ac-induced damping of a fluxon in a long Josephson junction

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We present a theoretical and experimental study of Josephson vortex (fluxon) moving in the presence of spatially *homogeneous* dc and ac bias currents. By mapping this problem to the problem of calculating the current-voltage characteristic of a small Josephson junction, we derive the dependence of the average fluxon velocity u on the dc bias current γ. In particular we find that the low frequency ac bias current results in an additional nonlinear damping of fluxon motion. Such ac induced damping crucially depends on the intrinsic damping parameter α and increases drastically as α is reduced. We find a good agreement of the analysis with both the direct numerical simulations and the experimentally measured $\gamma(u)$ curves of a long annular Josephson junction with one trapped fluxon.

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I. INTRODUCTION

Solitary waves have attracted a great attention for many years because they can be excited and observed in diverse nonlinear systems.[1,2] Well-known examples of such solitary waves are optical solitons, domain walls, kinks, and magnetic fluxons, just to name a few.

The latter case, a fluxon carrying the magnetic flux quantum Φ_0, has a form of a Josephson vortex moving along a long Josephson junction (LJJ). This system presents additional interest because the fluxon motion can be easily controlled by externally applied magnetic field and dc bias.[3–5] It is also well known that under these conditions the fluxon motion can be mapped to the classical mechanics of a macroscopic (relativistic) particle and such correspondence is practically exact.[2–5]

It was shown theoretically and experimentally that in the presence of both dc and ac bias the fluxon motion becomes more complex and it displays various fascinating phenomena.[3,6–10] Thus, as the frequency ω of ac bias is large, i.e., $\omega \simeq \omega_p$, where ω_p is the plasma frequency of a LJJ, the fluxon radiates plasma waves.[2,6,7] When the frequency ω is small ($\omega \ll \omega_p$) but *inhomogeneous* (static or dynamic) potential is present, the fluxon motion can be locked to the external ac bias. In this case the vertical steps (''Shapiro steps'') appear in the dependence of the average fluxon velocity u on the dc bias γ (analog of the *I-V* curve).[3,8,9] Moreover, in the same frequency region the resonant escape of a pinned fluxon has been observed.[10]

However, the fluxon motion in the presence of both *homogeneously* applied dc and ac bias for the frequency $\omega \ll \omega_p$, has not been investigated neither theoretically nor experimentally. In this Brief Report we present a theoretical and experimental study of this limit. We show that, unexpectedly, the influence of a small ac bias is rather large, and an additional ac induced damping increases as the standard damping characterized by parameter α reduces.

II. EQUATION OF MOTION AND THEORETICAL ANALYSIS

The dynamics of a fluxon in a long Josephson junction is described by a perturbed sin-Gordon equation for the Josephson phase $\varphi(x,t)$.[2,11] For a particular case of homogeneous dc (γ) and ac ($\varepsilon \sin \omega t$) bias currents this equation has the form[6,7]

$$\varphi_{xx} - \varphi_{tt} - \sin\varphi = \alpha\varphi_t - \gamma - \varepsilon \sin \omega t. \tag{1}$$

Here, the units of time and coordinate are correspondingly, the inverse plasma frequency $1/\omega_p$ and the Josephson penetration length λ_J. The dimensionless damping parameter α is determined by the direct losses that are due to the presence of quasiparticle (dissipative) current.

In the absence of perturbation [right-hand side of Eq. (1)] the solution of Eq. (1) is a fluxon moving with velocity u. In a general case we use the standard transformation of the Eq. (1) to the fluxon reference frame, where fluxon moves with an *average* velocity u. New variables are defined as

$$\xi = \frac{x-ut}{\beta}, \quad \tau = \frac{t-ux}{\beta}, \quad \beta = \sqrt{1-u^2}.$$

Equation (1) rewritten in the new variables is

$$\varphi_{\xi\xi} - \varphi_{\tau\tau} - \sin\varphi = -\frac{\alpha}{\beta}\varphi_\tau + \frac{\alpha u}{\beta}\varphi_\xi - \gamma - \varepsilon \sin\left(\omega \frac{\tau+\xi u}{\beta}\right). \tag{2}$$

Note here, that the fluxon, in its reference frame, ''feels'' the periodic coordinate and time dependent potential given by the last term of Eq. (2).

