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Magnetoresistance oscillations due to internal Landau band structure of a two-dimensional electron system in a periodic magnetic field

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We report alternative types of magnetoresistance oscillations in high mobility two-dimensional electron systems subjected to large amplitude one-dimensional periodic magnetic modulations, of period 500 nm to 1 μm. We observe Shubnikov–de Haas oscillations that are strongly modified in amplitude and phase, Hall resistance oscillations, and aperiodic magnetoresistance oscillations. These effects are shown to arise from the internal structure of overlapping Landau bands and are well accounted for by perturbation calculations.

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Two-dimensional electron gases (2DEG's) subject to uniform magnetic fields show a vast range of phenomena, including the quantum Hall effects, which arise from the formation of highly degenerate Landau levels. In the presence of periodic magnetic or electrostatic modulations, this degeneracy is removed resulting in Landau bands with an oscillatory bandwidth and a rich internal structure.[1–3] This is manifested in a number of magnetotransport effects, many of which are now well understood from a semiclassical perspective.[4–6] Some evidence has been found for an internal Landau band structure,[7–9] in particular for miniband formation in very short period one-dimensional (1D) and two-dimensional (2D) lateral superlattices.[10,11] However, a number of long standing predictions of complex quantum oscillations for longer period modulations have remained unconfirmed. Here we present results for high mobility 2DEG's subject to large amplitude 1D magnetic modulations, of period 500 nm to 1 μm. We are able to verify a number of predictions, and we observe phenomena arising from the internal structure of the Landau bands.

The allowed electron energy states of a 2DEG in a uniform perpendicular magnetic field B_z^0, are Landau levels separated in energy by $\hbar\omega_c$, where $\omega_c = eB_z^0/m^*$ and m^* is the electron effective mass. The presence of an additional, spatially periodic, magnetic field $\Delta B_z(x)$ results in an energy dispersion $E(k_y)$, where k_y is the electron wave vector in the y-direction [Fig. 1(a)]. The resulting Landau bands have van Hove singularities at the band edges and local *minima* at the band centers[1–3] [Fig. 1(b)]. The width of each Landau band, Γ_B, is an oscillatory function of B_z^0 as is illustrated in Fig. 1(c). These oscillations are due to commensurability between the cyclotron diameter, $2R_c$ and the modulation period, a. For $\Delta B_z \ll B_z^0$, the bandwidth at the Fermi energy, $\Gamma_B(E_F)$, is a minimum whenever $2R_c = (\lambda + 1/4)a$ for $\lambda = 1,2,3,\ldots,$ (the *flat band* condition).

In a uniform magnetic field, conductivity in a 2DEG is due to scattering and is given by the collisional conductivity tensor, σ^c. The dispersion $E(k_y)$, induced by a periodic magnetic modulation in the x direction, leads to an additional band conductivity σ_{yy}^h. For large $\omega_c\tau_s$, where τ_s is the scattering time, the resistance parallel to the modulation direction, R_{xx}, is proportional to σ_{yy}^h. The oscillations in $\Gamma_B(E_F)$ as a function of B_z^0 therefore result in $R_{xx}(B_z^0)$ commensurability oscillations (CO's), with R_{xx} minima at the flat band condition.[12–14] The Γ_B oscillations also cause CO's in the collisional conductivity σ_{xx}^c, leading to oscillations in the resistance perpendicular to the modulation direction, R_{yy}, which are in antiphase with the R_{xx} oscillations.[12] CO's in the Hall resistance, R_{xy}, arising from the modified density of states have also been predicted,[3] but until now have not been observed.

For large amplitude magnetic or electrostatic modulations, $\Gamma_B(E_F)$ may be larger than the Landau level broadening due to impurity scattering, $\Gamma_i(E_F)$. The Landau band structure should then be directly reflected in the Shubnikov-de Haas oscillations, (SdHO's). Figure 1(c) illustrates the particularly interesting situation in which, for large amplitude modulations, Γ_B becomes comparable with $\hbar\omega_c$

