Scaling of the conductivity in icosahedral Al-Pd-Re metallic samples

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(Received 13 March 2001; published 8 August 2001)

Temperature dependence of electrical conductivity σ is investigated for a set of icosahedral ($i-$) $Al_{70.5}Pd_{21}Re_{8.5}$ samples. We focus on ribbon samples with resistance ratios $R=\rho(4\ \mathrm{K})/\rho(300\ \mathrm{K})$ below $\simeq 20$. We show these samples are on the metallic side of a metal-insulator transition. We analyze our $\sigma(T)$ data between 400 mK and 300 K with the one-parameter scaling theory of the Mott-Anderson metal-insulator transition. To the best of our knowledge, this is the first overall approach covering all the metallic samples of this system and a large temperature range up to 300 K. This interpretation highlights the relevance of the critical regime close to the metal-insulator transition, regime that is characterized by an inelastic scattering length smaller than the metallic correlation length ξ. Consequences for insulating $i-$Al-Pd-Re samples are also discussed.

DOI: 10.1103/PhysRevB.64.094203 PACS number(s): 71.30.+h, 71.23.Ft, 72.15.−v

I. INTRODUCTION

Stable quasicrystals of high structural icosahedral order possess a long-range order that set them structurally apart from what are called disordered systems. Among these materials, ternary alloys based on Al and containing transition metals display spectacular electronic properties. Indeed, icosahedral alloys like $i-$Al-Pd-Mn, $i-$Al-Cu-Fe, and more significantly $i-$Al-Pd-Re, have values of the electrical resistivity ρ orders of magnitude higher than their constitutive metals. For example, the resistivity can reach[1] 10 m Ω cm at 4 K in $i-$Al-Cu-Fe and can exceed[2–4] 1000 m Ω cm at 4 K in $i-$Al-Pd-Re samples. It should be noted that in the $i-$Al-Pd-Re system, we have found that the value of $\rho(4\ \mathrm{K})$ can change by two orders of magnitude, even for *i*-samples of the same nominal composition and prepared in the same way.[5] This allows us to get samples in a wide range of resistivity values. Another remarkable property is the ''nonmetallic'' dependence of ρ with the temperature *T*. The resistivity can be at 4 K twice the room temperature value in $i-$Al-Cu-Fe and up to 200 times in some $i-$Al-Pd-Re samples.[4,6] By analogy with disordered systems, such values suggest the crossing of a metal-insulator (MI) transition in the $i-$Al-Pd-Re system. In fact, it can be shown[7] that the resistance ratio $R=\rho(4\ \mathrm{K})/\rho(300\ \mathrm{K})$ is correlated with the resistivity of the samples, the higher ρ, the higher *R*. This ratio thus makes up a good parameter in order to classify the $i-$Al-Pd-Re samples according to the MI transition. Recent studies[4,8–10] have tentatively concluded to the existence of a MI transition in this system for a value of *R* around 20. However, our preliminary structural and chemical analysis of $i-$Al-Pd-Re ribbons of various resistivities did not yield evidence for differences between samples.[11] As a matter of fact the actual difference between insulating and metallic $i-$Al-Pd-Re samples is poorly understood, and the ''microscopic'' origin of the MI transition in the $i-$Al-Pd-Re system remains an open question. We must stress that the MI transition is not only a band-structure property as it was observed in some crystalline alloys like Al_2Ru composed also of metallic elements.[2] Indeed, there is until now no experimental evidence for the opening of a true semiconducting gap in the electronic density of states (DOS) of $i-$Al-Pd-Re samples.

This question should be related more generally to the physical origin of the high resistivity values observed in a number of icosahedral alloys. The respective roles of quasiperiodicity, chemistry, disorder and electron-electron interactions are experimentally and theoretically not clarified, although signatures for these effects might be infered from the temperature and magnetic field dependence of the conductivity σ, or from the energy dependence of the one-electron DOS near the Fermi level. A comprehensive and coherent interpretation of simple measurements like the *T* dependence of σ is still lacking in the $i-$Al-Pd-Re system. Regarding the $\sigma(T)$ data, two kinds of approach have been proposed in the literature. On the one hand, the low-temperatures $\sigma(T,H)$ curves were analyzed. For the more metallic $i-$Al-Pd-Re samples, $\sigma(T,H)$ at temperatures below $\simeq 30$ K were interpreted[12] by quantum interference effects (QIE) taken into account both the weak localization and the electron-electron interactions contributions. Although this theory is perturbative, it seems to fit the data well, including samples with *R* up to 5. This analysis was forced up to $R \simeq 10$ but beyond it definitively breaks down. For systems close to the MI transition, the nonperturbative but controversial theory of McMillan[13] predicts a square-root *T* dependence for the conductivity at very low temperature. It was also used[8] for $i-$Al-Pd-Re samples on the metallic side and near the MI transition, below $\simeq 20$ K. The interesting point is that both QIE in the weak localization regime and McMillan prediction result from disorder-localization effects and electronic interactions. In the same vein, the $\sigma(T)$ laws of disordered insulators, like the Mott's variable range hopping (VRH) conductivity, were also used for $i-$Al-Pd-Re samples near the MI transition and deep in the insulating regime.[9,10,14–16]

