Invariance of charge of Laughlin quasiparticles

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A quantum antidot electrometer is used to measure the charge of bulk Laughlin quasiparticles on the $f=1/3$ fractional quantum Hall plateau. We also report experiments performed on the integer $i=1$ and 2 plateaus extending over a range of filling factors used to calibrate the electrometer, and to demonstrate independence of the charge measurement of filling factor. We find the charge of the Laughlin quasiparticles to be invariantly $e/3$, with a standard deviation of 1.2% and an absolute accuracy of 4%, independent of filling, tunneling current, and temperature.

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I. INTRODUCTION

The most conspicuous aspect of the quantum Hall effect[1] (QHE) is the constancy of the Hall conductance over a finite range of the filling factor ν. Indeed, this property defines the phenomenon of QHE; the *quantized* value of the Hall conductance σ_{xy} of a particular QH state in units of e^2/h, $f=h\sigma_{xy}/e^2$, is a principal quantum number of that quantum Hall (QH) state called "exact filling." Specifically, the electric charge of the quasiparticles is expected to be determined by the relevant quantum numbers, including f, and thus is not expected to vary on a QH plateau when ν is varied from the exact filling.[2,3] The exactness of quantization of the Hall conductance is understood as a consequence of the gauge invariance of electromagnetic field and the exact quantization of the charge of electrons.[2] On a plateau the charge of quasiparticles localized in the *interior* of a two-dimensional electron system (2DES) is well defined.[4] In the case of the integer QH plateau at exact filling $f=i$ $(i=1,2,\dots)$ the quasielectrons are simply electrons in the Landau level $i+1$, and the quasiholes are the holes in the ith level. It is easy to understand the properties of fractiona quantum Hall (FQH) quasiparticles using composite fermions.[5] In the case of the FQH plateau at $f=i^*/2pi^*+1$ quasielectrons are composite fermions (an electron binding $2p$ vortices) in the "Landau level" i^*+1 of composite fermions, and the quasiholes are the holes in the i^*th level. It has been predicted theoretically[6,7] that the electric charge of these quasiparticles is $q=e/(2pi^*+1)$. This fascinating fractional quantization of electric charge is a fundamental property of the strongly correlated FQH fluid.

However, the above-described "orthodox" theory has been questioned from two different directions. Jain has argued that since fractional quasiparticles and quasiholes are extended composite objects, their properties, and in particular charge, are not well defined away from exact filling, when their density is high and "they can certainly not be assigned a well-defined charge or statistics."[8] Even more recently, experiments measuring shot noise power in QH constrictions without antidots have been reported; these experiments, interpreted as a measurement of Laughlin quasiparticle charge, show variations of q by factors of up to three and even six depending on all: filling factor, two-terminal conductance, applied current, and temperature.[9] If the charge of quasiparticles is not a well-defined and invariant quantum number throughout a plateau, then the "orthodox" theoretical proof of the exactness of quantization of σ_{xy} in the QHE (Refs. 2 and 3) would *require* that there were corresponding large corrections to the Hall conductance too. Otherwise, if σ_{xy} is quantized exactly in some limit with only exponentially small corrections, as numerous experiments show, the validity of the orthodox theory is under suspicion and additional theory is required to explain the exact quantization of the Hall conductance even when the charge of quasiparticles is not well defined.

Although six years have passed since the first direct observation of $\frac{1}{3}e$ particles in quantum antidot (QAD) electrometer experiments,[10] one crucial aspect of theory remained untested: the invariance of charge at ν far from exact filling. In this paper we report experiments performed on the integer $i=1,2$ and fractional $f=\frac{1}{3}$ QH plateaus that extend over a filling factor ν range of 27% to 45%. The charge of the QAD-bound Laughlin quasiparticles has been measured to be constant, independent of ν over the entire plateau extent, with relative accuracy of $\pm1.2\%$ and absolute accuracy of 4%. In addition, we observe no variation of the quasiparticle charge upon a variation of temperature, tunneling conductance, or applied current in the experimentally accessible range.

