Interchannel electron transitions in a Luttinger liquid

I. V. Krive,[1,2] S. I. Kulinich,[1,2] L. Y. Gorelik,[1] R. I. Shekhter,[1] and M. Jonson[1]
[1]*Department of Applied Physics, Chalmers University of Technology and Göteborg University, SE-412 96 Göteborg, Sweden*
[2]*B.I. Verkin Institute for Low Temperature Physics and Engineering, 47 Lenin Avenue, 310164 Kharkov, Ukraine*
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Intermode electron transitions in a two-channel Luttinger liquid (LL) are considered. At first we study chiral LL and show that for a long-range Coulomb interaction even forward-scattering intermode electron transitions are strongly renormalized by interaction at low energies. For a short-range interaction the renormalization of the electron transition rates in a chiral LL do not depend on the energy (temperature, bias voltage) involved in the tunneling process. We show, however, that the interference effects are sensitive to the strength of electron interaction, and they are significantly enhanced for strongly correlated electrons. The analyses of the intermode electron transitions is generalized for a nonchiral two-channel LL. The forward- and the backward-scattering interchannel electron transition rates are calculated and their connection to the problem of x-ray singularity in a LL is elucidated.

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I. INTRODUCTION

Tunneling in one-dimensional systems of interacting electrons is drastically distinct from that for noninteracting particles. As was shown by Kane and Fisher,[1] the bare tunneling amplitude t_0 is strongly renormalized by interaction. At low temperatures $T \ll \varepsilon_F$ and for the large barriers $t_0/\hbar v_F \ll 1$, the tunneling rate for spinless electrons scales as a power-law function of temperature[1] $\Gamma(T) \propto |t_0|^2 (T/\varepsilon_F)^{2(g^{-1}-1)}$, where g is the correlation parameter of the Luttinger liquid ($g=1$ corresponds to the case of noninteracting electrons). Physically it means that the charge accumulated near the barrier cannot be redistributed along the one-dimensional (1D) wire on a time scale shorter than the characteristic ''thermal'' time $\tau_T \sim \hbar/T$. As a consequence, at $T \to 0$ and for sufficiently long wires even a weak bare potential is renormalized up to an infinitely high barrier which ''breaks'' the Luttinger liquid (LL) in the case of the charge transport into two semi-infinite subsystems.

This result was predicted for a purely 1D system. In real quantum wires (e.g., in carbon nanotubes[2]) there are as a rule several propagating longitudinal modes. Then the question arises — How do these extra modes influence the intramode and intermode tunneling of electrons?

The intramode tunneling in a multichannel LL was studied in Ref. 3, where it was shown that the additional modes effectively increase the ''stiffness'' of the system by redefining the correlation parameters g_j $(j=1,\ldots,N)$ which determine the tunneling rate $\Gamma_j^{(N)}$ in the jth mode of the N-channel LL. The problem of intermode tunneling and its influence on the Kane-Fisher effect is less well studied.

The interchannel electron transition can be regarded as a process of creating a dipole in the region of transition. This perturbation is less dramatic than the perturbation of charge densities in the uncoupled channels. For instance, in the case of equal Fermi velocities the two-channel spinless LL is equivalent to a single-channel spin-$\frac{1}{2}$ LL. Then the interchannel electron transition is just the spin-flip process which itself could not disturb the charge density. Therefore the forward spin-flip scattering is not affected by interactions. However, if the Fermi velocities in the channels are different we could expect the electron-electron interaction to influence the interchannel electron transitions.

In this paper we consider a two-channel LL and study intrachannel and interchannel electron transitions induced by a local scattering potential. The most harmless processes in a LL are those associated with forward scattering. So, at first the problem is treated for a chiral LL, where only forward-scattering processes are allowed. In a purely 1D system the forward scattering does not lead to a redistribution of the charge density along the wire and the corresponding characteristics (scattering phase) are not renormalized by interactions. For a multimode LL (see Ref. 3) one could consider channels with different Fermi velocities $v_F^{(j)}$. In this case even in a chiral LL the transition of electrons from one channel to another results in a redistribution of the charge density and the tunneling rates do get renormalized. Notice that in the LL model the electron spectrum is linearized and the velocities of the excitations are energy independent. Therefore the renormalization effects in a single chirality multimode LL depend only on the ratio of Fermi velocities of the channels. Contrary to the Kane-Fisher effect, the renormalized electron transition rates in a chiral multichannel LL are temperature- and voltage-bias independent.

We show, however, that one can introduce local quantities — densities of states (DOS) for interacting electrons, which are renormalized by the energy-dependent factor. These quantities determine the transitions of electrons ''dressed'' by the interchannel interaction to or from the vicinity of the Fermi energy. They are strongly energy dependent in a complementary manner. When one of the transition amplitudes is enhanced, the other is suppressed. The total electron transition rate $\Gamma_{12}(\omega)$ at frequency ω can be recast in the form of the convolution of the two mentioned transition amplitudes shifted by $\hbar\omega$ and, therefore, the interaction-induced energy-dependent renormalization is not manifest in $\Gamma_{12}(\omega)$. One may expect that the ''local'' (energy-dependent) electron transition amplitudes could be important in various interference effects in a two-channel LL. By considering the

interchannel electron transitions at two points spaced by the distance $d \gg a$ (where a is the lattice spacing) we demonstrate that the interference pattern is controlled by the parameters that determine the energy dependence of the local DOS.

For a short-range electron-electron interaction the renormalization factor in the expression for $\Gamma_{12}(\omega)$ depends only on the ratio of the Fermi velocities and the interaction strength. The suppression of electron transitions by the interaction in this case is pronounced only in the strong-coupling limit when the effective tunneling rate is inversely proportional to the square of the interaction constant. The dependence of electron tunneling rates on the energy in a chiral LL occurs for a long-range electron-electron interaction due to the dispersion of the plasmon velocities. In this case even for a weak interaction the transition rate $\Gamma_{12}(\omega)$ is strongly renormalized in the low-frequency limit $\omega \to 0$. This is the closest analog of the interaction-induced renormalization effects in a chiral LL to the Kane-Fisher effect. Notice, however, that the effect in the chiral model is much weaker than that caused by the backscattering of electrons (the Kane-Fisher effect). For instance, for an unscreened Coulomb interaction $\Gamma_{12}(\omega)$ scales with frequency ω only as the power of a logarithm, $\Gamma_{12}(\omega) \propto 1/\ln^2 \omega$.

A multichannel chiral LL [although it was realized in experiments as edge states in the fractional quantum Hall effect (FQHE) systems[4]] is a rather exotic object. In quantum wires electrons are nonchiral. So, the important question arises — Does the effects predicted for a chiral LL survive when the interaction between the right and left movers is switched on? At first we considered the situation when there is no backscattering in the channels (adiabatic transport). It is shown that the interchannel forward-scattering transition rate $\Gamma_{12}^{(f)}(\omega)$ is suppressed by the interaction as $\omega \to 0$ if $v_F^{(1)} \neq v_F^{(2)}$. However, the exponents that determine the energy behavior of the local DOS's vary with the interaction strength qualitatively in the same manner as in a chiral LL. So, our analysis of the interference effects, performed in a simple model of two-channel chiral LL, holds also for the quantum wires.

If there is a barrier that backscatters electrons in the channels, it ''breaks'' the LL wire into two semi-infinite parts for the charge transport. The tunneling of electrons across the weak link will always be strongly suppressed irrespective to which channel it occurs in. So, we could consider either the interchannel forward-scattering electron transitions induced independently in the left (right) part of the wire and/or the interchannel backward scattering at the position of the ''impurity.'' The last process seems to be the most interesting one since it is just the backscattering with a change of flavors (channel indices in our case). It is shown that this channel of backscattering is always suppressed by interactions. So, the barrier strongly reflect the electrons only to their own modes ($1_{L()} \leftrightarrow 1_{R(L)}$, $2_{L(R)} \leftrightarrow 2_{R(L)}$).

The paper is organized as follows. Section II deals with the detailed analysis of the interchannel electron transitions in a chiral LL. The interference effects induced by the electron transitions in a two-channel LL are considered in Sec. III. In Sec. IV the results derived in Sec. II are generalized to the case of long-range electron-electron interaction. In Sec. V the Hamiltonian of a two-channel LL is diagonalized with the help of canonical pseudoorthogonal transformation. The rates of the interchannel electron transitions in an infinite LL are evaluated in Sec. VI. Section VII deals with the problem of interchannel electron transitions at the end of the semi-infinite LL. The importance of the derived results for the optical transitions in a LL is briefly discussed in Sec. VIII.

II. TWO-CHANNEL CHIRAL LUTTINGER LIQUID

Consider a one-dimensional two-channel spinless electron liquid interacting via a density-density short-range interaction. The Hamiltonian of the system takes the form (for definiteness we will consider right-moving electrons)

$$\mathcal{H} = \pi\hbar \int dx \left\{ u_1 \rho_1^2(x,t) + u_2 \rho_2^2(x,t) + \frac{V_0}{\pi\hbar} \rho_1(x,t)\rho_2(x,t) \right\}, \tag{1}$$

where $\rho_{1,(2)}$ are the charge-density operators in the first (second) channel, V_0 is the interaction strength (V_0 is of the order of the electron charge squared, $V_0 \simeq e^2$),

$$u_{1(2)} = v_F^{(1),(2)} + \frac{V_0}{\pi\hbar}, \tag{2}$$

and $v_F^{(1),(2)}$ are the Fermi velocities. The charge-density operators obey the equal time commutation relations (see, e.g., Ref. 5) $[\rho_j(x), \rho_k(x')] = (\delta_{jk}/2\pi i)\, \partial_x\, \delta(x-x')$. The Hamiltonian (1) is easily diagonalized by the unitary transformation

$$\rho_1 = \tilde{\rho}_1 \cos\psi + \tilde{\rho}_2 \sin\psi; \quad \rho_2 = \tilde{\rho}_1 \sin\psi - \tilde{\rho}_2 \cos\psi, \tag{3}$$

where

$$\tan 2\psi = \frac{V_0}{\pi\hbar(u_1 - u_2)} = \frac{V_0}{\pi\hbar(v_F^{(1)} - v_F^{(2)})}. \tag{4}$$

In diagonal form, the Hamiltonian $\tilde{\mathcal{H}}$ describes two decoupled boson modes with the velocities

$$s_{1,(2)} = \frac{1}{2}(u_1 + u_2) \pm \sqrt{\left(\frac{u_1 - u_2}{2}\right)^2 + \left(\frac{V_0}{2\pi\hbar}\right)^2}. \tag{5}$$

We will describe the transition of an electron from one channel to another at point $x = 0$ by the tunneling Hamiltonian

$$\mathcal{H}_t = t_\omega \Psi_2^\dagger(0)\Psi_1(0) + \text{H.c.}, \tag{6}$$

where Ψ ($\Psi^\dagger$) is the electron annihilation (creation) operator at point $x = 0$; t_ω is the bare tunneling amplitude [in what follows we will only consider a harmonic perturbation, $t_\omega = t_0 \exp(i\omega t)$]. We will also assume that the transitions induced by the Hamiltonian (6) are weak ($t_0 \ll \hbar v_F^{(1),(2)}$) and will treat them perturbatively. By using the Fermi golden rule, the electron transition rate $\Gamma_{12}(\omega)$ can be represented as the Fourier transform of the four-point correlation function

$$\Gamma_{12}(\omega)=\int dt e^{i\omega t}\Gamma_{12}(t),$$

$$\Gamma_{12}(t)=\left|\frac{t_0}{\hbar}\right|^2\langle\Psi_1^\dagger(t,0)\Psi_2(t,0)\Psi_2^\dagger(0,0)\Psi_1(0,0)\rangle, \quad (7)$$

where the average is taken over the ground state of the interacting electron system described by the Hamiltonian (1).