On the other hand, less attention has been paid to the higher temperature part of the $\sigma(T)$ curves (i.e., $30\ \mathrm{K} \leqslant T \leqslant 300\ \mathrm{K}$). Noteworthy power laws were observed[4] on two orders of magnitude in *T* (from 6 K up to 600 K!) in highly resistive $i-$Al-Pd-Re samples. Some interpretations were

advanced, such as Refs. 17 and 18, and a common opinion is that these power laws could be the manifestation of the anomalous spreading of electronic wave functions theoretically predicted for quasiperiodic potentials. [19–21]

But to our knowledge, no coherent model has been advanced that can explain both the metallic and insulating $\sigma(T)$ regimes, at both low and intermediate temperatures. Without such a unified picture the situation will remain confused and no light can be shed on the MI transition in the $i-$Al-Pd-Re system. In this paper, we propose an analysis of our $\sigma(T)$ data below 300 K for a set of samples on the metallic side of the MI transition. This analysis brings out three important conclusions. First, the scaling theory developed for disordered systems near the Mott-Anderson MI transition can be applied successfully to the $\sigma(T)$ curves of metallic $i-$Al-Pd-Re. Second, the MI transition is located close to a ribbon sample of resistance ratio $R \simeq 20$. Third the critical regime is to be considered in order to understand the $\sigma(T)$ dependence in the whole T range near the MI transition. This last point should also be valid in the insulating phase.

II. SCALING THEORY, METALLIC AND CRITICAL REGIME

Abrahams *et al.* proposed[22] in 1979 a scaling theory of the Anderson MI transition, based on the renormalization-group analysis and on the pioneering work conducted by Thouless.[23,24] In this theory, the conductance G [or the dimensionless Thouless number $g=G/(e^2/\hbar)$] for a system of size L evolves in an universal way with L and this evolution depends only on the value of the conductance itself. We can distinguish different regimes for the dependence of g with L. We call g_0 the Thouless number at the microscopic scale that is typically the elastic mean free path L_{el}. The critical Thouless number g_c corresponds to the location of the MI transition and is of the order of unity. If $g_0 > g_c$, the system is on the metallic side of the MI transition and g increases with L. For large enough g, the ohmic regime is recovered and the conductance is $G=(e^2/\hbar)g=\sigma L$ at 3D. If $g_0 < g_c$, the system is on the insulating side of the MI transition and g decreases by increasing L. This is consistent with exponentially localized electronic states if $g \propto \exp(-L/\xi)$, where ξ is the localization length of the wave function. If $g_0 = g_c$, g is independent of L and adds to g_c. For g_0 close to g_c, g changes little with L $(g \simeq g_c)$ and this is only for very large L that it evolves towards a clear metallic or insulating behavior. This is the critical regime of the MI transition. In other words, in this regime, the metallic and insulator behavior are not fully developed in the system and cannot be distinguished. Therefore, the scaling theory introduces two important length scales. On the metallic side of the MI transition, this is the correlation length ξ. This length is the border line between the ohmic (or metallic regime, $L > \xi$) and the critical regime ($L < \xi$). On the insulating side of the MI transition, the localization length also noted ξ parts the exponential decrease of g (i.e., the insulating regime, $L > \xi$) from the critical one ($L < \xi$). The two lengths ξ are sample dependent and they diverge at the MI transition.

Now at a given temperature T, the maximum length L for the renormalization of g is the inelastic scattering length $L_{in}(T)$. Indeed, if $L > L_{in}(T)$, the quantum coherence of the wave function is lost and ohmic regime is believed to be valid.[25] The conductivity thus takes its macroscopic value as soon as L reaches $L_{in}(T)$ and $\sigma(T)$ is given by the relation:

$$\sigma(T) = \frac{e^2}{\hbar} \frac{g(L_{in}(T))}{L_{in}(T)}. \tag{1}$$

Let us consider the metallic side of the MI insulator transition. If $L_{in}(T) \ll \xi$ (ξ is the correlation length), the sample is in the critical regime. Then, $g \simeq g_c$, and

$$\sigma(T) \simeq \frac{e^2}{\hbar} \frac{B}{L_{in}(T)}, \tag{2}$$

where B is a constant of the order of unity. The T dependence of σ is given by the T dependence of $L_{in}(T)$. If $L_{in}(T) \gg \xi$ the sample lies in the metallic regime. Then, it can be shown[22] that the residual conductivity is

$$\sigma(T=0) \simeq \frac{e^2}{\hbar} \frac{A}{\xi}, \tag{3}$$

where A is another constant of the order of unity.

