Nonperturbative renormalization flow and essential scaling for the Kosterlitz-Thouless transition

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The Kosterlitz-Thouless phase transition is described by the nonperturbative renormalization flow of the two-dimensional φ^4 model. The observation of essential scaling demonstrates that the flow equation incorporates nonperturbative effects that have previously found an alternative description in terms of vortices. The duality between the linear and nonlinear σ model gives a unified description of the long-distance behavior for $O(N)$ models in arbitrary dimension d. We compute critical exponents in first order in the derivative expansion.

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I. INTRODUCTION

The Kosterlitz-Thouless (KT) phase transition[1] may describe the critical behavior of various two-dimensional systems. Some examples include thin superconductors,[2] superfluid films,[3] the melting of two-dimensional crystals,[4] arrays of Josephson contacts,[5] and thin liquid-crystal films.[6] The KT transition poses a challenge to our theoretical understanding due to several uncommon features. The low-temperature (LT) phase exhibits a massless Goldstone boson-like excitation despite the fact that the global $U(1)$ symmetry is not spontaneously broken by a standard-order parameter. In this phase the critical exponents depend on the temperature. In the high-temperature (HT) phase the approach to the transition is not governed by critical exponents but rather by essential scaling.

The transition is well understood by considering vortices as the dominant degrees of freedom.[1] It proceeds from a state of tightly bound vortex-antivortex pairs at low temperature to a plasma of interacting vortices at high temperature. On the other hand renormalization-group (RG) methods understand the critical phenomena in three-dimensional systems in terms of the universal behavior of $O(N)$ symmetric φ^4 theories. The success of the vortex picture in $d=2$, $N=2$ inspired many authors to search for a description in terms of vortices also for phase transitions in $d=3$ (Ref. 7). Thus in three dimensions there exists a dual view of criticality. In this note we try to explore this duality in the opposite direction. Modern nonperturbative RG methods give already a satisfactory qualitative picture of the phase below the critical KT temperature.[8] We extend this analysis to the essential scaling in the HT phase and provide for a quantitatively accurate discussion of the LT phase. These findings establish duality as a valid concept for $d=2$, $N=2$. We also present a universal RG description of models with $O(N)$ symmetry in arbitrary dimension. The good quantitative agreement of our results with theory and experiment for various d and N is encouraging with respect to further computations of universal properties like the critical equation of state. Moreover, the only missing ingredient for an examination of nonuniversal properties of a specific system is its translation into a microscopic action of the φ^4 type. In our language, this simply corresponds to an initial condition for the RG equations.

II. THE RG EQUATIONS

We employ the concept of the effective average action Γ_k (Ref. 9), which equals the effective action Γ apart from the fact that in the former only fluctuations with momenta larger than k are included. Thus Γ_k interpolates between microscopic and macroscopic scales as k moves from large values to zero. Since $\Gamma_0=\Gamma$ is the generating functional for the 1PI correlation functions it specifies directly the quantities of interest like the correlation length $\xi=m_R^{-1}$. The flow of Γ_k obeys an *exact* RG equation,[9]

$$\partial_t\Gamma_k[\varphi]=\frac{1}{2}\mathrm{Tr}\{(\Gamma_k^{(2)}[\varphi]+R_k)^{-1}\partial_t R_k\}. \tag{1}$$

Here $\Gamma_k^{(2)}$ is the second functional derivative, ∂_t denotes the logarithmic derivative $k\partial/\partial k$, and the trace (in momentum space) reads $\mathrm{Tr}=\Sigma_{a=1}^N\int d^dq/(2\pi)^d$. The cutoff $R_k(q^2)$ suppresses the low-momentum modes. We use a cutoff of the form

$$R_k(q^2)=Z_kq^2r(q^2/k^2)=\frac{Z_kq^2}{\exp(q^2/k^2)-1}, \tag{2}$$

where the wave-function renormalization Z_k will be fixed later. In order to solve Eq. (1) numerically, one has to truncate the most general form of Γ_k. We introduce dimensionless, renormalized fields, $\tilde\varphi_a=Z_k^{1/2}k^{(2-d)/2}\varphi_a$, $\tilde\rho=(1/2)\tilde\varphi_a\tilde\varphi_a$, and parametrize Γ_k in first order in a derivative expansion by

$$\Gamma_k=\int d^dx\Big\{k^du_k(\tilde\rho)+\frac{1}{2}k^{d-2}z_k(\tilde\rho)\partial_\mu\tilde\varphi_a\partial_\mu\tilde\varphi_a+\frac{1}{4}k^{d-2}\tilde y_k(\tilde\rho)\partial_\mu\tilde\rho\partial_\mu\tilde\rho+O(\partial^4)\Big\}. \tag{3}$$

The flow of Γ_k is then given by the flow of the dimensionless functions u, z, and $\tilde y$ that depend on the $O(N)$-invariant $\tilde\rho$ and on k. We denote by κ the running minimum of the potential $u_k(\tilde\rho)$ and fix Z_k by requiring $z_k(\kappa)=1$.

