Suzuki phase in two-dimensional sonic crystals

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In analogy with the structures discovered by Suzuki in alkali halides, in this article we introduce a two-dimensional acoustic system consisting of a periodic distribution of impurities (vacancies) in a host array. A triangular lattice of cylindrical sound scatterers is chosen as the host. The sonic crystal, called the Suzuki phase, shows extraordinary sound transmission properties: it holds the attenuation bands of the host structure and it also presents additional ones associated to the periodicity of the missing cylinders. The experiments agree with predictions based on the calculated acoustic band structure and they are explained by an analysis of eigenstates.

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It is known that in periodic composite media multiple scattered waves interfere to open frequency gaps where propagation is forbidden. This behavior is typical of any kind of wave: acoustic, elastic, electromagnetic, and even electrons inside crystalline materials. Although it is quite an old topic,[1] nowadays we have the appropriate tools to face the problem. In the field of acoustics, a periodic structure made of materials with different acoustic properties is called sonic crystal (SC).

The acoustic bands of infinite SC's are currently obtained by several theoretical approaches.[2–6] From the experimental side, experiments of sound transmission have been reported that confirm predictions.[5,7–9] The works focussed on the search of mechanisms for band gap control are specially interesting. Thus, Kuswaha and Halevi[10] proposed the fabrication of multiperiodic structures to open stop bands at desired ranges of frequencies with prefixed width. On the other hand, the work of Caballero *et al.*[11] was carried out in order to understand the role of symmetry in determining the bandgap width. It is clear that if we could control acoustic gaps we would be able to make a true engineering of the band gap and, consequently, to design simple structures that suit our necessities: acoustic screens, filters, or wave guides.

When looking for mechanisms of sound control, solid state physics is a very good source of ideas to export to the field of acoustics. Thus, Suzuki discovered that some alkali halide can be doped with divalent cations to produce a new ionic compound with periodically distributed vacancies and lattice parameter roughly twice the original one.[12] The compound was called the Suzuki phase. It retained properties of the initial compound and new properties arosed as a consequence of the translational symmetry imposed by the vacancies.

Following Suzuki's idea, we started with a triangular lattice of rigid cylinders in air and we made a rectangular array of vacancies taking out cylinders to arrive at the structure of Fig. 1(a). This new structure is called Suzuki phase because of the following features: (a) its lattice parameter is roughly twice the one of triangular lattice and (b) sound transmission experiments show attenuation bands around frequencies associated to the vacancies that were not observed in the starting triangular lattice.

The Suzuki phase defines a SC that can be described as a rectangular lattice with a basis of three cylinders in the primitive cell. In Fig. 1(a) the primitive vectors of the Bravais lattice are $\mathbf{a}_1 = 2a\hat{\mathbf{x}}$ and $\mathbf{a}_2 = a\sqrt{3}\hat{\mathbf{y}}$, where a is the distance between nearest neighbor cylinders. The positions of the three cylinders on the basis are $\mathbf{d}_1 = a(1,0)$, $\mathbf{d}_2 = a(\frac{3}{2}, \sqrt{3}/2)$, and $\mathbf{d}_3 = a(\frac{1}{2}, \sqrt{3}/2)$. Notice that the structure of the basis holds the triangular symmetry. To characterize this system we also need the radius of the cylinders R or the fraction of the unit cell occupied by the cylinders f_S (filling fraction). As long as the cylinders do not overlap each other, the three parameters are related by the expression $f_S = (\pi\sqrt{3}/2)(R/a)^2$. The reciprocal space and its corresponding Brillouin Zone (BZ), which is rectangular, are plot in Fig. 1(b). The four special points in the BZ are $\Gamma = (0,0)$,

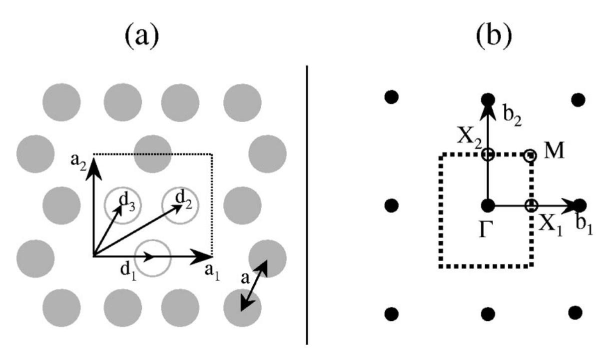

FIG. 1. (a) Schematic plot of the Suzuki phase structure under study in this work. The circles represent the cylinders, the dotted rectangle defines the unit cell, and the hollow circles are the basis in the cell. The parameters are explained in the text. (b) Its corresponding reciprocal space. The area inside the dotted rectangle is the Brillouin Zone. The letters define the high symmetry points.