Since the Hamiltonian, Eq. (1), when represented via the charge-density operators is quadratic, the correlation function $\Gamma_{12}(t)$ can be evaluated exactly by using bosonization techniques (see, e.g., Ref. 5). At first we represent the fermion operators in terms of boson fields $\varphi_j(x,t)$, where $\rho_j(x,t)=(1/\sqrt{\pi})\partial_x\varphi_j(x,t)(j=1,2)$:

$$\Psi_j(x,t)=\frac{U_j}{\sqrt{2\pi a}}\exp\{ik_F^{(j)}x+i\sqrt{4\pi}\varphi_j(x,t)\}. \quad (8)$$

Here a is a cutoff of the order of the lattice spacing and $k_F^{(j)}$ is the Fermi momentum in the first ($j=1$) or the second ($j=2$) channel (recall that for definiteness we consider right-moving fermions), and U_j is the unitary rising operator that provides the correct commutation relations for the Fermi fields Ψ_j in different channels. We will not specify the form of this operator since it disappears from the expression for the correlation function, Eq. (7). The standard procedure allows us to express the correlation function $\Gamma_{12}(t)$ as the exponential of the boson correlation functions

$$\Gamma_{12}(t)=\left|\frac{t_0}{2\pi\hbar a}\right|^2\exp\{4\pi(\langle\langle\varphi_1(t)\varphi_1\rangle\rangle+\langle\langle\varphi_2(t)\varphi_2\rangle\rangle-\langle\langle\varphi_2(t)\varphi_1\rangle\rangle-\langle\langle\varphi_1(t)\varphi_2\rangle\rangle)\}, \quad (9)$$

where $\langle\langle\varphi_i(t)\varphi_j\rangle\rangle\equiv\langle[\varphi_i(t,0)-\varphi_i(0,0)]\varphi_j(0,0)\rangle$. The boson fields $\varphi_j(x,t)$ in their turn are linear combinations [see the Bogoliubov transformation, Eq. (3)] of the ''diagonal'' fields $\tilde{\varphi}_j(x,t)$ with the well-known property (see, e.g., Ref. 5) $\langle\langle\tilde{\varphi}_j(t)\tilde{\varphi}_k\rangle\rangle=-(\delta_{jk}/4\pi)\ln[(a+is_kt)/a]$. After straightforward calculations the correlation function $\Gamma_{12}(t)$ takes the form

$$\Gamma_{12}(t)=\left|\frac{t_0}{2\pi\hbar}\right|^2(a+is_1t)^{-\lambda_-}(a+is_2t)^{-\lambda_+}, \quad (10)$$

where

$$\lambda_\pm=1\pm\sin 2\psi=1\pm\frac{\kappa}{\sqrt{1+\kappa^2}}. \quad (11)$$

Here $\kappa\equiv\tan 2\psi$ is determined by Eq. (4).

In the LL approach to the interacting fermions we are interested in the low-frequency limit $\omega\ll v_F^{(1),(2)}/a$. The desired result for the tunneling rate is

$$\Gamma_{12}(\omega)\simeq\Gamma_{12}^{(0)}(\omega)\frac{v_F^{(1)}v_F^{(2)}}{s_1s_2}\left(\frac{s_1}{s_2}\right)^\zeta, \quad \zeta\equiv\frac{\kappa}{\sqrt{1+\kappa^2}}. \quad (12)$$

Here $\Gamma_{12}^{(0)}$ is the corresponding tunneling rate for noninteracting electrons ($V_0=0$),

$$\Gamma_{12}^{(0)}(\omega)=\frac{|t_0|^2}{2\pi\hbar^2}\frac{\omega\theta(\omega)}{v_F^{(1)}v_F^{(2)}}, \quad (13)$$

where $\theta(\omega)$ is the Heaviside step function. The appearance of the step function in Eq. (13) reflects the evident fact that due to the Fermi statistics the electrons can tunnel only to unoccupied states with the energy $\omega>0(\varepsilon>\varepsilon_F)$.

According to Eq. (12) the tunneling rate is renormalized multiplicatively. Notice that for chiral fermions the renormalization factor does not depend on the energy $\hbar\omega$ involved in the transition process. For repulsive forces ($V_0>0$) the interaction suppresses the tunneling, and the attraction ($V_0<0$) leads to the enhancement of the tunneling rates. The effect is small in the weak-coupling limit $V_0/\pi\hbar v_F^{(1,2)}\ll 1$ when

$$\frac{\Gamma_{12}(\omega)}{\Gamma_{12}^{(0)}(\omega)}\simeq 1-\frac{V_0}{\pi\hbar}\left(\frac{1}{v_F^{(1)}}+\frac{1}{v_F^{(2)}}-\frac{1}{v_F^{(1)}-v_F^{(2)}}\ln\frac{v_F^{(1)}}{v_F^{(2)}}\right). \quad (14)$$

Notice that the expression within the parentheses is positively definite. In the strong-coupling limit $V_0/\pi\hbar v_F^{(1,2)}\gg 1$ the velocities $s_{1,2}$ are totally determined by the interaction $s_2\simeq s_1/3\simeq|V_0|/2\pi\hbar$, and the tunneling rate is strongly suppressed. For $v_F^{(1)}\simeq v_F^{(2)}=v_F\ll V_0/2\pi\hbar$

$$\Gamma_{12}(\omega)\simeq\left(\frac{2\pi\hbar v_F}{V_0}\right)^2\Gamma_{12}^{(0)}(\omega)\ll\Gamma_{12}^{(0)}(\omega). \quad (15)$$

We will see in what follows that for a long-range interaction the renormalization factor in Eq. (15) acquires an additional frequency-dependent coefficient and the problem of the interchannel electron tunneling ($\omega\to 0$) always is a problem of strong coupling.

One can readily generalize Eq. (12) to finite temperatures. At $T\neq 0$ the correlation function for the noninteracting boson fields reads

$$\langle\tilde{\varphi}_j(t)\tilde{\varphi}_k\rangle=-\frac{\delta_{jk}}{4\pi}\ln\left[\frac{(a+is_kt)}{a}\frac{\sinh(\pi\hbar^{-1}tT)}{\pi\hbar^{-1}tT}\right], \quad (16)$$

and then

$$\Gamma_{12}(T,\omega)\simeq\left|\frac{t_0}{2\pi\hbar}\right|^2 s_1^{-\lambda_-}s_2^{-\lambda_+}\times\int_{-\infty}^{\infty}dt\frac{e^{i\omega t}}{(\tau_1+it)^{\lambda_-}(\tau_2+it)^{\lambda_+}}\times\left[\frac{\pi\hbar^{-1}tT}{\sinh(\pi\hbar^{-1}tT)}\right]^2, \quad (17)$$

where $\tau_{1(2)}=a/v_F^{(1),(2)}$ and $\lambda_\pm$ is defined in Eq. (11). In the limit $\hbar\omega,T\ll\hbar/\tau_{1,2}$ the integral in Eq. (17) is determined by the long-t asymptotics of the integrand and, since $\lambda_++\lambda_-=2$, it is evident that the temperature contributions to Γ_{12} do

not depend on the interaction. The integral can be done analytically by the contour-integral method. The result is

$$\Gamma_{12}(T,\omega)=\frac{|t_0|^2}{2\pi\hbar^2}s_1^{\zeta-1}s_2^{-\zeta-1}\left\{\omega\theta(\omega)+\frac{|\omega|}{\exp(\hbar|\omega|/T)-1}\right\}. \tag{18}$$

So, if $T\ll\hbar|\omega|$ we reproduce Eqs. (12) and (13) up to exponentially small corrections. In the ''high-T'' limit $\hbar|\omega|\ll T\ll\hbar/\tau_{1,2}$ the electron transition rate in the main approximation is determined by temperature

$$\Gamma_{12}(T\gg\hbar\omega)\simeq\frac{|t_0|^2}{2\pi\hbar^3}s_1^{\zeta-1}s_2^{-\zeta-1}T. \tag{19}$$

We showed that the temperature- and interaction-dependent parts of the electron transition rate are factorized. In what follows we set $T=0$ and consider only the effects of electron-electron interaction.

III. ROTATED DENSITY OF STATES

It is useful to represent $\Gamma_{12}(\omega)$ Eqs. (7) and (10) in integral form, namely, as an integral over energy of the product of two local density of states (DOS)

$$\Gamma_{12}(\omega)=\frac{2\pi}{\hbar}|t_0|^2\int_{-\infty}^{\infty}d\varepsilon\tilde{n}_1^{(-)}(\varepsilon)\tilde{n}_2^{(+)}(\varepsilon+\hbar\omega), \tag{20}$$

where

$$\tilde{n}_j^{\pm}(\varepsilon)=\frac{1}{(2\pi)^2\hbar}\int_{-\infty}^{\infty}\frac{dt}{a+is_jt}\exp\left[\pm\frac{i\varepsilon t}{\hbar}\pm\sin 2\psi\int_0^{\infty}\frac{dk}{k}e^{-ak}(e^{-iks_jt}-1)\right] \tag{21}$$

and the angle ψ is determined by Eq. (4). It is easy to check that the introduced quantities possess the exact symmetry property $\tilde{n}_1^{(-)}[-\varepsilon,-\mathrm{sgn}(v_F^{(1)}-v_F^{(2)})]=\tilde{n}_2^{(+)}[\varepsilon,\mathrm{sgn}(v_F^{(1)}-v_F^{(2)})]$. Notice that the local DOS's, Eq. (20), were defined after the diagonalization of the boson fields φ_j and therefore they depend on the correlation function of the diagonal fields $\langle\langle\tilde{\varphi}_j(t)\tilde{\varphi}_j(0)\rangle\rangle$. We will call these quantities the *rotated* DOS's to distinguish them from the ordinary density of states

$$n_1^{(-)}(\varepsilon)=\frac{1}{2\pi\hbar}\int_{-\infty}^{\infty}dte^{-i\varepsilon t/\hbar}\langle\Psi_1^{\dagger}(t)\Psi_1(0)\rangle=n_{10}^{(-)}(\varepsilon)\frac{v_F^{(1)}}{s_1}\left(\frac{s_1}{s_2}\right)^{\sin^2\psi}, \tag{22}$$

$$n_2^{(+)}(\varepsilon)=\frac{1}{2\pi\hbar}\int_{-\infty}^{\infty}dte^{i\varepsilon t/\hbar}\langle\Psi_2^{\dagger}(t)\Psi_2(0)\rangle=n_{20}^{(+)}(\varepsilon)\frac{v_F^{(2)}}{s_2}\left(\frac{s_2}{s_1}\right)^{\sin^2\psi}, \tag{23}$$

where

$$n_{10}^{(-)}(\varepsilon)=\frac{\theta(-\varepsilon)}{2\pi\hbar v_F^{(1)}},\quad n_{20}^{(+)}(\varepsilon)=\frac{\theta(\varepsilon)}{2\pi\hbar v_F^{(2)}} \tag{24}$$

are the density of states of noninteracting chiral electrons ($V_0=0$).

