High-T_c phase diagram based on the SU(2) slave-boson approach to the *t-J* Hamiltonian

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Based on an improved SU(2) slave-boson approach showing coupling between the charge and spin degrees of freedom, we derive a phase diagram of high-T_c cuprates which displays both the superconducting and pseudogap phases in the plane of temperature vs hole doping rate. It is shown that phase fluctuations in the order parameters result in a closer agreement with the observed phase diagram of an arch shape, by manifesting the presence of an optimal doping rate closer to observation, compared to the U(1) slave-boson theory.

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High-T_c superconductivity arises as a consequence of hole(or electron) doping in the parent cuprate oxides which are Mott insulators with antiferromagnetic long-range order. The observed phase diagram[1,2] in the plane of temperature T vs hole doping rate δ shows the bose condensation (superconducting temperature) curve of an "arch" shape rather than the often predicted linear increase, by manifesting the presence of the optimal doping rate of $\delta=0.16$ to 0.2. On the other hand, the observed pseudogap temperature displays nearly a linear decrease with δ. The high-T_c cuprate of $Bi_2Sr_2Ca\,Cu_2O_{8+\delta}$ with a higher pseudogap (spin gap) temperature T^* is observed to have a higher superconducting transition temperature T_c than the cuprate of $La_{2-x}Sr_xCu\,O_4$ with a lower T^*[2]. Further we find from the observed phase diagrams[2] of both cuprates above that the two different high-T_c cuprates, $La_{2-x}Sr_xCu\,O_4$ and $Bi_2Sr_2Ca\,Cu_2O_{8+\delta}$ display an universal behavior of T^*/T_c as a function of hole (positive charge) doping δ/δ_o with δ_o, the optimal doping rate, as is shown in Fig. 1. The two observations manifest the presence of a relationship between the spin gap (relevant to the spin degree of freedom) and the superconductivity (related to the charge degree of freedom). Thus, the spinon pairing (spin singlet pairing) for pseudogap phase and the charge pairing (holon pairing) for superconductivity are not independent owing to the manifest presence of coupling between the charge and spin degrees of freedom.

Various U(1) slave-boson approaches to the *t-J* Hamiltonian were able to predict such a linear decrease in the pseudogap temperature as a function of δ.[3–6] In our earlier U(1) slave-boson study,[6] we presented a phase diagram based on the allowance of holon pairing channel, thus showing the feature of the holon-pair bose condensation temperature rather than the single-holon bose condensation temperature.[3–5] On the other hand, all of these theories failed to predict the experimentally observed bose condensation temperature T_c of the arch shape as a function of δ. Instead a linear increase of T_c with δ was predicted. Further the pseudogap phase was shown to disappear when the gauge fluctuations are introduced into the U(1) slave-boson mean field theory.[5] Most recently Wen and Lee proposed an SU(2) theory to readily estimate the low energy phase fluctuationof order parameters and questioned whether there exists a possibility of holon(boson) pair condensation.[7] In view of the failure of earlier theories in the correct prediction of the bose condensation temperature T_c in the phase diagram, in the present study we see a possibility of improvement by a rigorous treatment of the Heisenberg interaction term in the slave-boson representation which reveals the importance of boson (holon) contribution.

We realize from the aforementioned observation of the universality in T^*/T_c vs δ/δ_o for high-T_c cuprates that coupling between the spin (spinon) and charge (holon) degrees of freedom is essential for superconductivity. Our theoretical derivation from a rigorous use of the slave-boson theory for *t-J* Hamiltonian manifests this feature as is shown in the derived effective Hamiltonian Eq. (4) below. Comparison between the U(1) and SU(2) theories will be made to reveal the importance of the low energy phase fluctuations of the order parameters. The present work differs from our previous U(1) slave-boson study (of the phase diagram involving the holon-pair bose condensation),[6] and other earlier studies[3–5] (involving the single holon condensation) in that coupling between the holon and spinon degrees of freedom in the slave-boson representation of the Heisenberg term of the *t-J* Hamiltonian is no longer neglected. We find from the treatment of the coupling between the holon and spinon degrees of freedom that the predicted phase diagram displays the arch-shaped bose condensation curve (temperature T_c) as a function of hole doping rate in both the U(1) and SU(2) slave-boson approaches.[8]

