Generation of a dc voltage by an ac magnetic field in type-II superconductors

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We show that an ac external magnetic field can generate a dc voltage in type-II superconductors carrying a constant transport current. This rectifying effect occurs even at low temperatures where flux creep may be disregarded and even for arbitrarily small applied current. The dc signal appears when the magnitude of the applied ac field exceeds a threshold value which depends on the shape of the superconductor and on the applied current. Experiments on this subject are discussed.

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In recent experiments Huebener and his group[1,2] discovered a striking effect: If a large dc and small ac magnetic field are applied perpendicular to a type-II superconductor thin strip of Mo_3Si then this strip exhibits an Ohmic dc resistivity. This effect occurs even at low temperatures, where flux creep is negligible, and at dc measuring currents I much less than the critical current I_c of the strip. Therefore, neither flux creep nor flux flow played an essential role in these experiments and the superconductor is in the fully penetrated critical state where the current density has saturated to its critical value everywhere in the material. The observed voltage drop along the strip thus means that there is a continuous net transfer of Abrikosov vortices across the sample in the *critical state*. On the other hand, in the applied dc and ac field the critical state of the strip changes periodically with time t such that when averaged over one cycle, the time derivative of the magnetic induction B is zero. Therefore, no dc electric field should be expected, in contradiction to the experimental observation.

Although the authors of Refs. 1,2 did not present the correct explanation of the effect, they proposed simple formulas which describe all the features of the experimental data: The measured voltage drop U is proportional to both the amplitude and frequency of the ac field and to the applied transport current I along the strip, as long as this current is smaller than the critical current I_c. In addition to this, they experimentally proved that $U \propto I_c^{-1}$ and that the voltage U depends on the temperature T and on the value of the large external dc field B_0 only through $I_c(T,B)$. A *qualitatively* correct explanation of the effect can be derived from the results of Ref. 3, where a dc electric field was studied in the case of a slab in a *longitudinal* magnetic field.

In this paper we point out how a transfer of vortices across the superconductor can occur in its *critical state* and thus resolve the seeming contradiction. Based on this idea, a theory of the effect is developed for the real geometry of the experiments: thin strips in a *transverse* magnetic field. Our strip theory explains all the above-mentioned features of the observed dc voltage.

There remains, however, a puzzling problem. According to Faraday's law, the maximum electric field in the sample, even *before* averaging, is of the order of $E \sim (\partial B/\partial t)w \sim \omega B_1 w$, where ω and B_1 are frequency and amplitude of the ac field and w is a characteristic size of the sample. Surprisingly, the measured voltage drop is *larger* than this estimate and our theoretical result by two orders of magnitude. A similar discrepancy was seen in the theoretical estimates of Refs. 1,2, but there this discrepancy was removed by multiplication of the estimated dc voltage by a geometrical "enhancement factor" equal to the aspect ratio of the strip. This enhancement factor cannot be justified by the strict theory. We shall present a possible explanation of this quantitative discrepancy in terms of flux focusing and suggest new experiments which could test our explanation.

Consider a superconductor filling the space $|x| \leqslant w$, $|y| < \infty$, $|z| \leqslant d/2$, with a homogeneous magnetic field $B_a(t) = B_0 + B_1 \cos \omega t$ applied along z and a transport current I along y. If $d \ll w$ this geometry describes a thin strip in perpendicular field, and for $d \gg w$ a slab in parallel magnetic field, both with width $2w$. We first calculate the above effect for the slab and then for the strip. In both geometries we make the usual Bean assumptions that the critical current density j_c does not depend on the local induction B and that creep is negligible. Furthermore, since B_0 is much larger than the lower critical field B_{c1}, one may put $B = \mu_0 H$. We show now that with these simple assumptions the appearance of a dc voltage U even at small currents $I \ll I_c = 2wdj_c$ can be explained and the experimentally observed four dependences $U \propto B_1 \omega I/I_c$ are obtained.

