Dynamics and transformations of the Josephson vortex lattice in layered superconductors

D. A. Ryndyk,* V. I. Pozdnjakova, I. A. Shereshevskii, and N. K. Vdovicheva
Institute for Physics of Microstructures, Russian Academy of Sciences, GSP-105, Nizhny Novgorod 603600, Russia
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We consider the dynamics of a Josephson vortex lattice in layered superconductors with magnetic, charge (electrostatic), and charge-imbalance (quasiparticle) interactions between interlayer Josephson junctions taken into account. The macroscopic dynamical equations for the interlayer Josephson phase differences, intralayer charge, and electron-hole imbalance are obtained and used for numerical simulations. Different transformations of the vortex lattice structure are observed. It is shown that additional dissipation due to charge-imbalance relaxation leads to stability of the triangular lattice.

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The investigation of the resistive state of layered high-T_c superconductors (HTSC's) in an external magnetic field parallel to the layers is the important task of modern experiment and theory. The c-axis resistivity in this case is determined by Josephson flux flow. Thus, the nonstationary properties of HTSC's can be investigated in wide parameter range of temperatures, fields, and currents. In recent experiments[1–4] flux flow states in low magnetic fields are observed and in Refs. 5 and 6 in high fields up to 3.5 T. A number of peculiarities are discovered. In particular, in Refs. 2 and 4 multiple flux flow branches, associated with different vortex modes, are observed. In Refs. 3 and 4 it is shown that two universal flux flow regimes with different V/H take place at different magnetic fields and a transformation from the first state to the second is possible with a current rise at intermediate fields. Finally, in Ref. 6 broadband microwave emission is measured with frequencies much less than the Josephson frequency $\omega_J = 2eV/N\hbar$. All these phenomena are related to the dynamical transformations of the Josephson vortex lattice (JVL) and vortex interaction with linear plasma modes (see also Refs. 7–17).

On the other hand, there is the long-standing problem to obtain the coherent electromagnetic emission from a stacked Josephson junction. The most simple way to do this is to achieve an ‘‘in-phase’’ (square) arrangement of vortices. Although at zero external current the triangular JVL is favorable, it was shown for a two-junction stack theoretically and experimentally[11,14] and for a multijunction stack theoretically[9,10,13,15,16] that a moving square JVL can be stable. However, no indications of this in-phase regime were found in experiments with HTSC's. The reason for this discrepancy may be that only the magnetic coupling between layers has been taken into account in calculations.

In Refs. 18–23 it was also suggested that in the case of thin superconducting layers some nonequilibrium mechanisms are to be taken into account, namely, electrical charging of layers (charge effect) and nonequilibrium quasiparticles (charge-imbalance effect). To investigate this effects we obtain macroscopic dynamical equations for interlayer Josephson phase differences with magnetic, charge, and charge-imbalance interactions taken into account.

Following Refs. 7 and 8 we use the London expression for the longitudinal in-layer supercurrent, neglecting the longitudinal in-layer electric field $\mathbf{E}_n^{\parallel}$ and normal current [with an accuracy of $(\sigma_{ab}/\sigma_c)\gamma^{-2} \ll 1$, where $\gamma = \lambda_c/\lambda_{ab}$ is the anisotropy parameter]

$$\mathbf{j}_n^{\parallel}(\mathbf{r}) = -\frac{c\Phi_0}{8\pi^2\lambda^2}\left[\nabla_{\parallel}\theta_n(\mathbf{r}) + \frac{2\pi}{\Phi_0}\mathbf{A}_n(\mathbf{r})\right], \tag{1}$$

together with the equation $\mathbf{B} = \mathrm{rot}\mathbf{A}$ and the definition of the invariant Josephson phase $\varphi_{n,n+1} = \theta_{n+1} - \theta_n - (2\pi/\Phi_0)\int_{nd}^{(n+1)d} A_z dz$. Integrating $\mathbf{A}$ over a closed contour one obtains

$$\mathbf{B} \times \mathbf{z}_0 = \frac{\Phi_0}{2\pi d}\nabla_{\parallel}\varphi_{n,n+1} - \frac{4\pi\lambda^2}{cd}(\mathbf{j}_{n+1}^{\parallel} - \mathbf{j}_n^{\parallel}); \tag{2}$$

here, $\mathbf{B}$ is the averaged interlayer magnetic field, $\mathbf{z}_0$ is the unit vector perpendicular to the layer, and d is the interlayer distance, layers assumed to be thin ($d \ll \lambda_L$).