As $L_{in}(T)$ decreases when temperature increases, samples close enough to the MI transition (having large ξ) can leave from the metallic regime at low T into the critical regime at high enough T. A general law can be extrapolated in the intermediate temperature range:[26,27]

$$\sigma(T) = \frac{e^2}{\hbar} \left(\frac{A}{\xi} + \frac{B}{L_{in}(T)} \right). \tag{4}$$

Similar arguments can be developed on the insulating side of the MI transition, and an important prediction is that the critical-regime laws are valid on both sides of the transition.[27,28] This simply reflects the idea that, if a system is probed on length scales smaller than ξ, an insulator cannot be distinguished from a bad metal. In fact, these two states differ only in the low-temperature properties, and by definition in the zero-temperature conductivity (finite or equal to zero).

The relation between $L_{in}(T)$ and T depends upon the considered regime. If we introduce $\tau_{in}(T)$, which is the average time between two inelastic scattering events, we expect $L_{in}(T) \propto \tau_{in}(T)^{1/2}$ in the metallic regime and $L_{in}(T) \propto \tau_{in}(T)^{1/3}$ in the critical regime.[25] In general, $\tau_{in}(T)$ is proportional to T^{-p}. The value of p depends on the inelastic process (electron-electron inelastic scattering, electron-phonon inelastic scattering ...) and on the resistivity of the samples (the strength of disorder). Note that the dominant scattering process changes with temperature, so does p. All these features give complex $\sigma(T)$ laws, which evolve when approaching the MI transition.

From previous arguments, we can write a general relation for a given sample labeled by R between its conductivity

$\sigma_R(T)$, its inelastic scattering length $L_{in,R}(T)$ and the g function that following Abrahams *et al.*[22] should be the same for all the samples,

$$\sigma_R(T) = \frac{e^2}{\hbar}\frac{1}{L_{in,R}(T)} g\left(\frac{L_{in,R}(T)}{\xi_R}\right). \tag{5}$$

This relation is believed to be valid also in the presence of effective and finite interactions between electrons.[29] One way to extract the function g on the metallic side of the MI transition from the experimental $\sigma_R(T)$ curves is to use the following relations.[30] First, from Eqs. (3) and (2) we have $\sigma_R(0) = e^2/(\hbar\xi_R)$ and $\sigma_{Rc}(T) = e^2/[\hbar L_{in,R_c}(T)]$. The constants A and B have been omitted for simplicity, and R_c corresponds to a sample just at the MI transition. If we make furthermore the crude approximation that $L_{in}(T)$ is the same for all the samples (which is satisfactory only to describe general trends), then we get: [30]

$$\frac{L_{in}(T)}{\xi_R} = \frac{\sigma_R(0)}{\sigma_{Rc}(T)}, \tag{6}$$

$$g\left(\frac{L_{in}(T)}{\xi_R}\right) = \frac{\sigma_R(T)}{\sigma_{Rc}(T)}. \tag{7}$$

So the scaling hypothesis can be tested with a set of data $\sigma_R(T)$ and with the curve $\sigma_{Rc}(T)$ of a sample located exactly at the MI transition. It is worth noting that this approach is only valid as long as $L_{in}(T) > L_{el}$. Some recent analysis using Eqs. (6) and (7) was successfully performed in disordered systems like Nb-Si. [30]

III. RESULTS

The conductivity data are presented in this paper for $i-$Al-Pd-Re melt-spun ribbons prepared from the same ingot (nominal composition $Al_{70.5}Pd_{21}Re_{8.5}$) and with a maximal annealing temperature around 1000 °C. The structural quality of the samples was checked by powder x-ray diffraction, as well as microdiffraction at the ESRF Synchrotron. All the peaks of the patterns could be indexed by the icosahedral phase, with no secondary phase. The latter was also confirmed by scanning electron microscopy and microprobe analysis.[5,11]

A critical point in the scaling analysis near the MI transition is the accurate estimate of the absolute conductivity of the samples. Actually, a direct estimate of σ in our irregular-shaped small ribbons can lead to relative errors up to 20%. Thus, by analogy with what is done in disordered systems, we have used the resistance ratio $R = \rho(4\ \mathrm{K})/\rho(300\ \mathrm{K})$, that can be determined to a much higher accuracy (better than 1%). We have established an empirical relation between R and the actual σ value by taking an average curve from σ (R) measurements on a large number of samples. [11,32] As a consistency test, we have also checked that each R value is associated with a single $\sigma(T)$ behavior, and that the $\sigma(T)$ curves evolve smoothly and gradually with R. This confirms the idea that R is a relevant parameter to sort the ribbon samples according to their proximity to the MI transition. As previously discussed, we can obtain ribbons with resistance ratios between 2 and 200 from the same ingot.