We are interested in the regime of small frequencies of ac bias current, namely, $\omega \ll 1$ in the dimensionless units. In this limit we can neglect the interaction of the fluxon with the plasma oscillations.[6,7] The potential energy $E_{\rm pot}$ of the fluxon is

$$E_{\rm pot}=\int_{-\infty}^{+\infty}\left[\frac{1}{2}\varphi_{\xi}^{2}-\gamma\varphi+(1-\cos\varphi)-\varepsilon\varphi\sin\left(\omega\frac{\tau+\xi u}{\beta}\right)\right]d\xi. \tag{3}$$

Introducing the fluxon solution in the form $\varphi(\xi,\tau)=\varphi_0[\xi-X(\tau)]$, where $\varphi_0(\xi)$ is a well known Josephson vortex solution we derive the equation of motion for the coordinate $X(\tau)$ of the fluxon (in the reference frame):

$$\ddot{X}+\frac{\alpha}{\beta}\dot{X}+\frac{\alpha u}{\beta}=\frac{\pi}{4}\gamma-\frac{\pi\varepsilon}{4\cosh\left(\frac{u\omega\pi}{2\beta}\right)}\sin\left(\omega\frac{\tau+Xu}{\beta}\right). \tag{4}$$

Thus, the main result can be seen from this equation: the applied ac bias current [last term in Eq. (4)] excites fluxon librations of the frequency ω/β and, in turn, the interaction of such librations with the ac bias current leads to the additional nonlinear damping of a fluxon motion. The interesting feature of the Eq. (4) is that it can be mapped to the problem of calculating the I-V characteristic (IVC) of the small Josephson junction with a specific damping, in general, different from α. To see a precise mapping we introduce new variables

$$y=\frac{u\omega}{\beta}X, \quad \tau_0=\tau\frac{\omega}{\beta},$$

and obtain the equation

$$\ddot{y}(\tau_0)+\frac{\alpha}{\omega}\dot{y}(\tau_0)+i_c(u)\sin(y+\tau_0)=\frac{u\beta}{\omega}\left(\frac{\pi\gamma}{4}-\frac{\alpha u}{\beta}\right), \tag{5}$$

where $i_c(u)$ is a "critical current" of the small junction which depends on the velocity u of the fluxon from our problem and is given by

$$i_c(u)=\frac{\pi u\beta\varepsilon}{4\omega\cosh\left(\frac{u\omega\pi}{2\beta}\right)}. \tag{6}$$

The γ-u characteristics of moving fluxon can be found from this equation in the following form:

$$\gamma=\frac{4\alpha u}{\pi\beta}+\frac{4\omega}{\pi u\beta}\left[i(1)-\frac{\alpha}{\omega}\right], \tag{7}$$

where $i(v)$ is the nonlinear current-voltage characteristics of a small Josephson junction with the critical current $i_c(u)$ and the damping α/ω. In Eq. (7), the first term is the usual γ-u characteristic for a fluxon moving along LJJ and the second term corresponds to the correction due to the presence of ac bias current.

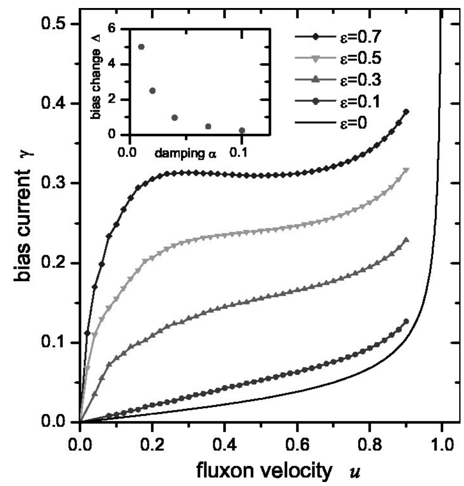

FIG. 1. The γ-u curves obtained by using Eq. (7) for various values of the ac bias amplitudes $\varepsilon=0$, 0.1, 0.3, 0.5, 0.7. The damping parameter α and ac frequency ω are, correspondingly, 0.04 and 0.03. The inset shows the normalized dc bias change Δ to the applied ac bias $\varepsilon=0.1$ at $u_0=0.3$, as a function of the damping parameter α.

The analytical expression for $i(v)$ dependence for small JJ at arbitrary values of damping and critical current is not known. Using numerically simulated $i(v)$ (Ref. 12) and Eq. (7), we obtain the $\gamma(u)$ dependence for various values of ε. Figure 1 shows γ-u characteristics calculated in this way for ε between zero and 0.7.