For the *slab* geometry, with these assumptions the slope of the field profile is $|dB/dx| = \mu_0 j_c$ everywhere since the field inside the superconductor is much larger than the field of full penetration, $B_p = \mu_0 j_c w$ when $B_0 \gg B_p$. The applied current I causes the profile $B(x)$ to become *asymmetric* since at the two surfaces $x = \pm w$ the field values are now different, $B(x = \pm w) = B_a \mp \mu_0 I/2d$. Due to the ac component $B_1 \cos \omega t$ of B_a these two field values and the resulting critical profiles $B(x,t)$ also oscillate. The two extreme profiles belonging to $B_a = B_0 \pm B_1$ are depicted in Fig. 1. When the ac amplitude B_1 is small, the oscillation of the profile occurs only close to the surface, but near the specimen center the

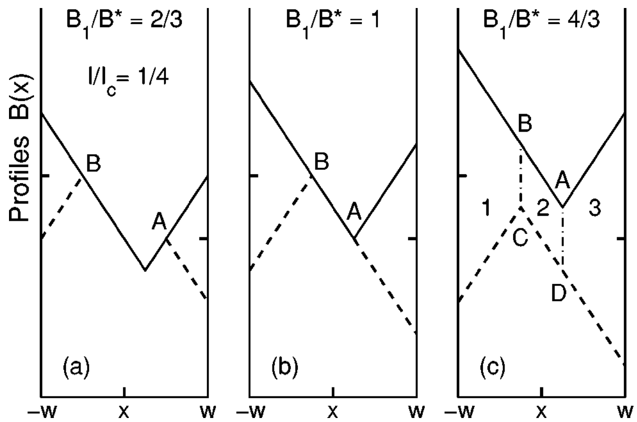

FIG. 1. Magnetic field profiles $B(x)$ in a *slab in longitudinal* dc and ac fields $B_a = B_0 + B_1 \cos \omega t$ and with applied transport current $I = I_c/4$ at various ac amplitudes: (a) $B_1 = 2B^*/3$, (b) $B_1 = B^*$, (c) $B_1 = 4B^*/3$. Bean model. Shown are two extreme profiles at $B_a = B_0 + B_1$ (solid lines) and $B_a = B_0 - B_1$ (dashed lines). In (a) and (b) no magnetic flux crosses the slab, since the central part AB of $B(x)$ remains frozen. In (c) during one half cycle, the flux in areas 1 and 2 enters from left, and the flux in 3 enters from right. In the other half cycle, flux 1 leaves at left and flux 2 and 3 at right. Thus, during each cycle the flux 2, contained in the parallelogram $ABCD$, crosses the slab from left to right.

profile remains frozen, see Fig. 1(a). Therefore, during one cycle the same amount of flux enters and leaves at each surface, and no flux lines can pass through the center. The time-averaged longitudinal voltage is thus zero.

When B_1 exceeds the threshold field

$$B^* = (1 - I/I_c) B_p, \tag{1}$$

where $I_c = 2wdj_c$ is the critical current and $B_p = \mu_0 j_c w$ the field of full penetration of the slab at $I = 0$, the oscillating parts of the flux density profiles penetrate deeper and meet each other. In this case magnetic flux is pumped across the slab as depicted in Fig. 1(c). This net transport of flux originates from the asymmetry that now, during each cycle, on one side more flux exits than enters, while on the other surface more flux enters than exits. The time-averaged electric field generated by this *vortex motion* is $E_{\rm av} = (\omega/2\pi) S_{ABCD}$ where $S_{ABCD} = (2I/dj_c)(B_1 - B^*)$ is the area of the parallelogram $ABCD$ in Fig. 1(c). One thus has[3]

$$E_{\rm av} = 0 \quad \text{for} \quad B_1 < B^*,$$
$$E_{\rm av} = \frac{I}{I_c} \frac{2w\omega}{\pi} (B_1 - B^*) \quad \text{for} \quad B_1 \geqslant B^*. \tag{2}$$

The obtained nonzero $E_{\rm av}$ does not contradict the time-averaged Maxwell equation $dE_{\rm av}/dx = -\langle \partial B/\partial t \rangle = 0$ because this finite $E_{\rm av}$ is constant at all points of the sample. This is so since each vortex which moves from one side of the slab to the other has to cross all points of the slab width. Note also that the magnitude of $E_{\rm av}$ is comparable with the amplitude $E_{\rm ac} \sim w\omega B_1$ of the ac electric field induced by the alternating magnetic field.