Then, using the Maxwell equation

$$\mathrm{rot}\,\mathbf{B} = \frac{\epsilon}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{j},$$

projected onto the z axes, we obtain

$$\frac{\epsilon}{c}\frac{\partial E_{n,n+1}}{\partial t} + \frac{4\pi}{c}j_{n,n+1} = \frac{\Phi_0}{2\pi d}\nabla_{\parallel}^2\varphi_{n,n+1} - \frac{4\pi\lambda^2}{cd}\nabla_{\parallel}(\mathbf{j}_{n+1}^{\parallel} - \mathbf{j}_n^{\parallel}). \tag{3}$$

Making use of the continuity equation

$$\frac{\partial \rho_n}{\partial t} + \nabla_{\parallel}\mathbf{j}_n^{\parallel} + \frac{j_{n,n+1} - j_{n-1,n}}{d_0} = 0$$

and the Maxwell equation $\epsilon\mathrm{div}\mathbf{E} = 4\pi\rho$ in integral form, $\rho_n = (\epsilon/4\pi d_0)(E_{n,n+1} - E_{n-1,n})$, one obtains finally the equation

$$\frac{c\Phi_0 d_0}{16\pi^2\lambda^2}\nabla_{\parallel}^2\varphi_{n,n+1} = \left(1 + \frac{dd_0}{2\lambda^2}\right)j_{n,n+1}^* - 0.5 \times (j_{n-1,n}^* + j_{n+1,n+2}^*), \tag{4}$$

$$j^*_{n,n+1}=j_{n,n+1}+\frac{\epsilon}{4\pi}\frac{\partial E_{n,n+1}}{\partial t}-j_{ext}$$
$$=j_{n,n+1}+\frac{\epsilon}{4\pi d}\frac{\partial V_{n,n+1}}{\partial t}-j_{ext}, \tag{5}$$

where j_{ext} is the external current in overlapping geometry (for details of introducing the external current in different geometries see Ref. 7).

To find the interlayer Josephson current $j_{n,n+1}$ we use the theory of nonequilibrium Josephson effect developed recently (Refs. 18–23 and references therein). We take into account that in the nonequilibrium state there is nonzero invariant potential

$$\Phi_n(t)=\phi_n+(\hbar/2e)(\partial\theta_n/\partial t), \tag{6}$$

where ϕ_n is the electrostatic potential and θ_n is the phase of the superconducting condensate; $\Phi=0$ in the equilibrium state.

An ordinary Josephson relation $(d\varphi/dt)=(2e/\hbar)V$ between the Josephson phase difference $\varphi_{n,n+1}$ and voltage $V_{n,n+1}=E_{n,n+1}d$ is violated. Instead, we have [from Eq. (6)]

$$\frac{\partial\varphi_{n,n+1}}{\partial t}=\frac{2e}{\hbar}V_{n,n+1}+\frac{2e}{\hbar}(\Phi_{n+1}-\Phi_n). \tag{7}$$

The charge density inside a superconducting layer is

$$\rho_n=-2e^2N_0(\Phi_n-\Psi_n), \tag{8}$$

where the first term is the charge of superconducting electrons determined by the shift of the condensate chemical potential $\delta\mu_s=-e\Phi$ and the second term is the quasiparticle charge, described by the potential Ψ_n (details can be found in Ref. 23). Finally, the equation for the charge imbalance can be written in the form corresponding to the generalized nonstationary Ginzburg-Landau theory,[23]

