Adiabatic steering and determination of dephasing rates in double-dot qubits

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We propose a scheme to prepare arbitrary superpositions of quantum states in double quantum dots irradiated by coherent microwave pulses. Solving the equations of motion for the dot-density matrix, we find that dephasing rates for such superpositions can be quantitatively inferred from additional electron-current pulses that appear due to a controllable breakdown of coherent population trapping in the dots.

I. INTRODUCTION

One key point for the implementation of quantum logic gates in quantum dots is the preparation of arbitrary superpositions $|\Psi_f\rangle$ of two (or more) electron eigenstates. Such superpositions would constitute a qubit basis in an artificial semiconductor structure that can be coupled to the external world by leads and therefore be accessed by transport spectroscopy. Transport experiments have already successfully revealed a number of quantum coherent effects due to static potential coupling[1–3] or coupling to microwave radiation[4–6] in double quantum dots, and their use as qubits that are controlled by gate voltages or magnetic fields has been suggested[7] recently. Unfortunately, at present there are no data at all for dephasing rates γ of superpositions $|\Psi_f\rangle$ in such systems. The knowledge of γ, in particular for lowest eigenstate superpositions that are expected to be most stable, is crucial to determine the feasibility of quantum-dot-based qubits and to choose the time scale for logic operations needed for, e.g., quantum computation.[8]

In this article, we suggest a scheme to *determine* γ from time-dependent transport measurements through coupled quantum dots. We show that it is indeed possible to prepare an arbitrary coherent superposition of ground states using stimulated Raman adiabatic passage[9] with two microwave pulses irradiated on a double dot in the Coulomb blockade regime. Solving the equations of motion for the dot-density matrix, it turns out that two subsequent pairs of microwave pulses filter out the degree of decoherence in the form of a detectable electron-current peak, the strength of which directly depends on γ.

Apart from conventional photon-assisted tunneling experiments with single microwave sources, recent *two-source* microwave techniques have turned out to be an extremely versatile tool to investigate both ground and excited states in single dots.[10] Furthermore, different groups[11–13] have already identified *three-level systems* as potential candidates to establish and control low-frequency coherence in solid-state structures.

II. ADIABATIC TRANSFER SCHEME

Our scheme to prepare and analyze superpositions is based on previously discussed dark resonance states in lateral

double dots[14] that can be probed by linear and nonlinear transport spectroscopy[2,3,5]: A left and a right dot are coupled to external leads with chemical potentials such that electrons can tunnel out to the right only via an excited state $|0\rangle$ in the right dot and tunnel in from the left and the right only via the two double-dot ground states $|1\rangle$ and $|2\rangle$, see Fig. 1. The latter are the symmetric and antisymmetric superpositions of the left and the right tunnel-coupled dot ground states for one additional electron. In the regime of weak coupling, this corresponds to ''ionic'' binding, while strong coupling can be termed ''covalent binding.'' Microwave-induced ''dark'' stable superpositions $|\psi_f\rangle$ of $|1\rangle$ and $|2\rangle$ then constitute the qubit and can be formed by adiabatic transfer as discussed in the following. In this work, we concentrate on Λ-type couplings in coupled N-dot devices for $N=2$ (double dots, Fig. 1 left and center). For multiple-dot devices with $N>2$ (Fig. 1 right), though more difficult to control, we expect even more possibilities to manipulate energy levels and the shape of wave functions.

We assume the dot initially prepared in $|1\rangle$, which can be easily achieved emptying level $|2\rangle$ by driving the $|2\rangle \leftrightarrow |0\rangle$ transition with a microwave field. Once preparation of the dot in $|1\rangle$ is achieved, the dot is set to a ground-state superposition of $|1\rangle$ and $|2\rangle$ by irradiation with two electric fields of the form

$$\mathbf{E}_i(t) = \vec{\mathcal{E}}_i(t)\cos(\omega_i t + \varphi_i), \quad i = 1,2, \tag{1}$$

with microwave frequencies ω_i of the order of the transition frequencies $\delta\varepsilon_i/\hbar$ and slowly varying pulse-shaped ampli-
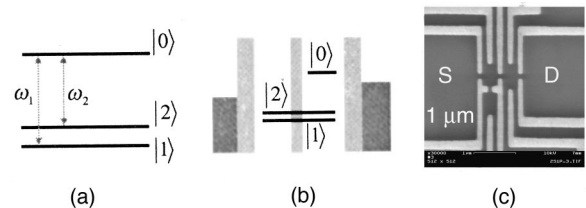

