivortex pairs) and the CCS distortions (dislocation loops). We have shown that at finite temperatures the transition to the ordered state is in the 3D XY universality class but the transition temperatures for the superconductivity and static lattice distortions can be very different because of the difference in the stiffnesses of the phase modes. In fact, we argue that static lattice distortions may be observable inside of the superconducting phase at very low temperatures. Although dynamic lattice distortions have been observed at temperatures below T_c [as given in Eq. (7.15)] we are not aware of observations of static distortions at low temperatures.[37] The search for such distortions would be a good test for this theory. At zero temperature we have shown that phase fluctuations prevent long range order to appear until the distance between stripes reaches the critical value N_c [given in Eq. (8.3)] where phase coherence is established. For $N_c < N < N_{sp}$ a crossover region appears in the phase diagram where incoherent bosons coexist with stripe fluctuations. The phase transition at zero temperature is in the 2D XY in a magnetic field universality class.

So far the discussion has been based on the idea of infinite 1D stripe segments. For this geometry it is not possible to have a true $d_{x^2-y^2}$ superconducting order parameter as it has been experimentally observed[73] because the Fermi surface is

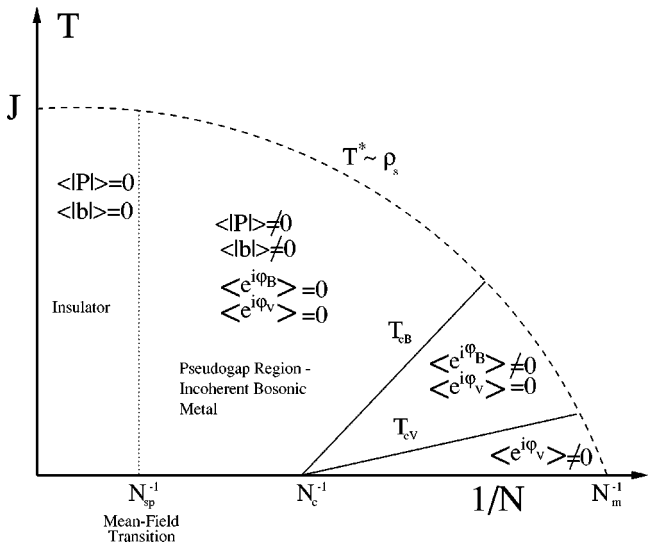

FIG. 15. Phase diagram of the model (the symbols are explained in the text).

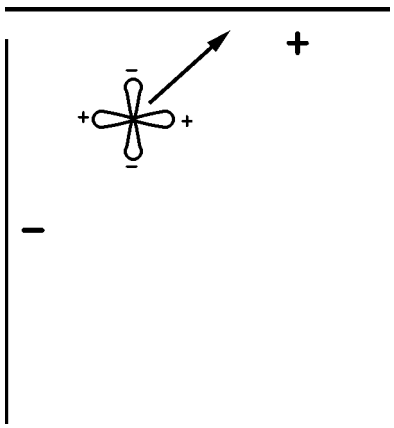

FIG. 16. Coupling generated by a $d_{x^2-y^2}$ boson in the presence of vertical and horizontal stripes.

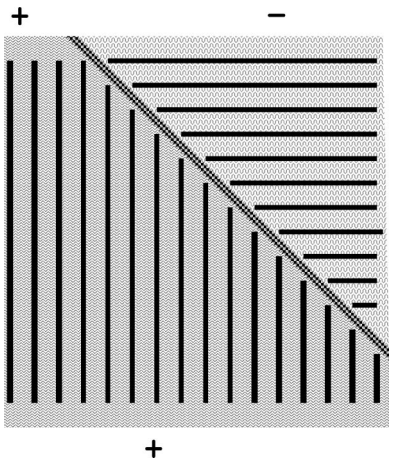

FIG. 17. Tunneling junction across a boundary between horizontal and vertical stripes.

flat. For a flat Fermi surface like the one shown in Fig. 4(a) the superconductivity would have *s*-wave symmetry since the whole Fermi surface would be gapped. In fact this is another major problem in approaches to the cuprate problem starting with 1D stripes, that is, how can one get a *d*-wave order parameter out of a 1D problem. In that aspect our theory indeed predicts a *s*-wave order parameter for infinite 1D stripes. If the small transverse tunneling $t_\perp(N)$ given in Eq. (3.1) is included, the fermion dispersion relation is modified to Eq. (3.3) and a curvature is introduced in the Fermi surface allowing for a very anisotropic *d*-wave state.[25] Observe, however, that because $t_\perp$ is exponentially small with N, changes in the Fermi surface shape only becomes observable when $N \approx N_m$ where the system is essentially two-dimensional. This is not sufficient to explain the isotropic *d*-wave state that is observed in the underdoped cuprates.