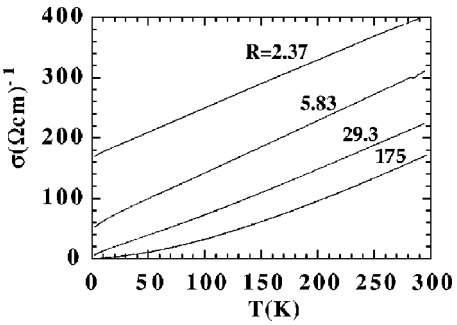

FIG. 1. Conductivity between 2.5 K and 300 K for two $i-$Al-Pd-Re metallic ribbons ($R = 2.37$ and 5.83) and two $i-$Al- Pd-Re insulating ribbons ($R = 29.3$ and 175).

The conductivity data in the range 2.5 K–300 K were obtained in a homemade He_4 flux cryostat. We used a four-aligned contact geometry, with a dc current source Keithley 224 and a nanovoltmeter Keithley 181. A few samples were also measured down to 0.4 K in a He_3 Oxford Instrument cryostat, with a low frequency ac bridge (Linear Research 700). In both cryostats, the current intensities were chosen low enough to prevent heating effects, and special care was taken for the thermalization of the samples. One ribbon with $R \simeq 20$, prepared from a different ingot but with the same process, was measured in a dilution fridge down to 10 mK. Details have already been reported,[14,31] and we are confident in the thermalization of this sample down to 20 mK.

In Fig. 1, typical $\sigma_R(T)$ curves for two metallic ribbons (with $R = 2.37$ and 5.83), and two insulating ribbons (with $R = 29.3$ and 175) are presented between 2.5 K and 300 K. All the curves are parallel around 300 K and the differences are more pronounced only at low temperatures. From at least 50 K up to 300 K, the temperature dependence of σ can be described by a power law, $\sigma(T) = \sigma_0 + \sigma_1 T^{\alpha}$. The exponent increases gradually from $\alpha = 0.97$ for $R = 2.37$ to $\alpha = 1.5$ for $R = 175$.

The determination of the zero T conductivity, and thus the precise location of the MI transition, is difficult to handle. Figure 2 is a zoom into the low temperature range (0.4–20 K) of $\sigma_R(T)$ for four samples near the MI transition ($R = 14.1$, 16.6, 21.8, and 29.3). Within the MI literature, two kinds of extrapolation laws are used: $T^{1/2}$ and $T^{1/3}\sigma(T)$ dependence.[33–36] If we limit our analysis above $\simeq 2.5$ K ($2.5^{1/2} \simeq 1.6$), $T^{1/2}$ law gives satisfactory results (see the middle panel of Fig. 2). But at lower temperature, the decrease of the conductivity seems to be stronger, at least for the $R = 14.1$, 16.6, and 21.8 samples. For these three samples, a $T^{1/3}$ dependence better fits the curves between 0.5 K and 5 K (bottom panel of Fig. 2). By choosing a $T^{1/2}$ law for the extrapolation, as was already done for metallic $i-$Al-Pd-Re samples,[8] we find that the MI transition is located at a R value just above $R = 21.8$ (compared with $R = 14.1$ if we have chosen the $T^{1/3}$ law). These results agree with similar recent studies in $i-$Al-Pd-Re ingots.[9]

However, measurement of a $R \simeq 20$ (see the top panel of Fig. 3) ribbon down to 10 mK provides interesting informa-