In slab geometry, the threshold B^*, Eq. (1), has a simple physical meaning. It is the field of full flux penetration into a slab with current I. If the threshold B^* and factor 2 are omitted, Eq. (2) agrees with a formula suggested in Ref. 1 to fit the measured $E_{\rm av}$. We shall show now that exactly the same expression (2) applies to thin strips but with a different threshold value.

The physics of the flux transport across the *strip* is essentially the same as for the slab. But for superconductor thin strips in perpendicular field, the Maxwell equations yield a *nonlocal* relation between the sheet current J (the current density integrated over the thickness d) and the normal component of the magnetic field B_z, and in general the critical state problem is highly *nonlinear*. In this *transverse geometry* the Bean critical state model was solved exactly for strips with applied transport current[4], applied perpendicular magnetic field,[5] and for both applied current and field,[6,7] see also Ref. 8. From this solution we obtain the following scenario.

When a large dc field B_0 and an arbitrary transport current I are applied, the strip is driven into the fully penetrated critical state where the sheet current J has saturated to its critical value $j_c d$ everywhere in the strip.[6,7] When B_0 and I are kept constant and an additional ac field is applied, the sheet current $J(x,t)$ and local induction $B_z(x,t)$ in general have complicated spatial and temporal dependences, which in principle can be calculated from equations of Refs. 6,7 or by computation.[9] However, as long as the ac amplitude B_1 is below some threshold value B^*, a region of *frozen flux* persists inside the strip. Therefore, vortices cannot cross the strip and the time-averaged electric field along the strip is strictly zero, as for the slab.

At larger amplitudes $B_1 \geqslant B^*$, the ac field penetrates the strip deeply such that at the maximum and minimum of the applied field $B_a = B_0 \pm B_1$, the sheet current saturates to $|J(x)| = j_c d$ almost everywhere and changes sign at only *one* position in the strip, at $x = \mp x_0$ with $x_0 = wI/I_c$; $I_c = 2wdj_c$ is the maximum supercurrent of the strip. The two extreme profiles of the magnetic field B_z for this piecewise constant $J(x)$ are, see Fig. 2,

$$B_z(x) = B_0 \pm B_1 \pm B_{\rm cr} \ln \frac{|x \mp x_0|}{(w^2 - x^2)^{1/2}}, \tag{3}$$

where $B_{\rm cr} = \mu_0 j_c d/\pi$ is the characteristic magnetic field for the strip[5] and the upper and lower signs correspond to the maximum and minimum of $B_a(t)$, respectively. The flux transported during one cycle across the strip is given by the area $ABCD$ denoted in Fig. 2(b). Calculating this area S_{ABCD} we find that the dc component of the electric field $E_{\rm av}$ is still described by Eq. (2) but with a different threshold value B^*,

$$B^* = \frac{B_{\rm cr}}{2} \left[\frac{1}{\tilde{I}} \ln \left(\frac{1 + \tilde{I}}{1 - \tilde{I}} \right) + \ln \left(\frac{1 - \tilde{I}^2}{4\tilde{I}^2} \right) \right], \tag{4}$$

where $\tilde{I} \equiv I/I_c$. Note that no ‘‘geometric enhancement factor’’ appears in Eq. (2), which applies to both slabs and