In reality, because of the presence of boundaries, we would expect domain formation with stripes running along the crystalographic directions [as it seems to be confirmed in neutron scattering experiments in YBCO (Ref. 74)]. The existence of microscopic twins and tweeds in the lattice structure of these systems seem to be intrinsic to the strong lattice constraints. This situation is quite similar to what occurs in some martensitic systems.[75] We can think of the boundaries as junctions between two superconductors. Since the boundary size, L, is very large (much larger than any of the superconducting length scales) each monodomain can be treated as a separate superconductor that is coupled to other domains through the boundary. Because the coherence length in these systems is of the order of the lattice spacing, the superconducting order parameter can be continuously depressed near the boundary leading to a situation similar to a superconducting-normal-superconducting junction. As we can see in Fig. 6(a) a $d_{x^2-y^2}$ boson couples the positive lobe of the wave function to the horizontal stripes. Thus, when phase coherence is achieved the whole Fermi surface in Fig. 4(a) have the same phase sign [say, positive like in Fig. 6(a)]. When the bosons cross the boundary and find a stripe oriented along the vertical direction the negative lobe of the wave function couples to the vertical stripe. This situation is shown in Fig. 16. Therefore adjacent domains have opposite phases across the boundary as shown in Fig. 17. Thus, if we superimpose the Fermi surfaces of the horizontal and vertical stripes we find a situation like in Fig. 18 where horizontal and vertical Fermi surfaces have opposite sign in their phases. The situation is identical to a π-junction between two superconductors[76] but the phase difference between the different domains is zero. It is the symmetry of the boson that determines the *d*-wave superconductor order parameter. Notice that in the presence of boundaries the electronic motion is actually 2D since stripe holes are being transferred from vertical to horizontal stripes via the bosons. Thus, there is true propagation along the diagonals. In the disordered phase this propagation is not coherent and the system remains essentially 1D. In the superconducting state, however, the coherence between the domains is attained and quasiparticles can propagate along the diagonals as in a ordinary superconductor. Thus, our conclusion is that in the presence of unavoidable microscopic boundaries no quasiparticle peak is possible in the pseudogap phase while in the superconducting state the quasiparticle peak should exist. This conclusion is in agreement with the ARPES data.[77] A consequence of this mechanism is that the size of the *d*-wave order parameter should change with the relative number of domains. In samples where there are equal amounts of vertical and horizontal domains the superconducting order parameter

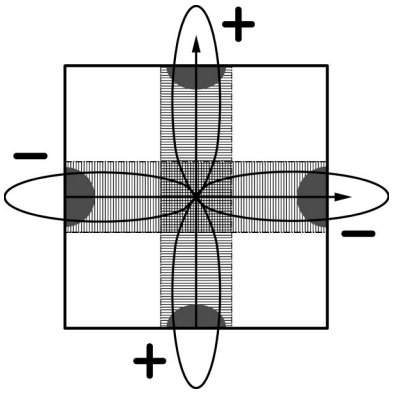

FIG. 18. Momentum space picture of the final *d*-wave state of the electrons.