Unlike ordinary DOS's, Eqs. (22) (23), the rotated ones even in chiral LL depend on energy in a nontrivial way. By doing the integrals in Eq. (21) one readily gets the low-energy asymptotics of Eq. (21),

$$\tilde{n}_1^{(-)}(\varepsilon)\simeq n_{10}^{(-)}(\varepsilon)\frac{v_F^{(1)}}{s_1\Gamma(1+\zeta)}\left|\frac{\varepsilon}{\Lambda_1}\right|^{-\zeta},\quad \varepsilon\ll\Lambda_1 \tag{25}$$

$$\tilde{n}_2^{(+)}(\varepsilon)\simeq n_{20}^{(+)}(\varepsilon)\frac{v_F^{(2)}}{s_2\Gamma(1-\zeta)}\left|\frac{\varepsilon}{\Lambda_2}\right|^{\zeta},\quad \varepsilon\ll\Lambda_2. \tag{26}$$

Here $\Gamma(x)$ is the gamma function, $\Lambda_j\equiv\hbar s_j/a$ $(j=1,2)$ are the cutoff energies, and ζ is defined in Eq. (12).

What is the physical meaning of the rotated DOS? Notice that due to the interchannel interaction the charge created, e.g., in the second channel interacts with the charge deficit left in the first channel. This dipolelike interaction ''dresses'' the electrons and strongly renormalizes the bare tunneling amplitude. Formally Eq. (21) can be interpreted as the ordinary tunneling DOS for dressed electrons $\tilde{\Psi}^{\pm}\sim\exp(i\sqrt{4\pi}\lambda_{\pm}\tilde{\phi}_j)$, where $\lambda_{\pm}$ is determined in Eq. (12). For dressed electrons the interaction is attractive if $\lambda_{\pm}>1$ and repulsive for $\lambda_{\pm}<1$. The integral representation, Eq. (20), for the tunneling rate allows us to rewrite the interchannel electron transition probability as the convolution of two energy-dependent transition amplitudes

$$\Gamma_{12}(\omega)=\int_{-\infty}^{\infty}\frac{d\varepsilon}{2\pi\hbar}t_1^{(-)}(\varepsilon)t_2^{(+)}(\varepsilon+\hbar\omega), \tag{27}$$

$$t_2^{(+)}(\varepsilon)=2\pi t_0\tilde{n}_2^{(+)}(\varepsilon),\quad t_1^{(-)}(\varepsilon)=2\pi t_0^*\tilde{n}_1^{(-)}(\varepsilon). \tag{28}$$

The amplitude $t_1^{(-)}(\varepsilon)$ determines the process of removing the dressed electron from the vicinity of the Fermi level ($\varepsilon\ll\Lambda_1$) in the first channel; $t_2^{(+)}(\varepsilon)$ describes the complementary process of adding the dressed electron to the region in the vicinity of the Fermi level ($\varepsilon\ll\Lambda_2$) in the second channel. At zero temperature the limits of integration in Eq. (27) are finite due to the presence of the step function in the definition of DOS. If $\hbar\omega$ is of the order of the energy cutoff Λ, $t_2^{(+)}$ will contribute to the integral Eq. (27) only in the lower limit ($\varepsilon\sim-\hbar\omega$) and $t_1^{(-)}$ only in the upper limit ($\varepsilon\sim0$). One may speculate that at least in the weak coupling limit ($\kappa\ll1$), when it is possible to use the one-particle concept of scattering data (see, e.g., Ref. 6) the electron transition amplitudes $t_{\mathrm{eff}}(\varepsilon\to\varepsilon+\hbar\omega)$ and $t_{\mathrm{eff}}(-\hbar\omega+\varepsilon\to\varepsilon)$ (see Fig. 1) are renormalized by interactions analogously to the rotated DOS,

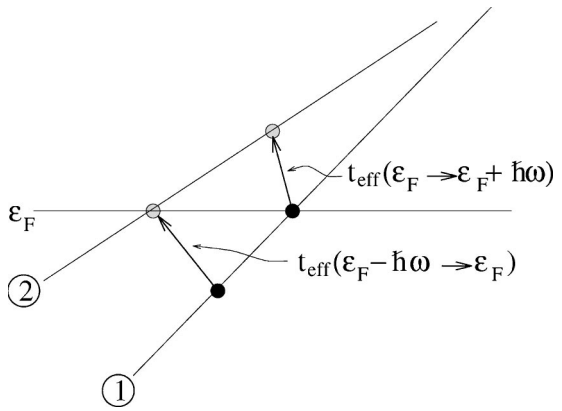

FIG. 1. ''Vertical'' intermode electron transition amplitudes. The curves 1 and 2 schematically represent electron energy bands $\varepsilon_j(p)$ in the first $(j=1)$ and second $(j=2)$ channels.

$$t_{\rm eff}(\varepsilon \to \varepsilon + \hbar\omega) \simeq t_f \theta(-\varepsilon) \left| \frac{\hbar\omega}{\varepsilon} \right|^{V_0/[\pi\hbar(v_F^{(1)} - v_F^{(2)})]}, \quad \varepsilon \ll \hbar\omega \tag{29}$$

$$t_{\rm eff}(-\hbar\omega + \varepsilon \to \varepsilon) \simeq t_f \theta(\varepsilon) \left| \frac{\varepsilon}{\hbar\omega} \right|^{V_0/[\pi\hbar(v_F^{(1)} - v_F^{(2)})]}, \quad \varepsilon \ll \hbar\omega, \tag{30}$$

where $t_f \ll 1$ is the bare forward- (f) scattering amplitude.

The above expressions indeed were derived in Ref. 7 in the weak-coupling approach to the 1D interacting electron systems proposed in Ref. 6. So our analysis supports this result and shows that even in chiral LL (without backscattering) there are ''local'' quantities [rotated DOS, Eqs. (25), (26)] which are strongly renormalized by interaction.

How could one probe the rotated DOS? It is clear that the different dependence of electron transition amplitudes on the energy could play an important role in the interference effects. Imagine that we induce electron transitions in a chiral LL by electromagnetic field at two ''points'' (say, $x=0$ and $x=d$; see Fig. 2). The local transition regions could be realized in a laterally confined two-dimensional (2D) electron gas by modulating the width of the quantum wire. Then the frequency of the ac field could be adjusted for resonant electron transitions only in the local regions (widenings) well separated from each other along the wire. We consider the influence of the interference effects on the power absorption $P(\omega) = \hbar\omega\Gamma_{12}^{(d)}(\omega)$. This measurable quantity can be analytically calculated and it does demonstrate the strong dependence on the interaction strength.

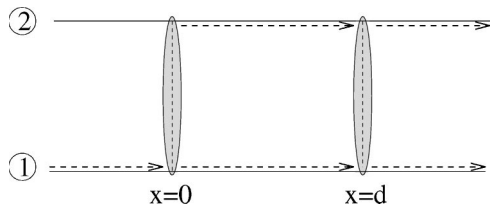

FIG. 2. Two-point electron ''splitter.'' The electromagnetic field induces electron transitions at points $x=0,d$ in a two-channel nonhomogeneous Luttinger liquid.

The evaluation of $\Gamma_{12}^{(d)}(\omega)$ results in the expression

$$\frac{\Gamma_{12}^{(d)}(\omega)}{2\Gamma_{12}(\omega)} = 1 + \Re\{\exp[i(k_F^{(1)} - k_F^{(2)} - s_1^{-1}\omega)d] \times {}_1F_1(1-\zeta, 2; -i\omega d(s_2^{-1} - s_1^{-1}))\}, \tag{31}$$

where $\Gamma_{12}(\omega)$ is determined by Eq. (12) and ${}_1F_1(a,c;z)$ is the confluent hypergeometric function. We consider the asymptotics of Eq. (31) when $x \equiv |\omega d(s_2^{-1} - s_1^{-1})| \gg 1$, which is the proper limit for the adiabatic channels ($x \sim d/\lambda_F \gg 1$ in the weak-coupling regime and $x \sim \varepsilon_F/E_c \gg 1$ for strong interaction; here λ_F is the Fermi wavelength and $E_c \simeq e^2/d$ is the ''Coulomb blockade'' energy). By making use of the asymptotic behavior of the confluent hypergeometric function,[8] we get the desired result

$$\frac{\Gamma_{12}^{(d)}(\omega)}{2\Gamma_{12}(\omega)} \simeq 1 + \frac{\sin\Phi_1}{\Gamma(1+\zeta)} x^{\zeta-1} - \frac{\sin\Phi_2}{\Gamma(1-\zeta)} x^{-\zeta-1}, \tag{32}$$

where

$$\Phi_j = (k_F^{(1)} - k_F^{(2)} - s_j^{-1}\omega)d + (\pi/2)\zeta, \qquad \zeta = V_0/\sqrt{V_0^2 + [\pi\hbar(v_F^{(1)} - v_F^{(2)})]^2}. \tag{33}$$

Notice that $\zeta = 0$ for noninteracting electrons $(V_0 = 0)$ and $\zeta \to 1$ in the strong-coupling regime. One can see from Eq. (33) that the interference effects in the power absorption do depend on the interaction and that they are significantly enhanced for strongly correlated electrons. The interaction-dependent parameters controlling the decay of the oscillating part of $\Gamma_{12}^{(d)}$ coincide with those who determine the energy dependence of the rotated DOS's, Eq. (25). A detailed analysis of the power absorption in the multichannel LL wire irradiated by electromagnetic field will be published elsewhere.

IV. LONG-RANGE INTERACTION

The interaction-induced energy dependence of the ''local'' tunneling characteristics like effective transmission amplitudes, Eq. (29), disappears in the ''global'' properties of electron tunneling in a two-channel chiral LL. Now we show that in a chiral electron liquid with Coulomb interaction the electron transition rate $\Gamma_{12}(\omega)$ is renormalized by interaction and that the renormalization factor depends on the frequency.

Let us replace the interaction term in the Hamiltonian, Eq. (1), by the Coulomb energy

$$\hat{V}_{int} = e^2 \int\int dx dy \frac{\rho_1(x)\rho_2(y)}{\sqrt{(x-y)^2 + d^2}}, \tag{34}$$

where d is the width of the wire (d is of the order of the UV cutoff a and in what follows we will set $d \simeq a$). The calculations analogous to that performed in Sec. II for LL (short-range interaction) results in the expression for the four-point electron correlation function as follows:

$$\Gamma_{12}(t)=\left|\frac{t_0}{2\pi\hbar}\right|^2\exp\left(\sum_{j=1,2}\int_0^\infty\frac{dk}{k}e^{-ak}(e^{-iks_j(k)t}-1)+\int_0^\infty\frac{dk}{k}\frac{\kappa_k}{\sqrt{1+\kappa_k^2}}e^{-ak}[e^{-iks_2(k)t}-e^{-iks_1(k)t}]\right),\tag{35}$$

where

$$\kappa_k=\frac{e^2V_k}{\pi\hbar(v_F^{(1)}-v_F^{(2)})},\quad V_k=\int_{-\infty}^{\infty}dx\frac{e^{ikx}}{\sqrt{x^2+a^2}}=2K_0(ak).\tag{36}$$

Here $K_0(x)$ is the modified Bessel function of the third kind (see, e.g., Ref. 9). The velocities $s_{1,2}(k)$ are determined by Eq. (5), where the interaction strength V_0 is replaced by e^2V_k. The integrals over momentum k in Eq. (35) can be done analytically in the limits of weak and strong interaction.