II. QUANTUM ANTIDOT ELECTROMETER

The QAD electrometer[10,11] is illustrated in Fig. 1. The antidot is defined lithographically in a constriction between two front gates in a 2DES. The antidot and the front gates create depletion potential hills in the 2DES plane and, in quantizing magnetic field B, the QHE edge channels are formed following equipotentials where the electron density n is such that $\nu=hn/eB$ is equal to integer i or fractional f exact filling. The edge channels on the periphery of the 2DES have a continuous energy spectrum, while the particle states of the edge channel circling the antidot are quantized by the Aharonov-Bohm condition that the state ψ_m with angular momentum $\hbar m$, in each Landau level, encloses $m\phi_0=m(h/e)$ magnetic flux. In other words, the semiclassical area of the state ψ_m is $S_m=m\phi_0/B$. The electrometer appli-

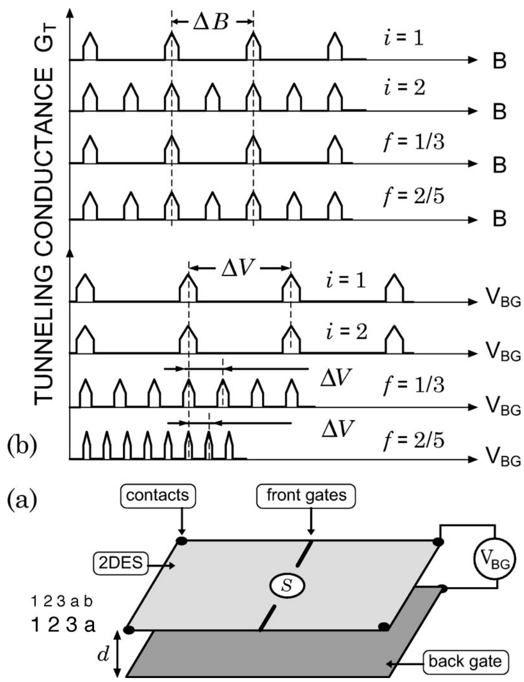

FIG. 1. (a) Schematic of the quantum antidot electrometer. (b) Idealized summary of experimental observations. There is one G_T peak on the $i=1$ and $f=\frac{1}{3}$ plateaus, and two peaks on the $i=2$ and $f=\frac{2}{5}$ plateaus observed per period ΔB, with the back gate voltage held constant $V_{BG}=0$. ΔB gives the size of the QAD: $S=\phi_0/\Delta B$. The same G_T peaks are also observed when a small perpendicular electric field V_{BG}/d is applied to 2DES by biasing the back gate. The period ΔV_{BG} then directly gives the charge q of the QAD-bound particles, in Coulombs, via $q=(\epsilon\epsilon_0\phi_0/d)\times(\Delta V_{BG}/\Delta B)$.

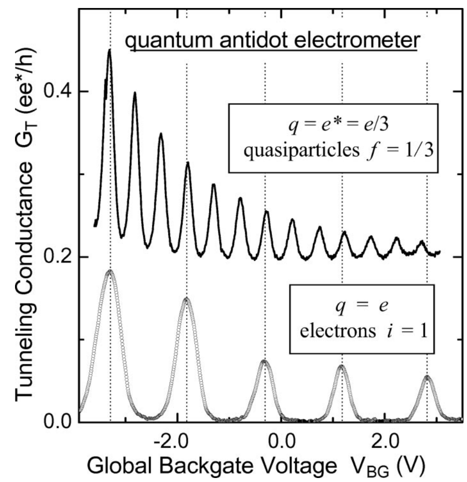

FIG. 2. In the experiment G_T peaks occur each time the occupation of the antidot changes by one particle, an electron for IQHE and a Laughlin quasielectron for the $f=\frac{1}{3}$ FQHE. The measured charge q of the particle is directly proportional to V_{BG}: it takes the same electric field V_{BG}/d to attract three $e/3$ quasiparticles as one electron. The upper G_T curve is offset vertically by $0.2e^2/3h$.

cation is made possible by a large, global "back gate" on the other side of the GaAs sample of thickness d. This gate forms a parallel plate capacitor with the two-dimensional (2D) electrons.