The functions u, z, and $\tilde y$ obey the partial differential equations[9–11]

TABLE I. Nonabelian nonlinear sigma model in $d=2$. We show the ratio between the renormalized mass m_R and the nonperturbative scale Λ_{ERGE} in comparison with the known ratio (Ref. 12) invoking $\Lambda_{\overline{MS}}$: $C_{ERGE}=m_R/\Lambda_{ERGE}$, $C_{\overline{MS}}=m_R/\Lambda_{\overline{MS}}$, $C_s=m_R/k_s$. We also display the expansion coefficients for the beta function.

N	C_{ERGE}	$C_{\overline{MS}}$	C_s	$\beta_\kappa^{(1)}$	$\beta_\kappa^{(2)}$
3	2.81 ± 0.30	2.94	1.00	1.00	0.79
9	1.22 ± 0.03	1.25	1.05	1.00	0.84
100	1.08 ± 0.04	1.02	1.06	1.00	0.87

$$\partial_t u=-du+(d-2+\eta)\tilde{\rho}u'+2v_d(N-1)l_0^d(w,z,\eta)+2v_d l_0^d(\tilde{w},\tilde{z},\eta), \tag{4}$$

$$\begin{aligned}\partial_t z=&\ \eta z+\tilde{\rho}z'(d-2+\eta)-(4v_d/d)\tilde{\rho}^{-1}\{m_{2,0}^d(w,z,\eta)-2m_{1,1}^d(w,\tilde{w},z,\tilde{z},\eta)+m_{0,2}^d(\tilde{w},\tilde{z},\eta)\}-2v_d(\tilde{z}-z)\tilde{\rho}^{-1}\{l_1^d(\tilde{w},\tilde{z},\eta)\\ &-(2/d)(\tilde{z}-z)l_2^{d+2}(\tilde{w},\tilde{z},\eta)\}-2v_d z'\{(N-1)l_1^d(w,z,\eta)-(8/d)n_{1,1}^d(w,\tilde{w},z,\tilde{z},\eta)+(5+2z''\tilde{\rho}/z')l_1^d(\tilde{w},\tilde{z},\eta)\\ &-(4/d)z'\tilde{\rho}l_{1,l}^{d+2}(w,\tilde{w},z,\tilde{z},\eta)\},\end{aligned} \tag{5}$$

$$\begin{aligned}\partial_t\tilde{z}=&\ \eta\tilde{z}+\tilde{\rho}\tilde{z}'(d-2+\eta)-2v_d(\tilde{z}'+2\tilde{\rho}\tilde{z}'')l_1^d(\tilde{w},\tilde{z},\eta)\\ &+8v_d\tilde{\rho}\tilde{z}'(3u''+2\tilde{\rho}u''')l_2^d(\tilde{w},\tilde{z},\eta)+4v_d(2+1/d)\tilde{\rho}(\tilde{z}')^2 l_2^{d+2}(\tilde{w},\tilde{z},\eta)\\ &-(8/d)v_d\tilde{\rho}(3u''+2\tilde{\rho}u''')^2\tilde{m}_4^d(\tilde{w},\tilde{z},\eta)-(16/d)v_d\tilde{\rho}\tilde{z}'(3u''+2\tilde{\rho}u''')\tilde{m}_4^{d+2}(\tilde{w},\tilde{z},\eta)-(8/d)v_d\tilde{\rho}(\tilde{z}')^2\tilde{m}_4^{d+4}(\tilde{w},\tilde{z},\eta)\\ &+(N-1)v_d\{-2[\tilde{z}'-\tilde{\rho}^{-1}(\tilde{z}-z)]l_1^d(w,z,\eta)-(8/d)\tilde{\rho}(u'')^2 m_4^d(w,z,\eta)-(16/d)\tilde{\rho}u''z'm_4^{d+2}(w,z,\eta)\\ &-(8/d)\tilde{\rho}(z')^2 m_4^{d+4}(w,z,\eta)+4(\tilde{z}-z)u''l_2^d(w,z,\eta)+4[z'(\tilde{z}-z)+(1/d)\tilde{\rho}(z')^2]l_2^{d+2}(w,z,\eta)\}.\end{aligned} \tag{6}$$

