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Dip problem of the electron mobility on a thin helium film

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Electrons floating above liquid helium form an ideal two-dimensional system with an extremely high mobility. However, the mobility can change substantially when decreasing the thickness of the helium film from bulk to a thin film of a few hundred Å. Furthermore it is observed that for certain film thicknesses there is a pronounced dip in the mobility. We present theoretical investigations and measurements concerning this problem. Taking into account the roughness of the substrate, which supports the helium film, we find theoretically a discontinuity in the chemical potential of the electrons which results in a diplike behavior in the electron current and hence in the electron mobility. This scenario is supported by direct measurements of the electron current on substrates with different roughness and at different electron densities.

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A two-dimensional (2D) electron sheet, localized on the surface of a liquid helium film, forms a well-defined Coulomb system. The mobility of such a charge system strongly depends on the thickness of the helium film. Whereas on bulk helium the electron mobility can reach extremely high values up to 10^4 $\mathrm{m^2/V\,s}$,[1] for a thin van der Waals film the mobility can drastically decrease and usually depends on the surface quality of the underlying substrate.

One of the qualitative questions in the 2D electron kinetics is the so-called dip problem in the electron mobility on a thin liquid helium film. The first indication of this phenomenon was given by mobility measurements of electrons on relatively thick helium films (around 10^{-4}cm) on a sapphire substrate.[2] Later, a similar behavior was observed in the mobility of electrons on very thin helium films (around 10^{-6} cm) adsorbed on a quench-condensed solid hydrogen substrate.[3,4] However, so far it is not clear whether there is any correlation between the data of Refs. 2 and Refs. 3,4.

There exist several interpretations for the dip effect. In a paper by Peeters and Jackson[5] the electron mobility on helium films above a *flat* substrate is calculated. It predicts the monotonic mobility decrease versus the coupling constant, which is sensitive to the helium film thickness, and the nonmonotonic mobility behavior due to the self-trapping effect. However, the authors conclude that self-trapping of the electrons can not explain the dip behavior in Ref. 2, because in this case the self-trapping energy is too small with respect to the experimental temperature. Various experiments with electrons on helium films (see the review article by Dahm[6]) show the monotonic decrease of electron mobility versus the coupling constant. But there is no indication of the dip effect, which should be more probable in the self-trapping scenario,[5] when the helium film thickness becomes small enough.

The second interpretation for the dip effect is given for a *nonflat* substrate. As shown in Refs. 4 and 7 a diplike behavior of the electron mobility versus helium film thickness can develop in the presence of substrate corrugation. The final expression for the dip effect looks quite reasonable. But there are still open questions. The conductivity term in Ref. 4 is sensitive to the wave number of the substrate corrugation.[8] Such a situation is only possible, if the corrugation is periodic. However, no comment on this issue is given, and from such scenario we cannot formulate a suitable expression of the conductivity for a randomly rough substrate.

So, it becomes clear that the existing explanations for the dip effect in the electron mobility versus helium film thickness are not unitary. In this paper we consider some aspects of the 2D kinetics of electrons on helium films above a randomly rough solid substrate. The amplitude of this roughness is not small with respect to the helium film thickness. Under these conditions a 2D electron system shows a two-fraction behavior,[9] which explains the mobility dip effect and is valid both for thin and thick He films. Measurements are presented which show the dip effect and support the above scenario.

We use a so-called two-fraction model for the density of an electron system above a thin liquid helium film adsorbed on a rough solid substrate.[9] One of the components of the electron density n_e corresponds to the free electron motion along the helium surface. The second one n_l represents possible localized electrons due to the potential wells caused by the roughness of the underlying solid substrate. It is evident that

$$n_e + n_l = n_s, \tag{1}$$

where n_s is the total 2D electron density. This is usually fixed by an applied holding field. However, the ratio between these fractions can vary. The filling factors n_e and n_l are controlled by the electrochemical potential μ. An investigation of the characteristics of the electrons shows a θ-function-like behavior of μ versus n_e under certain conditions (see Fig. 1), as discussed below.

The electronic Ohm law has the conventional structure

$$j_{xx} = e^{-1}\sigma_{xx}\frac{\partial \mu}{\partial x} \quad \text{with} \quad \mu = e\varphi + \mu_0(n_e), \tag{2}$$

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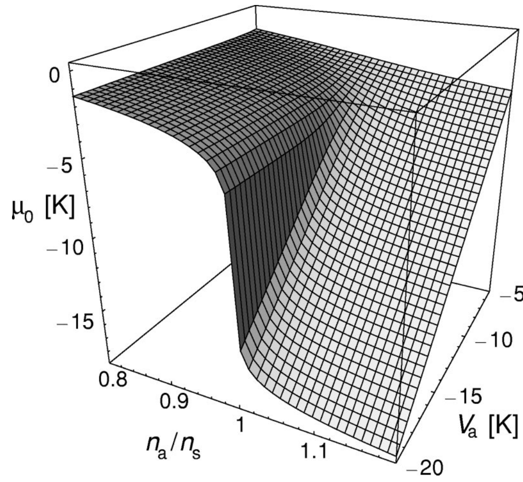

FIG. 1. Shown is the chemical potential μ_0 as function of the coupling energy V_a and the density ratio of the number of localized electrons due to the active tops n_a to the total number of electrons n_s. When $n_a=n_s$ then there is a sharp drop in μ_0, which increases as the coupling energy gets bigger.