FIG. 1. Λ configuration (left), three-level scheme in coupled two-dot system (center), and lateral three-dot device in a GaAs/AlGaAs heterostructure. The 1-μm mark gives a size scale for the structure.

tudes $\vec{\mathcal{E}}_i(t)$. The latter give rise to time-dependent matrix elements $\Omega_P(t) \propto |\vec{\mathcal{E}}_1(t)|$ and $\Omega_S(t) \propto |\vec{\mathcal{E}}_2(t)|$ for the transitions $0 \leftrightarrow 1$ (P) and $0 \leftrightarrow 2$ (S) that induce the adiabatic transfer of the qubit from the initial state to the desired superposition. Without loss of generality, we assume real Rabi frequencies[15]

$$\Omega_P(t) = \Omega^0 \sin \theta \, e^{-(t-\tau)^2/T^2}, \tag{2a}$$

$$\Omega_S(t) = \Omega^0 (e^{-t^2/T^2} + \cos \theta \, e^{-(t-\tau)^2/T^2}). \tag{2b}$$

Here, τ and T are the pulse delay and pulse duration, respectively. The precise Gaussian form of Eq. (2b) is not a strict requirement for the process of adiabatic transfer and has been chosen only for convenience.

The Stokes microwave pulse S couples $|2\rangle$ to $|0\rangle$, before a second pulse (the pump pulse P), partially overlapping with S, couples $|1\rangle$ to $|0\rangle$.[16,15] If the pulses terminate simultaneously with a constant ratio of their amplitudes, the dot is left in a superposition

$$|\psi_f\rangle = \cos \theta |1\rangle - \sin \theta |2\rangle, \tag{3}$$

where the mixing angle θ is determined by the ratio with which the pump and Stokes pulses terminate. The process is robust against experimental details such as the delay between pulses or pulse areas. The only strict requirement is two-photon Raman resonance $\delta_R \equiv \delta_2 - \delta_1$ where $\delta_j = \omega_j - \delta\epsilon_j/\hbar$ is the one-photon detuning. The phase difference $\varphi_1 - \varphi_2$ of the two electric fields is either 0 or π so that by changing $|\vec{\mathcal{E}}_1|$ and $|\vec{\mathcal{E}}_2|$ one covers the whole range of θ values.

III. MODEL

A. Interaction Hamiltonian and density-matrix equations

We now investigate the preparation of superpositions $|\psi_f\rangle$ and their stability with respect to dephasing numerically. In the dipole and rotating-wave approximation, as appropriate for near-resonant excitation, the time-dependent interaction Hamiltonian is

$$V_{\text{AL}}(t) = -\frac{\hbar}{2} [\Omega_P(t) e^{-i\omega_P t} |0\rangle\langle 1| + \Omega_S(t) e^{-i\omega_S t} |0\rangle\langle 2|]$$

$$+ \text{H.c.} \tag{4}$$

For simplicity, we assume identical tunneling rates Γ for tunneling of electrons into $|1\rangle$ and $|2\rangle$ and out of $|0\rangle$. The double dot is assumed to be in the strong Coulomb blockade regime where the charging energy is larger than the single-particle excitation energy. Electron tunneling is one by one, and the system can effectively be described by its three states 0, 1, 2, and the empty state e before or after one additional electron has tunneled. The resulting density-matrix equations are

$$\dot{\rho}_{1,1} = \alpha_1 \Gamma^0 \rho_{0,0} + \Gamma \rho_{e,e} + 2\gamma \rho_{2,2} + \text{Im}[\Omega_P(t) \tilde{\rho}_{1,0}], \tag{5a}$$

$$\dot{\rho}_{2,2} = \alpha_2 \Gamma^0 \rho_{0,0} + \Gamma \rho_{e,e} - 2\gamma \rho_{2,2} + \text{Im}[\Omega_S(t) \tilde{\rho}_{2,0}], \tag{5b}$$

$$\dot{\rho}_{0,0} = -(\Gamma + \Gamma^0) \rho_{0,0} - \text{Im}[\Omega_P(t) \tilde{\rho}_{1,0}] - \text{Im}[\Omega_S(t) \tilde{\rho}_{2,0}], \tag{5c}$$