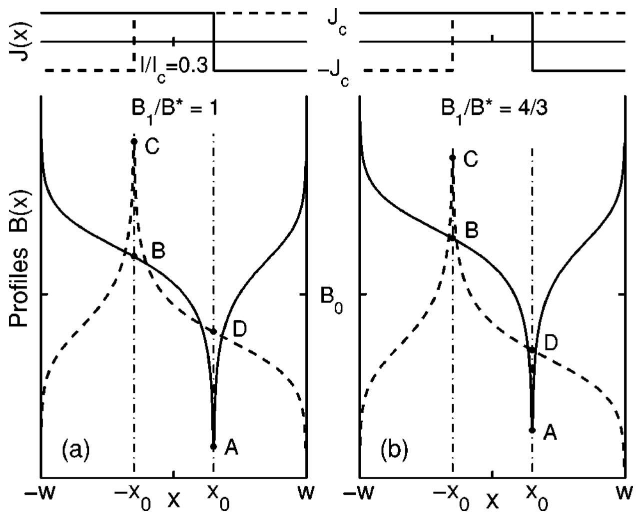

FIG. 2. As in Fig. 1 but for a *strip in transverse* dc and ac fields with current $I=0.3I_c$ at two ac amplitudes (a) $B_1=B^*$, (b) $B_1=4B^*/3$. In (a) the region of frozen flux has just shrunk to zero width and still no magnetic flux crosses the strip, since the area $ABCD$ is zero. In (b) during each cycle the flux contained in the area $ABCD$ crosses the strip from left to right and causes a dc voltage. The top of this figure shows the profile of the sheet current $J(x)$, which is the same for cases (a) and (b).

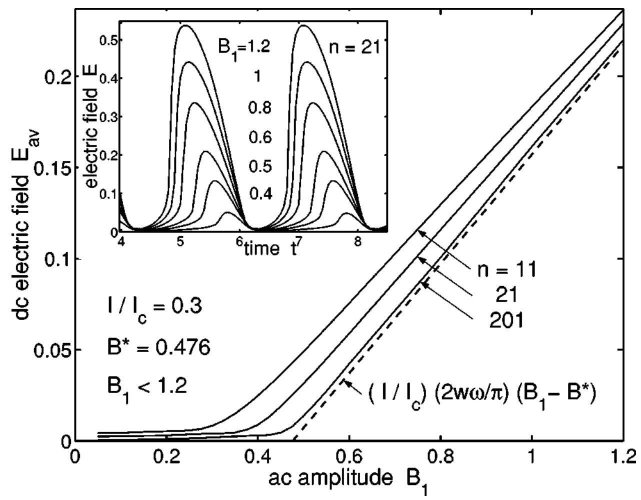

FIG. 3. Computed time averaged electric field $E_{\rm av}$ along a strip with current $I=0.3I_c$ versus the ac amplitude B_1 (in units $\mu_0 j_c d$) for creep exponents $n=11$, 21, and 201. The dashed line shows Eq. (2) with $B^*=0.476$ from Eq. (4) ($J_c=j_c d=1$, $w=1$, $E=J^n$, $B_a=B_0+B_1\cos\omega t$, $\omega=\pi/2$, $B_0=2$). The inset for $n=21$ shows one cycle (two pulses) of the time resolved electric field $E(t)$ averaged over the strip width at various ac amplitudes B_1.

strips. However, for strips the threshold (4) is of the order of $B_{\rm cr}=\mu_0 j_c d/\pi$ and is thus *considerably less* than the threshold for the slab, $B^*\approx B_p=\mu_0 j_c w$, Eq. (1), if one puts $d \ll w$.

According to Eq. (4), B^* is a decreasing function of $\tilde{I}$. A good approximation is $B^*\approx B_{\rm cr}\ln(e/2\tilde{I})$ for $\tilde{I}\leqslant 0.5$ and $B^*\approx 2(1-\tilde{I})B_{\rm cr}$ for $0.5\leqslant\tilde{I}<1$. At $\tilde{I}<d/2w$ or $I<j_c d^2$ the threshold B^* as usual should be cut off and is of the order of the field of full penetration of thin strips, $B_p=B_{\rm cr}\ln(2ew/d)$.[10] But as opposed to slabs, for finite current I the threshold B^* for strips, Eq. (4), does not coincide with the penetration field $B_p(I)$.

Our analytical theory is confirmed by computations of $B(x,t)$, $J(x,t)$, $E(x,t)$, and $E_{\rm av}$ using the method developed for superconductor thin strips in a perpendicular field[9] and generalizing it to the presence of a transport current. This *dynamic* method allows us to consider flux creep, described, e.g., by a constitutive law $E\propto(j/j_c)^n$ with creep exponent $n\gg 1$. For large $n\gg 1$ this computation indeed reproduces all the above results. For smaller n, flux creep increases the computed $E_{\rm av}(B_1)$ and rounds its sharp bend at $B_1=B^*$, see Fig. 3. These computations can be generalized to more complex situations. Namely, the general method[10] allows us to compute the dynamic ac and dc responses of strips with arbitrary thickness, accounting for any $j_c(B)$, for any law $E(j,B)$, for the lower critical field B_{c1},[11] and recently also for the nonzero London penetration depth λ.[12]

Formulas (2) and (4) explain the dependences of the experimental data[1,2] on the frequency ω and amplitude B_1 of the ac magnetic field and on the applied current I and critical current I_c of the superconductor. However, the measured dc voltages are *larger* than our result (and than simple estimates) by *two orders of magnitude*. We suggest the following explanation for this discrepancy.