We write the *t-J* Hamiltonian,

$$H=-t\sum_{\langle i,j\rangle}(c_{i\sigma}^{\dagger}c_{j\sigma}+\text{c.c.})+J\sum_{\langle i,j\rangle}[\mathbf{S}_i\cdot\mathbf{S}_j-(1/4)\,n_in_j]. \tag{1}$$

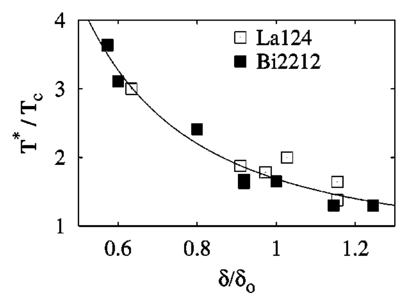

FIG. 1. T^*/T_c vs δ/δ_o for $La_{2-x}Sr_xCu\,O_4$ and $Bi_2Sr_2Ca\,Cu_2O_{8+\delta}$. The solid line represents a fitted curve by $T^*/T_c=(\delta/\delta_o)^a+b$ with $a=-1.86$ and $b=0.69$. Data points are taken from the paper of Nakano *et al.* (Ref. 2).

Here $\mathbf{S}_i$ is the electron spin operator at site i, $\mathbf{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} c_{i\beta}$ with $\boldsymbol{\sigma}_{\alpha\beta}$, the Pauli spin matrix element and n_i, the electron number operator at site i, $n_i = c_{i\sigma}^\dagger c_{i\sigma}$. We note from the use of $c_{i\sigma} = f_{i\sigma} b_i^\dagger$ for the local single occupancy constraint that $\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} n_i n_j = -\frac{1}{2}(c_{i2}^\dagger c_{j1}^\dagger - c_{i1}^\dagger c_{j2}^\dagger)(c_{j1}c_{i2} - c_{j2}c_{i1})$ leads to $-\frac{1}{2} b_i b_j b_j^\dagger b_i^\dagger (f_{i\downarrow}^\dagger f_{j\uparrow}^\dagger - f_{i\uparrow}^\dagger f_{j\downarrow}^\dagger)(f_{j\uparrow} f_{i\downarrow} - f_{j\downarrow} f_{i\uparrow})$ in the U(1) slave boson representation, that is, $P(\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} n_i n_j) P = -\frac{1}{2} b_i b_j b_j^\dagger b_i^\dagger (f_{\downarrow i}^\dagger f_{\uparrow j}^\dagger - f_{\uparrow i}^\dagger f_{\downarrow j}^\dagger)(f_{\uparrow j} f_{\downarrow i} - f_{\downarrow j} f_{\uparrow i})$ where P represents the projection operator onto the single occupied or empty site. It is of note that the spinon operator represents spin with no charge and the boson (or holon) operator represents charge with no spin. We stress that the presence of the boson (holon) operator in the Heisenberg term above is not redundant. This is because it represents the charge degree of freedom. This can be readily understood from the $n_i n_j$ term alone in the Heisenberg term; n_i here represents the electron (or physically the negative charge) number operator. Further for a rigorous treatment of the Heisenberg interaction term, fluctuations of charge density at each site, i.e., the on-site charge fluctuations should be allowed. For this reason the holon (boson) operator should be kept. In earlier studies of the slave-boson theory, it is often assumed that $b_i b_j b_j^\dagger b_i^\dagger = 1$. Strictly speaking, this is precise only at half-filling (or no hole doping). This is because charge fluctuations cannot readily occur owing to the prohibition of electron hopping from site to site in the t-J Hamiltonian.

By admitting the coupling between the charge and spin degrees of freedom in the SU(2) slave-boson representation,[7] the t-J Hamiltonian above can be written