$$\dot{\rho}_{e,e} = -2\Gamma \rho_{e,e} + \Gamma \rho_{0,0} \tag{5d}$$

$$\dot{\tilde{\rho}}_{1,0} = -\left[\frac{1}{2}(\Gamma + \Gamma^0) + i\delta_P\right]\tilde{\rho}_{1,0} + \frac{i}{2}\Omega_P(t)(\rho_{0,0} - \rho_{1,1})$$

$$-\frac{i}{2}\Omega_S(t)\tilde{\rho}_{1,2}, \tag{5e}$$

$$\dot{\tilde{\rho}}_{2,0} = -\left[\frac{1}{2}(\Gamma + \Gamma^0) + i\delta_S\right]\tilde{\rho}_{2,0} + \frac{i}{2}\Omega_S(t)(\rho_{0,0} - \rho_{2,2})$$

$$-\frac{i}{2}\Omega_P(t)\tilde{\rho}_{1,2}^*, \tag{5f}$$

$$\dot{\tilde{\rho}}_{1,2} = -(\gamma + i\delta_R)\tilde{\rho}_{1,2} + \frac{i}{2}\Omega_P(t)\tilde{\rho}_{0,2} - \frac{i}{2}\Omega_S(t)\tilde{\rho}_{1,0}, \tag{5g}$$

where α_1 and $\alpha_2 = 1 - \alpha_1$ determine the branching ratio for the decay of the excited state with rate Γ^0.

B. Parameters

Before discussing the numerical results, we address typical values of the parameters for lateral quantum dots in GaAs/Al$_x$Ga$_{1-x}$As. Coulomb charging energies are of a few meV and set the largest energy scale so that states with more than one additional transport electron can be neglected. Typical excitation energies $\delta\varepsilon_i$ from the ground states $|i\rangle$ ($i = 1, 2$) are of the order of a few hundred μeV, which corresponds to microwave frequencies up to the 100 GHz range. The decay of the excited state $|0\rangle$ then is due to acoustic phonon emission at typical rates of $\Gamma^0 \approx 10^9$ s^{-1}.

Microwave experiments so far have been performed in cw mode only, and we expect the pulse strengths to be difficult to control. The latter determine, together with the two dipole-matrix elements for the transitions $|i\rangle \rightarrow |0\rangle$, the parameters Ω_0 and θ, Eq. (2). Fortunately, control of the strength ratio Ω_P/Ω_S is sufficient for the scheme to work, apart from the requirement to generate pulsed electric fields $\mathbf{E}_i(t)$ in smooth forms.

Recently, it has been suggested that the dephasing of the optical coherence for transitions in (electron and hole) quantum dots is produced by elastic LO-phonon carrier collisions.[17] In our case, such processes result in a much smaller dephasing for ground-state electron superpositions because the quasidegenerate states forming the superpositions experience a similar phase shift while scattering.

Instead, there is strong experimental[3] and theoretical[18] evidence that the spontaneous emission of acoustical phonons is the most dominant process affecting ground-state superpositions $|\psi_f\rangle$ based on the electron charge for sub-Kelvin-regime temperatures. Microscopic coupling constants that determine the dephasing rate γ of the coherence $\tilde{\rho}_{1,2}$,

Eq. (5g), for the different dephasing channels (bulk and surface acoustic phonons[19]) are difficult to calculate, in particular, as there might be additional contributions from nonequilibrium phonons induced by the microwave radiation itself.[20] A decisive advantage of coupled dots, however, is the fact that $\gamma \propto T_c^2$ can be tuned to values that are orders of magnitude smaller than Γ^0 by varying the tunnel coupling T_c between the dots.

Here, we do not pursue a microscopic approach to the dephasing rate γ, but rather solve the dynamics of the dot for given γ numerically. As we show now, this allows us to infer the value of γ in a real experiment from another observable quantity, i.e., the electric current, much similar to a recent time-resolved experiment in a superconducting system.[21] Note that due to the Pauli blocking of the leads and the Coulomb blockade, electrons trapped in the coherent ground-state superposition cannot tunnel out of the dot and no second electron can tunnel in. This is why a coupling to external leads of the qubit does not introduce additional dephasing channels.[14]

IV. NUMERICAL RESULTS

We now discuss our results from the numerical solution of the equations of motion (5). For ideal adiabatic evolution and long-living ground-state superpositions ($\gamma = 0$), the pulses (2) prepare the dot in the superposition $|\psi_f\rangle = \cos\theta|1\rangle - \sin\theta|2\rangle$, Eq. (3). For finite γ, the adiabatic steering is disturbed and the superposition decays into a mixture.

