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Anisotropic magnetoresistance in the hopping regime: Low frequencies and dc limit

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The magnetic field dependence of the magnetoconductivity in the strongly localized regime is investigated at lowest frequencies, in the multiple hopping regime and in the dc limit. It is found that the magnetoconductivity of isotropic three-dimensional samples is anisotropic. It depends on the angle between the electric and the magnetic field. A simple relationship between the longitudinal part of the magnetoconductivity and the transverse part of the magnetoconductivity, valid for magnetic fields of any strength, is obtained. It turns out that the transverse part of the magnetoconductivity is always larger than the parallel part. As a function of the magnetic field the magnetoconductivity is a quadratic function for small magnetic fields, a nearly linear function for moderate magnetic fields, and saturates for high magnetic fields. Its frequency dependence agrees with that of the conductivity in the range of frequencies in question.

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I. INTRODUCTION

The investigation of the impact of magnetic fields on transport properties in the hopping regime has received much attention in recent years. Often spin effects can be ignored. Then the influence of a magnetic field on transport is governed by its impact on the resonance integrals. On the one hand, the magnetic field causes shrinkage, and thus leads to a reduction of the magnitude of the resonance integral. On the other hand, the magnetic field affects the phase, and therefore has impact on quantum interferences. The answer of the question what the dominating effect is depends on the magnitude of the dimensionless parameters which govern the impact. Since the impact of the magnetic field depends on the size of the characteristic area penetrated by flux the dimensionless parameters depend also on the size of the area. For shrinkage the relevant area is of the order of α^{-2}, where α^{-1} is the localization length, that is of atomic size. Accordingly, the dimensionless parameter governing the impact is given by $\kappa_1 = eH/(\alpha^2 hc)$, where e is the charge of the electron, h is Planck's constant and c is the velocity of light. If the magnetic field is low, that is if $\kappa_1 \ll 1$, shrinkage can be ignored. For quantum interferences the characteristic area S_c is determined by the positions of the initial and the final site of the hop, and the positions of intermediate scattering sites. Therefore, the dimensionless parameter is given by $\kappa_2 = eHS_c/(hc)$ in this case. Since the hopping length is large $\kappa_2 \gg \kappa_1$. Consequently, as first shown by Nguyen, Spivak, and Shklovskii,[1] there is a range of magnetic fields in which shrinkage can be ignored, but the impact of the magnetic field on quantum interferences is relevant. Within this range interferences have been studied in numerous papers both theoretically (see, e.g., Refs. 1–8) and experimentally (see, e.g., Refs. 9–16).

Related to the size of the area S_c is the question, what the effective scattering sites are. If the system is strongly localized, scattering at only one intermediate site is of most importance. Multisite scattering is strongly suppressed, since the corresponding transition probabilities are of higher order with respect to the ratio between resonance integral and level spread, which is small in the strongly localized regime. Consequently, in the strongly localized regime a triangle is associated to every bond, spanned by the initial, the final and the scattering site. The magnetic field affects only the quantum interferences in the plane transverse to the field, so that the impact of the magnetic field on the quantum interferences for hops transverse to the field is always larger than for hops into the direction of the magnetic field. Consequently, the effect is highly anisotropic on the microscopic level.

The situation in the strongly localized regime is to be distinguished from the situation close to the metal insulator transition. Here scattering at many intermediate sites is relevant, so that there is not a characteristic triangle associated to every bond. Rather, the initial and the final site are surrounded by a whole cloud of scattering sites, so that the area penetrable by flux is always as large for hops transverse to the magnetic field as for hops into the direction of the magnetic field. In this case the effect is nearly isotropic on the microscopic level.

An interesting question is, whether the anisotropy on the microscopic level in the strongly localized regime manifests itself also in the macroscopic properties of the sample. The current itself is a vector, and since for an isotropic sample in the presence of an electric and a magnetic field only two vectors are available, $\boldsymbol{E}$ and $\boldsymbol{H}$, the current has to lie within the vector space spanned by $\boldsymbol{E}$ and $\boldsymbol{H}$. Consequently, the symmetric part of the magnetic field induced change of the current with respect to the direction of the magnetic field has the form

$$\delta \boldsymbol{j} = \delta\sigma^{\perp}(H)\boldsymbol{E} + \frac{\delta\sigma^{||}(H) - \delta\sigma^{\perp}(H)}{H^2}(\boldsymbol{E}\boldsymbol{H})\boldsymbol{H}. \qquad (1)$$

If $\delta\sigma^{||} \neq \delta\sigma^{\perp}$ anisotropy is present. In this case the current is not always parallel to $\boldsymbol{E}$. In a recent paper[17] we have used the Holstein model to investigate the magnetoconductivity at high frequencies in the strongly localized regime. The model has the advantage that it can be solved exactly. Using the Holstein model we could show that

$$\delta\sigma^{\perp}(H)=\frac{1}{2H}\frac{d}{dH}[H^2\delta\sigma^{||}(H)]. \tag{2}$$

In the quadratic regime with respect to the magnetic field this relationship entails

$$\delta\sigma^{\perp}(H)=2\,\delta\sigma^{||}(H)+o(H^2), \tag{3}$$

so that for low magnetic fields the magnetoresistance is always larger if the electric field is applied transversely to the magnetic field than for parallel electric and magnetic fields. We do not expect that anisotropy can be produced merely by increasing the frequency. Therefore, we expect that anisotropy is also present at low frequencies and in the dc limit. It is the purpose of the present paper to investigate the situation at low frequencies and in the dc limit, and to examine the difference between the situation at high frequencies and the situation at low frequencies.

So far all calculations (e.g., Refs. 1–6) yield an isotropic magnetoresistance in the dc limit. Only in the numerical calculations of Ref. 18 anisotropy was detected, but reported to vanish with increasing sample size. No investigations of the magnetoresistance in the variable-range hopping (VRH) regime for low frequencies and in the multiple-hopping regime have been published so far.

In order to explain the isotropy of the magnetoresistance also in the strongly localized regime the authors usually focus on the properties of the critical resistor. The critical resistor of the percolation path is an object $f(\boldsymbol{S},\boldsymbol{H})$, which depends on the surface vector $\boldsymbol{S}$ and the magnetic field $\boldsymbol{H}$. The calculation of the configuration average amounts to an integration over all possible surface vectors. Thus, the angle between $\boldsymbol{S}$ and $\boldsymbol{H}$ is integrated out. Consequently, the result is a function of H only.

While this argument seems to be plausible it does not take into account the direction of the electric field. To understand how the electric field modifies this argument consider the Ohmic current. The Ohmic current is a function $\boldsymbol{j}(\boldsymbol{S},\boldsymbol{H},\boldsymbol{E})$. It is linear with respect to $|\boldsymbol{E}|$ but depends also on the angle between $\boldsymbol{S}$ and $\boldsymbol{E}$. In calculating the average we have to integrate over all directions of the surface $\boldsymbol{S}$. However, in doing so we have to take into account that the function does not only depend on the angle between $\boldsymbol{S}$ and $\boldsymbol{H}$, but also on the angle between $\boldsymbol{S}$ and $\boldsymbol{E}$. The latter angle is absent in previous theories, which focus on the critical resistor only. If this angle is taken into account the result of the integrations depends on the angle between $\boldsymbol{H}$ and $\boldsymbol{E}$, as shown in Ref. 17 in the limit of high frequencies.