The momentum integration of Eq. (1) is contained in the "threshold functions" l_{n_1,n_2}^d, m_{n_1,n_2}^d, $\tilde{m}_{n_1,n_2}^d$, and n_{n_1,n_2}^d defined by the integral

$$-\frac{1}{2}\int_0^\infty dy\, y^{(d/2)-1}\tilde{\partial}_t\left\{\frac{X}{[p(y)+w]^{n_1}[\tilde{p}(y)+\tilde{w}]^{n_2}}\right\}, \tag{7}$$

with $X=1$, $y(\partial_y p)^2$, $y(\partial_y\tilde{p})^2$, $y\partial_y p$ for l, m, $\tilde{m}$, n, respectively. We have defined $u'=\partial u/\partial\tilde{\rho}$ and we use the shorthands $v_d^{-1}=2^{d+1}\pi^{d/2}\Gamma(d/2)$, $w=u'$, $\tilde{w}=u'+2\tilde{\rho}u''$, $\tilde{z}=z+\tilde{\rho}\tilde{y}$, $p(y)=y[z+r(y)]$, and $\tilde{p}(y)=y[\tilde{z}+r(y)]$. The derivative $\tilde{\partial}_t$ only acts on the k dependence of the cutoff R_k, i.e., $\tilde{\partial}_t p(y)=-y[\eta r(y)+2y\partial_y r(y)]$ and we note that $\tilde{\partial}_t\partial_y p=\partial_y\tilde{\partial}_t p$. Finally we abbreviate $l_{n,0}^d=l_n^d$ etc., where l_0^d is defined by the rule $(p+w)^{-n}\to-\log(p+w)$. The expression for the anomalous dimension $\eta=-\partial_t\ln Z_k$ can be obtained from the identity $\partial_t z_k(\kappa_k)\equiv0$. For $N=1$ one has $\tilde{y}=0$, $z=\tilde{z}$.

III. RESULTS

These equations are valid in arbitrary dimension. In the very simple approximation $z=\tilde{z}=1$, $u=\frac{1}{2}\lambda(\tilde{\rho}-\kappa)^2$ these equations already give a correct qualitative picture for $O(N)$ symmetric models in arbitrary dimension.[10] The present version leads to quantitatively accurate results. For a numerical solution we specify the initial values at a microscopic scale $k=\Lambda$. For $\kappa\gg1$ the evolution is dominated by the $N-1$ Goldstone modes ($N>1$). More precisely, the threshold functions at the minimum vanish rapidly for $\tilde{w}=2\kappa u''(\kappa)\gg1$. For $N>2$ the coupling of the nonlinear σ model for the Goldstone bosons is given by κ^{-1}.

We concentrate first on $d=2$, where the universality of the β function for the nonlinear coupling implies for $\kappa\gg1$ an asymptotic form ($N\geq2$)

$$\partial_t\kappa=\beta_\kappa=\frac{N-2}{4\pi}+\frac{N-2}{16\pi^2}\kappa^{-1}+\mathcal{O}(\kappa^{-2}). \tag{8}$$

In the linear description one can easily obtain an equation for κ by using $\partial_t u_k(\kappa_k)=0$ together with Eq. (4). By evaluating the above equations for large κ it is possible to compare it with Eq. (8). Previously it was found[10] that in a much simpler truncation one already obtains the correct lowest order in the above expansion. On the other hand Eqs. (4)–(6) do not contain all contributions $\mathcal{O}(\kappa^{-2})$. In order to reproduce the exact two-loop result one has therefore to go even beyond the truncation (3).