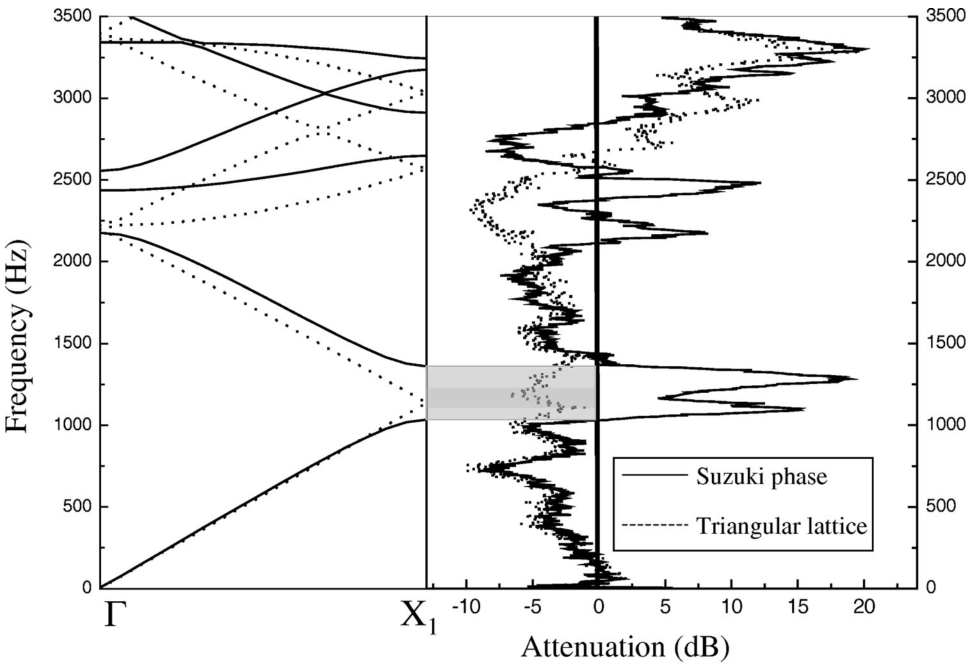

FIG. 2. (right panel) Sound attenuation vs. frequency along the ΓX_1 direction for the Suzuki phase (Fig. 1) and the triangular lattice. The cylinder radius R is 2 cm, and the parameter a is 6.35 cm (left panel). Acoustic bands of the corresponding infinite structures. The shadowed stripe represents the acoustic gap associated with the lattice of vacancies.

$X_1=(\pi/2a)(1,0)$, $X_2=(\pi/a)(0,1/\sqrt{3})$, and $M=(\pi/a)\times(\frac{1}{2},1/\sqrt{3})$.

With regards to experiments, they were performed inside an echo free chamber. The samples were made of aluminum cylinders 1 m long and 2 cm radius. They were hanged on a frame, which can rotate around the vertical axis in order to easily explore different directions of sound propagation. The lattice parameter of the triangular lattice were 6.35 cm ($a=6.35$ cm). The Suzuki phase structure [see Fig. 1(a)] was composed of 246 cylinders and its filling fraction is 0.27 ($R/a=0.31$). The triangular lattice was made with 311 cylinders and its corresponding filling fraction is $f_T=(2\pi/\sqrt{3})(R/a)^2=0.36$. Zero-order transmission experiments were performed to characterize both samples. Briefly, in the experiment the sound produced by a omnidirectional source was recorded by two microphones: the first one (direct microphone) received the signal straight from the source, while the second (interfered microphone) recorded the signal that crossed the sample. Both signals were compared in order to obtain the attenuation spectrum. Details of the experimental set up can be found in Refs. 5 and 9.

To calculate the acoustic dispersion relations infinite long cylinders are considered. The mathematical problem consists of solving the acoustic differential equation with periodic boundary condition in the two-dimensional space $\mathbf{r}=(x,y)$:

$$\nabla\left(\frac{\nabla p(\mathbf{r})}{\rho(\mathbf{r})}\right)+\frac{\omega^2}{\rho(\mathbf{r})c^2(\mathbf{r})}p(\mathbf{r})=0, \qquad (1)$$

where p is the pressure and ω is the angular frequency of the steady state. In addition, ρ and c are the mass density and the velocity of sound, respectively. They are functions of the position and contain the symmetry of the systems. Our structures are composed of air as background material ($\rho_{\text{air}}=1.3\times10^{-3}$ g cm^{-3}, $c_{\text{air}}=340$ m s^{-1}) and rigid cylinders made of aluminum ($\rho_{\text{alum}}=2.73$ g cm^{-3}, $c_{\text{alum}}=6800$ m s^{-1}). So, we are dealing with a composite having a huge density and sound velocity contrasts: $\rho_{\text{alum}}/\rho_{\text{air}}\approx2000$, $c_{\text{alum}}/c_{\text{air}}\approx20$.