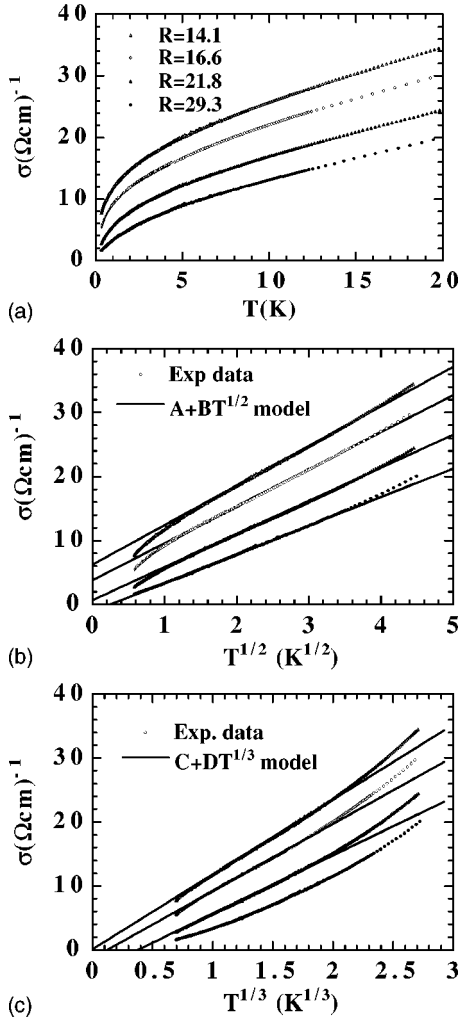

FIG. 2. Conductivity σ as a function of temperature T (top panel), $T^{1/2}$ (middle panel), and $T^{1/3}$ (bottom panel), for $i-$Al-Pd-Re ribbons near the MI transition. In each panel, from top to bottom: $R=14.1$, 16.6, 21.8, and 29.3. The temperature ranges between 0.4 K and 20 K in all panels. The $A+BT^{1/2}$ and $C+DT^{1/3}$ fits used to estimate the $\sigma_R(T=0)$ values are indicated by full lines.

tion. In the common T range of measurement, above $\simeq$400 mK, the normalized conductivity $\sigma(T)/\sigma(4\ \mathrm{K})$ is indistinguishable between the $R=20$ and $R=21.8$ samples (due to similar R), both samples having a $T^{1/3}$ dependence up to $\simeq$5 K. A drastic change in the T dependence of σ is observed only below $\simeq$200 mK. Indeed, the conductivity seems to saturate at a constant value notably different from the value extrapolated with the $T^{1/3}$ law. Similar trends were already observed[7] in other ribbons measured down to 40 mK. Other changes in the T dependence of σ are of course *a priori* possible at lower temperatures and thus, the actual extrapolation at $T=0$ is not an easy task, as also debated in many disordered systems near the MI transition (see, for example, Ref. 33).

Interesting enough the usual extrapolation of $\sigma(T)$ to $T=0$ with the $T^{1/2}$ power law gives a $\sigma(T=0)$ value close to the saturation value inferred from the measurement down to

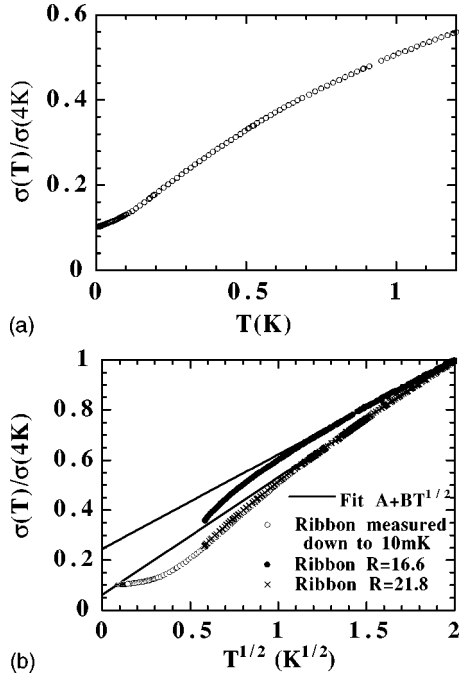

FIG. 3. Top panel: Very-low-temperature conductivity (T between 10 mK and 1.2 K) as a function of T for a ribbon of $R\simeq 20$. The conductivity is normalized at 4 K. Bottom panel: conductivity normalized at 4 K as a function of $T^{1/2}$ for different ribbons ($T<4$ K). Data are plotted for three samples: the sample measured down to 10 mK and two samples measured down to 400 mK ($R=16.6$ and $R=21.8$). The $A+BT^{1/2}$ fits from Fig. 2 are also plotted for the $R=16.6$ and $R=21.8$ as full lines.