From Eq. (8) we expect that κ will run only marginally at large κ. As a consequence the flow of the action follows a single trajectory for large $-t$ and can be characterized by a single scale. Notice that the perturbative β function of the

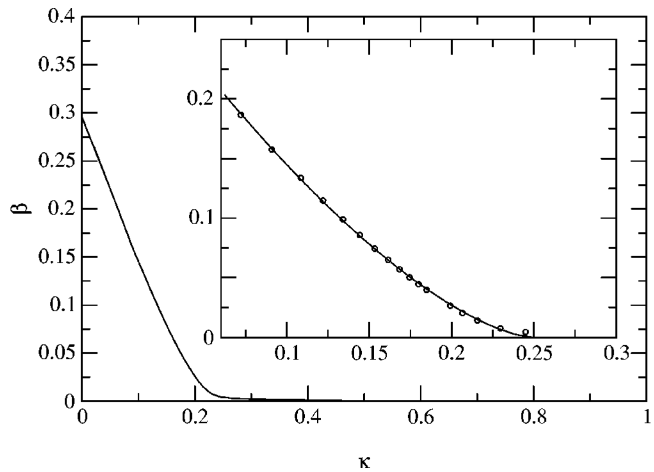

FIG. 1. The beta function for $d=2$, $N=2$. The inner plot shows a fit to Eq. (11).

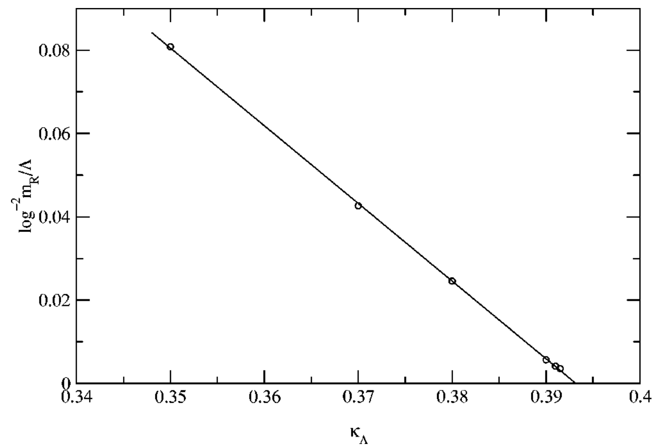

FIG. 2. Essential scaling for $d=2$, $N=2$. The renormalized mass m_R is plotted as a function of $\kappa_\Lambda=\kappa_{\Lambda *}-H(T-T_c)$.

nonlinear sigma model vanishes for $N=2$ since the Goldstone bosons are not interacting in the abelian case. Thus for large κ one expects a line of fixed points that can be parametrized by κ. This fact plays a major role in the discussion of the KT transition below. It is responsible for the temperature dependence of the critical exponents. We have evaluated $\beta_\kappa=\beta_\kappa^{(1)}(N-2)/(4\pi)+\beta_\kappa^{(2)}\kappa^{-1}(N-2)/(16\pi^2)+\cdots$ numerically from the solution of Eqs. (4)–(6) and extracted the expansion coefficients for large κ (see Table I).

For the nonabelian nonlinear sigma model in $d=2$, $N>2$ there exists an exact expression[12] for the ratio of the renormalized mass m_R and the scale $\Lambda_{\overline{MS}}$ that characterizes the two-loop running coupling in the $\overline{\mathrm{MS}}$ scheme by dimensional transmutation. The flow Eq. (1) together with a choice of the cutoff R_k and the initial conditions also defines a renormalization scheme. The corresponding parameter Λ_{ERGE} specifies the two-loop perturbative value of the running coupling κ^{-1} similar to $\Lambda_{\overline{MS}}$ in the $\overline{\mathrm{MS}}$ scheme. The numerical solution of the flow equation permits us to compute m_R/Λ_{ERGE}. (Two-loop accuracy would be needed for a quantitative determination of $\Lambda_{ERGE}/\Lambda_{\overline{MS}}$.) In Table I we compare our results with the exact value of $m_R/\Lambda_{\overline{MS}}$. We also report the ratio m_R/k_s with k_s defined by $\kappa(k_s)=0$.

The abelian case, $N=2$, exhibits the above-mentioned KT transition.[1] The characteristics of this transition are a massive HT phase and a LT phase with divergent correlation length but zero magnetization. The anomalous dimension η depends on T below T_c and is zero above. It takes the exact value $\eta_*=0.25$ at the transition. The most distinguishing feature is essential scaling for the temperature dependence of m_R just above T_c,

$$m_R \sim e^{-b/(T-T_c)^\zeta}, \quad \zeta=\frac{1}{2}. \tag{9}$$

We have already mentioned the existence of a line of fixed points for large values of κ, which is relevant for the LT phase. The contribution of a massless (Goldstone) boson in the RG equation $[w(k)=0]$ is responsible for the finite value of η. This in turn drives the expectation value of the unrenormalized field to zero (even for a nonvanishing renormalized expectation value κ),