A. Single pair of pulses

We solved the density-matrix equations (5) and calculated the fidelity of the preparation in $|\psi_f\rangle$,

$$F = \langle \psi_f | \rho | \psi_f \rangle, \qquad (6)$$

as a function of the interaction time. Numerical results for F for different values of the ground-state relaxation rate γ are reported in Fig. 2 together with a plot of the Rabi frequencies of the pulses, as from Eq. (2b). In all the numerical calculations $T = \tau = 100/\Gamma^0$. This corresponds to a pulse of the order of $1/10 \ \mu$s resulting in a frequency dispersion of the order of 10 MHz. Ground-state splittings larger than that are easily achieved in quantum dots by an appropriate tuning via gate voltages, so that the assumption that each microwave field couples only one transition is fully justified. Furthermore, the ground-state splitting has to be larger than the excited-state width to avoid unwanted couplings.

In the absence of ground-state relaxation processes, the evolution is analogous to what is known from atomic physics: the interaction with the light leads to the preparation of the dot in the desired superposition of states. After the pulse sequence, the dot stays in this superposition. For small but nonzero relaxation rate ($\gamma = 0.001\Gamma^0$), one recognizes that the microwave driving still results in the preparation of the dot in $|\psi_f\rangle$, but after the pulse sequence the fidelity degrades rapidly. For a larger relaxation rate ($\gamma \simeq 0.01\Gamma^0$), also the driving into the superposition is disturbed and the prepara-
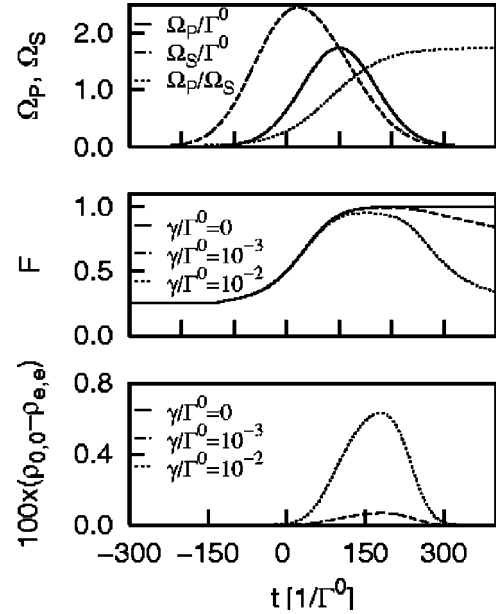

FIG. 2. Rabi frequencies, fidelity, and electric current as a function of the interaction time. Parameters of the calculations are $\Omega^0 = 2\Gamma^0$, $\Gamma = \Gamma^0/3$, $\theta = \pi/3$, $\alpha_1 = \alpha_2 = 1/2$.

tion in $|\psi_f\rangle$ is never complete. Furthermore, there is a fast degradation of the fidelity after the pulse sequence.

We now show how to get access to the ground-state evolution and to γ by a current measurement. As shown in Fig. 2, it is, in principle, possible to get an insight on the ground state dynamics by monitoring the time-dependent electric current through the dot,

$$I(t) = -e\Gamma[\rho_{0,0}(t) - \rho_{e,e}(t)], \qquad (7)$$

due to the flow of electrons with charge $-e < 0$ through the tunnel barrier connecting the dot into the right reservoir. However, for small relaxation rates, the current through the dot is weak; note that the scale in Fig. 2 is blown up by a factor of 100 and that a current $-e\Gamma/100$ with $\Gamma = 10^{-9} \ \text{s}^{-1}$ corresponds to 1.6 pA.

B. Double-pulse sequence

A more sensitive detection is obtained by letting the system evolve freely and then applying the microwave radiation once again. The two pulses that have been used for the preparation of the state $|\psi_f\rangle$ are then applied simultaneously at a second time $\Delta t > 0$ (i.e., after the first pair of pulses) with

$$\Omega_P(t) = \Omega_p \sin\theta \exp[-(t-\Delta t)^2/T_p^2], \qquad (8a)$$

$$\Omega_S(t) = \Omega_p \cos(\theta + \phi) \exp[-(t-\Delta t)^2/T_p^2] \qquad (8b)$$

and the ratio of their amplitudes corresponding to $|\psi_f\rangle$ ($\phi = 0$) or to its orthogonal state ($\phi = \pi$).