The experiment[1,2,13] at $T=1.2-4.2$ K used 100 nm thick superconducting a-Mo_3Si and $Nd_{1.85}Ce_{0.15}CuO_x$ films of size 1×1 cm^2, into which slits of about 20 μm width were etched to pattern a 20×200 μm^2 strip with two current leads and two voltage leads, see Fig. 4. For a small ac magnetic field, the large film area surrounding the strip *behaves as in the Meissner state*, in spite of the large dc field, since the flux lines in the film are pinned and the ac penetration depth[5,6] is much smaller than the film width.[14] The small ac field is thus *expelled* from the film by screening currents that flow mainly near the film edges. The presence of slits slightly reduces these screening currents but forces them now to flow also along the borders of the slits. This generates an ac field inside these slits and at the strip which is much larger than the applied ac field.

To estimate this field enhancement we consider two ideally screening parallel strips placed in the x,y plane at $-a\leqslant x\leqslant -b$ and $b\leqslant x\leqslant a$. If the slit width $2b$ is much smaller than the total width $2a$, a perpendicular magnetic field B_1 is concentrated in the slit, see Fig. 4,

$$B(x)\approx B_1^{\rm eff}(1-x^2/b^2)^{-1/2}, \quad |x|<b, \tag{5}$$

$$B_1^{\rm eff}=B(x=0)\approx B_1\frac{a/b}{\ln(4a/b)}\gg B_1. \tag{6}$$

Thus, the physically correct "enhancement factor" in this experiment is approximately $B^{\rm eff}/B_1$, Eq. (6), which is of the order of the ratio of the total film width to the width of the

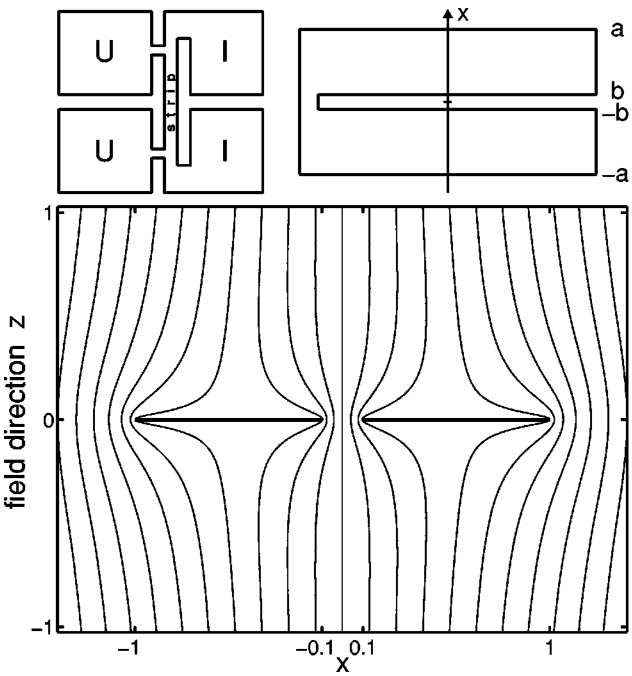

FIG. 4. Magnetic field lines near an ideally screening thin double strip ($b \leqslant |x| \leqslant a$, $a=1$, $b=0.1$, two thick lines) in a perpendicular magnetic field B_1. The field enhancement in the center of the gap is $10/\ln 40 \approx 3$ for $a/b=10$, cf. Eq. (6). Also shown are the schematic shape of the thin film specimen of the experiment (Refs. 1,2,14) (top left, *U* and *I* denote voltage and current leads) and the double strip used to estimate the field enhancement (top right).

slits that surround the strip, but is not related to the thickness or aspect ratio of the strip. This ''flux focusing'' effect may also explain why the threshold ac field in the experiment[1,2] is smaller than the theoretical thin-strip threshold B^*, Eq. (4). To test our suggestion, one may change the width of the slits in the film without changing the strip and the wiring. This should result in a change of the measured dc voltage.