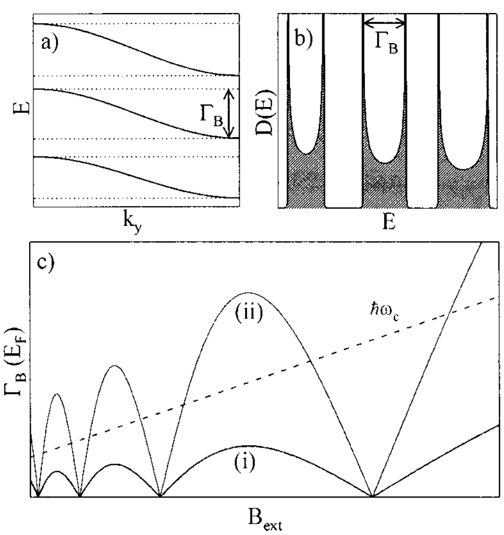

FIG. 1. (a) Energy dispersion curves of a 2DEG with periodic magnetic modulation; (b) density of states; (c) Landau bandwidth as a function of external field, for (i) weak and (ii) strong modulation, with the band separation $\hbar\omega_c$ shown by the dashed line.

for particular values of B_z^0 and the individual Landau bands overlap. In this regime, the overlapping van Hove singularities at the band edges may lead to a switching of SdHO minima from integer to half-integer Landau filling factors.[9,15] This effect has previously only been reported for electrostatic modulations and at high magnetic fields, beyond the range of the CO's.[9] Perturbation calculations are not applicable in this limit and comparison of experiment and theory is therefore problematic.[16,17]

Here we present low temperature magnetotransport results for 2DEG's subjected to large amplitude magnetic modulations ($\approx$25% of B_z^0), for which well-defined CO's and SdHO's coexist over a wide range in R_{xx}. This allows us to relate the observed SdHO switching to the Landau bandwidth. We show that the SdHO switching is also present in R_{yy}, for which there is no band conductivity contribution, and that this leads to aperiodic magnetoresistance oscillation for this geometry. The results are shown to be well described by perturbation calculations, which is surprising given the large modulation amplitude. Oscillations in R_{xy} are also observed that have additional structure to that predicted.

The samples studied are Hall bars fabricated from high mobility GaAs/Al$_x$Ga$_{1-x}$As heterojunctions, with the 2DEG located 35 nm below the surface. The modulations are produced by periodic arrays of Co stripes, fabricated on the surface of the devices by electron beam lithography and sputtering. We present results for two samples. Sample S1 has stripes oriented perpendicular to the current direction, i.e. across the Hall bar, of height 120 nm (in the *z*-direction), width 200 nm (parallel to the current direction), and period 500 nm. Sample S2 has stripes oriented parallel to the current (i.e., along the Hall bar) of height 60 nm, width 400 nm (perpendicular to the current direction), and period 1000 nm. At $T=4.2$ K, sample S1 (S2) has an electron density n_s of 3.9 (3.6)$\times 10^{15}$ m^{-2}, and a mobility μ of 70 (60) m^2 V^{-1} s^{-1}, corresponding to a mean free path of $\sim$7 (6) μm. The arrays cover the entire active area of the Hall bars, which are 50 μm wide with voltage probes separated by 130 μm. The Hall bars are orientated along the nonpiezoelectric GaAs [100] direction to minimize electrostatic modulation due to the different thermal contraction of Co and GaAs.[18] Resistance measurements were performed using an ac lock-in technique with an excitation current of 30 nA. For both samples, R_{xx} (R_{yy}) is defined to be the longitudinal resistance for transport perpendicular (parallel) to the magnetic stripes.

For zero applied field, the magnetization, **M**, of the ferromagnetic stripes will lie along their length due to the strong shape anisotropy and there will be no magnetic modulation at the 2DEG. Application of an external magnetic field B_{ext} perpendicular to the 2DEG plane rotates **M** into the *z*-direction and produces a periodic magnetic modulation of amplitude ΔB_z at the 2DEG. ΔB_z increases approximately linearly with B_{ext} up to a value of $\Delta B_z^{\mathrm{max}}$ at an external field $B_{\mathrm{ext}}^{\mathrm{sat}}$.[19] Assuming that the stripes have the saturation magnetization of bulk Co, we obtain $\Delta B_z^{\mathrm{max}}=170$ and 90 mT for samples S1 and S2, respectively.