Note, that the argument given above applies only to the strongly localized regime. Close to the metal insulator transition, where scattering at many intermediate sites is relevant, the initial and the final site are surrounded by a cloud of scattering sites. In this case we expect that the current is nearly independent of the angle between $\boldsymbol{S}$ and $\boldsymbol{H}$ before averaging. Consequently, the averaging procedure leads to an isotropic magnetoresistance in this case. Multisite scattering is, in principle, treated in Refs. 1, 5, and 18, so that the results of these papers are not in contradiction to those published here, if applied not too far from the metal insulator transition.

It should be mentioned that in those experiments, in which the problem of anisotropy is tackled, an isotropic magnetoresistance is found. Only in Ref. 10 anisotropy has been observed in a strong electric field. There $\delta\sigma^{\perp}/\delta\sigma^{||}=1.94$ was measured in n-type GaAs samples, which, in principle, fits well to Eq. (3). In our opinion, the reason for the absence of anisotropy is that the samples were too close to the metal insulator transition, so that multisite scattering was relevant.

Since the magnetoconductivity discussed here originates from the impact of the magnetic field on quantum interferences, one would also expect that the magnetoconductivity depends strongly on the frequency of the applied external field. In Ref. 17 we showed that $\mathrm{Re}\delta\sigma^{\perp/||}(\omega,H)/\mathrm{Re}\sigma(\omega,0)$ decreases with increasing frequency at high frequencies. At very high frequencies the ratio passes into a plateau. To complete the investigation of the frequency dependence we also investigate the behavior of the magnetoconductivity at lowest frequencies and in the multiple hopping regime. Below we will show, that for lowest frequencies and in the multiple hopping regime the above mentioned ratio is independent of frequency. The omitted corrections only lead to a weak reduction of the above mentioned ratio. This result is qualitatively in line with the analytical and numerical investigations on the frequency dependence of the magnetoconductivity in the nearest-neighbor hopping (NNH) regime of Ref. 8.

The paper is organized as follows. In Sec. III our basic equations are introduced. Here we discuss the transport equation and the main formula for the calculation of the magnetoconductivity. Section IV is devoted to the problem of anisotropy. Here the main relationship between longitudinal and transverse part of the magnetoconductivity is derived. In Secs. V and VI the dependence of the magnetoconductivity on the magnetic field and on the critical hopping length is examined in detail, both in the NNH and in the VRH regime. Based on the results of these sections the frequency dependence of the magnetoconductivity is studied in Sec. VII. Finally a discussion of the results is given in Sec. VIII.

II. BASIC EQUATIONS

We consider spinless electrons hopping in the narrow impurity band of a lightly doped semiconductor. If their motion is affected by a weak electric field $\boldsymbol{E}(t)$ suddenly switched on at $t=0$ the transport equation takes the form (see, e.g., Ref. 6)

$$sC_m(U_m+\boldsymbol{E}\boldsymbol{R}_m)=\sum_{m'}\Gamma_{m'm}(U_{m'}-U_m). \tag{4}$$

Here $C_m=f_m(1-f_m)$ (f_m: Fermi distribution with site energy ϵ_m), $s=-i\omega$ (ω: frequency of the applied electric field), and U_m is the local electrochemical potential at the site m with position vector $\boldsymbol{R}_m$. The quantities $\Gamma_{m'm}$ are the transition rates. In the absence of the magnetic field their calculation can be restricted to two-site processes, which describe only direct hops between the initial and the final site. Since these contributions are independent of the magnetic field the consideration of the effect of the magnetic field requires to go beyond this approximation, so that higher order processes have to be taken into account. To do so, we consider besides two-site contributions also contributions originating from three-site processes. The latter contributions result from interferences between the amplitude for the direct hopping path from the initial to the final site with amplitudes for alternative hopping paths via an intermediate third site. In that approximation the quantities $\Gamma_{m'm}$ can be decomposed into their two-site and their three-site parts according to

$$\Gamma_{m'm}=\Gamma^{(2)}_{m'm}+\Gamma^{(3)}_{m'm}(\boldsymbol{H}), \tag{5}$$

where $\Gamma^{(3)}_{mm'}$ has the structure

$$\Gamma^{(3)}_{mm'}=\sum_{n}\Gamma^{(3)}_{mnm'}\,. \tag{6}$$

Both for strong and weak electron-phonon coupling the three-site rates are small as compared to the two-site ones, so that

$$\Gamma^{(2)}_{m'm}\gg\Gamma^{(3)}_{m'm}(\boldsymbol{H}) \tag{7}$$

holds.

Explicit expressions for the transition rates were derived using the renormalized perturbation expansion in Ref. 2, the Konstantinov-Perel method in Ref. 6, and the nonequilibrium Greens function technique in Ref. 7. These expressions can be found in Appendix A.

III. THE EFFECTIVE CURRENT

In order to calculate the configuration averaged magnetoconductivity we take advantage of the inequality (7), which holds for magnetic fields of any strength. If we take into account this inequality we can restrict the consideration to the linear approximation of the magnetoconductivity with respect to $\Gamma^{(3)}$. In this case we can use the formula

$$\begin{aligned}\boldsymbol{j}(\boldsymbol{H},s)-\boldsymbol{j}(\boldsymbol{0},s)=&\frac{e^2}{2kT\Omega}\int d\rho_1 d\rho_2 d\rho_3 N(\epsilon_1)N(\epsilon_2)N(\epsilon_3)\\&\times\Gamma^{(3)}_{123}D^{-2}(\rho_1,\rho_2,\rho_3)(\boldsymbol{R}_{13}b_{123}-\boldsymbol{R}_{23}b_{213})\\&\times\{\boldsymbol{E}(\boldsymbol{R}_{13}b_{123}-\boldsymbol{R}_{23}b_{213})\}\end{aligned} \tag{8}$$

for the magnetic field induced change of the conductivity of Ref. 19. Here the quantities $\Gamma^{(3)}_{mnk}$ and $\Gamma^{(2)}_{mn}$ have to be considered as functions of their coordinates $\rho_i=(\boldsymbol{R}_i,\epsilon_i)$, and Ω is the volume of the system. Furthermore, $\boldsymbol{R}_{ik}=\boldsymbol{R}_i-\boldsymbol{R}_k$,

$$\begin{aligned}D(\rho_1,\rho_2,\rho_3)=&1+2f(\Gamma^{(2)}_{12}+\Gamma^{(2)}_{13}+\Gamma^{(2)}_{23})+3f^2(\Gamma^{(2)}_{12}\Gamma^{(2)}_{13}\\&+\Gamma^{(2)}_{12}\Gamma^{(2)}_{23}+\Gamma^{(2)}_{13}\Gamma^{(2)}_{23}),\end{aligned} \tag{9}$$

and

$$b_{123}=1+2f\Gamma^{(2)}_{23}+f\Gamma^{(2)}_{13}\,. \tag{10}$$

The frequency dependent parameter f is related to the frequency dependent critical hopping length $R_c(\omega)$ by the relationship[21]