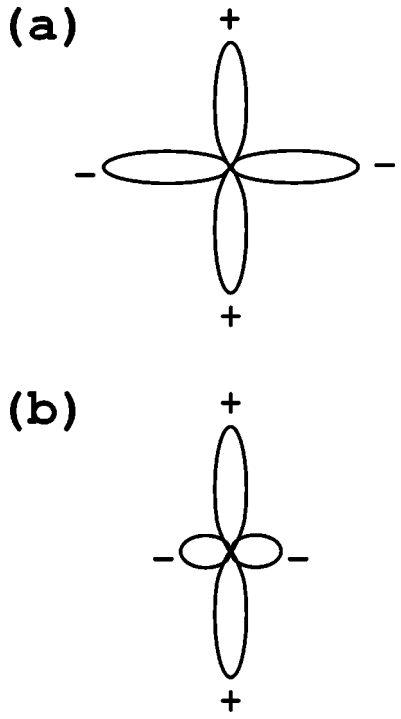

FIG. 19. (a) Superconducting order parameter for equal amounts of vertical and horizontal domains; (b) superconducting order parameter in the case where there is an excess of domains in one direction.

is symmetric [see Fig. 19(a)]. If there are more domains in one direction than in another the *d*-wave order parameter should be asymmetric with a larger lobe in one of the directions as shown in Fig. 19(b). In fact, an *s*-wave component for the order parameter should also develop since for a monodomain system we would predict a fully *s*-wave order parameter. We should also point out that *c*-axis coupling can lead to further increase of the condensation energy. In fact hopping along the *c* axis can help to stabilize the *d*-wave order parameter.[78]

Although T_c is weakly dependent on the boundaries in the sample, the transport properties, the critical currents and fields depend strongly on them. Let us consider a simple model for the junction between a vertical and a horizontal domain. The free energy for the junction is written as

$$\delta F = -\frac{I_c}{2e}\cos(\phi), \tag{9.1}$$

where I_c is the critical current and ϕ is the phase difference between domains. The critical current density is

$$J_c = \frac{I_c}{L l_c}, \tag{9.2}$$

where l_c is the distance between the CuO_2 planes (for simplicity we assume that $I_{c,a}=I_{c,b}=I_c$). In the presence of an electromagnetic vector potential **A** the Josephson current between two different domains is simply given by

$$I_J = 2e\frac{\partial \delta F}{\partial \phi} = I_c \sin\left(\phi - \frac{2\pi}{\Phi_0}\int_C d\boldsymbol{l}\cdot\mathbf{A}\right), \tag{9.3}$$

where C is the line that links the two domains and $\Phi_0 = c/(2e)$ is the magnetic flux quantum. Choosing a gauge such that $\phi=0$ at the junction, assuming that the boundary has width d and that the variations of the order parameter across the boundary are negligible, we have from Eqs. (9.2) and (9.3)

$$\mathbf{J} = -\frac{2\pi J_c d}{\Phi_0}\mathbf{A}, \tag{9.4}$$

that leads to a penetration depth, λ_J, given by

$$\lambda_J = \sqrt{\frac{c\Phi_0}{8\pi^2 d J_c}}. \tag{9.5}$$

Moreover, the variation of the phase across the boundary is π/d but because this variation occurs in a length scale of the correlation length induced by the Josephson coupling, ξ_J, we expect that

$$\xi_J \approx d. \tag{9.6}$$

Thus, the Ginzburg-Landau parameter, κ_J, is given by

$$\kappa_J = \frac{\lambda_J}{\xi_J} \approx \sqrt{\frac{c\Phi_0}{8\pi^2 d^3 J_c}}, \tag{9.7}$$

leading to a critical field, $H_{c1,J}$, for the field strength that is screened by surface currents given by

$$H_{c1,J} = \frac{\Phi_0}{4\pi\lambda_J^2}\ln(\kappa_J) \approx \frac{\pi d J_c}{c}\ln\left(\frac{c\Phi_0}{8\pi^2 d^3 J_c}\right). \tag{9.8}$$

Finally, the critical field for the penetration of one flux of quanta through the boundary is of order

$$H_{c2,J} = \frac{\Phi_0}{2\pi\xi_J^2} \approx \frac{\Phi_0}{2\pi d^2}. \tag{9.9}$$

Thus, we expect $H_{c2,J}$ to be much smaller than the upper critical field required for the extinction of long range order. It is clear that many macroscopic properties of the superconductor are determined by what happens at the internal boundaries.