A. Weak interaction

In the weak interaction limit the plasmon velocities are

$$s_j(k)=v_F^{(j)}+\frac{e^2V_k}{\pi\hbar}\tag{37}$$

and we have to take into account the velocity dispersion while evaluating the first two integrals in Eq. (35). The last two terms are already small ($\kappa_k\ll1$) and can be evaluated for unperturbed velocities $s_j(k)\simeq v_F^{(j)}$. The corresponding contributions are

$$\Gamma_{12}(t)=\left|\frac{t_0}{2\pi\hbar}\right|^2\frac{\exp[W_1(t)+W_2(t)]}{(a+iv_F^{(1)}t)(a+iv_F^{(2)}t)},\tag{38}$$

$$W_j\simeq-\frac{2e^2}{\pi\hbar a}\frac{t}{\sqrt{l_j^2+1}}[\sinh^{-1}(l_j)+i\pi/2]+(-1)^{j+1}\frac{e^2}{\pi\hbar(v_F^{(1)}-v_F^{(2)})}\sinh^{-1}(l_j)\times[\sinh^{-1}(l_j)+i\pi],\tag{39}$$

where $l_j=v_F^{(j)}t/a$. In the absence of interaction $W_j(t)=0$ and $\Gamma_{12}(t)$ is reduced to the well-known expression for noninteracting electrons.

To get the tunneling rate Γ_{12} we need to take the Fourier integral equation (7). In the LL approach we are interested in the low-frequency behavior, $a\omega\ll v_F^{(j)}$, of the tunneling rate $\Gamma_{12}(\omega)$. So we can approximate W_j by its long-time asymptotics $t\gg\tau_j\equiv a/2v_F^{(j)}$,

$$\sum_{j=1,2}W_j(t)\simeq-\frac{2e^2}{\pi\hbar}\left[\frac{1}{v_F^{(1)}}\ln\frac{it}{\tau_1}+\frac{1}{v_F^{(2)}}\ln\frac{it}{\tau_2}-\frac{1}{v_F^{(1)}-v_F^{(2)}}\ln\frac{v_F^{(1)}}{v_F^{(2)}}\ln\frac{it}{\tau_{12}}\right].\tag{40}$$

Here $\tau_{12}=a/2\sqrt{v_F^{(1)}v_F^{(2)}}$. With the help of Eq. (40) it is straightforward to evaluate the Fourier integral. In the low-frequency limit, $\omega\tau\ll1$, one gets

$$\frac{\Gamma_{12}(\omega)}{\Gamma_{12}^{(0)}(\omega)}\simeq1-\frac{2e^2}{\pi\hbar v_F^{(1)}}\ln\frac{1}{\omega\tau_1}-\frac{2e^2}{\pi\hbar v_F^{(2)}}\ln\frac{1}{\omega\tau_2}+\frac{2e^2}{\pi\hbar(v_F^{(1)}-v_F^{(2)})}\ln\frac{v_F^{(1)}}{v_F^{(2)}}\ln\frac{1}{\omega\tau_{12}}.\tag{41}$$

Actually, our assumption of weak interaction for Coulomb forces holds until $|e^2\ln(\omega\tau_j)|\ll\hbar v_F^{(j)}$, otherwise the interaction acts effectively as the strong one (see below). Equation (41) is an evident generalization of Eq. (14) for the case of long-range interaction. The additional appearance of the frequency dependence in Eq. (41) is totally due to the fact that for Coulomb interaction the plasmon velocities acquire in the long-wavelength limit the logarithmic corrections. As in the case of LL [see Eq. (14)] the interaction always suppresses the intermode electron tunneling. It is evident from Eq. (41) that the effect of suppression is pronounced only at small frequencies. If $\omega\ll\omega_c\sim\tau_j^{-1}\exp(-\pi\hbar v_F^{(j)}/2e^2)$ the influence of the interaction cannot be treated perturbatively and we are in the range of strong interaction.

B. Strong interaction: $e^2/\hbar v_F^{(j)}\gg1$

For simplicity we consider the strong-coupling limit in the model with equal Fermi velocities $v_F^{(1)}=v_F^{(2)}\equiv v_F$. In this case the renormalization of the electron tunneling rate is totally determined by the plasmon velocity dispersion. In the low-frequency limit the expression for $\Gamma_{12}(\omega)$ can be represented as

$$\Gamma_{12}(\omega)=\left|\frac{t_0}{2\pi\hbar a}\right|^2\int_{-\infty}^{\infty}dte^{i\omega t}\times\exp\left[2\int_0^\infty\frac{dk}{k}e^{-ak}(e^{-it\omega_p(k)}-1)\right],\tag{42}$$

where $\omega_p(k)=(3e^2/\pi\hbar)k\ln(1/ak)$ is the energy of the long-wavelength plasmon. The appearance of a logarithmic dependence of energy of the elementary excitations on the momentum is a common feature of the Coulomb effects in 1D. For a nonchiral LL we would get $\omega_p(k)\propto k\sqrt{\ln(1/ak)}$ for the plasmon excitations (see, e.g., Ref. 10). This results in a non-power-law decay of correlation functions and in additional strong suppression of electron tunneling through the impurity (see Refs. 11–15). For a chiral LL the effect of a long-range interaction is not so dramatic. Nevertheless, it strongly modifies the behavior of electron tunneling rates at $\omega\to0$. Indeed, by doing integrals in Eq. (42) in the limit $\hbar\omega\ll\varepsilon_c\equiv e^2/a$ one readily gets the desired result

$$\frac{\Gamma_{12}(\omega)}{\Gamma_{12}^{(0)}(\omega)}\simeq\left[\frac{3}{\pi}\frac{e^2}{\hbar v_F}\ln\left(\frac{\varepsilon_c}{\hbar\omega}\right)\right]^{-2}\ll1,\tag{43}$$

where $\Gamma_{12}^{0}(\omega)$ is the tunneling rate for noninteracting electrons, Eq. (13).

We see that in the case of strong interaction ($e^2/\hbar v_F \geqslant 1$) or in the limit $\omega \to 0$ for a weak interaction ($e^2/\hbar v_F \leqslant 1$) the long-range forces strongly suppress the electron tunneling in a two-channel chiral electron liquid. This result [see Eqs. (41) and (43)] could be regarded as the analog of the Kane-Fisher effect[1] for a forward scattering in a multichannel chiral system of interacting electrons. Notice that in the absence of backscattering the renormalization of the intermode tunneling is much weaker. Namely, the power-law dependence on frequency is replaced by the logarithmic one.

V. TWO-CHANNEL LUTTINGER LIQUID. DIAGONALIZATION

Until now we considered the electron tunneling in a two-channel chiral LL, where only forward-scattering processes are allowed. In this case the interaction could induce renormalization of the bare electron tunneling amplitudes if the velocities of the modes are different and interaction dependent. They are indeed different and therefore the excitation of plasmon modes during the intermode electron tunneling does result in the appearance of an interaction-dependent renormalization factor in the electron tunneling rates [see Eq. (12)]. We showed also that when the interaction is long ranged (e.g., an unscreened Coulomb interaction) the dispersion of plasmon velocities leads to the appearance of an energy-dependent renormalization factor. The novel feature of the charge-density dynamics in a two-channel chiral LL is the appearance of a dimensionless interaction strength which depends on the difference of the Fermi velocities in the channels. This quantity disappears from the electron tunneling rate induced by a local perturbation. But it is this effective interaction that determines the interference effects produced by the extended structures. Now we would like to understand: (i) Is there room for this quantity in a two-channel nonchiral LL? and (ii) How does interaction between right and left movers modify our results for a chiral model?

To be more concrete, in this section we consider how the *intermode* forward and backward scatterings in a two-channel LL are renormalized by the interaction. In Ref. 19 this problem was studied only for the case of *intramode* electron tunneling. It was shown[3,19] that for a repulsively interacting electron system the ''multimodeness'' effectively increases the ''rigidity'' of the electron liquid. It means that the effect of suppression of the intramode electron tunneling could be only enhanced by the presence of extra mode(s). Here we are interested in the influence of interaction on the *intermode* electron transitions.

The Hamiltonian that governs the quantum dynamics of interacting electrons in a two-channel LL takes the form

$$\mathcal{H}=\pi\hbar\int dx\Big\{u_1(\rho_{R1}^2+\rho_{L1}^2)+u_2(\rho_{R2}^2+\rho_{L2}^2)+\frac{V_1^{(e)}}{\pi\hbar}(\rho_{R1}\rho_{R2}+\rho_{L1}\rho_{L2})+\frac{V_2^{(i)}}{\pi\hbar}(\rho_{R1}\rho_{L1}+\rho_{R2}\rho_{L2})+\frac{V_2^{(e)}}{\pi\hbar}(\rho_{R1}\rho_{L2}+\rho_{R2}\rho_{L1})\Big\}. \tag{44}$$

Equation (44) is a straightforward generalization of the chiral model, Eq. (1). Here ρ_{Rj} and ρ_{Lj} are the chiral charge-density operators for the right (R) and left (L) moving particles in the first ($j=1$) and the second ($j=2$) channel. They obey anomalous Kac-Moody commutation relations (see, e.g., Ref. 5). The velocities $u_{1(2)}$ in Eq. (44) are the Fermi velocities of the channels renormalized by the intrachannel interaction ($V_1^{(i)}$), $u_{1,2}=v_F^{(1,2)}+V_1^{(i)}/2\pi\hbar$. In our model, Eq. (44), we assumed the electron-electron interaction to be short ranged (in a LL sense) and for the moment we distinguish the intrachannel interaction between electrons moving in the same ($V_1^{(i)}$) and in the opposite ($V_2^{(i)}$) directions and the interchannel ($V_1^{(e)}$, $V_2^{(e)}$) electron-electron interaction.