$$\begin{aligned}
H = &-(t/2) \sum_{\langle i,j\rangle \sigma} [(f_{\sigma i}^\dagger f_{\sigma j})(b_{1j}^\dagger b_{1i} - b_{2i}^\dagger b_{2j}) + (f_{\sigma j}^\dagger f_{\sigma i})(b_{1i}^\dagger b_{1j} - b_{2j}^\dagger b_{2i}) \\
&+ (f_{2i} f_{1j} - f_{1i} f_{2j})(b_{1j}^\dagger b_{2i} + b_{1i}^\dagger b_{2j}) + (f_{1j}^\dagger f_{2i}^\dagger - f_{2j}^\dagger f_{1i}^\dagger)(b_{2i}^\dagger b_{1j} + b_{2j}^\dagger b_{1i})] \\
&- (J/2) \sum_{\langle i,j\rangle} (1 - h_i^\dagger h_i)(1 - h_j^\dagger h_j)(f_{2i}^\dagger f_{1j}^\dagger - f_{1i}^\dagger f_{2j}^\dagger)(f_{1j} f_{2i} - f_{2j} f_{1i}) - \mu_0 \sum_i (h_i^\dagger h_i - \delta) \\
&- \sum_i [i\lambda_i^{(1)}(f_{1i}^\dagger f_{2i}^\dagger + b_{1i}^\dagger b_{2i}) + i\lambda_i^{(2)}(f_{2i} f_{1i} + b_{2i}^\dagger b_{1i}) + i\lambda_i^{(3)}(f_{1i}^\dagger f_{1i} - f_{2i} f_{2i}^\dagger + b_{1i}^\dagger b_{1i} - b_{2i}^\dagger b_{2i})]. \qquad (2)
\end{aligned}$$

Here $f_{\alpha i}$ ($f_{\alpha i}^\dagger$) is the spinon annihilation (creation) operator and $h_i \equiv \binom{b_{1i}}{b_{2i}}$ $[h_i^\dagger = (b_{1i}^\dagger, b_{2i}^\dagger)]$, the doublet of holon annihilation (creation) operators. $\lambda_i^{(1),(2),(3)}$ are the real Lagrangian multipliers to enforce the local single occupancy constraint in the SU(2) slave-boson representation.[7]

The Heisenberg interaction term [the second term in Eq. (2)] above can be decomposed into terms involving mean fields and fluctuations, respectively,

$$\begin{aligned}
&-(J/2)(1 - h_i^\dagger h_i)(1 - h_j^\dagger h_j)(f_{2i}^\dagger f_{1j}^\dagger - f_{1i}^\dagger f_{2j}^\dagger)(f_{1j} f_{2i} - f_{2j} f_{1i}) \\
&= -(J/2)\langle (1 - h_i^\dagger h_i)(1 - h_j^\dagger h_j)\rangle (f_{2i}^\dagger f_{1j}^\dagger - f_{1i}^\dagger f_{2j}^\dagger)(f_{1j} f_{2i} - f_{2j} f_{1i}) \\
&\quad - (J/2)\langle (f_{2i}^\dagger f_{1j}^\dagger - f_{1i}^\dagger f_{2j}^\dagger)(f_{1j} f_{2i} - f_{2j} f_{1i})\rangle (1 - h_i^\dagger h_i)(1 - h_j^\dagger h_j) \\
&\quad + (J/2)\langle (1 - h_i^\dagger h_i)(1 - h_j^\dagger h_j)\rangle \langle (f_{2i}^\dagger f_{1j}^\dagger - f_{1i}^\dagger f_{2j}^\dagger)(f_{1j} f_{2i} - f_{2j} f_{1i})\rangle \\
&\quad - (J/2)\big((1 - h_i^\dagger h_i)(1 - h_j^\dagger h_j) - \langle (1 - h_i^\dagger h_i)(1 - h_j^\dagger h_j)\rangle\big)\big((f_{2i}^\dagger f_{1j}^\dagger - f_{1i}^\dagger f_{2j}^\dagger)(f_{1j} f_{2i} - f_{2j} f_{1i}) \\
&\quad - \langle (f_{2i}^\dagger f_{1j}^\dagger - f_{1i}^\dagger f_{2j}^\dagger)(f_{1j} f_{2i} - f_{2j} f_{1i})\rangle\big). \qquad (3)
\end{aligned}$$