$$\frac{\partial\Psi_i}{\partial t}=(1-\Gamma)\frac{\partial\Phi_i}{\partial t}+2\nu_t\frac{\hbar}{2e}\left(\frac{\partial\varphi_{i-1,i}}{\partial t}-\frac{\partial\varphi_{i,i+1}}{\partial t}\right)$$
$$+2\nu_t(\Psi_{i-1}+\Psi_{i+1}-2\Psi_i)-\tau_q^{-1}\Psi_i, \tag{9}$$

where $\eta=2\nu_t\tau_q$ is the parameter of disequilibrium, τ_q is the well-known charge-imbalance relaxation time, $\nu_t=(4e^2N_0RSd_0)^{-1}$ is the "tunnel frequency," R is the normal resistivity of the tunnel junction, $V=Sd_0$ is the volume of the superconducting layer, $N_0=mp_F/2\pi^2\hbar^3$ is the density of states [or, instead, $N_f=N_0d_0$ can be considered as a two-dimensional (2D) density of states], and $\Gamma\ll 1$ is the parameter dependent on the details of microscopic theory.

In the same approximation we derive an expression for the nonequilibrium interlayer current:[23]

$$j_{i,i+1}=j_c\sin\varphi_{i,i+1}+\frac{\hbar}{2eR}\frac{\partial\varphi_{i,i+1}}{\partial t}+\frac{\Psi_i-\Psi_{i+1}}{R}. \tag{10}$$

Finally, in dimensionless form we obtain

$$\nabla^2_{\parallel}\varphi_{i,i+1}=j^*_{i,i+1}-s(j^*_{i-1,i}+j^*_{i+1,i+2}), \tag{11}$$

$$j^*_{i,i+1}=\beta\frac{\partial^2\varphi_{i,i+1}}{\partial\tau^2}+\frac{\partial\varphi_{i,i+1}}{\partial\tau}+\sin\varphi_{i,i+1}+\psi_i-\psi_{i+1}$$
$$+\beta\left(\frac{\partial\mu_i}{\partial\tau}-\frac{\partial\mu_{i+1}}{\partial\tau}\right)-j_{ext}, \tag{12}$$

$$\alpha'\frac{\partial\psi_i}{\partial\tau}+\psi_i+\eta(2\psi_i-\psi_{i-1}-\psi_{i+1})$$
$$=\eta\left(\frac{\partial\varphi_{i-1,i}}{\partial\tau}-\frac{\partial\varphi_{i,i+1}}{\partial\tau}\right)+\alpha'(1-\Gamma)\frac{\partial\mu_i}{\partial\tau}, \tag{13}$$

$$\mu_i+\zeta(2\mu_i-\mu_{i-1}-\mu_{i+1})=\psi_i+\zeta\left(\frac{\partial\varphi_{i-1,i}}{\partial\tau}-\frac{\partial\varphi_{i,i+1}}{\partial\tau}\right), \tag{14}$$

where $\mu(t)=\Phi(t)/V_c$, $\psi(t)=\Psi(t)/V_c$, $V_c=\hbar\omega_c/2e$, $\alpha'=\tau_q\omega_c$, $\beta=(\hbar\epsilon\omega_c^2S)/(8\pi edj_c)$, $\lambda_J^2=(\hbar c^2d_0)/[16\pi ej_c(\lambda^2+dd_0)]$, $\zeta=(\epsilon r_d^2)/(d_0d)$, $\omega_c=2eRj_c/\hbar$, $\tau=\omega_ct$, $\tilde{x}=x/\lambda_J$, and $s=0.5/[1+(dd_0)/(2\lambda^2)]$.

This system of equations is a generalization of a well-known magnetic coupling model of layered superconductors.[7,8] Neglecting electron-hole imbalance ($\eta=0$, $\Psi_n=0$), we derive a system of equations obtained in Ref. 15, and neglecting the charge effect ($\zeta=0$, $\Psi_n=\Phi_n$) we derive a system of equations similar to that of Refs. 18, 19, and 24.