As shown in Fig. 3, if $\gamma = 0$ and $\phi = 0$, nothing happens: the dot stays in the state $|\psi_f\rangle$ and the subsequent application of the probe pulses does not produce any current through the dot. On the contrary, for $\phi = \pi$ the probe pulses are in an-
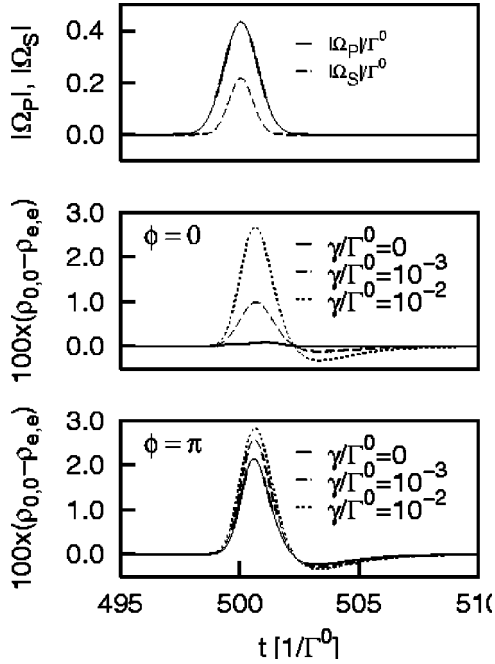
FIG. 3. Rabi frequencies and electric current as a function of the interaction time. Parameters of the calculations are $\Omega_p = 0.5\Gamma^0$, $T_p = 1/\Gamma^0$, and $\Delta t = 500/\Gamma^0$.

FIG. 4. Contrast C as a function of the ground-state relaxation rate (left) and time delay Δt (right) of the probe pulses. In the left plot squares correspond to $\Delta t = 500/\Gamma^0$. In the right plot squares correspond to $\gamma/\Gamma^0 = 10^{-3}$, circles to 5×10^{-3}, and triangles to 10^{-2}. Parameters for the probe pulses are the same as for Fig. 3.

tiphase with the ground-state superposition and a large current follows. For a nonzero relaxation rate γ, the superposition decays into a mixture on a time scale $1/\gamma$ and therefore the application of the probe pulses results in a current through the dot both for $\phi = 0$ and $\phi = \pi$. The larger the relaxation rate γ, the less sensitive is the current on the relative phase ϕ of the probe pulses. Therefore, the contrast

$$C = \frac{I_{\max}(\phi = \pi) - I_{\max}(\phi = 0)}{I_{\max}(\phi = \pi) + I_{\max}(\phi = 0)} \qquad (9)$$

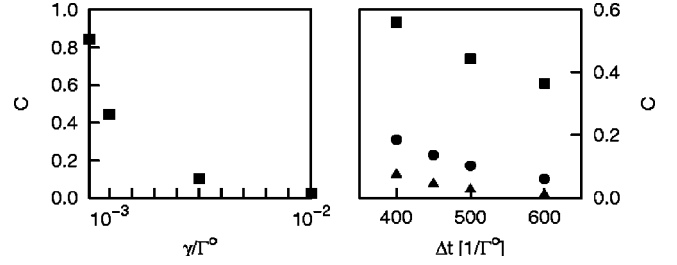is a good measure of the ground-state relaxation rate γ, as shown in Fig. 4.

The relaxation rate γ thus can be extracted experimentally in the following way: the dot is prepared in $|\psi_f\rangle$ as described and then probed at the instant Δt by pulses with a relative phase $\phi = 0$. The cycle preparation/probing is then repeated for $\phi = \pi$. Different measurements of C in this way are made for various choices of Δt. A plot of the contrast C as a function of Δt then allows the determination of γ.

V. CONCLUSIONS

In conclusion, we have shown that the time evolution of double dots in the regime of strong Coulomb blockade under coherent microwave pulses provides a scheme that allows both preparation and analysis of eigenstate superpositions $|\psi_f\rangle$. A ''read-out'' current pulse provides the information about the superposition dephasing rate γ. In double dots it is possible to fine tune γ by varying the interdot coupling by gate voltages, and to prepare $|\psi_f\rangle$ as a dark state that is protected deeply below the Fermi seas of the contact reservoirs by the Pauli principle and the Coulomb-blockade effect. This suggests that it might be worthwhile to further investigate (among other proposals based on, e.g., the electron spin[22,23]) such charge-based qubits that have been proven to be accessible to transport spectroscopy.

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