By introducing the Hubbard-Stratonovich fields, ρ_i^k, χ_{ij} and Δ_{ij} in association with the direct, exchange and pairing channels of the spinon, we obtain the effective Hamiltonian from Eq. (2),

$$\begin{aligned}
H_{\mathrm{eff}} = &[J(1-\delta)^2/2] \sum_{\langle i,j\rangle} \sum_{l=0}^{3} ((\rho_j^l)^2 - \rho_j^l (f_i^\dagger \sigma^l f_i)) + [J(1-\delta)^2/4] \sum_{\langle i,j\rangle} \big[|\chi_{ij}|^2 - \{f_{\sigma i}^\dagger f_{\sigma j} + [2t/J(1-\delta)^2](b_{1i}^\dagger b_{1j} - b_{2j}^\dagger b_{2i})\}\chi_{ij} \\
&- \text{c.c.}\big] + [J(1-\delta)^2/2] \sum_{\langle i,j\rangle} \big[|\Delta_{ij}|^2 - \{(f_{2i}^\dagger f_{1j}^\dagger - f_{1i}^\dagger f_{2j}^\dagger) - [t/J(1-\delta)^2](b_{1j}^\dagger b_{2i} + b_{1i}^\dagger b_{2j})\}\Delta_{ij} - \text{c.c.}\big] \\
&- (J/2) \sum_{\langle i,j\rangle} |\Delta_{ij}^f|^2 \left[\sum_{\alpha,\beta} b_{\alpha i}^\dagger b_{\beta j}^\dagger b_{\beta j} b_{\alpha i} - (h_j^\dagger h_j + h_i^\dagger h_i - 2\delta) - \delta^2\right] \\
&+ [t^2/J(1-\delta)^2] \sum_{\langle i,j\rangle} [(b_{1i}^\dagger b_{1j} - b_{2j}^\dagger b_{2i})(b_{1j}^\dagger b_{1i} - b_{2i}^\dagger b_{2j}) + (1/2)(b_{1j}^\dagger b_{2i} + b_{1i}^\dagger b_{2j})(b_{2i}^\dagger b_{1j} + b_{2j}^\dagger b_{1i})] \\
&+ [J(1-\delta)^2/2] \sum_{i,\sigma} (f_{\sigma i}^\dagger f_{\sigma i}) - \mu_0 \sum_i (h_i^\dagger h_i - \delta) - \sum_i [i\lambda_i^1 (f_{1i}^\dagger f_{2i}^\dagger + b_{1i}^\dagger b_{2i}) + i\lambda_i^2 (f_{2i} f_{1i} + b_{2i}^\dagger b_{1i}) \\
&+ i\lambda_i^3 (f_{1i}^\dagger f_{1i} - f_{2i} f_{2i}^\dagger + b_{1i}^\dagger b_{1i} - b_{2i}^\dagger b_{2i})], \qquad (4)
\end{aligned}$$

where $\Delta_{ij}=\langle (f_{1i}f_{2j}-f_{2i}f_{1j})-[t/J(1-\delta)^2](b^\dagger_{2i}b_{1j}+b^\dagger_{2j}b_{1i})\rangle=\Delta^f_{ij}-[t/J(1-\delta)]\chi^b_{ij;12}$, with $\chi^b_{ij;12}=\langle b^\dagger_{2i}b_{1j}+b^\dagger_{2j}b_{1i}\rangle$ and δ, hole doping rate. In Eq. (4) above we introduced $\langle (f^\dagger_{2i}f^\dagger_{1j}-f^\dagger_{1i}f^\dagger_{2j})(f_{1j}f_{2i}-f_{2j}f_{1i})\rangle\approx\langle (f^\dagger_{2i}f^\dagger_{1j}-f^\dagger_{1i}f^\dagger_{2j})\rangle\langle (f_{1j}f_{2i}-f_{2j}f_{1i})\rangle=|\Delta^f_{ij}|^2$ and $\langle (1-h^\dagger_i h_i)(1-h^\dagger_j h_j)\rangle\approx\langle (1-h^\dagger_i h_i)\rangle\langle (1-h^\dagger_j h_j)\rangle=(1-\delta)^2$ and neglected the last term in Eq. (3) above.