$$f\nu_{p,e}=\exp(2\alpha R_c). \tag{11}$$

Here $\nu_{p,e}$ is the attempt-to-escape frequency of the transition rates for strong (p) and weak (e) electron-phonon-coupling (see Appendix A). In the dc limit the critical hopping lengths are given by $R_c=(2\alpha)^{-1}(T_0/T)^{1/4}$ in the VRH regime, where T_0 is the characteristic temperature in Mott's law, and by $R_c=0.866n^{-1/3}$ in the NNH regime, where n is the concentration of sites. At lowest frequencies and in the multiple hopping regime it satisfies the equation[21]

$$2\alpha(R_c(0)-R_c(\omega))\exp(2\alpha[R_c(0)-R_c(\omega)])=i\frac{\omega}{\omega_0}, \tag{12}$$

where ω is the frequency of the electric field and ω_0 is a characteristic frequency of the order of the critical hopping probability.[21] In the VRH regime it is given by $\omega_0=\sigma\alpha^2/(4e^2N_F(2\alpha R_c)^{12})$ (N_F, density of states at the Fermi surface; σ, conductivity). For NNH the characteristic frequency depends on the concentration n of sites according to $\omega_0=4\pi/3\,nR_c^3\exp(-2\alpha R_c)$.

A detailed derivation of Eq. (8) is given in Ref. 19. The approximations used in this derivation can easily be translated into the language of percolation theory. In percolation theory the consideration of the interference contributions amounts to the consideration of additional resistors. If scattering at only one intermediate site is taken into account every simple resistor of the percolation path is replaced by two resistors switched parallel, one of them being the two-site contribution and the other the three-site contribution. Since the system is strongly localized the three-site resistors are large as compared to the two-site ones, so that the current is governed by the two-site resistors only. Consequently, the percolation path is at most slightly changed. The approximations used in our effective description amount to neglect this small magnetic field induced change of the percolation path. Within our formulation the two-site resistors are determined by the critical hopping length. To neglect the magnetic field induced change of the percolation path means to neglect the field dependence of the critical hopping length, so that R_c is still determined by the two-site contributions.

IV. ANISOTROPY

The further investigation of the magnetoconductivity requires the performance of the integrations. However, before tackling this problem, we note that Eq. (8) is very similar to the expression for the magnetic field induced change of the current in the Holstein model.[17] In fact, both expressions agree with each other if the parameter f in Eq. (8) is replaced by $1/s$, and $C_1=C_2=C_3$ is set in the determinant D in Eq. (22) of Ref. 17. If we use this observation we see that Eq. (8) reduces to the exact result at high frequencies. The symmetries of the integrand are not affected by this replacement, so that the integrations over the orientations of the surface normal of the characteristic triangle can be performed exactly and in the same way as in Ref. 17. Consequently, we also find the same result. If we use the results of Ref. 17 we obtain

$$\delta\sigma^{\|,\perp}(H)=\frac{4\pi^2e^2}{3kT}\int_0^\infty dR_1dR_2\int_{|R_1-R_2|}^{R_1+R_2}dR_3\int d\epsilon_1 \times d\epsilon_2 d\epsilon_3 R_1R_2R_3N(\epsilon_1)N(\epsilon_2)N(\epsilon_3) \times\gamma(\rho_1,\rho_3,\rho_2)D^{-2}(\rho_1,\rho_2,\rho_3)g_{\|,\perp}(h) \times\{(b_{123}-b_{213})(R_1^2b_{123}-R_2^2b_{213})+R_3^2b_{123}b_{213}\}. \tag{13}$$

Here

$$\gamma(\rho_1,\rho_2,\rho_3)=\frac{\Gamma^{(3)}(\rho_1,\rho_2,\rho_3)}{[\cos(e\boldsymbol{H}[\boldsymbol{R}_{13}\times\boldsymbol{R}_{23}]/(2\hbar c))-1]}. \tag{14}$$

Furthermore, different integration variables have been introduced. Here $\rho_i=(R_i,\epsilon_i)$, with $R_1=|\boldsymbol{R}_{23}|$, $R_2=|\boldsymbol{R}_{13}|$, and $R_3=|\boldsymbol{R}_{13}-\boldsymbol{R}_{23}|$. In Eq. (14) first the fraction is formed and then the integration variables are changed.

The function $g_{\|,\perp}(H)$ determines the magnetic field dependence of the conductivity. Its longitudinal part is given by

$$g_{\|}(h)=\frac{3}{2}\left(1+\frac{d^2}{dh^2}\right)\frac{\sin(h)}{h}-1. \tag{15}$$

Longitudinal and transverse part are related by the relationship

$$g_{\perp}(h)=\frac{1}{2h}\frac{d}{dh}(h^2g_{\|}(h)). \tag{16}$$

The dimensionless magnetic field

$$h=\frac{eH\sqrt{4R_1^2R_2^2-(R_1^2+R_2^2-R_3^2)^2}}{4\hbar c}=\frac{eHS}{\hbar c} \tag{17}$$

in Eqs. (15) and (16) is equal to the number of flux quanta penetrating the area S of the triangle formed by the sides R_1, R_2, and R_3. If we use the relationship (16) we again find that the longitudinal part of the magnetoconductivity and the transverse part of the magnetoconductivity are related by Eq. (2). Consequently, the anisotropy is not affected by frequency. We conclude that Eq. (2) describes the anisotropy in the whole range of frequencies.

Below we restrict our consideration to the calculation of the longitudinal part of the conductivity. The corresponding expressions for the transverse part can easily be obtained by means of equation (2). Both longitudinal and transverse part, as calculated below and from Eq. (2), are depicted in Fig. 1 and Fig. 2.

V. MAGNETOCONDUCTIVITY IN THE NNH REGIME

A. Strong electron-phonon coupling

Characteristic for the NNH regime is that the impurity band width is small as compared to the thermal energy, so that the spread of the energy levels can be neglected if compared to kT. In this situation the energy integrations are determined by the maximum of the density of states. To model this situation we assume that the maximum is located at $\epsilon=0$, and put

$$N(\epsilon)=n\,\delta(\epsilon). \tag{18}$$

Here n is the density of sites of the system. A redefinition of the position of the maximum of the density of states amounts to a redefinition of the Fermi energy.

If we use Eq. (18) the energy integrations can easily be performed. In this case, in order to simplify the integrations over the side lengths of the triangle, we can take advantage of the fact that the integrand of Eq. (13) is symmetric with respect to ρ_1 and ρ_2. If we introduce integration variables r_1, r_2, and r_3, according to $2R_1=r_1+r_2$, $2R_2=r_2+r_3$, and $2R_3=r_1+r_3$, then Eq. (13) takes the form

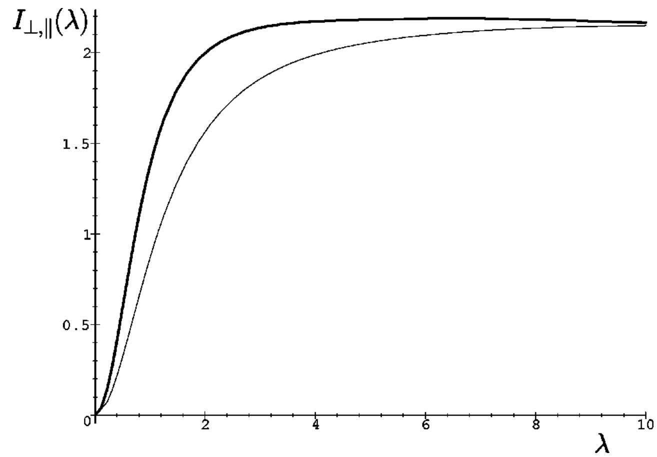

FIG. 1. Magnetic field dependence of the magnetoconductivity in the NNH regime. Thick line: transverse part; thin line: parallel part.