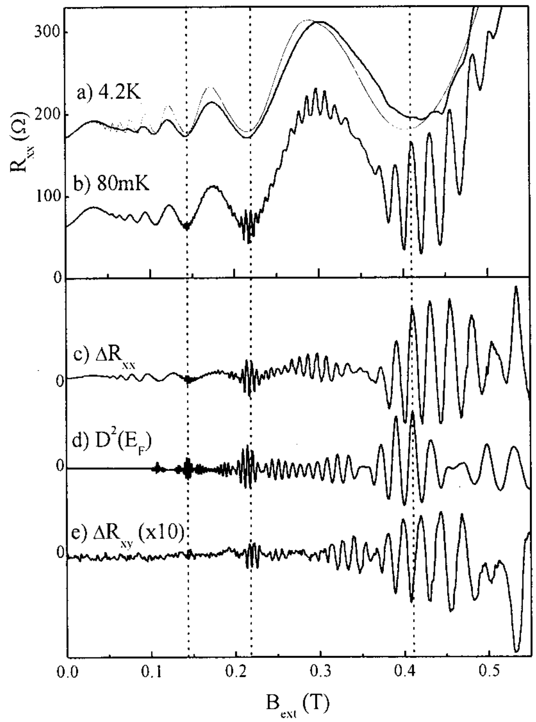

FIG. 2. (a) and (b) magnetoresistance perpendicular to the stripes, R_{xx}, at 4.2 K and 80 mK respectively. The former is offset by 100Ω. The semiclassical calculation of R_{xx} is shown by the thin line in (a). (c) $\Delta R_{xx}=R_{xx}(80\,\mathrm{mK})-R_{xx}(4.2\,\mathrm{K})$. (d) $D_T^2(E_F)$ calculated for sample S1 at 80 mK. (e) R_{xy} after subtraction of a linear background. Flat band conditions are marked by vertical dashed lines.

The measured $R_{xx}(B_{\mathrm{ext}})$ of sample S1 at 4.2 K and at 80 mK are shown in Fig. 2(a) and (b). Large amplitude CO's are observed with the last CO minimum at $B_{\mathrm{ext}}=0.4$ T. The positive magnetoresistance observed below $\sim$25 mT is due to channeling of electrons along open ''snake orbits'' that are perpendicular to the current direction.[4] The experimental results are compared to the results of a semiclassical numerical calculation of $R_{xx}(B_{\mathrm{ext}})$,[20] which assumes that ΔB_z increases linearly with B_{ext} up to $B_{\mathrm{ext}}^{\mathrm{sat}}$ (consistent with the Stoner-Wohlfarth model). The calculation then has two adjustable parameters, $B_{\mathrm{ext}}^{\mathrm{sat}}$ and V_0, the amplitude of the residual electrostatic modulation. For $V_0=0.8$ meV and $B_{\mathrm{ext}}^{\mathrm{sat}}=1.4$ T (a reasonable value for this geometry),[19] good agreement between the experimental and calculated results is obtained for the low field positive magnetoresistance, the CO positions, and the amplitude of the last CO. At lower fields, the measured CO amplitudes are smaller than calculated due to the influence of small angle scattering.[21]

At 80 mK CO's and SdHO's coexist over a wide range of B_{ext} and the envelope of the SdHO's is strongly modulated. The modifications to the SdH oscillations are particularly clear in $\Delta R_{xx}=R_{xx}(80\,\mathrm{mK})-R_{xx}(4.2\,\mathrm{K})$, shown in Fig. 2(c) since the CO's are only weakly temperature dependent over this range,[22] while the SdHO's are almost absent at 4.2 K below 0.5 T. The observed SdHO amplitudes are largest at the CO minima, shown by the dashed lines as found in Refs.