The four boson term in the fourth term of Eq. (4) allows holon pairing and a scalar boson field, $\Delta^b_{ij;\alpha\beta}$ is introduced for the holon pairing between the nearest neighbor b_α and b_β single bosons with the boson index, $\alpha,\beta=1$ or 2.[7] Using the saddle point approximation, we obtain from Eq. (4) the mean field Hamiltonian,

$$\begin{aligned}H^{\mathrm{MF}}=&[J(1-\delta)^2/2]\sum_{\langle i,j\rangle}[|\Delta^f_{ij}|^2+(1/2)|\chi_{ij}|^2+(1/4)]+(J/2)\sum_{\langle i,j\rangle}|\Delta^f_{ij}|^2\left[\sum_{\alpha,\beta}|\Delta^b_{ij;\alpha\beta}|^2+\delta^2\right]\\&-[J(1-\delta)^2/2]\sum_{\langle i,j\rangle}[\Delta^{f*}_{ij}(f_{1j}f_{2i}-f_{2j}f_{1i})+\mathrm{c.c.}]-[J(1-\delta)^2/4]\sum_{\langle i,j\rangle}[\chi_{ij}(f^\dagger_{\sigma i}f_{\sigma j})+\mathrm{c.c.}]\\&-(t/2)\sum_{\langle i,j\rangle}[\chi_{ij}(b^\dagger_{1i}b_{1j}-b^\dagger_{2j}b_{2i})-\Delta^f_{ij}(b^\dagger_{1j}b_{2i}+b^\dagger_{1i}b_{2j})]-\mathrm{c.c.}-\sum_{\langle i,j\rangle,\alpha,\beta}(J/2)|\Delta^f_{ij}|^2[\Delta^{b*}_{ij;\alpha\beta}(b_{\alpha i}b_{\beta j})+\mathrm{c.c.}]\\&-\sum_i[\mu_i(h^\dagger_i h_i-\delta)+i\lambda^1_i(f^\dagger_{1i}f^\dagger_{2i}+b^\dagger_{1i}b_{2i})+i\lambda^2_i(f_{2i}f_{1i}+b^\dagger_{2i}b_{1i})+i\lambda^3_i(f^\dagger_{1i}f_{1i}-f_{2i}f^\dagger_{2i}+b^\dagger_{1i}b_{1i}-b^\dagger_{2i}b_{2i})]\\&-(t/2)\sum_{\langle i,j\rangle}(\Delta^f_{ij}-(f_{1j}f_{2i}-f_{2j}f_{1i}))\chi^{b*}_{ij;12}-\mathrm{c.c.}+[t^2/2J(1-\delta)^2]\sum_{\langle i,j\rangle}|\chi^b_{ij;12}-(b^\dagger_{2i}b_{1j}+b^\dagger_{2j}b_{1i})|^2\\&+[t^2/J(1-\delta)^2]\sum_{\langle i,j\rangle}(b^\dagger_{1i}b_{1j}-b^\dagger_{2j}b_{2i})(b^\dagger_{1j}b_{1i}-b^\dagger_{2i}b_{2j}),\end{aligned}\tag{5}$$

where $\chi_{ij}=\langle f^\dagger_{\sigma j}f_{\sigma i}+[2t/J(1-\delta)^2](b^\dagger_{1j}b_{1i}-b^\dagger_{2i}b_{2j})\rangle$, $\Delta^f_{ij}=\langle f_{1j}f_{2i}-f_{2j}f_{1i}\rangle$, $\Delta^b_{ij;\alpha\beta}=\langle b_{i\alpha}b_{\beta j}\rangle$ and $\mu_i=\mu_0-(J/2)\Sigma_{j=i\pm\hat{x},i\pm\hat{y}}|\Delta^f_{ij}|^2$. The Hubbard Stratonovich field $\rho^{k=1,2,3}_i=\langle \frac{1}{2}f^\dagger_i\sigma^k f_i\rangle$ for direct channel is taken to be 0[5] and $\rho^{k=0}_i=\frac{1}{2}$. Owing to the energy cost the exchange interaction terms [the last two positive energy terms in Eq. (5)] is usually ignored.[5–7]

We now introduce the uniform hopping order parameter, $\chi_{ij}=\chi$, the d-wave spinon pairing order parameter, $\Delta^f_{ij}=\pm\Delta_f$ with the sign $+(-)$ for the nearest neighbor link parallel to $\hat{x}$ ($\hat{y}$) and the s-wave holon pairing order parameter, $\Delta^b_{ij;\alpha\beta}=\Delta^b_{\alpha\beta}$ with the boson indices α and β. For the case of $\Delta^b_{\alpha\beta}=0$, $\lambda^{(k)}=0$ and $\Delta^f\leqslant\chi$, the b_1 bosons are populated at and near $k=(0,0)$ in the momentum space and the b_2 bosons, at and near $k=(\pi,\pi)$[7]. Pairing of two different ($\alpha\neq\beta$) bosons (holons) gives rise to the nonzero center of mass momentum. On the other hand, the center of mass momentum is zero only for pairing between identical ($\alpha=\beta$) bosons. Thus writing $\Delta^b_{\alpha\beta}=\Delta_b(\delta_{\alpha,1}\delta_{\beta,1}-\delta_{\alpha,2}\delta_{\beta,2})$(Ref. 7) for pairing between the identical holons and allowing the uniform chemical potential, $\mu_i=\mu$, the mean field Hamiltonian from Eq. (5) is derived to be