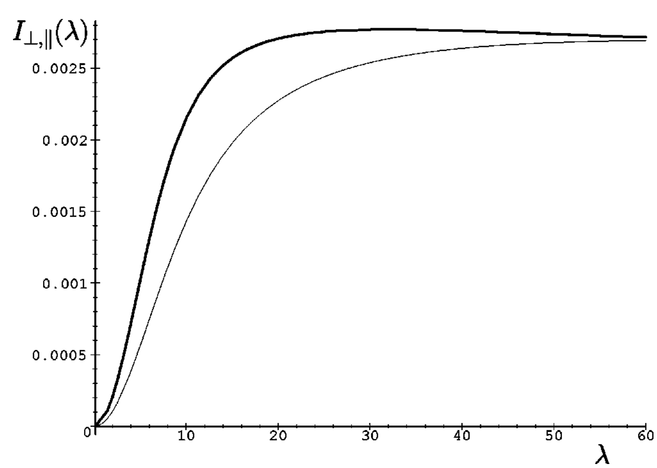

FIG. 2. Magnetic field dependence of the magnetoconductivity in the VRH regime. Thick line: transverse part; thin line: parallel part.

$$\delta\sigma^{\|}(H)=\frac{\pi^2e^2n^3}{96kT}\int_0^\infty dr_1dr_2dr_3(r_1+r_2)(r_1+r_3)(r_2+r_3)\times\gamma(r_1,r_2,r_3)D^{-2}(r_1,r_2,r_3)g_{\|}(h)\times\{(b_{123}-b_{213})[(r_1+r_3)^2b_{123}-(r_2+r_3)^2b_{213}]+(r_1+r_2)^2b_{123}b_{213}\}. \tag{19}$$

In order to perform the spatial integrations we restrict the consideration to strongly localized systems with $2\alpha R_c\gg1$. Furthermore, we take advantage of the fact that the integrand of Eq. (19) has two maxima, located at $r_1=2R_c, r_2=r_3=0$ and at $r_2=2R_c, r_1=r_3=0$. Since the contributions of these maxima are equal to each other we restrict our consideration to one maximum and multiply the result by two. On the maximum we have $f\Gamma_{12}=f\Gamma_{13}=1, f\Gamma_{23}=\exp(2\alpha R_c)\gg1$. To perform the integrations we neglect small contributions of the order $(f\Gamma_{23})^{-1}$, and take into account only the leading contribution of $f\Gamma_{23}$ in the determinant D. Doing so, we obtain

$$\delta\sigma^{\|}(H)=-\frac{4\pi^2e^2n^3\nu_pR_c^4}{9kT\alpha^4}\frac{J_0}{E_a}\tanh\left(\frac{\epsilon_F}{2kT}\right)\times\exp(-2\alpha R_c)I_{\|}\left(\frac{eHR_c}{2\hbar c\alpha}\right), \tag{20}$$

where h is given by $h\simeq eHR_c\sqrt{r_2r_3}/(2\hbar c)$, and

$$I_{\|}(\lambda)=-\int_0^\infty dxdy\frac{xg_{\|}(\lambda\sqrt{xy})}{\exp x+\exp y}. \tag{21}$$

The function $I_{\|}$ determines the dependence of the magnetoconductivity on the magnetic field. Its analytical calculability is restricted to small magnetic fields. For small magnetic fields $[eHR_c/(2\hbar c\alpha)\ll1]$ Eq. (21) can be expanded with respect to H^2. Doing so we obtain

$$I_{\|}=\frac{C}{10}\left(\frac{eHR_c}{2\hbar c\alpha}\right)^2, \tag{22}$$

where $C=15.0577$. To calculate the ratio $\delta\sigma^{\|}(H)/\sigma$ we use the expression

$$\sigma=\frac{2\pi}{15}\frac{e^2n^2}{kT}\nu_pR_c^5\exp(-2\alpha R_c) \tag{23}$$

for the conductivity[20,21] in the absence of the magnetic field and obtain

$$\frac{\delta\sigma_{\|}(H)}{\sigma}=-\frac{\pi C\eta^3}{24}\frac{J_0}{E_a}\tanh\left(\frac{\epsilon_F}{2kT}\right)(\alpha R_c)^{-2}\left(\frac{eH}{2\hbar c\alpha^2}\right)^2. \tag{24}$$

Consequently, the effect decreases with increasing temperature. Furthermore, if we replace ϵ_F by $-\epsilon_F$ the magnetoconductivity changes sign, so that Eq. (24) exhibits a p-n anomaly.

For moderate and high magnetic fields Eq. (21) can only be calculated numerically. Results are depicted in Fig. 1. In Fig. 1 it can be seen, that for moderate magnetic fields the magnetoconductivity is approximately a linear function of the magnetic field. For high magnetic fields the magnetoconductivity saturates.

B. Weak electron-phonon coupling

Due to the singular structure of the three-site part of function $\Gamma^{(3)}$ Eq. (A5) for weak electron-phonon coupling strength the same approximations cannot be applied in this situation. Furthermore, if the site energies were put to zero only in the exponents of the function $\Gamma^{(3)}$ (A5) and in the exponents of the quantities $\Gamma^{(2)}$ the magnetoconductivity would vanish. Consequently, in the limit of weak electron-phonon coupling strength the consideration of the influence of the magnetic field on the conductivity requires the consideration of the small energy contributions in the exponents of the resistors. To this end we write the two-site rates in the form

$$\Gamma_{ik}^{(2)}=\nu_e\exp(-2\alpha|\boldsymbol{R}_{ik}|)(1+\beta_{ik}), \tag{25}$$

and the function γ in the form

$$\gamma(\rho_1,\rho_2,\rho_3)=\nu_eJ_0\left(\frac{1}{\epsilon_1-\epsilon_3}+\frac{1}{\epsilon_2-\epsilon_3}\right)\times\exp(-\alpha(|\boldsymbol{R}_{12}|+|\boldsymbol{R}_{13}|+|\boldsymbol{R}_{23}|))(1+\beta_{12}), \tag{26}$$

where

$$\beta_{ik}=\frac{2|\epsilon_F|-|\epsilon_F-\epsilon_i|-|\epsilon_F-\epsilon_k|-|\epsilon_k-\epsilon_i|}{2kT}\ll1. \tag{27}$$