Since we are using the acoustic equation, the shear waves inside the cylinders are ignored. This approximation is good enough because the density contrast between air and aluminum is so high that the penetration of acoustic waves inside the cylinders can be neglected.[6] Equation (1) is solved by a variational method in which the pressure was expanded in a linear combination of Bloch functions built from spline-based polynomials.[5] Satisfactory results are obtained with 100 functions in the linear combination, but we can easily go as far as 1500 function in order to ensure accurate results.

Figures 2 and 3 show the comparison between theory and experiment for the systems studied. The left panels display the dispersion relations along the ΓX_1 and ΓX_2 directions for the sample parameters. The right panels plot the measured attenuation spectra. In the Suzuki phase case (continuous lines), the agreement between predicted gaps and attenuation peaks is remarkable in the low frequency region ($\nu<2400$ Hz) in both directions. The attenuation bands observed at higher frequencies (where bands exist) are produced by other mechanisms, such as the existence of deaf bands[5] and/or by and energy transfer to higher Bragg orders when the sound leaves the sample. For the full triangular array (dotted lines) the acoustic bands are plotted in the BZ defined by the Bravais lattice of the Suzuki phase for an easy comparison. In this case, along ΓX_2 (Fig. 3) the attenuation band observed in frequencies 2200–3400 Hz agrees fairly well with the gap predicted at the very same frequency region. On the other hand, along ΓX_1 the attenuation measured at frequencies higher than 2400 Hz are originated by the mechanisms mentioned above.

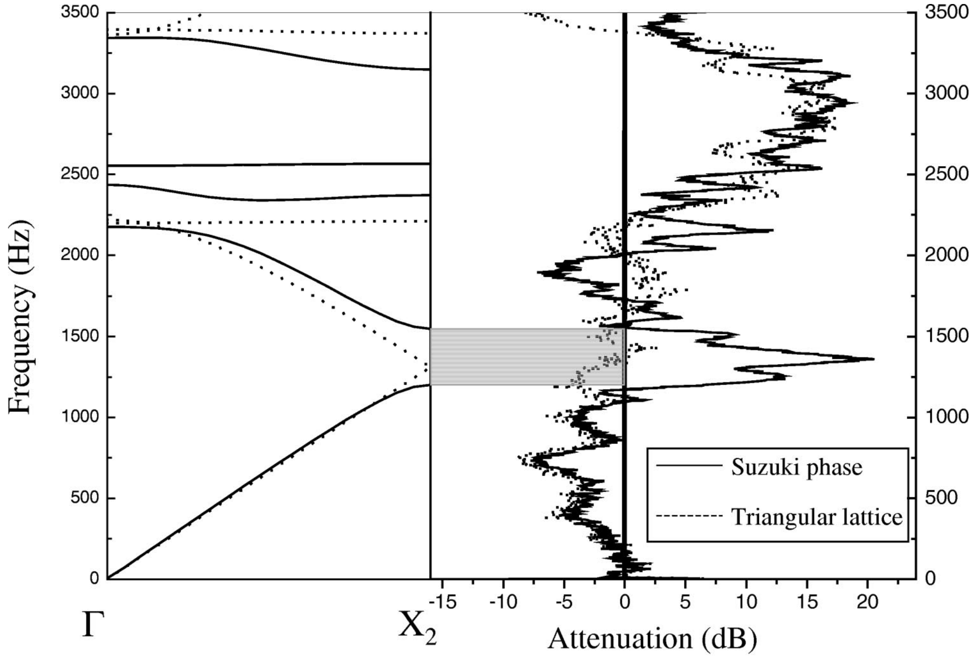

FIG. 3. (right panel) Sound attenuation vs frequency along the ΓX_2 direction for the Suzuki phase (Fig. 1) and the triangular lattice. The cylinder radius R is 2 cm, and the parameter a is 6.35 cm (left panel). Acoustic bands of the corresponding infinite structures. The shadowed stripe represents the acoustic gap associated with the lattice of vacancies.