In summary, we have shown that a continuous transfer of vortices, and thus a dc voltage, can occur in the critical state of type-II superconductors with transport current less than the critical current when dc and ac magnetic fields are applied simultaneously. For this transfer to occur, the amplitude of the ac field must exceed a certain threshold B^*, defined by Eqs. (1) and (4) for slabs and strips, respectively. This vortex transfer generates a dc voltage, even though the flux averaged over one cycle of the ac field is constant in the superconductor. The amplitude of this dc voltage is not small but of the order of the usual ac signal. The time resolved voltage drop shows *two* equal pulses per cycle, Fig. 3. Our results well describe the experimental dependences[1,2] of the electric dc field on the amplitude and frequency of the ac magnetic field and on the transport current and critical current in the the sample. As to the absolute value of the electric field, we propose that in the experiments[1,2] flux focusing strongly enhanced the applied ac field in the narrow slits of the superconducting film surrounding the strip, leading to a much larger observed signal and smaller threshold than our theoretical result.

Our results also shed light on the origin of another practically important and puzzling effect observed recently in superconductors.[15,16] The application of a weak ac magnetic field *perpendicular* to the main dc field leads to a fast decay of critical currents. This ''vortex shaking'' dramatically extends the reversible domain in the *B*-*T* phase diagram of superconductors and allows accurate measurements of the superconducting parameters and of possible phase transitions. We think that although the dc and ac magnetic fields are not parallel in these experiments, the observed decay here also originates from the generation of a dc electric field in the superconductor.

[1] M.P. Risse, M.G. Aikele, S.G. Doettinger, R.P. Huebener, C.C. Tsuei, and M. Naito, Phys. Rev. B **55**, 15 191 (1997).

[2] M.G. Aikele, R.P. Huebener, D. Weischer, and C.C. Tsuei, Physica C **290**, 109 (1997).

[3] The slab result (2) was obtained by V.V. Andrianov *et al.*, Sov. Phys. JETP **31**, 815 (1970); see also J.M. Delrieu, Solid State Commun. **11**, 1557 (1972); **12**, 881 (1973).

[4] W.T. Norris, J. Phys. D **3**, 489 (1970).

[5] E.H. Brandt, M.V. Indenbom, and A. Forkl, Europhys. Lett. **22**, 735 (1993).

[6] E.H. Brandt and M.V. Indenbom, Phys. Rev. B **48**, 12 893 (1993).

[7] E. Zeldov *et al.*, Phys. Rev. B **49**, 9802 (1994).

[8] E.H. Brandt, Rep. Prog. Phys. **58**, 1465 (1995).

[9] E.H. Brandt, Phys. Rev. B **49**, 9024 (1994); A. Gurevich and E.H. Brandt, Phys. Rev. Lett. **73**, 178 (1994); T.H. Schuster *et al.*, *ibid.* **73**, 1424 (1994).

[10] E.H. Brandt, Phys. Rev. B **54**, 4246 (1996); **58**, 6506 (1998); **58**, 6523 (1998).

[11] E.H. Brandt, Phys. Rev. B **59**, 3369 (1999); **60**, 11 939 (1999).

[12] E.H. Brandt and G.P. Mikitik, Phys. Rev. Lett. **85**, 4164 (2000); E. H. Brandt, Phys. Rev. B **64**, 024505 (2001).

[13] M. P. Risse, Diplomarbeit, University of Tübingen, 1996; M. G. Aikele, Diplomarbeit, University of Tübingen, 1997.

[14] A similar screening effect strongly enhances the resonance frequency of superconducting vibrating reeds: a small perpendicular ac field generated by periodic tilting is expelled from the reed even in a large dc field, see E.H. Brandt, P. Esquinazi, H. Neckel, and G. Weiss, Phys. Rev. Lett. **56**, 89 (1986).

[15] L.M. Fisher *et al.*, Physica C **278**, 169 (1997).

[16] M. Willemin *et al.*, Phys. Rev. Lett. **81**, 4236 (1998); M. Willemin *et al.*, Phys. Rev. B **58**, R5940 (1998).