Before further study, Eqs. (11)–(14) must be added with boundary conditions, which we write as

$$j^*_{-1,0}=j^*_{N,N+1}=0,\quad \frac{\partial\varphi_{i,i+1}}{\partial x}\Big|_{0,L}=B,\quad \psi_0=\psi_N=0,$$
$$\frac{\partial\psi_{i,i+1}}{\partial x}\Big|_{0,L}=0,\quad \mu_0=\mu_N=0. \tag{15}$$

For numerical solution of Eqs. (11)–(14) with boundary conditions (15) we introduce the new functions $\hat{\varphi}$, v, and z, defined by the relations

$$\hat{\varphi}_i(x,t)=\varphi_{i,i+1}(x,t)-Bx,$$
$$v_i(x,t)=\frac{\partial\hat{\varphi}_i(x,t)}{\partial t}+\psi_i(x,t)-\psi_{i+1}(x,t),$$
$$z_i(x,t)=\varepsilon\frac{\partial\hat{\varphi}_i(x,t)}{\partial t}+\alpha'[\psi_i(x,t)-\psi_{i+1}(x,t)],$$
$$i=0,\dots,N-1,\quad \varepsilon=\alpha'(1-\Gamma). \tag{16}$$

Let us denote now by U a column vector function of the kind

$$U=\begin{pmatrix}\hat{\varphi}\\ v\\ z\end{pmatrix}, \tag{17}$$

by $\hat{A}$ an operator matrix of the kind

$$\hat{A}=\begin{pmatrix} 0 & \dfrac{\alpha'\beta}{\alpha'-\varepsilon} & -\dfrac{\beta}{\alpha'-\varepsilon} \\ 0 & -1+\zeta\Delta_d & 0 \\ 0 & -\varepsilon\left(1-\dfrac{\beta}{\alpha'-\varepsilon}\right)-\dfrac{\beta\gamma\varepsilon}{\alpha'-\varepsilon}\nabla_{\parallel}^2+\beta\eta\Delta_d & \dfrac{\beta\gamma}{\alpha'-\varepsilon}\nabla_{\parallel}^2-\dfrac{\beta}{\alpha'-\varepsilon} \end{pmatrix} \tag{18}$$

[here Δ_d is the difference Laplacian, $(\Delta_d f)_i = f_{i-1} - 2f_i + f_{i+1}$], and by F a column vector of kind

$$F=\begin{pmatrix} 0 \\ (1-\zeta\Delta_d)[j^* + j_{ext} - \sin(\varphi)] \\ \varepsilon[j^* + j_{ext} - \sin(\varphi)] \end{pmatrix}. \tag{19}$$

Then we can rewrite the above system of equations in the ‘‘operator’’ form

$$\beta\frac{\partial U}{\partial t} = \hat{A}U + F,$$

$$(s\Delta_d + 2s - 1)j^* = -\nabla_{\parallel}^2\hat{\varphi}, \tag{20}$$

with appropriate homogeneous boundary conditions. For numerical solution of Eq. (20) we use, after sampling in the x direction, a semi-implicit scheme of the form

$$U(t+\Delta t) = 2\left(E - \frac{\Delta t}{2\beta}\hat{A}\right)^{-1}\left(U(t) + \frac{\Delta t}{\beta}F(t)\right) - U(t) - \frac{\Delta t}{\beta}F(t), \tag{21}$$

in which all needed inverse operators can be easily evaluated by the standard sweep method. The transformation from vector U to initial variables is evident.

The developed program gives us the possibility to observe the state of system during the calculation process in a convenient graphical form. We can therefore find many different variants of the system dynamics and vortex lattice structures.

The largest part of the simulations has been carried out for the case of high magnetic fields $H > H^* = \Phi_0/\gamma t^2$, where $\Phi_0 = \pi\hbar c/e$ is the flux quantum, $\gamma = \lambda_c/\lambda_{ab}$ is the anisotropy parameter, and $t = d + d_0$ is the period of the structure. At this field a triangular vortex lattice is formed in the static case.

The results of numerical simulations can be shortly described in the following way. At small currents the vortex lattice remains triangular. At high currents the situation depends on dissipation. If the interlayer dissipation is strong enough $(\beta < 1)$, then the triangular lattice is stable at all currents. Otherwise, we observed various transformations of the lattice structure (further $\beta = 100$ and $s = 0.48$ are considered).