When the constriction is on a quantum Hall plateau, particles can tunnel resonantly via the QAD-bound states giving rise to quasiperiodic tunneling conductance G_T peaks,[12] see Fig. 1. A peak in G_T occurs when a QAD-bound state crosses the chemical potential and thus marks the change of QAD occupation by one particle. The experimental fact that the G_T peaks are observed implies that the charge induced in the QAD is quantized, that it comes in discrete particles occupying the antidot-bound states; the G_T peaks mark the change of the population of the QAD by one particle per peak. Measuring G_T as a function of B gives the area of the QAD-bound state through which the tunneling occurs, $S=\phi_0/\Delta B$, where ΔB is the quasiperiod in magnetic field. On the $\nu\approx i$ plateau there are i peaks per ΔB because i Landau levels are occupied. The above discussion is easy to generalize for $f=i^*/(2pi^*+1)$ FQH plateaus by considering "Landau levels" of composite fermions.

First, we neglect the small ($\sim 10\%$) change of the QAD area S upon change of B. The charge of the QAD-bound particles is then determined directly from the separation of the same G_T peaks as a function of the back-gate voltage V_{BG}, Fig. 2. The back gate produces uniform electric field $E_\perp=V_{BG}/d$ which induces a small change of $\epsilon\epsilon_0E_\perp$ in the 2DES charge density.[13] Classical electrostatics states[14] that the charge induced in the QAD is exactly equal to the 2D charge density induced *far* from the QAD times the area of the QAD, $\epsilon\epsilon_0E_\perp S$. The classical correction due to finite size of the antidot is very small: $S/\pi d^2\sim 10^{-6}$. The quantum capacitance correction due to finite compressibility of 2DES (Refs. 15 and 16) is very small too: $\sim 10^{-5}$. Thus, the charge of one particle q is directly given by the electric field needed to attract one more particle in the area S: $q=\epsilon\epsilon_0 S\Delta V_{BG}/d$, where ΔV_{BG} is the change of the global gate voltage between two consecutive conductance peaks.[17] An absolute and more accurate determination of q uses direct measurement of area $S=\phi_0/\Delta B$ for each QH plateau, as described in Sec. IV.

III. RESONANT TUNNELING EXPERIMENTS

We use low disorder GaAs heterojunction material where 2DES (density 1×10^{11} cm^{-2} and mobility 2×10^6 cm^2/V s) is prepared by exposure to red light at 4.2 K. The antidot-in-a-constriction geometry (somewhat different from that of Refs. 10 and 11) was defined by electron-beam lithography on a pre-etched mesa with Ohmic contacts. After ≈ 150 nm of chemical etching, Au/Ti front gate metalization was deposited in the etched trenches. Samples were mounted on sapphire substrates with In metal which serves as the global back gate. All data presented in this paper were taken at 12-mK bath temperature with the sample immersed in a ^{3}He-^{4}He mixture. Extensive cold filtering cuts the electromagnetic background incident on the sample to 5×10^{-17} W, which allows us to achieve a record low effective electron temperature of 18 mK reported for a mesoscopic sample.[18]

Figures 3–5 show the directly measured four-terminal R_{xx} vs B data for three QAD plateaus: $i=2$, $i=1$, and $f=\frac{1}{3}$. We