$$\begin{aligned}H^{MF}=&NJ(1-\delta)^2[\tfrac{1}{2}\chi^2+\Delta_f^2+\tfrac{1}{4}]+NJ\Delta_f^2(2\Delta_b^2+\delta^2)\\&+\sum_k E^f_k(\alpha^\dagger_{k1}\alpha_{k1}-\alpha_{k2}\alpha^\dagger_{k2})\\&+\sum_{k,s=1,2}[E^b_{ks}\beta^\dagger_{ks}\beta_{ks}+\tfrac{1}{2}(E^b_{ks}+\mu)]+\mu N\delta.\end{aligned}\tag{6}$$

Here E^f_k and E^b_{ks} are the quasiparticle energies of spinon and holon, respectively. $\alpha_{ks}(\alpha^\dagger_{ks})$ and $\beta_{ks}(\beta^\dagger_{ks})$ are the annihilation (creation) operators of the spinon quasiparticles and the holon quasiparticles, respectively.

From the diagonalized Hamiltonian Eq. (6), we readily obtain the total free energy

$$\begin{aligned}F=&NJ(1-\delta)^2[\tfrac{1}{4}+\Delta_f^2+\tfrac{1}{2}\chi^2]\\&-2k_BT\sum_k\ln[\cosh(\beta E^f_k/2)]+NJ\Delta_f^2(2\Delta_b^2+\delta^2)\\&+k_BT\sum_{k,s}\ln[1-e^{-\beta E^b_{ks}}]+\sum_{k,s}[(E^b_{ks}+\mu)/2]+\mu N\delta.\end{aligned}\tag{7}$$

The chemical potential is determined from the number constraint of doped holes,

$$\begin{aligned}-\frac{\partial F}{\partial\mu}=\sum_k&\left[\frac{1}{e^{\beta E^b_{k1}}-1}\frac{-\epsilon^b_k-\mu}{E^b_{k1}}+\frac{1}{2}\left(\frac{-\epsilon^b_k-\mu}{E^b_{k1}}-1\right)\right.\\&\left.+\frac{1}{e^{\beta E^b_{k2}}-1}\frac{\epsilon^b_k-\mu}{E^b_{k2}}+\frac{1}{2}\left(\frac{\epsilon^b_k-\mu}{E^b_{k2}}-1\right)\right]-N\delta=0,\end{aligned}\tag{8}$$

and the Lagrangian multipliers are determined by the following three constraints imposed by the SU(2) slave-boson theory,

$$\frac{\partial F}{\partial\lambda^{(k)}} = -\sum_k \tanh\frac{\beta E_k^f}{2}\frac{\partial E_k^f}{\partial\lambda^{(k)}} + \sum_{k,s}\frac{e^{\beta E_{ks}^b}+1}{2(e^{\beta E_{ks}^b}-1)}\frac{\partial E_{ks}^b}{\partial\lambda^{(k)}} = 0,$$

$$k=1,2,3. \tag{9}$$

It can be readily proven from Eq. (9) above that $\lambda^{(k)}=0$ satisfies the three constraints above.