Since the quantities β_{ik} are small we only take into account their linear contributions to the magnetoconductivity. In this approximation the calculation can be performed as in the strong coupling limit, up to additional energy integrations over the quantities β_{ik}. After a lengthy but straightforward calculation we obtain

$$\delta\sigma^{\|}(H)=-\frac{\pi^2e^2n^3J_0\nu}{3(kT)^2}\frac{R_c^4}{\alpha^4}\exp(-2\alpha R_c)I_{\|}\left(\frac{eHR_c}{2\hbar c\alpha}\right)\mu(\epsilon_F), \tag{28}$$

where $I_{\|}(\lambda)$ is given by Eq. (21), and $\mu(\epsilon_F)$ has the form

$$\mu(\epsilon_F)=n^{-3}\int_0^\infty d\epsilon_1d\epsilon_2d\epsilon_3N(\epsilon_1)N(\epsilon_2)N(\epsilon_3)\times\frac{|\epsilon_F-\epsilon_3|+|\epsilon_2-\epsilon_3|}{\epsilon_1-\epsilon_3}. \tag{29}$$

For symmetric densities of states the quantity $\mu(\epsilon_F)$ can be replaced approximately by sgn(ϵ_F). In this case the p-n anomaly of the magnetoconductivity is reobtained. If we use Eq. (23) we obtain in the quadratic approximation with respect to the magnetic field

$$\frac{\delta\sigma_{\parallel}(H)}{\sigma} = -\frac{\pi C \eta^3}{32}\frac{J_0}{kT}(\alpha R_c)^{-2}\left(\frac{eH}{2\hbar c \alpha^2}\right)^2 \mu(\epsilon_F). \tag{30}$$

Note that both in the case of weak electron-phonon coupling strength and in the case of strong electron-phonon coupling strength the magnetoconductivity changes sign if the sign of the resonance integral or the sign of the Fermi energy is changed. This is to be contrasted, with the situation for multisite scattering, where the magnetoconductivity is always positive (see, e.g., Refs. 1,5).

C. Comparison with results of the standard effective medium theory

The fact that the most important contributions to the current originate from triangles with $r_1 = 2R_c$ and $r_2 = r_3 = 0$ entails that the most important contributions to the integrations arise from nonsymmetric triangles. Two sides of the triangles are of the order of the critical hopping length R_c. The contributions from the third site are from the interval $(0, \alpha^{-1})$. Since one side (R_3) always lies on the percolation path the critical part of an infinite cluster contains always the intermediate third site. This fact justifies posteriorly the averaging procedure over the intermediate third site used in Ref. 6. Furthermore, it determines the agreement of the results of the present paper and the results of Ref. 6 in two important points. First, since in both papers the most important contributions to the current arise from triangles with small areas, which vary between 0 and $\alpha^{-1}R_c$, the dimensionless magnetic field $h = eHR_c/(2\hbar c \alpha)$, the number of flux quanta penetrating the critical configuration, is the same. Secondly, due to the fact that the area of the critical triangle is small the dimensionless magnetic field is also small. Therefore, the magnetoconductivity does not exhibit quantum oscillations depending on the strength of the applied magnetic field. This can also be seen in Fig. 1, which shows a saturation for high magnetic fields. In that the magnetoconductivity in the NNH regime differs from the Hall effect in the NNH regime. There the characteristic configurations are given by equilateral triangles with large areas and side length of the order of the critical hopping length, which manifest themselves in quantum oscillations of the Hall conductivity in strong magnetic fields.[19] In the case of the magnetoconductivity quantum oscillations can only be obtained for moderate, that is for not too large, critical hopping lengths. If the critical hopping length is large, but not large enough to justify the restriction to the leading asymptotic contribution to the integrals, corrections to the asymptotic calculations of the integrals have to be taken into account. In this case both triangles with small and triangles with large areas contribute, and the latter manifest themselves in quantum oscillations.

A further comparison of our results with that of the standard effective medium approximation of Ref. 6 shows that for weak electron-phonon coupling the results of both calculations agree. For strong electron-phonon coupling the results of both papers differ by a factor $(\alpha R_c)^{-1}$. Obviously, the agreement of both results in the weak-coupling limit is purely by chance. The consideration of its origin reveals that in contrast to the present paper, where the characteristic spread of the energies in the singular part of the denominator of the three-site resistance is of the order $\langle(\epsilon_1 - \epsilon_3)\rangle \propto (kT)^{-1}$, the averaging procedure in Ref. 6 yields $\langle(\epsilon_1 - \epsilon_3)\rangle \propto (kT\alpha R_c)^{-1}$. This compensates the differences in the averaging over the coordinates. Despite these differences, it can be checked numerically that both results are in good agreement with the numerical calculations published in Ref. 6. To do so, one has to take into account that the consideration in Ref. 6 is restricted to the transverse contribution and involves furthermore a suitably chosen constant.

VI. MAGNETOCONDUCTIVITY IN THE VRH REGIME

A. Temperature and field dependence

If we look on Eq. (13) we see that, contrary to the NNH regime, in the VRH regime the role of the three sites in the hopping process is determined in advance. The leading contributions to the integrations are obtained if the energies of the sites 1 and 2 are close to the Fermi surface and the energy of site 3 is far away from it. To check this assertion note that the magnitude of the integrand of Eq. (13) increases with decreasing Γ_{12} and Γ_{23} for fixed Γ_{12} and γ, which corresponds to an increase of $|\epsilon_F - \epsilon_3|$ for fixed values of ϵ_1, ϵ_2, r_1, r_2, and r_3. Owing to this fact Γ_{13} and Γ_{23} may be put 0. Since furthermore the most important contributions to the integrations over the energies of the sites 1 and 2 result from the vicinity of the Fermi energy both ϵ_1 and ϵ_2 can be put equal to ϵ_F in preexponential factors. Doing so, we assume that the density of states has no peculiarities in the region under consideration. In the course of this procedure the expression for the current simplifies considerably. We obtain

$$\delta\sigma^{\parallel}(H) = \frac{\pi^2 e^2 N_F^2 \nu_{p,e}\Lambda_{p,e}}{96kT}\int_0^{\infty} dr_1 dr_2 dr_3 (r_1 + r_3)(r_2 + r_3) \times (r_1 + r_2)^3 g_{\parallel}(h)\int d\epsilon_1 d\epsilon_2 \frac{\exp(-\alpha r_3)\Gamma_{12}}{(1 + 2f\Gamma_{12})^2}. \tag{31}$$

According to Eqs. (A4) and (A5) $\Lambda_{p,e}$ is given by

$$\Lambda_p = \frac{J_0}{E_a}\int d\epsilon_3 N(\epsilon_3)\tanh\left(\frac{\epsilon_F - \epsilon_3}{2kT}\right) \tag{32}$$

for strong electron-phonon coupling and by

$$\Lambda_e = 2J_0 \int d\epsilon_3 \frac{N(\epsilon_3)}{\epsilon_F - \epsilon_3} \tag{33}$$

in the limit of weak electron-phonon coupling. Note that in both cases Λ changes sign if the sign of the Fermi energy is changed, if the density of states is symmetric.