Let us stress that the attenuation spectra of both lattices, triangular and Suzuki phase, practically coincide in the high frequency region. That is also true for the acoustic band structure; though gaps are opened in the low frequency region, at high frequencies similar dispersion relation is obtained. This result shows the dual behavior of the SC defined by the Suzuki phase. Since the Suzuki phase presents attenuations not shown in the triangular array, we can assign those features to the rectangular lattice of vacancies. This assignment is further supported by the Bragg peaks of a rectangular array of scatterers in air placed at the vacancy positions. Their values along ΓX_1 and ΓX_2 directions $[\omega(X_i) \approx c_{\rm air} k_{X_i}]$ are, respectively, 1339 and 1546 Hz, which agree with the experiments (see the shadowed stripes in Figs. 3, 4).

The discussion above permits to obtain the following conclusion regarding the sound transmission properties of the Suzuki phase. At wavelengths larger than, or comparable to, the periodicity of this SC, the vacancies behave as if they were a true lattice of sound scatterers in a background. On the other hand, at lower wavelengths (large frequencies) the sound transmission properties are controlled by the symmetry of the basis in the unit cell, which in our case is triangular. This result is the key point of this work, and it can be generalized to any structure based on similar principles.

The behavior of the gaps in the Suzuki phase as a function of the ratio R/a has also been theoretically analyzed. The dispersion relations were calculated for f_S ranging from 0 to 55 % (almost touching cylinders); i.e., $0 \leqslant R/a \leqslant 0.45$. Figure 4 depicts the map for the gaps between the first and second bands. A full band gap exists for $R/a \geqslant 0.36$, which, in comparison with that of the triangular lattice, is completely new and appears at frequencies half of the gap found in the triangular symmetry. On the other hand, it is worthy to note that pseudogaps exist for very low ratio R/a and they spread and overlap as this parameter increases. What is more, pseudogaps are opened between every band at the special points of the first BZ, no matter how low f_S is. This property is characteristic of the rectangular symmetry and it can be predicted by group theory: none of the point groups of the special points of the BZ have got irreducible representations of dimension greater than 1.[13] So we expected that all accidental degeneracies at the boundary of the BZ split up, even with very thin cylinders.

Finally, in what follows theoretical arguments are used in order to understand the dual behavior of the Suzuki phase. They are based on the analysis of the eigenstates of the SC.

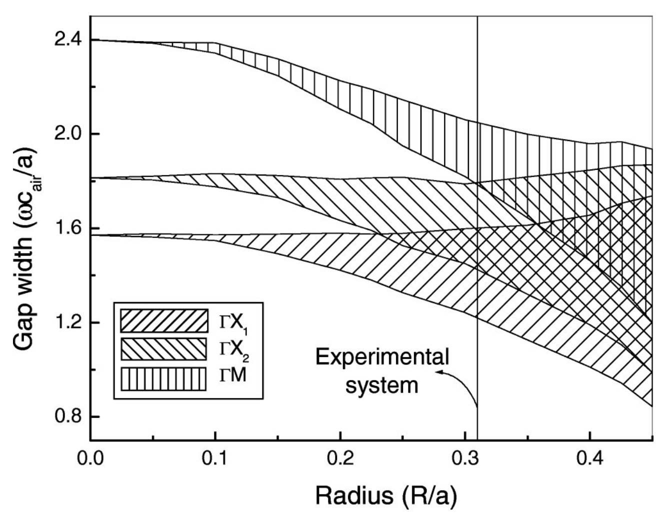

FIG. 4. Gap map (in reduced units) for the Suzuki phase structure (see Fig. 1). Only pseudogaps between the first and second bands are plotted as a function of the ratio between cylinder radius (R) and nearest neighbor distance (a). A full gap develops for $R/a \geqslant 0.36$

Let us recast Eq. (1) in the following operator form:

$$\hat{H}p(\mathbf{r}) = -\omega^2 \hat{S}p(\mathbf{r}); \quad \hat{H} \equiv \nabla\left(\frac{\nabla}{\rho(\mathbf{r})}\right); \quad \hat{S} \equiv \frac{1}{c^2(\mathbf{r})\rho(\mathbf{r})}, \tag{2}$$

where ω^2 is the eigenvalue and p the eigenstate. It is easy to demonstrate that operator $\hat{S}$ is definite positive and $\hat{H}$ is Hermitian.

A variational principle can be extracted from the Eq. (2) as it was done in electromagnetism for photonic crystals.[14] If eigenstates $p(\mathbf{r})$ are normalized, it can be stated that the eigenfrequencies of the system are the stationary values of the functional

$$\int \frac{1}{\rho(\mathbf{r})} |\nabla p(\mathbf{r})|^2 \, d\mathbf{r}. \tag{3}$$

This functional establishes that lower frequency eigenstates have got maximum variation of pressure inside high density regions. In our system it means that variation of pressure must be confined inside the cylinders. This feature will help to understand the pressure pattern of eigenstates in Eq. (2) and their dominant symmetry characters.