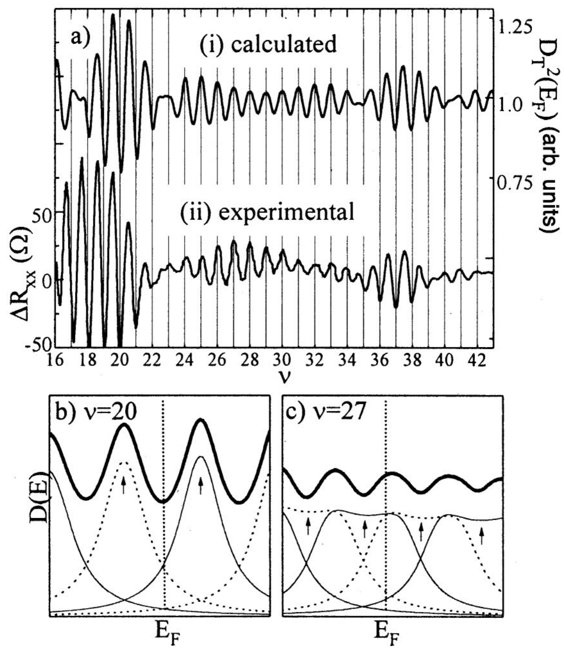

FIG. 3. (a) Calculated $D_T^2(E_F)$ (i), and experimental SdHOs (ii), versus Landau filling factor ν. (b) and (c) calculated $D(E)$ around E_F, at a CO minimum ($\nu=20$) and at a CO peak ($\nu=27$). The contributions from the individual Landau bands are shown alternately with solid and dashed lines, and the band centers are marked by vertical arrows.

23 and 24. Moving away from the minima, the SdHO's are suppressed before re-emerging at the CO peaks.

ΔR_{xx} is plotted against the Landau level filling factor $\nu = hn_s/2eB_{\text{ext}}$ in Fig. 3(a). Near the CO minima, SdHO minima occur at integer values of ν, i.e. the resistance is a minimum when the Fermi level lies between Landau levels, as is always observed in uniform magnetic fields. However, the SdHO minima around the CO peaks occur at half integer values of ν, i.e. the resistance is a *minimum* when E_F is at the *center* of a Landau band. The suppression of the SdHO amplitudes marks the transition between the two modes of behavior. This effect is not related to spin splitting, which is too small to be resolved at these low magnetic fields. At higher fields, beyond the last CO minimum, the SdHO minima switch to half integer ν at $B_{\text{ext}}\approx 0.5$ T before returning to integer ν just before the onset of the quantum Hall regime at $B_{\text{ext}}\approx 2$ T, similar to the behavior reported in Ref. 9.

The SdHOs can be described by the quantity

$$D_T^2(E_f) = \int \left(\frac{df(E)}{dE}\right)[D(E)]^2 dE,$$

where $D(E)$ is the density of states and $f(E)$ is the Fermi-Dirac distribution function. We have calculated $D_T^2(E_F)$ for sample S1 at 80 mK, within first order perturbation theory following the procedure outlined in Ref. 3. We again use $B_{\text{ext}}^{\text{sat}} = 1.4$ T and $V_0 = 0.8$ meV to describe the modulation amplitude. The additional parameter required is the electron scattering time. The value of 1.9×10^{-12} s used was obtained from an analysis of the SdHO amplitudes[25] for a Hall bar from the same wafer but without a periodic modulation. The calculated $D_T^2(E_F)$ is compared to the experimentally observed ΔR_{xx} in Figs. 2 and 3. The B_{ext} values at which the switching occurs are in good agreement. In addition, the reduction of the oscillation amplitudes near the CO peaks (e.g. around $\nu=30$) is reproduced. This shows that the collisional contribution to the SdHO's dominates over the band contribution, which is predicted to give SdHO enhancement at the CO peaks.[1–3]

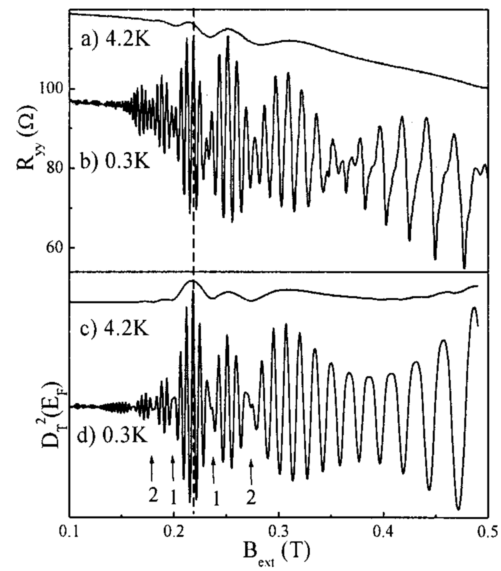

FIG. 4. (a) and (b) magnetoresistance parallel to stripes, R_{yy}, for sample S2 at 4.2 K and 300 mK respectively; (c) and (d) calculated $D_T^2(E_F)$ at 4.2 K and 300 mK for sample S2. The vertical line marks the last flat band condition. The arrows labeled 1 and 2 mark the points where $\Gamma_B(E_F) = \hbar\omega_c$ and $2\hbar\omega_c$, respectively.