$$\rho_0=\kappa/Z_k \sim \kappa(k/\Lambda)^\eta. \tag{10}$$

We can observe this line of fixed points to a good approximation (cf. Fig. 1), although the vanishing of β_κ is not exact [we find $\beta_\kappa \approx -4\cdot 10^{-5}\kappa^{-1}+\mathcal{O}(\kappa^{-2})$]. The line of fixed points ends at a critical value κ_* at which the phase transition occurs. There exists a critical trajectory towards this point corresponding to an initial value $\kappa_{\Lambda *}$. In order to verify essential scaling we have to examine the flow for values of κ_Λ just below that point, $\kappa_\Lambda=\kappa_{\Lambda *}+\delta\kappa_\Lambda$ [for sufficiently small $\delta\kappa_\Lambda$ we have $\delta\kappa_\Lambda \sim -(T-T_c)$]. Then κ_k crosses zero at the scale k_s and we find the mass by continuing the flow in the symmetric regime (minimum at $\kappa=0$). In Fig. 2 we plot $[\ln(m_R/\Lambda)]^{-2}$ against κ_Λ and find excellent agreement with the straight line (9).

How does β_κ have to look like in order to yield essential scaling? Since there is only one independent scale near the transition, one expects $m_R(T)=C_s k_s(T)$, where k_s denotes the scale at which κ vanishes, i.e., $\kappa(k_s,T)=0$. For κ close to and below κ_* we parametrize β_κ (this approximation is not valid for very small κ)

$$\beta_\kappa=\frac{1}{\nu}(\kappa_*-\kappa)^{\zeta+1}. \tag{11}$$

For conventional scaling one expects $\zeta=0$ and the correlation length exponent is given by ν. Integrating Eq. (11) yields for $\zeta\neq 0$, $\delta\kappa=\kappa-\kappa_*$:

TABLE II. Critical exponents ν and η for $d=2$. We compare each value with the exact result.

N	ν		η	
0	0.70	0.75	0.222	0.2083 . . .
1	0.92	1	0.295	0.25
2			0.287	0.25

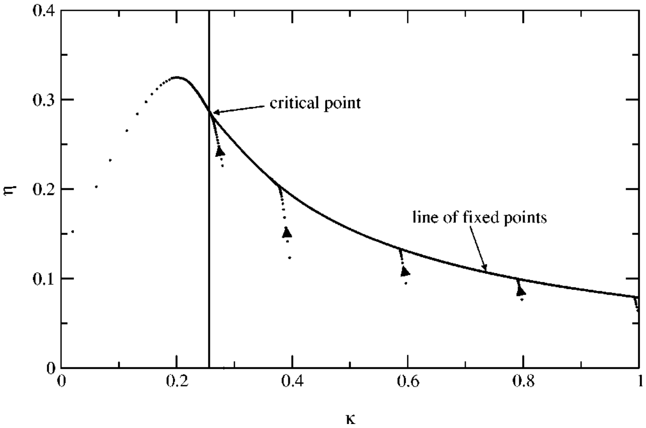

FIG. 3. Temperature dependence of the anomalous dimension η for the LT phase, $d=2$ and $N=2$. The line of fixed points is characterized by κ and ends in the critical point for the KT phase transition. We also show the flow towards the line of fixed points and the flow in the HT phase away from the critical point (left). The spacing between the points indicates the speed of the flow.

$$\ln(k/\Lambda)=\frac{\nu}{\zeta}\left(\frac{1}{(-\delta\kappa)^{\zeta}}-\frac{1}{(-\delta\kappa_{\Lambda})^{\zeta}}\right). \quad (12)$$

For $k=k_s$ the first term $\sim(-\delta\kappa)^{-\zeta}$ is small and independent of T [since $-\delta\kappa(k_s)=\kappa_*$] and Eq. (12) yields the essential scaling relation (9) for $\zeta=1/2$. Usually, the microscopic theory is such that one does not start immediately in the vicinity of the critical point and the approximation (11) is not valid for $k\approx\Lambda$. However, if one is near the critical temperature the trajectories will stay close to the critical one, $\kappa_c(t)$, with $\kappa_c(0)=\kappa_{\Lambda *}$. This critical trajectory converges rapidly to its asymptotic value κ_* and β_κ gets close to Eq. (11) at some scale $\Lambda'<\Lambda$. As a result, one may use Eq. (12) only in its range of validity $(k<\Lambda')$ and observe that $\delta\kappa_{\Lambda'}$ is also proportional to T_c-T. The numerical verification of Eq. (11) is quite satisfactory: Fitting our data yields $\kappa_*=0.248$, $\zeta=0.502$, and $\nu^{-1}=2.54$. The uncertainty for ζ is approximately ± 0.05. The numerical values of β_κ and the approximation (11) are shown in Fig. 1.