The first problem is to diagonalize the Hamiltonian, Eq. (44). It is reasonable to parametrize the canonical transformation we are looking for by the product of trigonometric and hyperbolic functions. Besides, the transformations should respect the left-right symmetry of our model. The desired pseudoorthogonal transformation is

$$\begin{aligned}
\rho_{R1}&=\cosh\vartheta_1\cos\psi\tilde{\rho}_{R1}+\sinh\vartheta_1\cos\psi\tilde{\rho}_{L1}-\cosh\vartheta_2\sin\psi\tilde{\rho}_{R2}-\sinh\vartheta_2\sin\psi\tilde{\rho}_{L2},\\
\rho_{L1}&=\sinh\vartheta_1\cos\psi\tilde{\rho}_{R1}+\cosh\vartheta_1\cos\psi\tilde{\rho}_{L1}-\sinh\vartheta_2\sin\psi\tilde{\rho}_{R2}-\cosh\vartheta_2\sin\psi\tilde{\rho}_{L2},\\
\rho_{R2}&=\cosh\vartheta_3\sin\psi\tilde{\rho}_{R1}+\sinh\vartheta_3\sin\psi\tilde{\rho}_{L1}+\cosh\vartheta_4\cos\psi\tilde{\rho}_{R2}+\sinh\vartheta_4\sin\psi\tilde{\rho}_{L2},\\
\rho_{L2}&=\sinh\vartheta_3\sin\psi\tilde{\rho}_{R1}+\cosh\vartheta_3\sin\psi\tilde{\rho}_{L1}+\sinh\vartheta_4\cos\psi\tilde{\rho}_{R2}+\cosh\vartheta_4\cos\psi\tilde{\rho}_{L2}.
\end{aligned} \tag{45}$$

Notice that in the limit $\vartheta_j=0$ ($j=1-4$) Eqs. (45) are reduced to the unitary transformation Eq. (3). For $\psi=0$ we reproduce the standard Bogoliubov transformation for the two uncoupled channels.

It is easy to check that the pseudoorthogonal transformation Eq. (45) will be canonical if

$$\vartheta_1-\vartheta_3=\vartheta_2-\vartheta_4. \tag{46}$$

So we are left with $16-10=6$ equations for the determination of four independent ''rotation angles.'' For our parametrization, Eq. (45), these equations are reduced to four different transcendental equations which, together with Eq. (46), form the complete set of equations for our diagonalization problem (see the Appendix).

In the two limiting cases the solution of the set of equations is obvious. When we set $V_2^{(i)}=V_2^{(e)}=0$ we reproduce the chiral model $\tan 2\psi=V_1^{(e)}/\pi\hbar(v_F^{(1)}-v_F^{(2)})$, $\vartheta_j=0$, $j=1,\ldots,4$. If $V_1^{(e)}=V_2^{(e)}=0$, then we are dealt with the two uncoupled channels ($\psi=0$) and

$$\tanh 2\vartheta_{1(4)} = -\frac{V_2^{(i)}}{2\pi\hbar v_F^{1,(2)} + V_1^{(i)}}, \quad \vartheta_3 = \vartheta_1, \quad \vartheta_2 = \vartheta_4. \tag{47}$$

For the interacting nonchiral channels the most important case is the limit when all coupling constants are equal $V_1^{(i)} = V_2^{(i)} = V_1^{(e)} = V_2^{(e)} \equiv V_0$. In this case the exact solution of the set of equations presented in the Appendix takes the form

$$\vartheta_1 = \frac{1}{2}\ln g_1, \quad \vartheta_2 = \frac{1}{2}\ln\left(\frac{v_F^{(1)}}{v_F^{(2)}} g_2\right),$$
$$\vartheta_3 = \frac{1}{2}\ln\left(\frac{v_F^{(2)}}{v_F^{(1)}} g_1\right), \quad \vartheta_4 = \frac{1}{2}\ln g_2, \tag{48}$$

$$\tan 2\psi = \frac{2V_0\sqrt{v_F^{(1)} v_F^{(2)}}}{(v_F^{(1)} - v_F^{(2)})[V_0 + \pi\hbar(v_F^{(1)} + v_F^{(2)})]}, \tag{49}$$

where $g_j = v_F^{(j)}/s_j$ $(j=1,2)$ are the correlation parameters of the two-channel LL and the plasmon velocities $s_{1(2)}$ are

$$s_1 = v_F^{(1)}\left\{\cos^2\psi + \left(\frac{v_F^{(2)}}{v_F^{(1)}}\right)^2 \sin^2\psi + \frac{V_0}{\pi\hbar v_F^{(1)}}\left(\cos\psi + \sqrt{\frac{v_F^{(2)}}{v_F^{(1)}}}\sin\psi\right)^2\right\}^{1/2}, \tag{50}$$

$$s_2 = s_1(\psi \leftrightarrow -\psi, v_F^{(1)} \leftrightarrow v_F^{(2)}). \tag{51}$$

Notice that in the limit $v_F^{(1)} = v_F^{(2)} = v_F$ the two-channel LL of the spinless electron is equivalent to a single-channel LL of spin-$\frac{1}{2}$ electrons. In this case $\sin 2\psi = 1$ and we easily rederive the well-known expression (see, e.g., Refs. 5 and 19)

$$\vartheta_1 = \vartheta_3 = \frac{1}{2}\ln g_c, \quad \vartheta_2 = \vartheta_4 = 0, \quad g_c = \left(1 + \frac{2V_0}{\pi\hbar v_F}\right)^{-1/2}. \tag{52}$$

Here g_c is the correlation parameter of spin-$\frac{1}{2}$ LL in the charge sector.

VI. INTERCHANNEL ELECTRON TRANSITIONS

The interchannel electron transitions are described by the tunnel Hamiltonian Eq. (6), where electron operators Ψ_j in the bosonization technique are represented via ''chiral'' boson fields $\varphi_{R(L)}(x,t)$ by the standard formula (see, e.g., Ref. 5)

$$\Psi_j(x,t) = \Psi_{Rj}(x,t) + \Psi_{Lj}(x,t) = \frac{1}{\sqrt{2\pi a}}\{U_{R,j}\exp[ik_F^{(j)}x + i\sqrt{4\pi}\varphi_{Rj}(x,t)] + U_{L,j}\exp[-ik_F^{(j)}x - i\sqrt{4\pi}\varphi_{Lj}(x,t)]\}, \tag{53}$$

where $U_{R(L),j}$ is the unitary rising operator that increases the number of electrons on the $R(L)$ branch by one and does not affect the bosonic excitations. The boson operators $\varphi_{R(L)j}(x,t)$ [notice that $\rho_{R(L)j} = (1/\sqrt{\pi})\partial_x\varphi_{R(L)j}$] satisfy the following commutation relation (see, e.g., Ref. 5):

$$[\varphi_{Rj}(x), \varphi_{Rk}(0)] = -[\varphi_{Lj}(x), \varphi_{Lk}(0)] = \delta_{jk}\frac{i}{4}\mathrm{sgn}(x),$$
$$[\varphi_{Rj}(x), \varphi_{Lk}(0)] = \delta_{jk}\frac{i}{4}. \tag{54}$$

The calculation of the correlation function $\Gamma_{12}(t)$, Eq. (7), for a two-channel LL is much more lengthy than that for a chiral model. It is clear that now Γ_{12} can be represented as a sum of 16 terms which contain only chiral fields. To simplify the calculations we make use of the L-R symmetry of our model. It allows us to reduce the number of different terms in $\Gamma_{12}(t)$. As a direct consequence of L-R symmetry, we have the following identities:

$$\begin{aligned}
&\langle\Psi_{R1}^\dagger(t)\Psi_{R2}(t)\Psi_{R2}^\dagger(0)\Psi_{L1}(0)\rangle = -\langle\Psi_{L1}^\dagger(t)\Psi_{L2}(t)\Psi_{L2}^\dagger(0)\Psi_{R1}(0)\rangle,\\
&\langle\Psi_{R1}^\dagger(t)\Psi_{R2}(t)\Psi_{L2}^\dagger(0)\Psi_{R1}(0)\rangle = -\langle\Psi_{L1}^\dagger(t)\Psi_{L2}(t)\Psi_{R2}^\dagger(0)\Psi_{L1}(0)\rangle,\\
&\langle\Psi_{R1}^\dagger(t)\Psi_{L2}(t)\Psi_{R2}^\dagger(0)\Psi_{R1}(0)\rangle = -\langle\Psi_{L1}^\dagger(t)\Psi_{R2}(t)\Psi_{L2}^\dagger(0)\Psi_{L1}(0)\rangle,\\
&\langle\Psi_{R1}^\dagger(t)\Psi_{L2}(t)\Psi_{L2}^\dagger(0)\Psi_{L1}(0)\rangle = -\langle\Psi_{L1}^\dagger(t)\Psi_{R2}(t)\Psi_{R2}^\dagger(0)\Psi_{R1}(0)\rangle,
\end{aligned} \tag{55}$$

and therefore eight terms in the sum cancel each other. The left eight symmetric terms due to L-R symmetry can be represented by four independent contributions to $\Gamma_{12}(t)$,

$$\Gamma_{12}(t) = 2[\Gamma_f(t) + \Gamma_b(t) + \Gamma_+(t) + \Gamma_-(t)], \tag{56}$$

where

$$\begin{aligned}
\Gamma_f(t) &= \langle\Psi_{R1}^\dagger(t)\Psi_{R2}(t)\Psi_{R2}^\dagger(0)\Psi_{R1}(0)\rangle = \langle\Psi_{L1}^\dagger(t)\Psi_{L2}(t)\Psi_{L2}^\dagger(0)\Psi_{L1}(0)\rangle,\\
\Gamma_b(t) &= \langle\Psi_{R1}^\dagger(t)\Psi_{L2}(t)\Psi_{L2}^\dagger(0)\Psi_{R1}(0)\rangle = \langle\Psi_{L1}^\dagger(t)\Psi_{R2}(t)\Psi_{R2}^\dagger(0)\Psi_{L1}(0)\rangle,\\
\Gamma_+(t) &= \langle\Psi_{R1}^\dagger(t)\Psi_{L2}(t)\Psi_{R2}^\dagger(0)\Psi_{L1}(0)\rangle = \langle\Psi_{L1}^\dagger(t)\Psi_{R2}(t)\Psi_{L2}^\dagger(0)\Psi_{R1}(0)\rangle,\\
\Gamma_-(t) &= \langle\Psi_{R1}^\dagger(t)\Psi_{R2}(t)\Psi_{L2}^\dagger(0)\Psi_{L1}(0)\rangle = \langle\Psi_{L1}^\dagger(t)\Psi_{L2}(t)\Psi_{R2}^\dagger(0)\Psi_{R1}(0)\rangle.
\end{aligned} \tag{57}$$

It is easy to show by direct evaluation of $\Gamma_\pm(t)$ that both of these correlation functions contain the overall factor $R_\pm = \exp\{-2\pi\langle[\varphi_{R1}(0) + \varphi_{L1}(0) \pm \varphi_{R2}(0) + \varphi_{L2}(0)]^2\rangle\}$. By using the canonical transformation, Eq. (45), to represent the inter-

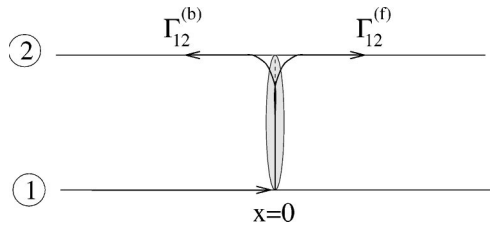

FIG. 3. The forward- (Γ_{12}^{f}) and the backward- (Γ_{12}^{b}) scattering interchannel electron transition rates.