10 mK (see the bottom panel of Fig. 3), and indeed much different from the one deduced with $T^{1/3}$ law [$\sigma(T=0)$ negative]. We would expect a similar feature for the other metallic samples in view of the similar $\sigma(T)$ behavior (see for instance $R=16.6$ plotted on the same scale). Consequently we eventually deduce $\sigma_R(T=0)$ values of our metallic samples from the $T^{1/2}$ law drawn as straight lines in Fig. 2. The quality of the scaling (see Fig. 4) obtained with

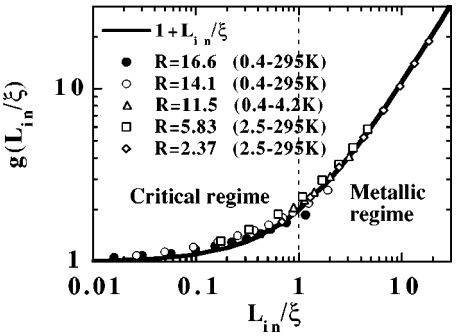

FIG. 4. Scaling function g on the metallic side of the MI transition as a function of the parameter $L_{in}(T)/\xi$. For each $i-$Al-Pd-Re ribbon, the resistance ratios R and the temperature range of conductivity measurements are indicated. The theoretical curve $1+L_{in}/\xi$ is represented as a full line. The critical and the metallic regimes are also located. They are separated by the dotted line $L_{in}(T)/\xi=1$.

TABLE I. Parameters extracted from the scaling analysis (see text).

R	$\sigma_R(0)(\Omega\ \mathrm{cm})^{-1}$	T_1 K	L_{in} (4 K)/ξ_R	ξ_R Å
2.37	162	200	15	6.8
5.83	41.2	40	3.7	27
11.5	10.0	4	0.97	100
14.1	6.21	1	0.57	170
16.6	3.77	0.5	0.35	290

this choice of $\sigma_R(T=0)$ confirms *a posteriori* that this extrapolation value is meaningful.

We now come to the scaling model. Following the above procedure, we have deduced $\sigma_R(T=0)$ for a number of ribbons on the metallic side of the MI transition, with $R=2.37$, 5.83, 11.5, 14.1, and 16.6 (see Table I). As explained above, we have chosen the $R=21.8$ ribbon as the critical sample, i.e., the closest to the MI transition. We have finally plotted on a double-logarithmic scale g versus $L_{in}(T)/\xi$, from Eqs. (6) and (7). Each sample is represented in Fig. 4 by a different symbol. For each sample, only data at the highest and the lowest T range of measurements and a few points at intermediate temperatures are plotted, not to overload the figure. We can see that in a first approximation, all the samples collapse on a single curve. Moreover, this curve is very close to the theoretical one expected for disordered systems near the Mott-Anderson transition, namely, $1+L_{in}(T)/\xi$ (full line in Fig. 4). The small deviations in the low-T range of $\sigma_R(T)$ near the MI transition could be attributed to several factors: the uncertainty in the actual $\sigma(T=0)$ values, the fact that the $R=21.8$ sample is probably not exactly critical and the possible variations of $L_{in}(T)$ values with R. Note that we have also tested the scaling model with a $T^{1/3}$ extrapolation law for $\sigma_R(T)$ but in that case, the different samples do not collapse on a single curve. This gives an order of magnitude about the sensitivity of the scaling to its input parameters, like the values of $\sigma(T=0)$. Once again this scaling picture is quantitatively and qualitatively similar to what is experimentally found in disordered systems on the metallic side of the Mott-Anderson MI transition.[30]

We can deduce from Fig. 4 the temperature, that we call T_1, such as $L_{in}(T_1)=\xi$ [actually, of the order of ξ because the ratio A/B was neglected in equation (6)]. By definition of ξ, T_1 is the limit between the critical regime ($T>T_1$) and the metallic regime ($T<T_1$). The T_1 values reported in Table I decrease from 200 K for the the $R=2.37$ ribbon down to 0.5 K for the $R=16.6$ ribbon near the MI transition. In the last column of Table I, we tentatively give an estimate of ξ_R for the different samples, by assuming[12] an inelastic mean-free path at 4 K of 100 Å. We find a maximal value of 290 Å for the sample closest to the MIT ($R=16.6$). This value matches with a direct estimate from the exact formulation of equation (3), namely,[34] $\sigma(T=0)=e^2/(3\pi^2\hbar\xi)$, which gives $\xi_R=220$ Å for $\sigma(T=0)=3.77\ (\Omega\ \mathrm{cm})^{-1}$.

IV. DISCUSSION

The first result of Fig. 4 is that the conductivity of our $i-$Al-Pd-Re ribbons with $R\leqslant 22$ follows the one-parameter scaling theory of the Mott-Anderson MI transition. This is actually another proof for the existence of a MI transition in this system. The previous evidence for a MI transition came from the direct analysis of the conductivity dependence on temperature. This was done on the one hand by an approach from the metallic side [$\sigma(T)=\sigma_0+m\sqrt{T}$]. The transition is there estimated [8] to occur for the sample having $\sigma_0=0$, as we did in Fig. 2. But as it has already been mentioned, this analysis suffers from possible deviations at lower temperature (see Figs. 2 and 3). Moreover, it is also bound to the actual interpretation of the temperature dependence. Indeed we believe that these extrapolation laws should not be used in principle to locate the MI transition since they are taken in the temperature range of the critical regime (see below). Moreover, the very low temperature $\sigma(T)$ could eventually saturate, even for nonmetallic samples, the reason being in some cases an improper thermalization of the samples, as explained elsewhere.[11,31] On the other hand, insulating samples were considered. The insulating character was shown[9,10,14,15] by an exponential T dependence of $\sigma(T)$. However, some concerns were risen because of the small T_0 value,[14] the presence of an additional constant term[9,15,16] or in some case a small fitting range.[32]