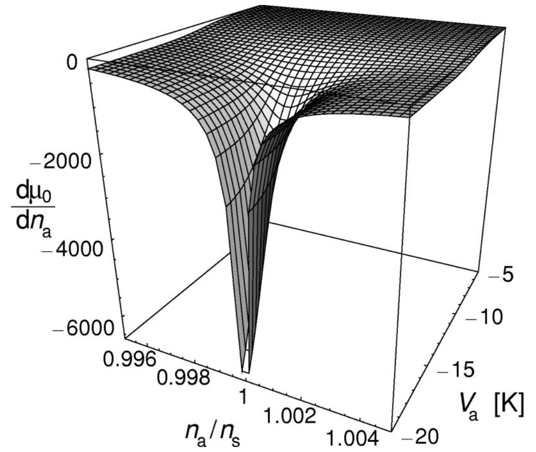

FIG. 2. Shown is the derivative of chemical potential against density of active tops $\partial\mu_0/\partial n_a$, as function of the density ratio n_a/n_s and the coupling energy V_a. The pronounced dip at $n_a=n_s$ causes a drop in the electron current, see definition (2), and hence in the mobility of the electrons.

where σ_{xx} is the diagonal conductivity of the 2D system, φ is the electric potential, and μ_0 is the chemical part of μ. The diffusive contribution in definition (2)

$$\frac{\partial\mu_0}{\partial x}\cong\frac{\partial\mu_0}{\partial n_e}\frac{\partial n_e}{\partial x} \tag{3}$$

has an opposite sign to the electric part $\sigma_{xx}\partial\varphi/\partial x$ of the total current j_{xx} (this follows from general quasiequilibrium thermodynamics) and usually is quite small. If the derivative $\partial\mu_0/\partial n_e$ from Eq. (3) gets big enough, then the compensation between the two contributions becomes essential and results in a diplike feature of j_{xx}.

The details of the behavior of electrons floating on top of a helium film, which is adsorbed on a substrate with given roughness, are presented in Ref. 9. Here we indicate only the existence of two essential parameters: (a) the effective density of "active tops" (those roughness peaks of the substrate which cause localisation of electrons, for details see Ref. 9) n_a and (b) the coupling energy for electrons on these tops V_a. n_a is defined by the number of electrons which are localized by the roughness peaks of the substrate. V_a is the coupling energy of the 2D electrons to the roughness peaks of the solid substrate. Both n_a and V_a continuously grow as the helium film decreases and can be regarded as phenomenological features.

For the definition of the chemical potential we follow the same procedure as in semiconductors.[10] So regarding definition (1), with the components n_l and n_e versus μ_0, we have

$$n_l+n_e=n_s,\qquad n_l=\frac{n_a}{\exp[(V_a-\mu_0)/T]+1},\qquad V_a<0,$$

$$n_e=\frac{n_0^e}{\exp(-\mu_0/T)+1},\qquad n_0^e=\frac{mT}{2\pi\hbar^2}. \tag{4}$$

Here m is the electron mass, T the temperature, and n_0^e the free electron density of states. Thus Eq. (4) describes the definition of μ_0 versus V_a, T, and n_s. Introducing the notation $x=\exp(-\mu_0/T)$ and $\epsilon=\exp(V_a/T)$ for $V_a<0$, we get for x the expression

$$2\epsilon n_s x=[\epsilon(n_0^e-n_s)+(n_a-n_s)]+\sqrt{[\epsilon(n_0^e-n_s)+(n_a-n_s)]^2+4\epsilon n_s(n_0^e+n_a-n_s)}. \tag{5}$$

The second term in Eq. (5) is always positive. The first term, however, changes sign (in the limit $\epsilon\to 0$) when n_a crosses the value of n_s. Before this point there is a strong compensation between these two terms. So at the condition

$$n_a=n_s, \tag{6}$$

the compensation stops, resulting in a jump of the chemical potential, see Fig. 1.

From Fig. 2 we can see that the derivative

$$\frac{\partial\mu_0}{\partial n_a}=\frac{\partial\mu_0}{\partial n_e}\frac{\partial n_e}{\partial n_a}$$

has the mentioned diplike behavior if the system crosses the point $n_a=n_s$. Under the usual conditions that n_s is constant the situation in Eq. (6) can be crossed during a continuous change of the thickness of the helium film, see Fig. 3 (details of this coupling are not presented here).