Let us analyze some limits. For small fluxon velocity, namely $u\ll\max\{\alpha,\omega\}/\varepsilon$, $\gamma(u)$ looks similar to a straight line, but its slope is steeper than in the autonomous case $\varepsilon=0$ (see Fig. 1):

$$\gamma=\frac{4\alpha u}{\pi\beta}\left[1+\frac{1}{2}\left(\frac{\pi\varepsilon}{4}\right)^2\frac{\beta^2}{\alpha^2+\omega^2}\right]. \tag{8}$$

In the opposite case $u\gg\max(\alpha,\omega)/\varepsilon$, the γ-u dependence just shifts by the constant value $\varepsilon\ll 1$ from the autonomous curve. Thus, ac bias induced damping is highly nonlinear, and the γ-u dependence displays an inflection point for moderate values of u (see Fig. 1).

In the region of small frequencies $\omega\simeq\alpha$ as the damping coefficient α decreases, the response of the fluxon to the ac bias current (which is proportional to $1/\alpha^2$) drastically increases [see Eq. (8)]. In order to analyze this effect in more precise terms we calculate the change of dc bias $\Delta=[\gamma(u_0)-\gamma_0(u_0)]/\gamma_0(u_0)$ as a function of the damping parameter α, for particular values of fluxon velocity $u_0=0.3$ and ac bias $\varepsilon=0.1$, where $\gamma_0(u_0)$ is the value of dc bias current in the absence of ac bias. The results are presented in the inset of Fig. 1.

III. NUMERICAL SIMULATIONS

In order to verify the analysis presented above and predictions made we performed direct numerical integration of the PDE (1). The periodic boundary conditions

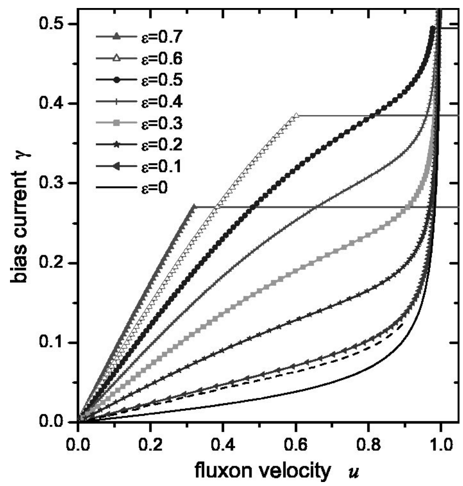

FIG. 2. Numerically simulated dc driving force vs velocity $\gamma(u)$ curves for various amplitudes of ac drive ε indicated in the legend. For comparison, dashed line shows the theoretical curve for $\varepsilon=0.1$ taken from Fig. 1.

$$\varphi(l,t)=\varphi(0,t)+2\pi, \tag{9}$$

$$\varphi_x(l,t)=\varphi_x(0,t) \tag{10}$$

were imposed and the normalized length of the LJJ l was taken to be 10. We used the damping coefficient $\alpha=0.04$ and the ac bias frequency $\omega=0.03$ as these values are very close to the experimental ones (see the next section). The obtained $\varphi(x,t)$ profile was used to find the voltage (proportional to the fluxon velocity u) across the junction. The averaging of velocity was performed over the time interval which is a multiple of the period of the ac frequency. We used two programs with different numerical schemes and averaging algorithms, and obtained identical results.

The γ-u curves for different values of the ac bias amplitude ε are shown in Fig. 2. The main effect of the ac induced damping which depends on the fluxon velocity is clearly present in numerical data.

Moreover, as the ac bias is small ($\varepsilon \leqslant 0.2$) the qualitative agreement between analytical results obtained above using the perturbation theory and direct numerical simulation is rather good practically in a whole region of fluxon velocities $u<1$ (see Fig. 2). However, as ε increases the theoretical analysis *overestimate* the ac induced damping in the region of small u and *underestimate* it in the opposite regime of large u. Most probably, the reason for that is that we neglect the interaction of the fluxon with the plasma oscillations. The direct numerical simulations also show that for high ac bias ($\varepsilon=0.5$,0.6, and 0.7) the state of moving fluxon becomes unstable, and the simulated curves in their upper range end with a point of switching to high voltages.

IV. EXPERIMENT

To observe the predicted effect we carried out real measurements using Nb/Al-AlO_x/Nb long annular Josephson junction[13] with a trapped magnetic fluxon. The junctions were underdamped with the typical value of $\alpha\approx 0.04$ at $T=4.2$ K. The junction has the mean radius $R=46$ μm and width $w=5$ μm. The critical current density j_c was about 100 A/cm^2, which corresponds to the normalized junction length l of about 10. A single magnetic fluxon (Josephson vortex) was trapped in the junction by applying a small dc bias current during the transition from the normal to the superconducting state while cooling down the sample.[5,10]