The second result of Fig. 4 is that we could locate the MI transition for a sample of ratio $R_c\simeq 20$. The value determined this way is in good agreement with previous estimates on the same ribbon-type samples, from magnetoresistance measurements.[4] The magnetoresistance $\delta\rho(H)/\rho$ cannot be fitted even qualitatively[12] by the quantum-interference effects in the metallic regime for ribbon samples of $R\geqslant 23$. Also $\delta\rho(H)/\rho$ goes through a maximum around $R=20$–30, which was early identified[4] as the MI transition by analogy with disordered systems. Finally this R_c value is also in agreement with other estimates from $\sigma(T)$ measurements on ingots[9] or thin-film samples.[10]

The third conclusion that stems from Fig. 4 is the importance of the critical regime for all the metallic ribbons measured. For example, from Fig. 4 and Table I, the sample $R=16.6$ lies for all the T range of measurements inside the critical regime. The $R=5.83$ sample enters the critical regime above $\simeq 40$ K. Only the more metallic sample of this series ($R=2.37$) is in the critical regime in a small T range. Then, our estimate of $\sigma(T=0)$ cannot be physically justified *a priori*, because by definition in the critical regime a system has not experienced its metallic or insulating nature, so critical laws cannot give precisely $\sigma(T=0)$.

The different curvatures of the temperature dependence $\delta\sigma(T)$ can be readily interpreted by considering the critical regime ($T>T_1$) and the lower-temperature metallic regime ($T<T_1$). From Table I, the samples closest to the MI transition ($R\geqslant 14$) are in the critical regime for almost the entire T range. In this regime the temperature dependence of the conductivity is $\delta\sigma(T)\propto\tau_{in}(T)^{-1/3}$. The metallic regime should be recovered only at the lowest temperatures, for instance below 200 mK for the $R=20$ sample shown in Fig. 3. Above T_1, the $\delta\sigma(T)\propto T^{1/3}$ (see bottom panel of Fig. 2) can be explained in the critical regime by an inelastic scattering time $1/\tau_{in}(T)\propto T^p$, from which $p=1$ is deduced. A straight-

forward interpretation is the occurrence of low T electron-electron interactions effects,[37] which are actually expected and observed at the lowest temperatures in disordered systems. By increasing temperature, the main inelastic scattering mechanism in disordered systems becomes electron-electron diffusions with[34] $1/\tau_{in}(T) \propto aT^{3/2} + bT^2$. So in the critical regime, we expect $\delta\sigma(T) \propto aT^{1/2} + bT^{2/3}$. The $\sqrt{T}$ observed in the middle panel of Fig. 2 can thus be readily accounted for. At higher temperature but below the Debye temperature Θ_D ($\Theta_D \approx 450$ K in $i-$Al-Pd-Re from specific heat measurements[2]), τ_{in} is dominated by electron-phonon inelastic processes. In a disordered metal, the exponent p lies between 3 and 4 and increases with the strength of disorder,[38] i.e., p increases by decreasing σ. This yield a power law for $\delta\sigma(T)$ with an exponent between 1 and 4/3. Such values correspond quite well to our observations above $\simeq 50$ K (see Fig. 1). Indeed, the exponent in the higher temperature range is just below 1 for the ribbon $R = 2.37$ and is increased to 1.13 for the less conducting $R = 21.8$ one (all the present samples are in the critical regime at high temperature).

Considering now the more conducting samples, they suffer a change of regime from metallic to critical at much higher temperature (see Table I). This does not affect the scattering mechanism but its relation to the conductivity: $\delta\sigma(T) \propto \tau_{in}(T)^{-1/2}$ in the metallic regime compared to a power $-1/3$ in the critical regime. Similarly as above, we thus expect in the metallic regime $\delta\sigma(T)$ to evolve from a $\sqrt{T}$ (low T electron-electron interaction effects) at the lowest temperatures, to power laws intermediate between $T^{3/4}$ and T (exponent just below or equal to one) at higher temperature (electron-electron inelastic diffusions), and stronger exponent ($T^{3/2}$ and T^2) at higher temperature (electron-phonon inelastic diffusions). These laws are well observed in our samples (see Fig. 1) for the higher-conducting samples up to the crossover temperature T_1. Actually, the crossing from one dominant scattering mechanism to another is very smooth, so we do not expect to observe more than a slight curvature change in the $\delta\sigma(T)$ curves.