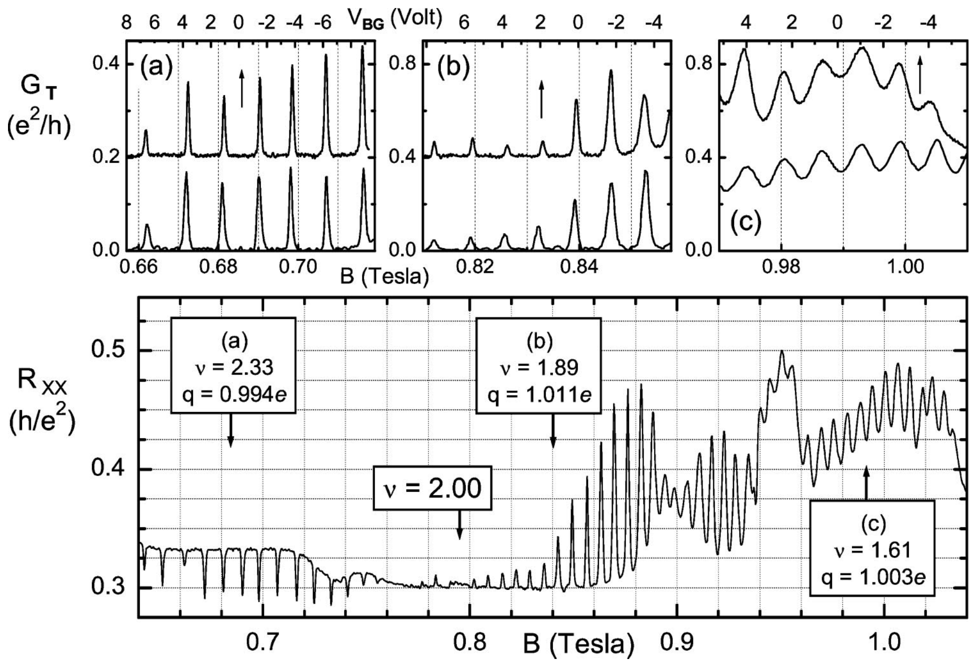

FIG. 3. R_{xx} vs magnetic field B for the antidot filling ν on the $i=2$ plateau. The upper panels (a)–(c) give the tunneling conductance measured both as a function of B (back-gate voltage is held constant $V_{BG}=0$), and as a function of V_{BG} (B is held constant, shown by arrows in the lower panel). The G_T vs V_{BG} curves in panels (a)–(c) are offset vertically.

use ν to denote the filling factor in the constriction region. The front gates are biased negatively in order to bring the edges closer to the antidot to increase the amplitude of the tunneling peaks to a measurable level. This results in ν being smaller than ν_B in the rest of the sample (''the bulk''). A QHE sample with two ν_B regions separated by a lower ν region, if no tunneling occurs, has $R_{xx}=R_L\approx R_{xy}(\nu)-R_{xy}(\nu_B)$. The equality is exact if both ν and ν_B are on a plateau, where the Hall resistances of all regions acquire quantized values. Thus, several R_L plateaus (neglecting tunneling peaks) are seen in Figs. 3–5. The tunneling peaks are superimposed on the smooth R_L background, and we calculate G_T as described previously.[18] In some data (Figs. 3 and 5) we observe both R_{xx} peaks for $\nu<i$ (''backscattering'') and dips for $\nu>i$ (''forward scattering'').[12,19] Details of this behavior will be presented elsewhere.

IV. QUASIPARTICLE CHARGE

At several ν on each plateau we took high resolution B sweeps at $V_{BG}=0$, and, having put the superconducting magnet in the persistent current mode to fix B, we took corresponding sweeps of V_{BG}. This constitutes an accurate measurement of QAD-bound quasiparticle charge, as described in Sec. II. Representative G_T vs B and V_{BG} data are shown in the upper panels of Figs. 3–5. Note that the negative V_{BG} axis direction corresponds to the increasing B. This is so because increasing B results in incremental depopula-