acting fields $\varphi_{L(R)}^{(j)}$ via the ''diagonal'' fields $\tilde{\varphi}_{L(R)}^{(j)}$ and by remembering that for noninteracting boson fields $\langle\tilde{\varphi}_{R1}^2(0)\rangle = \langle\tilde{\varphi}_{L1}^2(0)\rangle = \langle\tilde{\varphi}_{R2}^2(0)\rangle = \langle\tilde{\varphi}_{L2}^2(0)\rangle = (1/4\pi)\ln(l/a)$ (l is the length of the system), it is easy to prove that $\Gamma_{\pm}(t)$ vanishes in the thermodynamic limit ($l\to\infty$) $\Gamma_{\pm}(t)\propto(a/l)^{\nu_{\pm}}\to 0$. Here the exponents $\nu_{\pm}$ are always the positive quantities

$$\nu_{\pm}=e^{2\vartheta_1}\left(\cos\psi\pm\sqrt{\frac{v_F^{(2)}}{v_F^{(1)}}}\sin\psi\right)^2 + e^{2\vartheta_4}\left(\cos\psi\mp\sqrt{\frac{v_F^{(1)}}{v_F^{(2)}}}\sin\psi\right)^2>0. \tag{58}$$

So, we are left with two correlation functions which represent the forward (f) interchannel scattering (Γ_f) and the backward (b) interchannel scattering (see Fig. 3). The corresponding transition rates $\Gamma_{12}^{(\alpha)}(\omega)$ ($\alpha=f,b$) take the form

$$\begin{aligned}\Gamma_{12}^{(\alpha)}(\omega)&=\left|\frac{t_0}{2\pi\hbar}\right|^2 a^{\lambda_{1\alpha}+\lambda_{2\alpha}-2}\int_{-\infty}^{\infty}dt e^{i\omega t}(a+is_1t)^{-\lambda_{1\alpha}}\\ &\times(a+is_2t)^{-\lambda_{2\alpha}}\\ &\simeq\frac{\Gamma_{12}^{(0)}}{\Gamma(\lambda_{1\alpha}+\lambda_{2\alpha})}\left[\frac{v_F^{(1)}}{s_1}\right]^{\lambda_{1\alpha}}\left[\frac{v_F^{(2)}}{s_2}\right]^{\lambda_{2\alpha}}\\ &\times(\omega\tau_1)^{\lambda_{1\alpha}-1}(\omega\tau_2)^{\lambda_{2\alpha}-1}.\end{aligned} \tag{59}$$

Here $\Gamma_{12}^{(0)}(\omega)$ is the tunneling rate for noninteracting electrons, $\Gamma(x)$ is the gamma function, $\tau_{1(2)}=a/v_F^{(1),(2)}$, $s_{1,(2)}$ are the plasmon velocities [see Eqs. (50) and (51)], and

$$\begin{aligned}\lambda_{1f}&=\cosh(\vartheta_1+\vartheta_3)(\cosh\delta-\sin 2\psi)\\ &\quad+\sinh\delta\cos 2\psi\sinh(\vartheta_1+\vartheta_3),\\ \lambda_{1b}&=\cosh(\vartheta_1+\vartheta_3)\cosh\delta+\sinh(\vartheta_1+\vartheta_3)\sin 2\psi\\ &\quad+\sinh\delta\cos 2\psi\sinh(\vartheta_1+\vartheta_3),\\ \lambda_{2f}&=\cosh(\vartheta_2+\vartheta_4)(\cosh\delta+\sin 2\psi)\\ &\quad-\sinh\delta\cos 2\psi\sinh(\vartheta_2+\vartheta_4),\\ \lambda_{2b}&=\cosh(\vartheta_2+\vartheta_4)\cosh\delta-\sinh(\vartheta_2+\vartheta_4)\sin 2\psi\\ &\quad-\sinh\delta\cos 2\psi\sinh(\vartheta_2+\vartheta_4),\end{aligned} \tag{60}$$

where $2\delta=\ln(v_F^{(1)}/v_F^{(2)})$. The total electron tunneling rate is $\Gamma_{12}(\omega)=2\Gamma_{12}^{(f)}(\omega)+2\Gamma_{12}^{(b)}(\omega)$. Equations (59), (60), and (48)–(51) are the solution to our problem in the most general case. The expressions for the exponents $\lambda_{j\alpha}$ are too cumbersome and therefore in what follows we will analyze the rates of electron interchannel transitions in different limits.

At first we consider the limit $\vartheta_j=0$ ($j=1,\ldots,4$) which is realized when $V_2^{(i)}=V_2^{(e)}=0$, i.e., the interaction potential has no components which couple ''left'' to ''right'' densities. Physically this case corresponds to the situation when left- and right-moving electrons are separated in the space. It could be the edge states on the opposite edges of the wide channel in 2D electron gas under the conditions of FQHE. In the studied limit $\Gamma_{12}^{(f)}$ coincides with the tunneling rate for a chiral model (as it should be), Eq. (12), and $\Gamma_{12}^{(b)}(\omega)$ is reduced to a simple expression

$$\Gamma_{12}^{(b)}(\omega)=\Gamma_{12}^{(0)}(\omega)\frac{v_F^{(1)}v_F^{(2)}}{s_1s_2}<\Gamma_{12}^{(0)}(\omega), \tag{61}$$

where $s_{1(2)}$ are determined by Eqs. (50) and (51). So, in the absence of coupling between the ''left'' and ''right'' movers the dependence of the tunneling rates on the frequency ω disappears. The tunneling rates of interacting electrons are suppressed due to the different values of the plasmon velocities ($s_1\neq s_2$) in the channels (for repulsive interaction $s_1>v_F^{(1)}$, $s_2>v_F^{(2)}$).

The opposite limit $\psi=0$ is realized when $V_1^{(e)}=V_2^{(e)}=0$. It describes the uncoupled channels. This situation pertains, for instance, to two 1D wires situated far apart from each other. Now both electron transitions are strongly renormalized by interaction for $\omega\to 0$

$$\Gamma_{12}^{(b)}(\omega)=\Gamma_{12}^{(f)}(\omega)\sim\Gamma_{12}^{(0)}(\omega)(\omega\tau_1)^{\lambda_{KF}^{(1)}}(\omega\tau_2)^{\lambda_{KF}^{(2)}}. \tag{62}$$

Here $\lambda_{KF}^{(1,2)}=\cosh 2\vartheta_{1,4}-1=(1/2)(g_{1,2}+g_{1,2}^{-1}-2)\geqslant 0$. This is nothing but the manifestation of the Kane-Fisher effect.[1] The transitions are strongly suppressed ($\lambda_{KF}^{(j)}\gg 1$) for a strong repulsive interaction [$V_2^{(i)}\simeq V_1^{(i)}\gg\hbar v_F$, see Eq. (47)].

Let us introduce two exponents $\lambda_\alpha=\lambda_{1\alpha}+\lambda_{2\alpha}-2$ ($\alpha=f,b$), which determine the low-frequency behavior of electron tunneling rates. They can be represented in the following form:

$$\begin{aligned}2\lambda_{f,b}&=[g_1+g_1^{-1}+g_2+g_2^{-1}-4]+\sin^2\psi[(r^{-2}-1)(g_1\\ &\quad+g_2^{-1})+(r^2-1)(g_2+g_1^{-1})]\mp\sin 2\psi[r^{-1}(g_1\\ &\quad\mp g_2^{-1})-r(g_2\mp g_1^{-1})],\end{aligned} \tag{63}$$

where the upper sign in Eq. (63) corresponds to λ_f and the lower sign to λ_b. Here $r=\sqrt{v_F^{(1)}/v_F^{(2)}}$ and the correlation parameters g_j are determined by Eqs. (50) and (51).

The last formula, Eq. (63), is the main result of this section. It shows how the electron transition rate $\Gamma_{12}^{(f,b)}(\omega)\sim\omega^{1+\lambda_{f,b}}$ scales with frequency. What is the physical meaning of the terms in Eq. (63)? It is easy to get from the set of equations presented in the Appendix that if the intrachannel

$(V_1^{(i)}=V_2^{(i)}\equiv V_i)$ and the interchannel $(V_1^{(e)}=V_2^{(e)}\equiv V_e)$ interaction strengths are different, $(V_i\neq V_e)$, the ''rotation angle'' ψ takes the form

$$\tan 2\psi=\frac{2V_e\sqrt{v_F^{(1)}v_F^{(2)}}}{(v_F^{(1)}-v_F^{(2)})[V_i+\pi\hbar(v_F^{(1)}+v_F^{(2)})]}. \tag{64}$$

So, in the limit of uncoupled channels $V_e\to 0$ the last two terms in Eq. (63) (they are proportional to $\sin\psi$) vanish and the suppression of electron transitions is totally due to the LL effects in the wires $\gamma_{LL}=(g_1+g_1^{-1}+g_2+g_2^{-1}-4)/2$.

To understand the physical meaning of the terms arising from the interchannel interaction it is useful to consider the limit $v_F^{(2)}\gg v_F^{(1)}$. In this case our problem can be reduced to the problem of the x-ray response of the LL in the model considered in Ref. 16, where the core level was represented by a forward-scattering short-range potential. To get this mapping one has to disregard the effects associated with the band structure of one of the two Luttinger liquids (the first channel if the limit $v_F^{(1)}\to 0$ is considered). Then, the second term in Eq. (63) is reduced to

$$\alpha_{f,b}=\frac{\sin^2\psi}{2}[(r^{-2}-1)g_2^{-1}+(r^2-1)g_2]$$

$$\overset{v_F^{(1)}\to 0}{\Rightarrow}\ \frac{1}{2}\left(\frac{V_e}{\pi\hbar v_F^{(2)}}\right)^2 g^3, \tag{65}$$

where $g^{-1}=\sqrt{1+V_i/\pi\hbar v_F^{(2)}}$ is the correlation parameter of the spinless LL. The exponent $\alpha_{f,b}$ represents the so-called ''orthogonality catastrophe.''[17] The last term in Eq. (63) can be attributed to the ''exitonic'' effect.[17] In its truncated form it looks like

$$\beta_{f,b}=\frac{\sin 2\psi}{2}[(rg_2)^{-1}\pm rg_2]\overset{v_F^{(1)}\to 0}{\Rightarrow}-\frac{V_e}{\pi\hbar v_F^{(2)}}g. \tag{66}$$

So, the relation between the exponents $\alpha_{f,b}$ and $\beta_{f,b}$ is exactly that found in Ref. 16, namely $\alpha_{f,b}=(g/2)\,\beta_{f,b}^2$. Notice that for the noninteracting band $(g=1)$ this formula reproduces the correct expression for the exponents in the case of *spinless* electrons.[18,17] When $V_i\sim V_e$ and $v_F^{(1)}\sim v_F^{(2)}$ the simple classification adopted from the problem of the x-ray response loses its precise meaning since the effects of interchannel and intrachannel interactions have to be treated on the same footing. Anyway, one can see from Eq. (63) that the low-ω behavior of the interchannel electron transition rates strongly differs from the naive expectation that the corresponding exponent in the frequency dependence of $\Gamma_{12}^{(f,b)}(\omega)$ is a mere sum of the Kane-Fisher's exponents $\lambda_{KF}^{(j)}$ [see Eq. (62)] of the channels.