Important conclusions can be drawn from the validity of this scaling analysis for the insulating samples too. For highly resistive samples, the critical regime should dominate the T dependence of σ, down to low T near the MI transition. This remark naturally explains the continuity and similarities of the $\sigma(T)$ curves around 300 K independent of their metallic and insulating character that was already noticed in Fig. 1. We will propose in a forthcoming paper a coherent interpretation of the whole T range of insulating $\sigma(T)$ curves based on this idea. Note in particular that the same scaling analysis can be used in principle for insulating samples too.

It is not common to observe such a scaling behavior in an alloy of metals, which puts emphasis on the quasicrystalline nature of the material. The reduced density of states and the very low conductivity are to be understood in the framework of theories dealing with this specific structure (role of quasiperiodicity, icosahedral atomic environments, defects). However, we have shown that the $\sigma(T)$ curves ($T < 300$ K) of $i-$Al-Pd-Re ribbons of $R <\simeq 20$ can be interpreted consistently with theories developed for the Mott-Anderson MIT in disordered systems with no need for quasiperiodic specific laws. Indeed in quasiperiodic systems an anomalous spreading of the wave function was numerically observed,[19–21] and a diffusive regime is predicted to be recovered beyond a time τ, given by $1/\tau = 1/\tau_{el} + 1/\tau_{in}(T)$. It can be shown that $\sigma \propto \tau^{2\beta-1}$ with $0 < \beta < 1$ depending on the quasiperiodic Hamiltonian parameters and the energy considered. If $\tau_{in}(T) < \tau_{el}$ [$L_{el} > L_{in}(T)$], the T dependence of the conductivity is determined by the law[17] $\sigma(T) \propto \tau_{in}(T)^{2\beta-1}$, which can give temperature power laws with *a priori* unexpected exponents. In the reverse case $L_{in} > L_{el}$, it is expected to find the usual laws of disordered systems for the temperature dependence of conductivity. Within this simplified description, the quasicrystalline and disordered-system pictures can be viewed as complementary by supposing that L_{el} is not too large in these systems so that the relation $L_{in} > L_{el}$ is valid in a wide T range. In any case, the microscopic relations between conductivity and defects, for instance, should be addressed by the quasiperiodic laws through $\sigma \propto \tau_{el}^{2\beta-1}$. It should also be emphasised that these systems are not ordinary disordered systems, for which a very short mean free path would simply result in a trivial amorphous metallic-alloy behavior of much larger conductivity. Actually, a small L_{el} remains surprising in a structurally ordered alloy. In this regard, the determination of the L_{el} value in $i-$Al-Pd-Re samples, and more generally in highly resistive icosahedral alloys, would be a key element.

V. SUMMARY

In conclusion, a one-parameter scaling analysis is proposed for the first time in order to explain the temperature dependence of the conductivity of quasicrystals. The analysis includes all our $i-$Al-Pd-Re ribbon samples on the metallic side of the MI transition (resistance ratios below 20) and spreads over a temperature range from 0.4 K up to 300 K. Within this approach, firstly the transition has been located to occur for a critical sample of $R \simeq 20$. Secondly, the entire T range of the present $\sigma(T)$ measurement can be interpreted, together with the evolution of the residual resistivity. Thirdly, our analysis brings out the importance of the critical regime near the MI transition, characterized by a specific T dependence of σ. This regime should also be considered in insulating $i-$Al-Pd-Re samples in order to understand the conductivity behavior in the whole temperature range. It would be now interesting to perform the scaling analysis on the other set of $i-$Al-Pd-Re samples, like ingots, in order to confirm its universality in this system. Last but not least, our data are similar to what is found experimentally and predicted theoretically in disordered systems near the Mott-Anderson MI transition. This is an interesting issue in a structurally ordered system composed only of metallic elements. This point deserves further theoretical and experimental works. Indeed, disorder and electron correlations are the driven parameters of the MI transition in disordered systems, but their actual role in highly resistive $i-$Al-Pd-Re samples is still unclear.

ACKNOWLEDGMENTS

We wish to acknowledge G. Fourcaudot and J. C. Grieco for their help in preparing the samples, and J. P. Brison for the conductivity measurements down to 10 mK. M. Capitan and J. Alvarez are warmly thanked for the high-resolution diffraction experiments at the European Synchrotron Radiation Facility. We also want to thank T. Grenet for his fruitful comments on the paper.

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