(i) At pure magnetic coupling $(\zeta = 0,\ \eta = 0)$ we obtain the typical picture of JVL transformations. An example is shown in Fig. 1. We consecutively change the current from zero to some high value (a)–(f) and then decrease it up to zero again (g)–(l). The square lattice is clear seen in Figs. 1(d), 1(e), 1(i), and 1(j). Low current regimes, Figs. 1(a) and 1(l), as well as high current ones, Figs. 1(f) and 1(g), are triangular (or close to it). The regimes in Figs. 1(b), 1(c), and 1(k) are ‘‘inhomogeneous,’’—velocities of vortex chains are different in different layers. These regimes are also ‘‘breathing’’ due to strongly excited nonlinear plasma waves. We find that such breathing modes can be triangular or square on average. Note that different regimes exist at the same current that can lead to hysteresis on VCC.

(ii) If we take charge coupling into account (Fig. 2, $\zeta = 1,\ \eta = 0$), the picture of JVL transformations is qualitatively the same but the regimes with a square lattice are shifted to higher currents.

(iii) For charge-imbalance (quasiparticle) coupling the result is qualitatively different. We have found that if the time

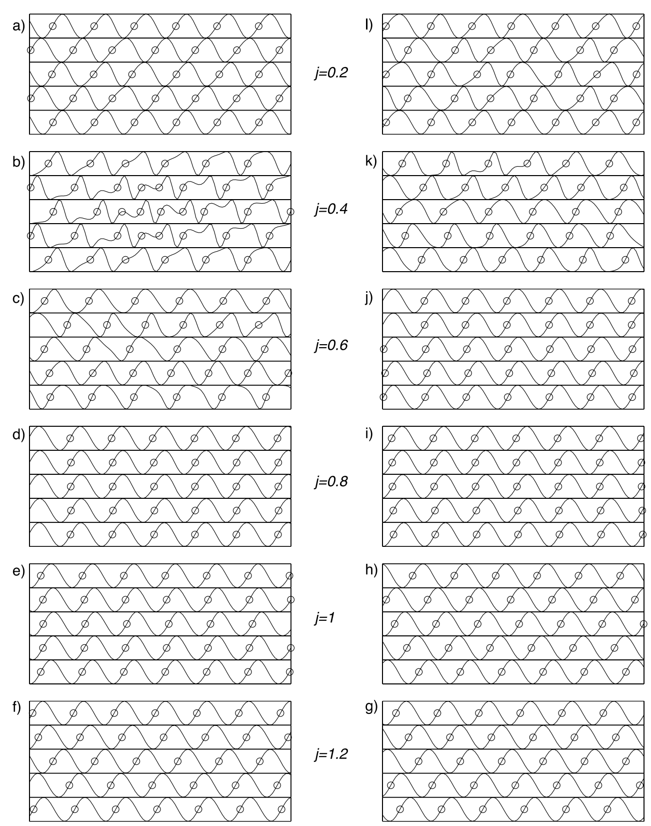

FIG. 1. The structure of the Josephson vortex lattice ($\sin\varphi_n$ are shown) with $\beta = 100$, $s = 0.48$, $\eta = 0$, $\zeta = 0$, and $j = 0.2, 0.4, 0.6, 0.8, 1, 1.2$. The current is increased (a)–(f) and then decreased (g)–(l).