The integrations over the energies can be performed exactly. They yield

$$\delta\sigma^{\parallel}(H)=\frac{\alpha\pi^2e^2N_F^2\nu_{p,e}\Lambda kT}{48\alpha^8}\int_0^\infty dr_1dr_2dr_3$$
$$\times(r_1+r_3)(r_2+r_3)(r_1+r_2)^3g_{\parallel}(h)$$
$$\times\exp(-\rho_c-r_3)\ln(1+2\exp(\rho_c-r_1-r_2)). \tag{34}$$

Here $a=4(=3/2)$ for strong (weak) electron-phonon coupling.

The remaining integrations can be performed in the limit of large $\rho_c=2\alpha R_c$, where we can take advantage of the fact that the leading contributions to the integrations in Eq. (34) arise from $0<r_1,r_2<\rho_c$, $0<r_3<1$. If we use this fact the integrations in Eq. (34) can be simplified by means of the relationship

$$\int_0^\infty dr\,\phi(r)\ln(1+\exp(\rho_c-r))\simeq\int_0^{\rho_c}dr\int_0^r dr'\,\phi(r')$$
$$=\int_0^{\rho_c}dr(\rho_c-r)\phi(r). \tag{35}$$

Thereafter Eq. (34) takes the form

$$\delta\sigma^{\parallel}(H)=-\frac{a\pi^2e^2N_F^2\nu_{p,e}\Lambda kT\rho_c^8}{48\alpha^8}e^{-\rho_c}I_{\parallel}\left(\frac{eH\rho_c^{3/2}}{\sqrt{2}\hbar c\alpha^2}\right), \tag{36}$$

where

$$I_{\parallel}(\lambda)=-\int_0^\infty dr_3e^{-r_3}\int_0^1dr_1\int_0^{r_1}dr_2(1-r_1)(r_1-r_2)$$
$$\times r_2r_1^3g_{\parallel}(\lambda\sqrt{r_1r_2(r_1-r_2)r_3}). \tag{37}$$

The remaining integrations cannot be performed in closed form. However, the threefold integral in Eq. (36) can be expanded with respect to the parameter $eH\rho_c^{3/2}/\hbar c\alpha^2$. Doing so, we obtain for the longitudinal part of the magnetoconductivity

$$\delta\sigma^{\parallel}(H)=-\frac{a\pi^2e^2N_F^2\nu_{p,e}\Lambda kT\rho_c^8}{32\alpha^8}e^{-\rho_c}$$
$$\times\sum_{k=1}^{\infty}\frac{(-1)^k(k+1)!}{((2k+3)!!)^2(3k+7)(3k+8)}\left(\frac{eH\rho_c^{3/2}}{8\hbar c\alpha^2}\right)^{2k}. \tag{38}$$

To calculate the ratio $\delta\sigma^{\parallel}/\sigma$ we use the expression

$$\sigma=\frac{a\pi e^2\nu_{p,e}kTN_F^2\rho_c^7}{1260\alpha^5}\exp(-\rho_c) \tag{39}$$

for the conductivity in the absence of the magnetic field,[21] which yields

$$\frac{\delta\sigma_{\parallel}(H)}{\sigma}=-\frac{7\pi}{2200}\frac{\Lambda}{\alpha^3}\frac{T_0}{T}\left(\frac{eH}{8\hbar c\alpha^2}\right)^2 \tag{40}$$

in the quadratic approximation with respect to the magnetic field. For moderate magnetic fields the integrals can be calculated numerically. Results are shown in Fig. 2. Again, for not too large magnetic fields the magnetoconductivity is approximately a linear function with respect to the magnetic field, so that a linear dependence on the magnetic field can also be obtained without logarithmic averaging.

If we look on our result we again find that the sign of the result depends on the sign of the Fermi energy and the sign of the resonance integrals. Again, this is to be contrasted with the situation for multisite scattering, where the magnetoconductivity is always positive (see, e.g., Refs. 1,5).

B. Comparison with results of standard effective-medium theory

If we compare the results of the calculation with those of the standard effective medium theory of Ref. 7 we see that in contrast to the standard effective medium theory, which yields $\delta\sigma(H)/\sigma\propto\rho_c^5H^2$,[7] the other method leads to $\delta\sigma(H)/\sigma\propto\rho_c^4H^2\propto T^{-1}H^2$. As for NNH the difference originates from the averaging over the positions of the sites. Furthermore, the dimensionless critical hopping length in the standard effective medium theory differs from the critical parameter ρ_c in that $\rho_c=(T_0/T)^{1/4}$ is replaced by $\rho_c=(T_0/T)^{2/5}$, which also sets the results of the standard effective medium theory apart from that of percolation theory, which also leads to $\rho_c=(T_0/T)^{1/4}$.

In the framework of two-site transition probabilities discrepancies between results of the standard effective medium theory with those of percolation theory have been discussed in Refs. 22 and 23. There it was pointed out that agreement between the results of both methods can be achieved by incorporating aspects of percolation theory into the standard effective medium theory. A similar situation occurs in the presence of the magnetic field. Since the two-site effective medium approximation does not take into account the additional intermediate third site properly, it has to be supplemented by additional assumptions. So it was assumed in Refs. 2–6 that the virtual path involving the intermediate third site does not coincide with a percolation path. In the framework of the random resistor network this requirement entails that both $Z_{13}>Z_{12}$ and $Z_{23}>Z_{13}$, where Z_{ik} are random resistors, and the intermediate site is labeled by 3. Furthermore, as in Ref. 1, the average has been taken over the logarithm of the phase factor in Refs. 2–4, which leads to a linear dependence of the magnetoconductance with respect to the magnitude of the magnetic field for small magnetic fields.

In contrast to the effective medium methods of Refs. 2–8, the effective method used here does not rely on these assumptions. All three sites enter the averaging procedure equally. The method itself leads automatically to a determination of the most important configurations for the formation of the current. According to our calculation these configurations are given by nonsymmetric triangles. Two sides of the triangles are of the order of the critical hopping length, and the length of the third side varies between 0 and α^{-1}. The site energies of the initial and the final site are close to the Fermi energy. The scattering site is far away from the Fermi energy.

VII. FREQUENCY DEPENDENCE OF THE MAGNETOCONDUCTIVITY

In our approach the frequency dependence of the current is entirely determined by the frequency dependence of the critical hopping length. The dispersion of the critical hopping length is given by Eq. (12) for lowest frequencies and in the multiple hopping regime. Since for such frequencies

$$|\rho_c(0)-\rho_c(s)|\ll\rho_c(0), \tag{41}$$

the frequency dependence of the preexponential factor is negligible. Consequently, since the magnetoconductivity and the conductivity have the same exponential dependence on the dimensionless critical hopping length, the ratio $\delta\sigma^{\|,\perp}(H,\omega)/\sigma(\omega)$ is independent of frequency, in the first approximation.

To investigate this point further we introduce the quantities

$$S_H(\omega)=\frac{\delta\sigma(H,\omega)}{\delta\sigma(H,0)} \tag{42}$$

and

$$S(\omega)=\frac{\sigma(\omega)}{\sigma(0)}. \tag{43}$$

Here the superscripts $\|$ and $\perp$ have been omitted, for brevity. From the results obtained above, it follows that for low magnetic fields

$$S_H(\omega)=S(\omega)\left(1-\frac{\rho_c-\rho_c(\omega)}{\rho_c}\right)^n, \tag{44}$$

where $n=1$ for NNH and 4 for VRH.