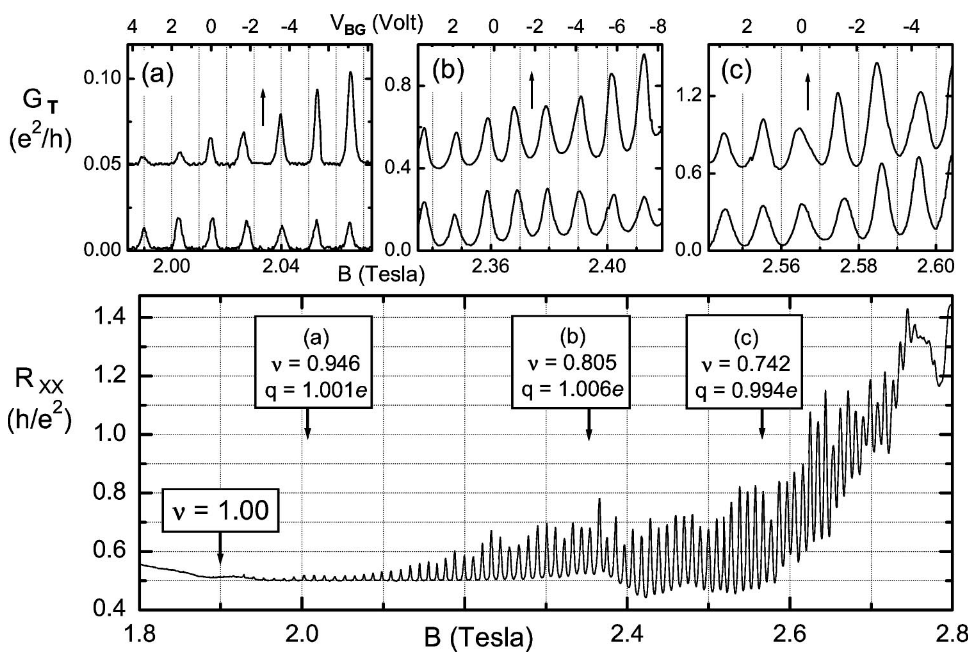

FIG. 4. R_{xx} vs magnetic field B for the antidot filling ν on the $i=1$ plateau. The upper panels (a)–(c) give the tunneling conductance measured both as a function of B (back-gate voltage $V_{BG}=0$) and as a function of V_{BG} (B is held constant).

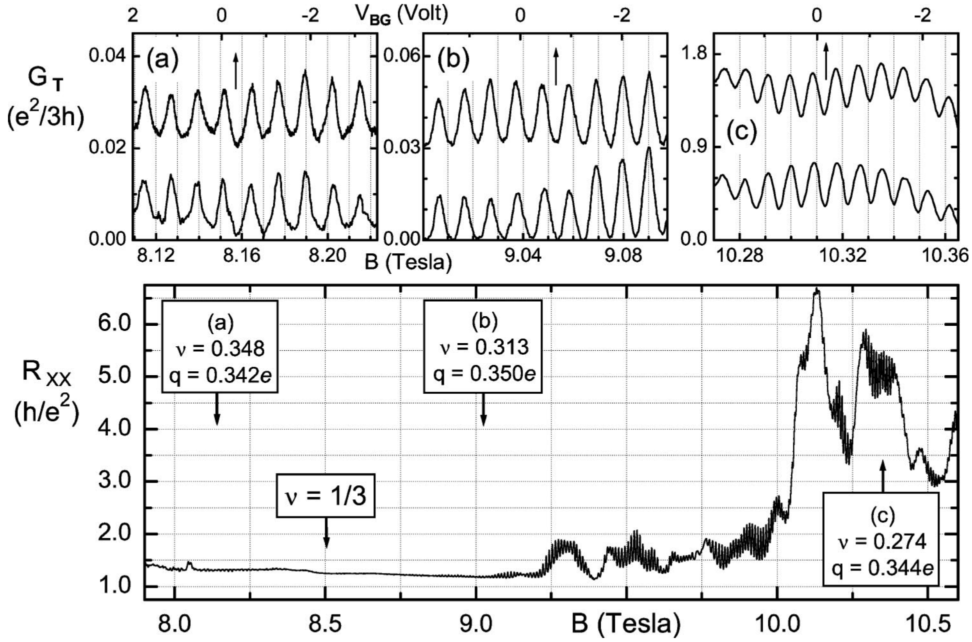

FIG. 5. R_{xx} vs magnetic field B for the antidot filling ν on the $f=\frac{1}{3}$ plateau. The upper panels (a)–(c) give the tunneling conductance measured both as a function of B (back-gate voltage $V_{BG}=0$), and as a function of V_{BG} (B is held constant).

tion of QAD, independent of the sign of the particle charge: the states ψ_m move closer to the center of the QAD, that is, states move above the chemical potential and become unoccupied. Negative V_{BG} depopulates the QAD if the charge of the particles is negative. Thus the QAD electrometer measures not only the magnitude of q, but also its sign.