One can use the information from Figs. 1 or 2 in order to determine κ_* and therefore $\eta_*=\eta(\kappa_*)$, the anomalous dimension at the transition. Note that from Fig. 2 one can determine $\kappa_{\Lambda *}$. Then κ_* can be found by following the corresponding trajectory towards the line of fixed points. We plot η against κ in Fig. 3. One reads off $\eta_*=0.287\pm 0.007$ where the error reflects the two methods used to compute κ_* and does not include the truncation error. For $\kappa_\Lambda>\kappa_{\Lambda *}$ or $T<T_c$ the running of κ essentially stops after a short "initial running" towards the line of fixed points. One can infer from Fig. 3 the temperature dependence of the critical exponent η for the LT phase. In the HT phase the positivity of β_κ drives the system in the symmetric regime and η vanishes. Thus η jumps from η_* to 0 as we cross the critical temperature from below. In summary all the relevant characteristic features of the KT transition are visible within our approach. Further quantities like the jump in superfluid density[15] involve composite derivative operators not investigated here. Progress in this direction should be very interesting for a comparison with universal results in the topological approach.

We end this note by reporting the values of the critical exponents obtained in our approximation (4)–(6) for the "standard" second-order phase transitions for $d=2$, $N=0,1$, and $d=3$, $N\geqslant 0$. In Tables II and III they are compared with exact results or "averages" (only for the simplicity of the display) of results from various other methods.[14] The agreement is very satisfactory. We also characterize in Tables IV and V the scaling solution relevant for the second-order transition by quoting κ_*, $\lambda_*=u''_*(\kappa_*)$, $u_{3*}=u'''_*(\kappa_*)$ as well as $z'_*(\kappa_*)$ and $\tilde{z}_*(\kappa_*)$. We conclude that the first order in the derivative expansion of the exact flow equation for the effective average action gives a quantitatively accurate picture of all phase transitions of scalar models in the $O(N)$ universality class for arbitrary dimension $2\leqslant d\leqslant 4$.

Our findings suggest that many interesting statistical systems with $O(N)$ symmetry and effective long-range translation and rotation symmetry could be translated to the language of φ^4-type models by computing the effective action Γ_Λ at some short-distance scale Λ. This first step does not

TABLE III. Critical exponents ν and η for $d=3$ (see Ref. 13 for $N=1$). For comparison we list in the third and fifth column an "average value" from various other methods (Ref. 14).

N	ν		η	
0	0.590	0.5878	0.039	0.292
1	0.6307	0.6308	0.0467	0.0356
2	0.666	0.6714	0.049	0.0385
3	0.704	0.7102	0.049	0.0380
4	0.739	0.7474	0.047	0.0363
10	0.881	0.886	0.028	0.025
100	0.990	0.989	0.0030	0.003

TABLE IV. Couplings for the scaling solution for $d=2$ and $N=0,1$.

N	κ_*	λ_*	u_{3*}	$z'_*(\kappa_*)$
0	0.151	5.33	61.6	−0.085
1	0.265	5.88	65.4	0.868

TABLE V. Couplings for the scaling solution for $d=3$ and various N.

N	κ_*	λ_*	u_{3*}	$z'_*(\kappa_*)$	$\tilde{z}_*(\kappa_*)$
0	0.03009	7.399	78.84	−0.192	
2	0.05984	6.769	51.25	−0.0415	1.0602
3	0.07651	6.256	39.46	−0.0920	1.0695
4	0.09414	5.752	30.52	−0.1107	1.0789
10	0.2162	3.365	8.17	−0.0584	1.1144
100	2.2313	0.3779	0.0947	−0.000759	1.1468

involve the complications of long-distance coherent fluctuations. Our method then permits a detailed computation of the free energy (which is directly related[9] to $u_{k\to 0}$) in dependence on the temperature and ‘‘microscopic couplings’’ for an arbitrary shape of local short-distance interactions (parametrized by u_Λ). This procedure can be applied for arbitrary dimension d and arbitrary N, and nearby or at a phase transition as well as away from it. A unified description of many statistical-models emerges that goes beyond the universal critical behavior.

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