The most important property of the quantity $S_H(\omega)/S(\omega)$ is its nonexponential dependence on the parameter $[\rho_c(0)-\rho_c(\omega)]/\rho_c(0)$, which describes the decrease of the critical hopping length with increasing frequency.[20] As mentioned above this parameter is small in range of frequencies under consideration. Only for high frequencies, within the range of applicability of the two-site model, where the critical hopping length decreases down to α^{-1}, this parameter approaches 1. Consequently, the relationship (44) shows that the ratio $\delta\sigma(H,\omega)/\sigma(\omega)$ depends only weakly on frequency within the range of frequencies under consideration.

If we take into account the frequency dependence of the preexponential factor and use $\rho_c(0)-\rho_c(\omega)=\ln S(\omega)$, then the frequency dependence of the quantity $S_H(\omega)$ can be cast into the form

$$S_H(\omega)\simeq S(\omega)-i\frac{n}{\rho_c}\frac{\omega}{\omega_0}, \tag{45}$$

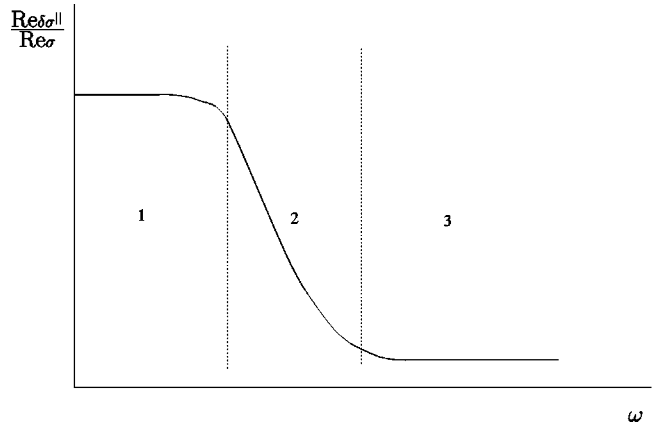

FIG. 3. Schematic sketch of the frequency dependence of the magnetoconductivity in the VRH regime.

which shows that the frequency dependence of the quantity $\mathrm{Re}\, S_H(\omega)$ differs only weakly from that of the quantity $\mathrm{Re}\, S(\omega)$.

For high frequencies, in the range of applicability of the Holstein-model, $\mathrm{Re}\;\delta\sigma(\omega,H)/\mathrm{Re}\;\sigma(\omega)$ decreases like

$$\frac{\mathrm{Re}\;\delta\sigma(\omega,H)}{\mathrm{Re}\;\sigma(\omega)}\propto\ln^4\left(\frac{\nu_e}{\omega}\right) \tag{46}$$

for $\omega\ll\nu_e$ in the quadratic regime with respect to the magnetic field,[17] and reaches the plateau for $\omega\gg\nu_e$. On the plateau we have[17]

$$\frac{\delta\sigma^{\|}(H,\infty)}{\sigma(\infty)}=\frac{1287}{120}\pi^2\frac{\Lambda}{\alpha^3}\frac{e^2H^2}{\hbar^2c^2\alpha^4}+o\left(\frac{e^2H^2}{\hbar^2c^2\alpha^4}\right), \tag{47}$$

so that

$$\frac{\mathrm{Re}\;\delta\sigma^{\|}(H,0)}{\sigma(0)}\;\frac{\sigma(\infty)}{\mathrm{Re}\;\delta\sigma^{\|}(H,\infty)}=\frac{7}{1510080}\rho_c^4. \tag{48}$$

Taking into account this fact, the frequency dependence of the magnetoconductivity in the limit of large ρ_c is as sketched in Fig. 3. Note, that the step is not of exponential height.

VIII. RESULTS

We have studied the magnetoconductivity of three-dimensional, strongly localized systems, far from the metal-insulator transition, where multisite scattering is irrelevant. The calculation shows that in this limit the magnetoconductivity has peculiarities, e.g., anisotropy, the p-n anomaly, and the frequency dependence. This sets the situation deep in the strongly localized regime, where multisite scattering is irrelevant, apart from that close to the metal-insulator transition, where multisite scattering is relevant.

Deep in the strongly localized regime we expect that at most scattering at one intermediate site is important, since the transition probabilities for scattering events at many intermediate sites are proportional to higher powers of the ratio between resonance integral and level spread, which is small in the strongly localized regime. Due to this fact we can imagine that for every initial and final site there is one scattering site, so that the positions of these sites span a triangle. Consequently, every bond is associated with a triangle. Since the impact of the magnetic field on the quantum interferences is governed by the flux penetrating the triangle the effect is highly anisotropic on the microscopic level.

To see, whether this anisotropy manifests also in the macroscopic properties of the sample we focused on the configuration average of the current. Also in the linear approximation with respect to the electric field, the current is a function $\boldsymbol{j}=\boldsymbol{j}(\boldsymbol{E},\boldsymbol{S},\boldsymbol{H})$, which depends on the surface vector $\boldsymbol{S}$ of the characteristic configuration, the electric field $\boldsymbol{E}$ and the magnetic field $\boldsymbol{H}$. In calculating the average one has to integrate over all directions of the surface normal. However, in doing so, it has to be taken into account that the current is not only a function of the angle between $\boldsymbol{S}$ and $\boldsymbol{H}$, but also of the angle between $\boldsymbol{S}$ and $\boldsymbol{E}$. Therefore, a dependence on the angle between $\boldsymbol{E}$ and $\boldsymbol{H}$ remains. Formula (2), which relates the longitudinal part of the magnetoconductivity to the transverse part, derived for lowest frequencies and in the multiple-hopping regime in the present paper, entirely agrees with the corresponding formula obtained in the Holstein-model in Ref. 17, which shows that anisotropy is not produced merely by increasing frequency. Consequently, Eq. (2) describes the anisotropy in the whole frequency range. As already pointed out in the introduction the transverse part of the magnetoconductivity is always larger than the parallel part of the magnetoconductivity for small magnetic fields.

Note that, the argumentation given above does not apply to the situation close to the metal-insulator transition, since multisite scattering is relevant there. If multisite scattering is relevant we cannot imagine that every bond is associated with a triangle. There every initial and final site is surrounded by a whole cloud of scattering sites, so that the current is nearly independent of the angle between $\boldsymbol{S}$ and $\boldsymbol{H}$ before averaging. Consequently, the magnetoresistance is isotropic in this case. Multisite scattering is, in principle, investigated in Refs. 1, 5, and 18, so that the results of these papers are not in contradiction to those published here.

As mentioned before in most experiments isotropic magnetoresistance is observed, which shows, in our opinion, that these experiments were performed in the vicinity of the metal-insulator transition. Only in Ref. 10 anisotropy was observed in a strong electric field. There $\delta\sigma^{\perp}/\delta\sigma^{\parallel}=1.94$ was measured in n-type GaAs samples in a strong electric field in the quadratic regime with respect to the magnetic field. Equation (2) yields $\delta\sigma_{\perp}/\delta\sigma_{\parallel}=2$, so that, in principle, this result fits well to the experimental data. However, since the experiments were performed in the non-Ohmic regime, they cannot be considered as verification for our prediction.