Figures. 5 show the contour lines of two eigenstates of the Suzuki phase: Fig. 5(a) presents the one at the X_2 point which belongs to the second band (i.e., at the end of the first attenuation peak) while Fig. 5(b) is the one also in the second band, but at the Γ point. In this kind of plot the density of lines is a measure of the gradient's modulus: the denser the lines, the higher the gradient. So that, in accordance with the variational picture, contour lines tend to be localized inside the cylinders, and so it is the pressure gradient. This feature is present on both patterns.

Now, with the help of the variational principle, let us discuss why the eigenstate in Fig. 5(a) has the character of the rectangular symmetry imposed by the vacancies while the one in Fig. 5(b) has a predominant triangular character. Since the frequency associated with the state in Fig. 5(a) is lower than the one of Fig. 5(b), we expect pressure's gradient to be more localized inside the cylinders than outside. This is clear at vacancy places because the contour lines are more separated in Fig. 5(a) (the low frequency state) than in Fig. 5(b), where the contour lines approach each other. In other words, the pressure lines in Fig. 5(b) behave as if there were an increase of density in the vacancy positions. This last feature let us conclude that eigenstate in Fig. 5(b) has a triangular character because it has a pressure pattern similar to the one expected in arrays of sound scatterers with such symmetry. Also notice that above this state attenuation of both structures, Suzuki phase and triangular, coincides.

In summary, we have reported astonishing sound transmission properties of a new SC consisting of a rectangular lattice of vacancies embedded in a triangular array of sound scatterers in air. This SC, which has been called Suzuki phase, presents stop gaps for sound transmission at frequencies related with the symmetry imposed by the vacancies. At the same time, attenuation bands of the underlying triangular lattice still remain in the spectra at higher frequencies. An intuitive picture derived from the fundamental properties of the wave equation for sound propagation has helped us to understand the inner processes responsible for this unexpected result. We conclude that lattices of vacancies embedded in a SC can be used as a smart mechanism of sound control in these materials.

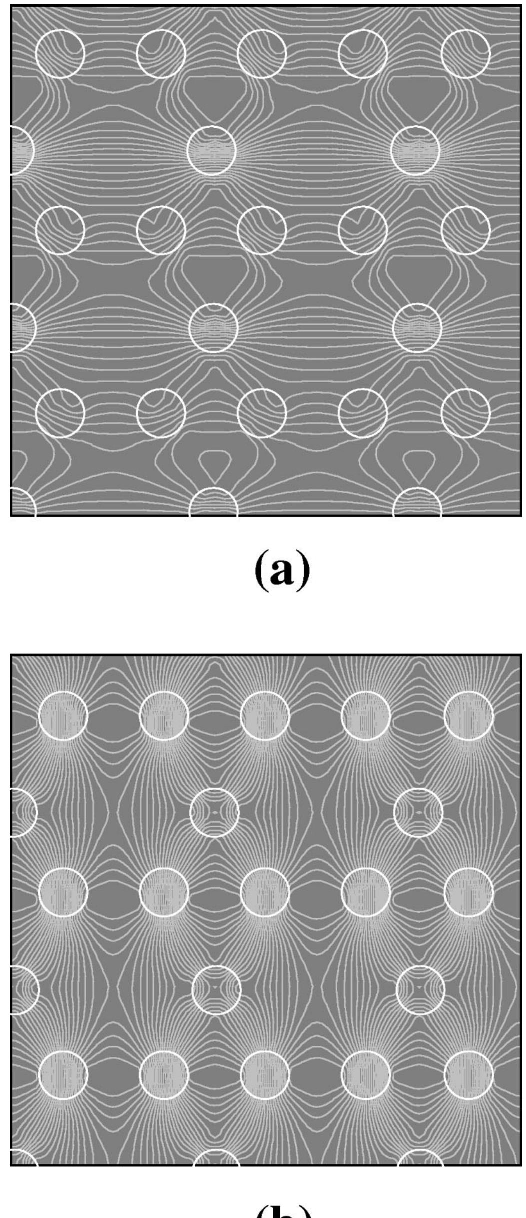

FIG. 5. Surface line plot of two band-edge states in the acoustic band structure of Fig. 3. (a) At the X_2 point of the second band. (b) At the Γ point of the second band. The white circumferences define the cylinder positions. Notice that the density of lines is proportional to the pressure's gradient.

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