The physical origin of the SdHO switching to half integer ν is illustrated in Figs. 3(b) and (c). Figure 3(b) shows the calculated $D(E)$ close to E_F at a CO minimum. Here, Γ_B is small and the total $D(E)$ has large amplitude peaks at the Landau band centers. Therefore, R_{xx} is a minimum for integer values of ν at the CO minima. Figure 3(c) shows $D(E)$ close to E_F at a CO peak. Here, Γ_B is larger than the band separation $\hbar\omega_c$, and the total $D(E)$ has minima at the band centers, due to a superposition of adjacent band edges. Therefore, at the CO peaks, R_{xx} is a minimum when ν is a half integer, as observed experimentally. The positions of the minima in the total $D(E)$ with respect to the band centers and the resultant SdHO switching, are a direct consequence of the van Hove singularities, which lead to a greatly increasing $D(E)$ away from the band centers.

Figure 4 shows $R_{yy}(B_{\text{ext}})$ at 4.2 K and at 80 mK for sample S2. At 4.2 K, a series of oscillations is present, which are not periodic in $1/B_{\text{ext}}$ and which have no simple relationship to the predicted positions of minimum bandwidth (the last minimum is marked by a vertical dashed line in Fig. 4). At 300 mK, the magnetoresistance is dominated by SdHO's, which are again strongly modified. Between $B_{\text{ext}}=0.1$ and 0.4 T, the SdHO envelope has several nodes at which the SdHO minima switch between integer and half integer ν.

Above 0.4 T, more complicated multipeak structures are observed. There is a clear correspondence between the SdHO envelope at 300 mK and the aperiodic oscillations at 4.2 K.

For large $\omega_c \tau_s$, R_{yy} is proportional to σ_{xx}, and the additional band conductivity contribution σ_{yy}^b due to the $E(k_y)$ dispersion is negligible. The observed structure in R_{yy} is from the collisional conductivity and so should reflect the structure in $D_T^2(E_F)$.[1–3] For weak modulations this gives rise to CO's in R_{yy} that are in antiphase with those in R_{xx}.[12] The calculated $D_T^2(E_F)$ for S2 is shown versus $B_{\rm ext}$ in Fig. 4. The values of $B_{\rm ext}^{\rm sat}$ and V_0 are the same as used for S1. The 80 mK result is in good agreement with the experimental SdHO's. The maximum in the SdHO amplitude at 0.22 T corresponds to a minimum in Γ_B, but the other features all arise from overlapping Landau bands. The nodes in the SdHO envelope and the associated phase switches that occur on either side of 0.22 T correspond to $\Gamma_B = \hbar\omega_c$ [1 in Fig. 4(d)]. The next set of nodes then corresponds to $\Gamma_B = 2\hbar\omega_c$ [2 in Fig. 4(d)]. At this point, three adjacent bands overlap and the SdHO minima switch back to integer ν, as has been observed in R_{xx} for large $B_{\rm ext}$ in Ref. 9. The new oscillations observed at 4.2 K are also reproduced in the calculated $D_T^2(E_F)$ at this temperature. The oscillations arise due to thermal averaging of the modulated density of states, with magnetoresistance minima occurring whenever the Landau bandwidth is an integer multiple of $\hbar\omega_c$.

Figure 2(e) shows the measured Hall resistance, R_{xy}, of sample S1 after subtraction of the linear term. The envelope of the R_{xy} oscillations is similar to that of the SdHO's in R_{xx} close to the CO minima, but is significantly different close to the CO maxima. Similar modulated R_{xy} oscillations are also observed for sample S2. In Refs. 1–3, CO's are predicted in R_{xy}, which are in phase with the CO's in R_{xx}, but reduced in amplitude by an order of magnitude. We find no evidence of these oscillations in our data. Instead, the observed R_{xy} oscillations appear to be related to the predicted oscillations in the collisional conductivity, with large amplitude modulations at the flat band condition. More theoretical work is required to understand this effect.

By careful sample design, we have been able to access experimentally the regime in which CO's and SdHO's coexist, and the induced Landau bandwidths are so large that adjacent Landau bands can overlap. The resultant magnetoresistances have been shown to have a rich oscillatory structure in this regime. We find good agreement between the measured magnetoresistance and the results of first order perturbation calculations of the density of states. This is perhaps surprising given the strength of the magnetic modulation. There is clearly a need for transport calculations that go beyond perturbation theory, to appreciate the full implications of these results.

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