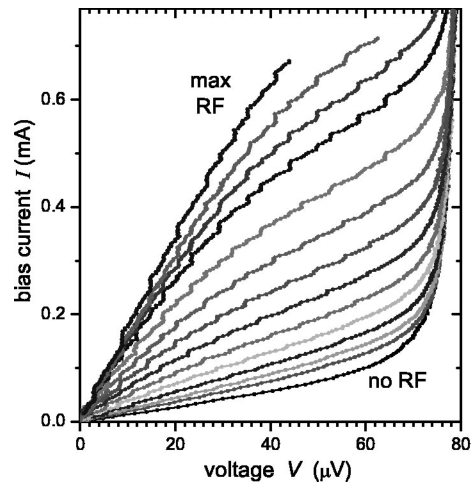

FIG. 3. The experimentally measured I-V curves of an annular LJJ with trapped fluxon for different values of microwave ac drive of the frequency 1.4 GHz at power levels varying from 4.3 to 20.6 dBm. The lowest curve corresponds to the autonomous I-V curve (zero ac power).

For the external ac bias the microwave radiation with different frequencies in the range from 1 to 20 GHz was used. Here we present the results only for $f\equiv\omega/(2\pi)=1.4$ GHz which corresponds to the $\omega\simeq 0.03\omega_p$. The microwave signal was applied by an antenna placed close to the sample, so that ac bias current was induced by the antenna in the bias leads of the junction. The typical power W of the applied microwave radiation was ranging from few pW to several tens of mW, referenced to the top flange of the sample holder.

We measured I-V curves of the annular Josephson junction with one trapped magnetic fluxon at different ac power levels W. These I-V are shown in Fig. 3. The measured I-V curves qualitatively look very similar to the $\gamma(u)$ dependencies obtained for the fluxon in Secs. II and III. Clearly, ac induced voltage-dependent damping is observed (see Figs. 1 and 2).

V. DISCUSSION

The I-V curves shown in Fig. 3 display many small steps which we explain by the presence of small nonuniformity of the LJJ. In fact these steps are nothing else but conventional Shapiro steps induced by ac drive. The imperfection of the junction can be due to either self-field effects, influence of the dc bias leads, or small technological inhomogeneities.

To check the influence of nonuniformity on simulated characteristics, we performed additional numerical simulations with a tiny external magnetic field h applied in the junction plane. It is a well known that a such magnetic field creates the coordinate dependent potential for the fluxon,[3–5] and therefore breaks the translational symmetry of the system. This leads to the change of γ-u curve such that the fluxon rotation frequency in the annular system gets locked to the frequency of ac drive in some ranges of γ. Physically, this effect corresponds to the appearance of ac-induced voltage locking steps known as Shapiro steps for Josephson junctions.

Figure 4 clearly shows that in the presence of magnetic field there are numerous Shapiro steps of a small magnitude that appear in the simulated γ-u curve. Yet the pronounced effect of ac induced fluxon damping remains very evident.

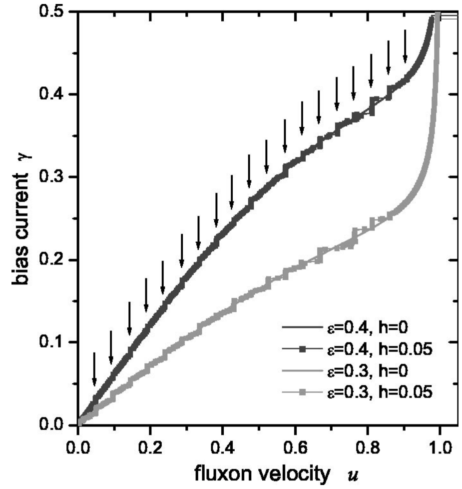

FIG. 4. The numerically simulated γ-u curves in the presence of magnetic field $h=0.05$. The amplitude of ac drive was chosen $\varepsilon=0.3$ and 0.4, and other parameters are the same as in Fig. 1. Arrows identify Shapiro steps.

VI. CONCLUSIONS

We have reported a theoretical and experimental study of the fluxon motion in the presence of dc and ac bias. By making use of the analysis and the direct numerical simulations we obtain the ac induced nonlinear damping of a fluxon. We also measured the current-voltage characteristics of an annular LJJ with the trapped fluxon. We observed a such ac induced damping as a large ascent of the I-V curve in the presence of externally applied microwave radiation. Because the response to the ac bias increases as the intrinsic damping parameter α reduces [Eq. (8) and Fig. 2], the observed effect can be useful as a method to detect weak microwave radiation. In addition, this effects allows to accurately measure the damping parameter α when it gets extremely small at low temperatures.

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