As discussed in Sec. II, the charge of one QAD-bound particle q is directly given by the electric field needed to attract one more particle in the area S: $q=\epsilon\epsilon_0 S\Delta V_{BG}/d$, where ΔV_{BG} is the change of the global gate voltage between two consecutive conductance peaks.[10,11] The magnitude of the charge q of the QAD-bound particles is then given by

$$q=\frac{\epsilon\epsilon_0\phi_0}{d}\frac{\Delta V_{BG}}{\Delta B} \quad \text{in Coulombs,} \tag{1}$$

using the low-temperature GaAs dielectric constant $\epsilon=12.74$ (Ref. 20) and the measured thickness of the sample $d\approx 0.430$ mm. The average of the 11 values obtained for the $i=1,2$ plateaus $\langle q\rangle_{\text{integer}}=0.9651e$ is off by 3.5%, more than the standard deviation of $0.0070e$ for the combined data,[21] and similar to the results from another electrometer device reported in Refs. 10 and 11. We then *normalize* values of q by setting $\langle q\rangle_{\text{integer}}=e$. Thus determined q are shown for (a)–(c) data in Figs. 3–5, and summarized in Fig. 6. The striking feature of the data of Fig. 6 is that the values of q are constant to a relative accuracy of at least $\pm 1.2\%$ throughout the plateau regions where it was possible to measure the particle charge. The range of ν is about 45% for the $i=2$ plateau, 27% for the $i=1$ plateau (the combined normalized ν/i range is 57%), and also 27% for the $f=\frac{1}{3}$ plateau.

For $\nu=0.274$ on the $f=\frac{1}{3}$ plateau [inset (c) in Fig. 5], the density of quasiholes is $3(1-3\nu)\approx 0.53n$; that is, on the average there is one quasihole per two 2D electrons. The average separation between these quasiholes is $7l_0$, where l_0 is the magnetic length. Given that $23l_0^2$ is the average area per electron, and that the characteristic "radius" of an $e/3$ quasihole[1] is $\sqrt{6\pi}l_0\approx 4l_0$, we conclude that under these conditions the quasiholes do overlap; yet, as our data show, their charge is still well defined, being quantized to at least 3%. Thus Laughlin quasiparticles at low energies do behave as just particles. We should add that FQH quasiparticles are the elementary charged excitations of the FQH condensate, a distinct state of matter that is necessarily separated from all noninteracting electron states by a phase transition. Therefore, FQH quasiparticles are composite objects vastly different from electrons, and are *not* adiabatic images of electrons as are quasiparticle excitations of metals and band insulators such as IQHE states.

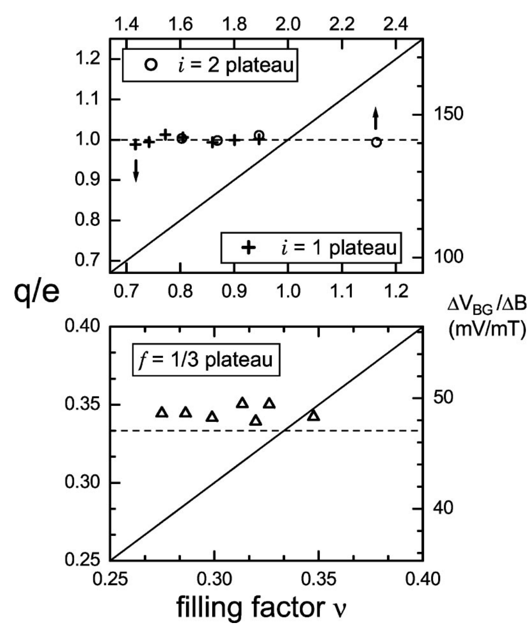

FIG. 6. Summary of the measured QAD-bound quasiparticle charge q in units of e. The horizontal dashed lines give the values e and $e/3$. The solid lines have the unit slope proportional to ν.