In particular, for a two-channel LL with equal Fermi velocities one easily gets, by using Eq. (52), that the forward-scattering electron transitions are not at all renormalized by interaction $\lambda_f(v_F^{(1)}=v_F^{(2)})=0$. In the terms of spin-$\frac{1}{2}$ LL this statement looks physically evident. Indeed, the spin-flip process itself does not lead to the redistribution of electric charge in the channels and therefore the spin-flip *forward* electron scattering is not affected by the electron-electron interaction $\Gamma_{12}^{(f)}(\omega)=\Gamma_{12}^{(0)}(\omega)$.

As for the backward interchannel transitions, one could expect, according to the ordinary picture of the charge transport in the LL, that $\Gamma_{12}^{(b)}(\omega)$ will be enhanced at low frequencies, at least in the case when the Fermi velocities in the channels are of the same order. From Eq. (63) one easily gets that in the limit $v_F^{(1)}=v_F^{(2)}$ the backward-scattering interchannel electron transitions are strongly enhanced by interaction at $\omega\to 0$: $\lambda_b(v_F^{(1)}=v_F^{(2)})=g_c-1<0$

$$\Gamma_{12}^{(b)}(\omega)=\Gamma_{12}^{(0)}(\omega)\frac{g_c^{g_c}}{\Gamma(1+g_c)}(\omega\tau)^{-(1-g_c)}, \tag{67}$$

where g_c is determined by Eq. (47). Recall that Eq. (67) was derived in a perturbation theory and it is valid until the renormalized backward-scattering electron rate is small. So, this expression indicates that at $\omega\to 0$ the perturbation approach for the evaluation of $\Gamma_{12}^{(b)}(\omega)$ does not work. One can speak only on the tendency of the enhancement of the backscattering processes at low frequencies.

If $v_F^{(1)}\neq v_F^{(2)}$ the exponent λ_f is positive and hence the forward-scattering transitions are always suppressed by interaction. In the weak-coupling limit $V_0/2\pi\hbar v_F^{(1,2)}\ll 1$

$$\lambda_f\simeq\frac{1}{2}\left(1-\frac{v_F^{(1)}}{v_F^{(2)}}\right)^2\left[1+\left(\frac{v_F^{(2)}}{v_F^{(1)}}\right)^2\right]\left[\frac{V_0}{2\pi\hbar(v_F^{(1)}+v_F^{(2)})}\right]^2, \tag{68}$$

$$\lambda_b\simeq-\frac{2V_0}{\pi\hbar(v_F^{(1)}+v_F^{(2)})}.$$

We see that $|\lambda_b|\gg\lambda_f$ and hence for repulsive interaction the backward-scattering interchannel electron transitions will be dominant at $\omega\to 0(\Gamma_{12}^{(b)}\gg\Gamma_{12}^{(f)})$ provided they are allowed for the noninteracting electrons.

In the regime of strong coupling we have

$$\lambda_f\simeq\frac{(\sqrt{v_F^{(1)}}-\sqrt{v_F^{(2)}})^4}{2\sqrt{v_F^{(1)}v_F^{(2)}}(v_F^{(1)}+v_F^{(2)})}+\frac{1}{2}\left(\frac{v_F^{(1)}-v_F^{(2)}}{v_F^{(1)}+v_F^{(2)}}\right)^2\left[\frac{V_0}{\pi\hbar(v_F^{(1)}+v_F^{(2)})}\right]^{-1/2}, \tag{69}$$

$$\lambda_b\simeq\frac{1}{2}\frac{v_F^{(1)}+v_F^{(2)}}{\sqrt{v_F^{(1)}v_F^{(2)}}}-2+\frac{1}{2}\left[\frac{V_0}{\pi\hbar(v_F^{(1)}+v_F^{(2)})}\right]^{-1/2}. \tag{70}$$

Notice that λ_b Eq. (70) changes its sign $(\lambda_b>0)$ if $v_F^{(1)}\gg v_F^{(2)}$ (or vice versa). It means that in the strong-coupling limit and for the channels with strongly different Fermi velocities (numerically $v_F^{(2)}/v_F^{(1)}<0.07$) both interchannel electron transitions (forward and backward) are suppressed by interaction. However, these conditions are unlikely to be satisfied in the real systems.

By making use of Eq. (20) it is easy to evaluate the rotated DOS for a two-channel nonchiral LL model. They are represented by the expression analogous to Eq. (25),

$$\tilde{n}_{\alpha}^{(j)}(\varepsilon)=n_0^{(j)}\frac{v_F^{(j)}}{s_j}\frac{1}{\Gamma(1-\lambda_{j\alpha})}\left|\frac{\varepsilon}{\Lambda_j}\right|^{\lambda_{j\alpha}-1}, \tag{71}$$

where $j=1,2$; $\alpha=f$, b, and $\lambda_{j\alpha}$ are determined by Eq. (60). As was shown in Sec. III, the exponent, which controls the low-energy behavior of the rotated DOS, determines also in the weak-coupling limit $|\vartheta_{1,2}|\ll 1, |\psi|\ll 1$ the energy behavior of interchannel ''vertical'' (see Fig. 2) electron transitions. For a chiral model the only possible transition was the forward-scattering process described by the transmission amplitude t_{12}. Now we could consider both the interchannel transmission (t_{12}) and reflection (r_{12}) amplitudes. According to Eqs. (71), (60), and (48)–(51), they can be represented as follows:

$$\begin{aligned}
t_{12}(\varepsilon\to\varepsilon+\hbar\omega)&=t_{12}^{(0)}\left(-\frac{\varepsilon}{\hbar\omega}\right)^{\lambda_F^{(-)}},\\
r_{12}(\varepsilon\to\varepsilon+\hbar\omega)&=r_{12}^{(0)}\left(-\frac{\varepsilon}{\hbar\omega}\right)^{\lambda_B^{(-)}}, \quad \varepsilon<0\\
t_{12}(-\hbar\omega+\varepsilon\to\varepsilon)&=t_{12}^{(0)}\left(\frac{\varepsilon}{\hbar\omega}\right)^{\lambda_F^{(+)}},\\
r_{12}(-\hbar\omega+\varepsilon\to\varepsilon)&=r_{12}^{(0)}\left(\frac{\varepsilon}{\hbar\omega}\right)^{\lambda_B^{(+)}}, \quad \varepsilon>0.
\end{aligned} \tag{72}$$

Here $t_{12}^{(0)}\ll 1, r_{12}^{(0)}\ll 1$ are the interchannel transmission and reflection amplitudes for noninteracting electrons and the exponents $\lambda_{F,B}^{(\pm)}$ in the weak-coupling limit are

$$\lambda_F^{(-)}\simeq-\frac{V_0}{\pi\hbar(v_F^{(1)}-v_F^{(2)})}+\frac{1}{2}\left(\frac{v_F^{(1)}-v_F^{(2)}}{v_F^{(1)}+v_F^{(2)}}\right)^2\left(\frac{V_0}{2\pi\hbar v_F^{(1)}}\right)^2, \tag{73}$$

$$\begin{aligned}
\lambda_B^{(-)}\simeq&-\frac{V_0}{\pi\hbar(v_F^{(1)}+v_F^{(2)})}\\
&+\frac{1}{2}\left[1+\left(\frac{2v_F^{(1)}}{v_F^{(1)}+v_F^{(2)}}\right)^2-\frac{4v_F^{(1)}}{v_F^{(1)}-v_F^{(2)}}\right]\left(\frac{V_0}{2\pi\hbar v_F^{(1)}}\right)^2,
\end{aligned}$$

$$\lambda_{F,B}^{(+)}=\lambda_{F,B}^{(-)}(v_F^{(1)}\leftrightarrow v_F^{(2)}).$$

So, we showed that at least for a weak electron-electron interaction $V_0/\pi\hbar v_F^{(1,2)}\ll 1$ the coupling between left- and right-moving electrons does not change the prediction of the chiral model (Sec. III) for the transmission amplitudes, Eq. (29). One can see also from Eq. (73) that the exponent λ_B in the energy dependence of the reflection amplitude in the first order on the interaction strength depends on the sum of the velocities (like the exponent in the Kane-Fisher effect[1]) and it satisfies the physically evident property $\lambda_B^{(\mp)}\simeq\lambda_F^{(\mp)}(v_F^{(2)}\to -v_F^{(2)})$.

As was demonstrated in Sec. III, the energy dependence of the rotated DOS determines the frequency dependence of the quantum interference effects in the absorption of photons by a two-channel chiral LL. The evaluation of the power absorption performed in Sec. III can be easily generalized to a nonchiral LL. Since the optical electron transitions represent the forward-scattering events one could expect that the parameters λ_{1f} and λ_{2f} will control the frequency dependence of the oscillating part $\delta\Gamma_{12}^{(d)}(\omega)$ of the interchannel electron transitions induced by the extended structure shown on Fig. 3. The expression for $\delta\Gamma_{12}^{(d)}(\omega)$ is analogous to Eq. (31), where now the exponents, who control the decay of the oscillating terms $\sin(\Phi_j)/x^{\lambda}$, are indeed λ_{jf}, Eq. (60). In the regime of strong coupling $\lambda_{1f}\to 0$ and λ_{2f} takes the finite positive value

$$\lambda_{2f}\simeq\frac{v_F^{(1)}+v_F^{(2)}}{2\sqrt{v_F^{(1)}v_F^{(2)}}}+\frac{2\sqrt{v_F^{(1)}v_F^{(2)}}}{v_F^{(1)}+v_F^{(2)}}\geqslant 2. \tag{74}$$

It means that (i) only one of the two oscillating contributions in Eq. (31) survives in the case of strongly interacting electrons, and (ii) the interaction significantly enhances the interference effects.

VII. LUTTINGER LIQUID WITH OPEN BOUNDARIES

So far, we studied the interchannel electron transition in a LL assuming that the *intrachannel* backscattering of electrons is absent or weak. However, it is well known that backscattering is a relevant perturbation in a LL and weak bare potential is renormalized up. At temperature $T\to 0$ it ''splits'' the LL wire for the charge transport into two semi-infinite systems. Now we consider how strong electron backscattering modifies our results for the interchannel electron transitions.

At first we assume that the ''impurity'' affects the electron dynamics in both the channels. The Luttinger liquids to the left and to the right of the impurity ($x=0$) are weakly connected and the electron transitions between them can be described by the tunnel Hamiltonian analogous to Eq. (6). It is clear that any kind of electron transition *across* the weak link leads to the redistribution of the charge density and hence the forward-scattering electron transitions will be strongly suppressed at $\omega\to 0$. In particular, for equal Fermi velocities one can readily get from the symmetry arguments that $\lambda_{11}^{(f)}=\lambda_{12}^{(f)}$. For the different velocities the two exponents under study are of the same order (they are approximately equal in the strong-coupling limit.) As a consequence, the backscattering into the same channel ($1_{R(L)}\leftrightarrow 1_{L(R)}$, $2_{R(L)}\leftrightarrow 2_{L(R)}$) is strongly enhanced at low frequencies.[1,5] Here we ask the question—is the electron backscattering with the change of flavor (channel index), $1_{R(L)}\leftrightarrow 2_{L(R)}$, enhanced or suppressed?