A further condition for $\partial\mu_0/\partial n_a$ resulting in such an impressive peak, see Fig. 2, is that V_a is not sensitive to the roughness details. By definition $V_a\propto -e^2/4d_a$ with d_a as the local helium film thickness above the roughness tops. The estimation of this value follows from

$$\frac{\Delta C_3}{d_a^3}\approx\rho g(h+\langle d\rangle+\delta d_a)$$

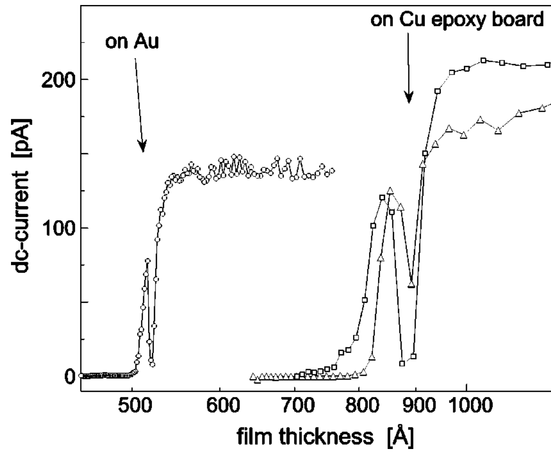

FIG. 3. Shown are directly measured currents of electrons traveling along a helium film adsorbed on a Cu or Au substrate. These currents represent a direct determination of the electrons involved during a dc transport experiment along a source-gate-drain electrode setup, see the inset of Fig. 5 and Ref. 11. The different current magnitudes are due to different electron densities. In the case of the rough Cu substrate the dip in current appears always at a thick He film, in contrast to the smoother Au evaporated on glass, as expected.

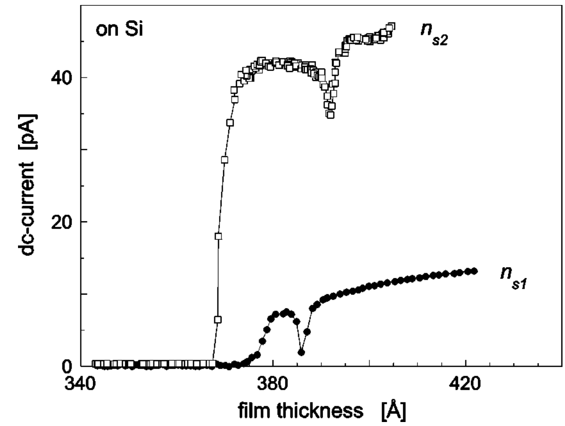

FIG. 4. Shown are electron currents along a helium film adsorbed on a Si wafer. In the case of a smoother Si surface the drop in current occurs at thinner helium films than on Au or Cu, see Fig. 3. The different curves are due to different electron densities, i.e., $n_{s2} > n_{s1}$.

if $h \gg \langle d \rangle$. ΔC_3 is the Hamaker constant, ρ the liquid helium density, h the distance between the helium film and the level of the bulk helium, $\langle d \rangle$ the average thickness of the helium film, and δd_a the fluctuation of the local helium film thickness. δd_a can be comparable to $\langle d \rangle$, but in any case $\delta d_a \ll h$. Therefore V_a is not dependent on these fluctuations.

So concluding from the above consideration at least two predictions can be made. (a) The position of the dip is sensitive to the level of roughness of the solid substrate and (b) sensitive to the equilibrium electron density.

To verify our model we have performed measurements to this problem. The electrons are continuously generated by a tungsten filament and float above a thin helium film covering a substrate at $T \approx 1.4$K. This substrate consists either of several in-plane metal electrodes or a rectangular silicon wafer which is contacted on two opposite sides forming source and drain. The principle experimental setup is sketched in the inset of Fig. 5. Applying ascending potentials on the single metal electrodes, or in the case of the Si substrate a potential difference directly across the wafer (which is barely conducting at low temperature), we transport the electrons above the helium surface over a distance of several millimeters. This "horizontal" dc current is detected with an additional pick-up electrode. The dc method is described in detail in Ref. 11.

Maintaining a certain configuration of potentials, we establish a constant dc-electron current. By varying the bulk helium level below the substrate we can directly measure the dependence of this current on the He-film thickness. The helium level is controlled by a cylindrical capacitor with a resolution of ≈ 60 μm. So the changes in helium film thickness can be measured to an accuracy of a few nm for films ≈ 100 nm thickness and to a few Å for films ≈ 40 nm. However, there is an uncertainty in the absolute He level (hence the error bars in Fig. 5).