We also note that the tunneling current $I_t \approx I G_T/\sigma_{xy}$ is proportional to G_T and thus varies much for peaks of different amplitude. Here I is the applied current used to measure R_{xx}; the $f=\frac{1}{3}$ plateau data of Fig. 5 was taken with $I = 50$ pA, and we have measured $\Delta V_{BG}/\Delta B$ with I up to 1 nA, which, combined with the variation in *peak* G_T by a factor of ≈ 100 gives the range of 5×10^{-13} A $\leqslant I_t \leqslant 2\times 10^{-10}$ A. Furthermore, $\Delta V_{BG}/\Delta B$ has been measured in the temperature range 12 mK $\leqslant T \leqslant 70$ mK.[16,18] Under these conditions we observe no change in the value of $q=e/3$ within our experimental accuracy of a few percent. These results are in stark disagreement with recent reports of experiments[9] measuring shot noise power in QH constrictions without antidots, which, interpreted as a measurement of charge, show variations of q by factors of up to three and even six depending on all ν, I_t, G_T, and T. Thus, based on our experimental results, we conclude that the charge of the bulk quasiparticles is indeed a well-defined quantum number characterizing a particular QHE state.

ACKNOWLEDGMENTS

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[1] For reviews see *The Quantum Hall Effect*, 2nd ed., edited by R. E. Prange and S. M. Girvin (Springer, New York, 1990); *Perspectives in Quantum Hall Effects*, edited by S. Das Sarma and A. Pinczuk (Wiley, New York, 1997); S.M. Girvin, *The Quantum Hall Effect*, Les Houches Lecture Notes (EDP Sciences, Paris, 1999).

[2] R.B. Laughlin, Phys. Rev. B **23**, 5632 (1981).

[3] B.I. Halperin, Phys. Rev. B **25**, 2185 (1982).

[4] The excitation spectrum of QH edge channels is gapless, thus excitations of arbitrary charge can be created; see X.-G. Wen, Int. J. Mod. Phys. B **6**, 1711 (1992). These edge excitations are a sort of 1D charge-density fluctuations in the direction transverse to the edge, and generally are not Laughlin quasiparticles. A weakly coupled QAD serves as a ‘‘charge filter’’ for interedge tunneling.

[5] J.K. Jain and V.J. Goldman, Phys. Rev. B **45**, 1255 (1992).

[6] R.B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983).

[7] F.D.M. Haldane, Phys. Rev. Lett. **51**, 605 (1983); B.I. Halperin, *ibid.* **52**, 1583 (1984).

[8] J.K. Jain, Adv. Phys. **41**, 105 (1992); in *Perspectives in Quantum Hall Effects*, Ref. 1.

[9] D. Glattli *et al.*, Physica E (Amsterdam) **6**, 22 (2000); T. Griffiths *et al.*, Phys. Rev. Lett. **85**, 3919 (2000).

[10] V.J. Goldman and B. Su, Science **267**, 1010 (1995).

[11] V.J. Goldman, Physica E **1**, 15 (1997); measurement of charge e/5 quasiparticles in the $f=2/5$ data are given in Surf. Sci. **361/362**, 1 (1996).

[12] J.K. Jain and S.A. Kivelson, Phys. Rev. Lett. **60**, 1542 (1988).

[13] Even on a QHE plateau it is possible to change 2DES density in the ‘‘incompressible’’ interior with a gate so long as it is done slowly on the time scale of $\tau = RC \approx \epsilon\epsilon_0 L^2/(d\sigma_{xx})$, where L is the size of 2DES, because of finite diagonal conductivity σ_{xx} at a finite temperature.

[14] L.D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media* (Pergamon Press, NY, 1984) Chap. 1, Sec. 3.

[15] A.L. Efros, Solid State Commun. **65**, 1281 (1988).

[16] I.J. Maasilta and V.J. Goldman, Phys. Rev. B **57**, R4273 (1998).

[17] A small front gate that *defines* the antidot [as in J. Franklin *et al.*, Surf. Sci. **361/362**, 17 (1996)] clearly does not produce a uniform $E_\perp$.

[18] I.J. Maasilta and V.J. Goldman, Phys. Rev. B **55**, 4081 (1997); Phys. Rev. Lett. **84**, 1776 (2000).

[19] C.J.B. Ford *et al.*, Phys. Rev. B **49**, 17 456 (1994).

[20] G.A. Samara, Phys. Rev. B **27**, 3494 (1983).

[21] Unintentional impurities near the antidot can change their charge state as B or V_{BG} are swept. Such events, often reproducible, cause ‘‘phase slips’’ of the G_T peaks; we exclude such data from the charge analysis.