As we have argued the formation of the stripes is driven by the gain in kinetic energy and therefore favors antiphase domain walls as shown in Fig. 3(a). Thus, the magnetic fluctuations are incommensurate with the lattice as it has been observed for a long time in all the cuprates.[2] However, we have found that there are lattice distortions that we can associate with the O motion as shown in Fig. 9. If the stripes are site centered then the distortion does not affect the incommensurate spin order since the Cu atoms are unaffected. However, if the stripes are bond centered (as some numerical[39] as well as analytical works[79] indicate) then the dimerization produces *commensurate* magnetic response. This can be seen in a schematic way in Fig. 20 where a

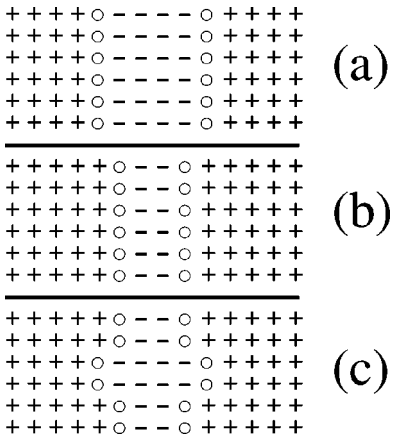

FIG. 20. + sign indicates an up staggered magnetization and − indicates a down staggered magnetization: (a) normal phase; (b) dimerized phase; (c) topological defect of a bond centered stripe.

dislocation loop of the CCS produces excess of one magnetic domain (a domain with a given staggered magnetization) over the other.[80] Thus, for bond centered stripes the dislocation loops produce a dynamic commensurate response. The presence of dynamical commensurate spin fluctuations have been observed in YBCO,[81] but not in LSCO. It might well be that stripes are bond centered in YBCO and site centered in LSCO. This would explain the difference between these two materials in regards to the presence of commensurate spin fluctuations at low temperatures.

It is clear from our theory that the vibrations are affected by an isotope effect [see Eq. (5.6)]. Moreover, because a static lattice distortion is involved we also expect the critical temperature for the distortions, $T_{c,V}$, to be strongly affected by changes of O isotopes (changes of O^{16} by O^{18}). There is strong evidence for the O isotope effect in LSCO and other cuprates that support our theory.[82] Moreover, we expect the isotope effect to be stronger at the $T=0$ transition to the superconducting state at $N=N_c$ where the static lattice distortions start to appear. However, the superconducting transition itself should be very weakly dependent on the isotope effect because the binding mechanism only involves the lattice in a indirect way (in other words, the stiffness of the superfluid is only weakly renormalized by the lattice). The experimental data in cuprates have indeed shown an unusual isotope effect where the critical superconducting temperature is not correlated with isotope effect that becomes stronger at the quantum critical point associated with superconductivity.[83] In fact, optimally doped cuprates show very weak signs of the isotope effect. This has been used as an argument against phonon mechanisms for superconductivity. In our theory at $T=0$ a static lattice distortion appears together with the superconductivity (see Fig. 15). It is the transition into this dimerized stripe phase that is strongly affected by the isotope effect, not the superconducting one. Thus, we can explain the unusual isotope effect in cuprates as a consequence of the unavoidable coupling between lattice and the superconducting condensate.

It is clear from the phase diagram shown in Fig. 15 that our theory does not describe the so-called overdoped region of the cuprates. When the distance between the stripes becomes of the order of the lattice spacing the system becomes homogeneous. The antiferromagnetic correlation length, for instance, is short and the bound states disappear from the spectrum by merging with the lower Hubbard band (this coincides with the vanishing of the pseudogap energy scale). Beyond this point we believe there is a crossover to a conventional BCS behavior with a well defined Fermi surface and therefore to Fermi liquid behavior in the normal phase.[84] This smooth crossover is possibly the same one that occurs between a Bose-Einstein system and a BCS superconductor.[85]

In summary, we have presented a model for a collective electronic state of the cuprates where the elementary excitations change from place to place in real space. We show that the decaying processes among these elementary excitations produce superconducting correlations even when the interactions are repulsive. We show that the *d*-wave nature of the order parameter is associated with the *d*-wave nature of bosons that exist due to the magnetic confinement. We have shown that phase fluctuations are responsible for the quantum disorder and that the phase diagram depends strongly on how vortices couple to dislocation loops. We have explained various different experimental facts of the cuprates and predicted effects that might prove or disprove our theory.

Note added. After this paper was completed I became aware of Ref. 86, where electron-phonon coupling is discussed in the context of ARPES and neutron scattering experiments.

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