By minimizing the free energy, the order parameters χ, Δ_f and Δ_b are numerically determined as a function of temperature and doping rate. In Fig. 2 the mean field results of the U(1) (dotted line) and SU(2) (solid line) slave-boson theories are displayed for $J=0.2t$. The predicted pseudogap (spin gap) temperature, $T^f_{SU(2)}$ is consistently higher than $T^f_{U(1)}$, the U(1) value. $T^b_{SU(2)}$ at optimal doping is predicted to be lower than the value of $T^b_{U(1)}$ predicted by the U(1) theory. The predicted optimal doping rate is shifted to a larger value, showing closer agreement with observation[1,2] than the U(1) mean field treatment. Such discrepancies are attributed to the phase fluctuations of order parameters, which were not treated in the U(1) mean field theory. We note from the four boson operator $-(J/2)|\Delta^f_{ij}|^2 b^\dagger_{\alpha i} b^\dagger_{\beta j} b_{\beta j} b_{\alpha i}$ in the fourth term of Eq. (4) that the strength of holon pairing depends on the spinon pairing amplitude (order parameter) Δ^f_{ij}. Accordingly the predicted holon pair condensation temperature (superconducting transition temperature) $T^b_{SU(2)}$ depends on the spin gap (pseudogap) temperature T^*; $T^b_{SU(2)}$ decreases with T^* in the overdoped region. Indeed it is shown in Eq. (2) that the predicted holon pair bose condensation at $T_c(=T^b_{SU(2)})$ is not independent of the spin gap (pseudogap) formation at T^*, by exhibiting the diminishing trend of superconducting temperature T_c as the spin gap temperature T^* decreases in the overdoped region. This is consistent with an experimental observation of the universal behavior of T^*/T_c as a function of hole doping rate δ/δ_o for different high-T_c cuprates, as is shown in Fig. 1. Although not shown, the higher the J value, the predicted T_c's are consistently higher than the case of $J=0.2t$.

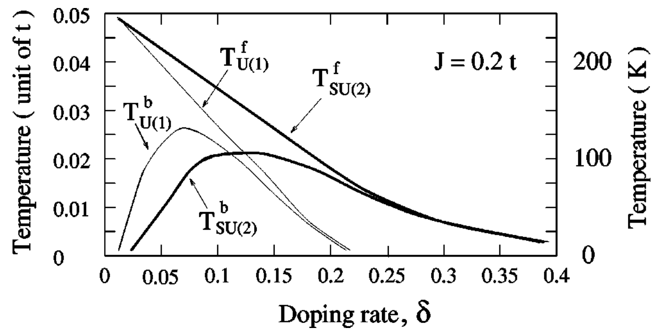

FIG. 2. Computed phase diagrams with $J=0.2t$. $T^f_{\mathrm{SU(2)}}(T^f_{\mathrm{U(1)}})$ denotes the pseudogap temperature and $T^b_{\mathrm{SU(2)}}(T^b_{\mathrm{U(1)}})$, the holon pair bose condensation temperature predicted from the SU(2) (solid lines) and [U(1)] (dotted lines) slave-boson theories, respectively. The scale of temperature is based on $t=0.44$ eV (Ref. 9).

In summary, based on the SU(2) slave-boson symmetry conserving t-J Hamiltonian which shows coupling between the charge and spin degrees of freedom, we derived a phase diagram of high-T_c cuprates which displays the bose condensation temperature of an arch shape as a function of hole doping rate. Unlike other previous studies which predicted a linear increase with the hole doping rate, this result is consistent with observation. We showed that the low energy fluctuations cause a shift of the optimal doping rate to a larger value and a suppression of the holon pair bose condensation temperature, thus allowing a closer agreement with observation compared to the U(1) case.

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[1] H. Yasuoka, Physica C **282-287**, 119 (1997), references therein.
[2] T. Nakano *et al.*, J. Phys. Soc. Jpn. **67**, 8 (1998); N. Momono *et al.*, J. Low Temp. Phys. **117**, 353 (1999), references therein.
[3] G. Kotliar and J. Liu, Phys. Rev. B **38**, 5142 (1988), references therein.
[4] Y. Suzumura *et al.*, J. Phys. Soc. Jpn. **57**, 2768 (1988).
[5] M. U. Ubbens and P. A. Lee, Phys. Rev. B **46**, 8434 (1992); **49**, 6853 (1994), references therein.
[6] T.-H. Gimm *et al.*, Phys. Rev. B **60**, 6324 (1999).
[7] X. G. Wen and P. A. Lee, Phys. Rev. Lett. **76**, 503 (1996); **80**, 2193 (1998), references therein.
[8] S.-S. Lee and Sung-Ho Suck Salk, Int. J. Mod. Phys. B **13**, 3455 (1999); Physica C (to be published); cond-mat/9907226 (unpublished).
[9] M. S. Hybertsen *et al.*, Phys. Rev. B **41**, 11 068 (1990).