A further peculiarity of the magnetoconductivity in the strongly localized regime is the p-n anomaly. While the magnetoconductivity is always positive if multisite scattering is relevant,[1,5,18] the sign of the magnetoconductivity deep in the strongly localized regime depends on the sign of the resonance integral and the sign of the Fermi energy. If the sign of resonance integrals is changed also the magnetoconductivity changes sign. A change of the sign of the Fermi-energy results also in a change of the sign of the magnetoconductivity, if the density of states is symmetric. That is, for symmetric densities of states the magnetoconductivity exhibits a p-n anomaly, as discussed also in Ref. 8.

The unusual behavior of the magnetoconductivity in the strongly localized regime manifests also in its frequency dependence. As shown above, the ratio between magnetoconductivity and conductivity is nearly independent of frequency for lowest frequencies and in the multiple-hopping regime. Consequently, the impact of such frequencies on the interferences in the strongly localized regime is also weak. Only at high frequencies the magnetoconductivity decreases appreciably with increasing frequency.[5] Since the magnetoconductivity discussed here is due to quantum interferences, one would have expected a strong dependence on frequency. Therefore, the weak dependence on frequency, as already the possibility to change the sign of the magnetoconductivity by changing the sign of the resonance integral or the Fermi energy for symmetric densities of states, is a further hint on that the role of quantum interferences deep in the strongly localized regime is very different from that close to the metal-insulator transition or in weak localization physics.

Despite all these peculiarities the magnetoconductivity deep in the strongly localized regime also has much in common with the situation for multisite scattering. As mentioned above, the existing theories describe the temperature and magnetic field dependence observed in the experiments fairly well. It turns out that the results derived in our paper do so also. As can be seen from our figures, the magnetoconductivity calculated in this paper is a quadratic function with respect to the magnetic field for small magnetic fields, a linear function of the magnetic field for moderate fields, and saturates for high magnetic fields, like in most other calculations.

The quadratic regime with respect to the magnetic field has been observed in many experiments (see, e.g., Refs. 12–16). In many cases the data were presented in the form $\delta\sigma(H)/\sigma \propto T^{-\gamma}H^2$. It was found, e.g., that $\gamma=1{,}22$ in GaAs in Ref. 12, $\gamma=1{,}32$ for CdSe in Ref. 13, $\gamma=0.93$ in thin films of In_2O_{3-x} in Refs. 11 and 14 with decrease to $\gamma=0{,}76$ with increasing thickness, $\gamma=0{,}75$ for $T<4$ K with increase up to $\gamma=1{,}25$ for $T>4$ K in $CuInSe_2$ in Ref. 15. Our calculation yields $\gamma=1$.

According to Eq. (38) the deviations from the quadratic behavior are governed by the parameter $eH\rho_c^{3/2}/(\hbar c\alpha^2)$. Consequently, such deviations have to be taken into account for fields of the order of $H_m \sim \hbar c\alpha^2\rho_c^{-3/2}/e \propto T^{3/8}$. The same result was found in Ref. 1, where the magnetic field dependence is governed by the same parameter. It also agrees well with the numerical simulations of Ref. 7, where the crossover was also observed. Experimentally the crossover from quadratic to quasilinear dependence with increasing magnetic field has been observed in numerous samples (see, e.g., Refs. 16, 13, and 15). So it was found that $H_m \propto T^{0.75}$ in CdSe samples in Ref. 13, $H_m \propto T^{3/8}$ for $T<4$ K and $H_m \propto T^{1.05}$ for $T>4$ K in $CuInSe_2$ samples in Ref. 15.

The quasilinear dependence of the magnetoconductivity has been observed in most experiments referred to (see, e.g., Refs. 13–16). Here in many cases the temperature dependence is also written in the form $\delta\sigma \propto HT^{-\lambda}$. For the exponent λ 3/4 was obtained in Ref. 1 and 7/8 in Ref. 2. If in the quasilinear regime we replace the series in Eq. (38) by a linear function we obtain $\lambda = 5/8$. Experimentally, e.g., $\lambda = 0.76$ was observed in Ref. 11 in In_2O_{3-x} and $\lambda = 0.63$ in n-type $CuInSe_2$ for temperatures below 4 K in Ref. 15. Consequently, we conclude that our calculations can account for the temperature dependence and the field dependence observed in the experiments as well.

APPENDIX A: TRANSITION RATES

The two-site conductances of the transport equation (4) are given by[6]

$$\Gamma_{ik} = \nu_{p,e} \exp\left(-2\alpha |\boldsymbol{R}_{ik}| - \frac{|\epsilon_F - \epsilon_i| + |\epsilon_F - \epsilon_k| + \lambda |\epsilon_i - \epsilon_k|}{2kT} \right). \tag{A1}$$

Here α^{-1} is the localization length, ϵ_F is the Fermi energy, and $\lambda = 1$ (0) for strong (weak) coupling with phonons. For weak electron-phonon coupling the preexponential factor ν_e is a constant of the order of the characteristic phonon frequency. For strong electron-phonon coupling the preexponential factor ν_p is given by

$$\nu_p = \frac{\sqrt{\pi}}{2\hbar} \frac{J_0^2}{\sqrt{E_a kT}} \exp\left(-\frac{E_a}{kT} \right), \tag{A2}$$

where E_a is the activation energy for a polaronic hop and J_0 is the preexponential factor of the resonance integral.

Since we are only interested in effects symmetric in $\boldsymbol{H}$ in the linear approximation with respect to $\Gamma^{(3)}$ we can restrict the consideration to the symmetric part of the function $\Gamma^{(3)}$ with respect to the direction of the magnetic field. Both for strong and weak electron-phonon coupling the three-site contribution has the structure[6]

$$\Gamma^{(s)}_{m_1 m_2 m_3} = \gamma_{m_1 m_2 m_3} \left(\cos \frac{e\boldsymbol{H}[\boldsymbol{R}_{13} \times \boldsymbol{R}_{23}]}{2\hbar c} - 1 \right). \tag{A3}$$

For strong electron-phonon coupling the function γ is given by

$$\gamma_{m_1 m_2 m_3} = \nu_p \frac{J_0}{E_a} \tanh\left(\frac{\epsilon_F - \epsilon_3}{2kT} \right) \times \exp\left(-\alpha(|\boldsymbol{R}_{12}| + |\boldsymbol{R}_{23}| + |\boldsymbol{R}_{13}|) - \frac{|\epsilon_F - \epsilon_1| + |\epsilon_F - \epsilon_2|}{2kT} \right). \tag{A4}$$

For weak electron-phonon coupling it has the form

$$\gamma_{m_1 m_2 m_3} = \nu_e J_0 \left(\frac{1}{\epsilon_1 - \epsilon_3} + \frac{1}{\epsilon_2 - \epsilon_3} \right) \times \exp(-\alpha(|\boldsymbol{R}_{12}| + |\boldsymbol{R}_{23}| + |\boldsymbol{R}_{13}|)) \times \exp\left(-\frac{|\epsilon_F - \epsilon_1| + |\epsilon_F - \epsilon_2| + |\epsilon_1 - \epsilon_2|}{2kT} \right). \tag{A5}$$

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