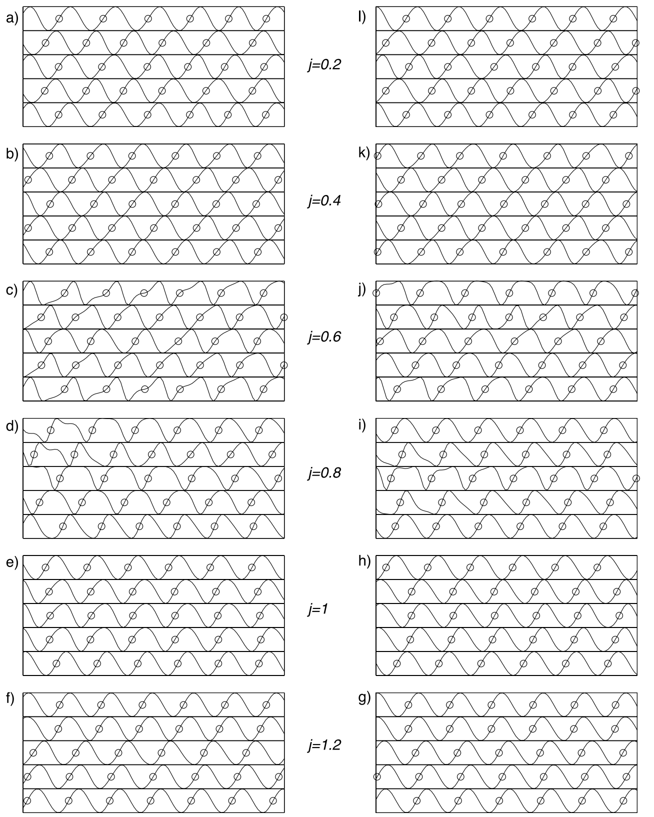

FIG. 2. The structure of the Josephson vortex lattice with $\beta=100$, $s=0.48$, $\eta=0$, and $\zeta=1$.

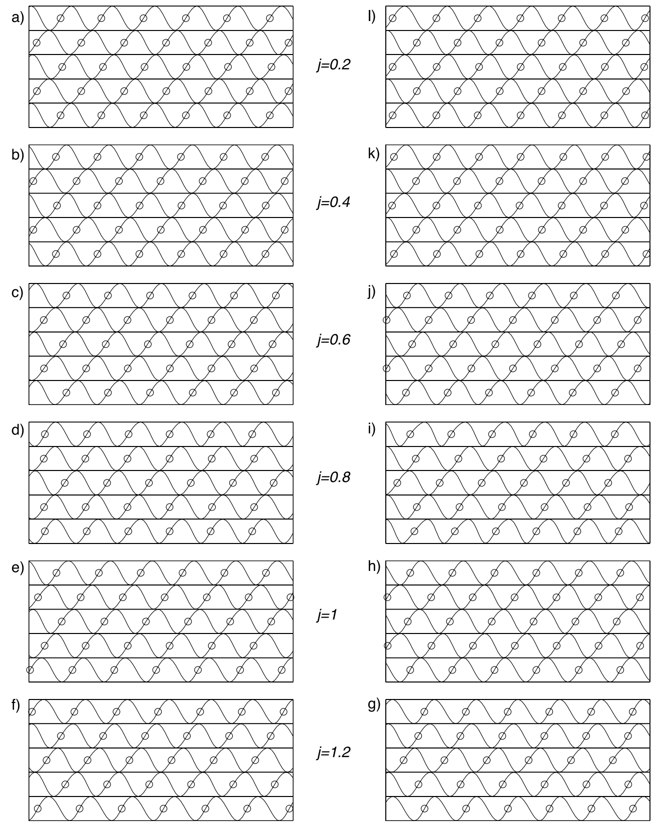

FIG. 3. The structure of the Josephson vortex lattice with $\beta=100$, $s=0.48$, $\eta=0.1$, $\zeta=0.1$, $\alpha'=100$, and $\Gamma=0.01$.

of charge-imbalance relaxation is large enough (approximately $\alpha'=\tau_q\omega_c>1$), then the triangular lattice becomes stable even if the parameter of disequilibrium η is small. An example is shown in Fig. 3 ($\zeta=0.1$, $\eta=0.1$, $\alpha'=100$). There are no JVL transformations at these parameters although both charge and charge-imbalance couplings are weak. The origin of this effect is the additional dissipation due to charge-imbalance relaxation. The detailed theory will be considered in another publication.

In conclusion, we derived the macroscopic dynamical equations for layered superconductors with magnetic, charge, and nonequilibrium quasiparticle interactions taken into account. Many dynamical regimes are observed, in particular triangular, square, inhomogeneous, and breathing modes. We established that additional dissipation due to the charge-imbalance relaxation can prevent JVL transformations and make the triangular lattice stable at all currents.

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*Electronic address: ryn@ipm.sci-nnov.ru

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