Since in the absence of perturbation the electrons cannot pass through the impurity, we have to treat LL with open boundary conditions, which can be accounted for by the requirement that the displacement field $\varphi_j(t,x)=\varphi_{jL}+\varphi_{jR}$ is pinned at the origin $x=0$ (see, e.g., Refs. 20–22 and 5) $\varphi_{jL}(t,x=0)+\varphi_{jR}(t,x=0)=0$, $j=1,2$. This requirement is

met if the ''diagonalized'' fields $\tilde{\varphi}_{j,R(L)}$ satisfy the same boundary condition. The restriction imposed modifies the expression for the two-point correlation function $\langle\langle \varphi_{i,R(L)}(t)\, \varphi_{j,R(L)}(0)\rangle\rangle$. The straightforward calculations result in Eq. (59), where now the exponents λ_{jb} take the form

$$\lambda_{1b}=g_1^{-1}\left(\cos\psi-\sqrt{\frac{v_F^{(1)}}{v_F^{(2)}}}\sin\psi\right)^2,$$
$$\lambda_{2b}=g_2^{-1}\left(\cos\psi+\sqrt{\frac{v_F^{(2)}}{v_F^{(1)}}}\sin\psi\right)^2. \tag{75}$$

The low-energy behavior of the backscattering transition rate is governed by the exponent $\lambda_b=\lambda_{1b}+\lambda_{2b}-2$

$$\lambda_b=g_1^{-1}+g_2^{-1}-2+\frac{v_F^{(1)}g_1^{-1}-v_F^{(2)}g_2^{-1}}{\sqrt{v_F^{(1)}v_F^{(2)}}}\times\left[\frac{v_F^{(1)}-v_F^{(2)}}{\sqrt{v_F^{(1)}v_F^{(2)}}}\sin^2\psi-\sin 2\psi\right]. \tag{76}$$

The asymptotics of Eq. (76) in the weak- and strong-coupling regimes are

$$\lambda_b(V_0\ll\hbar v_F^{(1,2)})\simeq\frac{V_0}{2\pi\hbar v_F^{(1)}}\frac{(v_F^{(1)}-v_F^{(2)})^2}{v_F^{(2)}(v_F^{(1)}+v_F^{(2)})},$$
$$\lambda_b(V_0\gg\hbar v_F^{(1,2)})\simeq\left[\left(\frac{v_F^{(1)}}{v_F^{(2)}}\right)^{1/4}-\left(\frac{v_F^{(2)}}{v_F^{(1)}}\right)^{1/4}\right]^2\geqslant 0. \tag{77}$$

So, for the repulsive electron-electron interaction the backscattering with the change of the channel will be suppressed at $\omega\to 0$ if the Fermi velocities of the channels are different. The suppression is pronounced in the strong-coupling regime and for the strong difference in the Fermi velocities $v_F^{(1,2)}\gg v_F^{(2,1)}$. In this case

$$\frac{\Gamma_{12}^{(b)}(\omega)}{\Gamma_{12}^{(0)}(\omega)}\simeq\frac{1}{\Gamma(\sqrt{v_F^{(1,2)}/v_F^{(2,1)}})}\left(\frac{\omega a}{\sqrt{v_F^{(1)}v_F^{(2)}}}\right)^{\sqrt{v_F^{(1,2)}/v_F^{(2,1)}}}\ll 1, \tag{78}$$

where $\Gamma(x)$ is the gamma function.

At equal velocities one gets from Eq. (76) $\lambda_b(v_F^{(1)}=v_F^{(2)})=0$. It means that the spin-$\frac{1}{2}$ electron backscattering with the spin-flip is not affected by the interaction. In the language of the spin-$\frac{1}{2}$ LL this statement looks evident since the spin-flip process does not lead to the redistribution of the electric charge density in the channels.

Now we consider the situation when impurity blocks the charge transport only in one of the channels (let it be for definiteness the first channel). Then, only the φ_1 field is pinned at the position of impurity $x=0$ and the boundary condition for the ''diagonalized'' fields $\tilde{\varphi}_{j,R(L)}$ looks as follows:

$$\cos\psi e^{\vartheta_1}(\tilde{\varphi}_{1R}+\tilde{\varphi}_{1L})+\sin\psi e^{\vartheta_2}(\tilde{\varphi}_{2R}+\tilde{\varphi}_{2L})=0. \tag{79}$$

With the help of Eq. (79) it is straightforward to recalculate the transition rate $\Gamma_{12}^{(b)}$ for the case when electrons are strongly backscattered only in the first channel. The corresponding expressions for the exponent λ_{1b} are determined by Eq. (75) while λ_{2b} takes a rather cumbersome form

$$\begin{aligned}\lambda_{2b}=e^{-2\vartheta_1}\{&\sin^2\psi\cosh 2(\vartheta_1-\vartheta_2)-\sin 2\psi[\cosh(\vartheta_1-\vartheta_2)\\&\times(e^{\vartheta_1}\sinh\vartheta_4+\tan^2\psi e^{\vartheta_2}\cosh\vartheta_3)-\sinh(\vartheta_1-\vartheta_2)\\&\times(e^{\vartheta_1}\cosh\vartheta_4+\tan^2\psi e^{\vartheta_2}\cosh\vartheta_3)]\\&+\cos^2\psi[(e^{\vartheta_1}\sinh\vartheta_4+\tan^2\psi e^{\vartheta_2}\cosh\vartheta_3)^2\\&+(e^{\vartheta_1}\cosh\vartheta_4+\tan^2\psi e^{\vartheta_2}\cosh\vartheta_3)^2]\}.\end{aligned} \tag{80}$$

However, it is easy to verify that $\lambda_b(v_F^{(1)}=v_F^{(2)})=0$ (recall that $\lambda_\alpha=\lambda_{1\alpha}+\lambda_{2\alpha}-2$) irrespective to the interaction strength. For $v_F^{(1)}\neq v_F^{(2)}$ Eq. (80) is simplified in the limit of strong interaction when it is reduced to a simple expression

$$\lambda_b\simeq\frac{1}{2}\sqrt{\frac{v_F^{(1)}}{v_F^{(2)}}}\left(1+\frac{v_F^{(2)}}{v_F^{(1)}}\right)^2-2. \tag{81}$$

Since the different boundary conditions for the electron field in the ''open'' and in the ''blocked'' channels break the flavor symmetry, the result, Eq. (81), is not symmetric under the interchange of the channel indexes $1\leftrightarrow 2$. The suppression of $\Gamma_{12}^{(b)}(\omega)$ will be maximum when the Fermi velocity of the blocked channel is much bigger than that in the open channel. In this case $\lambda_b\simeq(1/2)(v_F^{(2)}/v_F^{(1)})^{3/2}\gg 1$.

VIII. CONCLUSION

In the last decade of increasing interest in the LL effects in low-dimensional systems of strongly correlated electrons small attention had been paid to the problem of interchannel electron transitions in a multimode Luttinger liquid. The main interest of theoreticians was concentrated around the problem of charge transport through the impurity in LL. This is the process that determines the current-voltage characteristics in the long quantum wires and it is known to be strongly affected by interaction. Indeed, impurity induces electron backscattering that results in a strong redistribution of charge density around the scattering region. On the contrary, the forward interchannel electron scattering in adiabatic channels (induced, for instance, by an electromagnetic field) does not lead to the redistribution of charge density along the 1D wires and thus it has no influence on the longitudinal conductivity of the system. We would like to note that the forward interchannel electron transitions, having nothing to do with the resistivity of the LL wire, are important processes when investigating the optical properties of a multimode Luttinger liquid.

We showed that these transitions are controlled by the dimensionless interaction strength which depends on the *difference* of the Fermi velocities in the channels $\kappa=V_0/\pi\hbar(v_F^{(1)}-v_F^{(2)})$. This effective coupling could be either positive or negative, depending on the sgn$[V_0(v_F^{(1)}$

$-v_F^{(2)})]$ and therefore the interchannel electron-electron interaction could either enhance or suppress the effects of the intrachannel interaction. We have studied the interchannel electron transitions in a two-channel chiral LL and demonstrated that although the total electron transition rate $\Gamma_{12}(\omega)$ for a short-range interaction is not renormalized in an energy-dependent way, the coupling κ does control the interference effects in a multichannel wire. From a general point of view the interchannel electron transitions, by creating a dipolelike charge-density perturbation that can freely move along the wire, could be crucial for understanding the optical properties of a multimode LL.

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APPENDIX

The set of equations for the ''rotation angles'' Θ_j $(j=1,\ldots,4)$ and ψ that diagonalize the Hamiltonian for a two-mode Luttinger liquid take the form

$$
\begin{aligned}
&2\pi\hbar(v_F^{(1)}\sinh 2\vartheta_1\cos^2\psi+v_F^{(2)}\sinh 2\vartheta_3\sin^2\psi)\\
&\quad=-\cos^2\psi(V_1^{(i)}\sinh 2\vartheta_1+V_2^{(i)}\cosh 2\vartheta_1)\\
&\qquad-\sin^2\psi(V_1^{(i)}\sinh 2\vartheta_3+V_2^{(i)}\cosh 2\vartheta_3)\\
&\qquad-\sin 2\psi[V_1^{(e)}\sinh(\vartheta_1+\vartheta_3)+V_2^{(e)}\cosh(\vartheta_1+\vartheta_3)],\\
&2\pi\hbar(v_F^{(1)}\sinh 2\vartheta_2\sin^2\psi+v_F^{(2)}\sinh 2\vartheta_4\cos^2\psi)\\
&\quad=-\cos^2\psi(V_1^{(i)}\sinh 2\vartheta_4+V_2^{(i)}\cosh 2\vartheta_4)\\
&\qquad-\sin^2\psi(V_1^{(i)}\sinh 2\vartheta_2+V_2^{(i)}\cosh 2\vartheta_2)\\
&\qquad+\sin 2\psi[V_1^{(e)}\sinh(\vartheta_2+\vartheta_4)+V_2^{(e)}\cosh(\vartheta_2+\vartheta_4)],\\
&\pi\hbar\sin 2\psi[v_F^{(1)}\sinh(\vartheta_1+\vartheta_2)-v_F^{(2)}\sinh(\vartheta_3+\vartheta_4)]\\
&\quad=-\sin 2\psi\sinh(\vartheta_1-\vartheta_3)[V_1^{(i)}\cosh(\vartheta_2+\vartheta_3)\\
&\qquad+V_2^{(i)}\sinh(\vartheta_2+\vartheta_3)]+\cos 2\psi[V_1^{(e)}\sinh(\vartheta_2+\vartheta_3)\\
&\qquad+V_2^{(e)}\cosh(\vartheta_2+\vartheta_3)],\\
&\pi\hbar\sin 2\psi[v_F^{(1)}\cosh(\vartheta_1+\vartheta_2)-v_F^{(2)}\cosh(\vartheta_3+\vartheta_4)]\\
&\quad=-\sin 2\psi\sinh(\vartheta_1-\vartheta_3)[V_1^{(i)}\sinh(\vartheta_2+\vartheta_3)\\
&\qquad+V_2^{(i)}\cosh(\vartheta_2+\vartheta_3)]+\cos 2\psi[V_1^{(e)}\cosh(\vartheta_2+\vartheta_3)\\
&\qquad+V_2^{(e)}\sinh(\vartheta_2+\vartheta_3)],\\
&\qquad\vartheta_1-\vartheta_3=\vartheta_2-\vartheta_4.
\end{aligned}
$$

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