In Fig. 3 we show typical results for the floating electron current on an standard copper epoxy board as substrate and for gold evaporated on glass. Figure 4 represents the current behavior for a smooth Si substrate. We always observe three features: (1) the signal does not appear before a certain film thickness is achieved, (2) then the current increases rapidly to a certain value followed by a smaller increase as the helium film gets thicker, and (3) it shows a dip which results in a dramatic decrease of electron current at the critical film thickness.

We start with a dry substrate and then continuously grow a thin helium film on it. At the beginning most electrons are lost to the substrate.[12] In addition, due to scattering on rough spots the mobility is very low. This is the first feature. But as the helium film gets thicker, motion across the surface becomes possible and so the signal at the pick-up electrode builds up–the second feature. The third feature agrees with the theoretical argument from above. We keep in mind that the filament generates electrons continuously. When the mobility decreases at a critical film thickness because of the roughness of the substrate, i.e., $n_l \cong n_{\mathrm{stable}} \cong n_s$, then there are less electrons in time which can reach the pick-up electrode and hence the electron current decreases. The subsequent increase in current is due to a further increase in the thickness of the He film.

Although all these measurements show this dip, the depth of each dip is different. This can be explained by different electron densities and substrate roughness.[11] The Cu substrate is rougher than the silicon wafer; a consequence is the lower value of the stable electron density on it.[12] The filament ensures a steady supply of electrons, but if they are not stable, the dip in $\partial\mu_0/\partial n_a$ suffices for vanishing of the signal. In contrast to this feature, we have less localized electrons on the silicon wafer. In general a pure diffusive transport is not possible as we always have an inhomogeneous

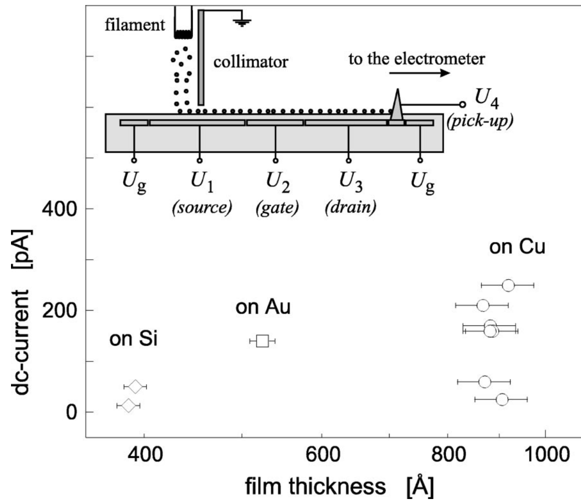

FIG. 5. Summarized is the measured dc-current dependence as function of the helium film thickness at the dip and function of the roughness of the underlying substrate. The inset shows the experimental setup schematically.

roughness distribution and therefore still some freely moving electrons even if $\partial\mu_0/\partial n_a$ has a minimum, and so the signal does not completely vanish.

The dependence of the maximum current on the underlying substrate, i.e., roughness, and the thickness of the He film at the dip is shown in Fig. 5. From our model we expect that as the roughness of the substrate gets smoother the dip moves to thinner He films. This is verified by the experimental data, see Fig. 5.

If the total electron density grows then we need more active tops to fulfill condition (6). Because $\partial n_a/\partial d<0$ a density increase, say $n_{s2}>n_{s1}$, results in the condition

$$d_{s1}>d_{s2}, \tag{7}$$

where d_{s1} and d_{s2} follow from Eq. (6); index "s" reflects the properties of the *real* helium film with electrons on top. The thickness of this film is sensitive to the electron density (see, e.g., Ref. 13)

$$d_s^{-1}=(d^{-3}+\alpha n_s^2)^{1/3}, \qquad \alpha=2\pi e^2/(\Delta C_3), \tag{8}$$

where d is the helium thickness of the uncharged film. Combining Eqs. (7) and (8) gives the inequality

$$d_1^{-3}-d_2^{-3}<\alpha(n_{s2}^2-n_{s1}^2), \tag{9}$$

which certainly "works" for $d_1>d_2$. But, when $n_s\neq 0$, the condition $d_1<d_2$ becomes possible. Such a situation is energetically favorable if the electron density is finite and large. Hence for the same roughness the dip moves to a thinner *real* helium film (d_s), but the measured d increases with n_s. This behavior is observed in the mobility measurements[2] and in our recent direct 2D current data, see Figs. 3 and 4.

In conclusion, we investigated the problem of the dip in the electron mobility on a thin helium film by taking into account the influence of the random surface roughness of the substrate, which supports the thin helium film. When the number of "active" roughness peaks is of the same size as the number of electrons floating above the helium film then one finds a discontinuity in the chemical potential of the electrons. This results in a diplike behavior in the electron current and hence in the mobility. We performed experiments on this problem which support the theoretical scenario. For that purpose the electron current is directly determined, on substrates of different roughness and at different electron densities.

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