One-dimensional electron gas interacting with a Heisenberg spin-1/2 chain

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We analyze a model of a one-dimensional electron-gas interacting with an antiferromagnetic Heisenberg spin-1/2 chain via the spin-exchange interactions. Using a solution at a special limit, we characterize the gapless modes of the spin-gap fixed point at weak coupling $J_K \ll J_H, E_F$. We show that the only gapless pairing mode with divergent susceptibility is a composite odd-parity/odd-frequency singlet-pairing order parameter, while the ordinary BCS even-parity singlet-pairing mode is incoherent. For two-leg ladder systems, we note that it is possible to have a range of doping where the chemical potential cuts only the antibonding band while the bonding band remains half filled. We propose that in such a state the two-leg ladder is effectively realizing the one-dimensional Kondo-Heisenberg model.

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The one-dimensional Kondo-Heisenberg model (K-H model) describes an incommensurate one-dimensional electron-gas (1DEG) interacting with a Heisenberg chain of spins 1/2 via spin-exchange interaction. For the K-H model, we show that the *only* gapless pairing mode with divergent susceptibility is a composite odd-parity/odd-frequency singlet-pairing order parameter, while the ordinary BCS even-parity singlet-pairing mode is incoherent. In addition, we find that the generalized Luttinger's theorem of Yamanaka *et al.*[1] is satisfied only by the introduction of a new composite-charge density wave (CDW). The composite CDW has power-law correlations with a ''large-Fermi-sea'' characteristics, while conventional CDW correlations decay exponentially.

We discuss the significance of our results in several contexts: First, our analysis sheds light on the relations between previous treatments[2,3] of the one-dimensional Kondo-Heisenberg model. Second, we discuss the possibility of effective realization of K-H model physics in two-leg ladder systems (This possibility was missed in all previous studies of doped two-leg ladders[4]). Third, we criticize previous suggestions regarding the relevance of K-H model to ''stripes theories'' of high-Tc superconductors.

The core of our analysis is based on the derivation of a special solvable limit of the K-H model, and the meaning of such a solution within the renormalization-group (RG) framework. The previous perturbative RG analysis of the K-H model[2] has shown that the spin-exchange interaction flows to some strong-coupling fixed point suggesting the formation of a spin-gap phase with enhanced pairing correlations. For particular value of parameters we obtain a well-controlled analytical solution that enables us to enumerate and characterize quantum numbers of *all* gapless modes. Gapless modes are properties of the fixed point. This means that the same gapless modes characterize all models that flow to the same fixed point. In particular, if there is only one fixed point to which all weak coupling K-H models flow, then our analysis is valid for all of them.

The K-H model (1) consists of two *inequivalent* interacting chains; one is a one-dimensional electron gas (described by the Hamiltonian H^{1DEG}[5]), and the other an antiferromagnetic Heisenberg chain of localized spins 1/2, $\{\vec{\tau}_j\}$. The chains interact via a spin-exchange interaction with an antiferromagnetic coupling constant $J_K>0$.

$$H = H^{1DEG} + H^{Heis} + H_K, \tag{1}$$

$$H^{Heis} = J_H \sum_j \vec{\tau}_j \cdot \vec{\tau}_{j+1}, \quad H_K = J_K \sum_j \vec{\tau}_j \cdot \vec{s}(x_j), \tag{2}$$

where $\vec{\mathbf{s}}(x_j) = \psi^\dagger_\alpha(x_j)(\boldsymbol{\sigma}_{\alpha\beta}/2)\psi_\beta(x_j)$ is the electron-gas spin-density operator at position x_j of the local spin $\vec{\tau}_j$ of the Heisenberg chain. We focus on the low-energy and long-distance behavior of the electron's correlation functions by taking the continuum limit of the electron gas and linearizing the 1DEG dispersion relation about the fermi points, $\pm k_F$, with corresponding right and left going electron fields, R_σ and L_σ; $\psi_\sigma(x) = R_\sigma(x)e^{+ik_F x} + L_\sigma(x)e^{-ik_F x}$, where $\sigma = \uparrow, \downarrow$.

The 1DEG spin currents are decomposed into forward- and back-scattering parts;

$$\mathbf{s}(x) = \psi^\dagger_\alpha(x_j)\frac{\vec{\boldsymbol{\sigma}}_{\alpha\beta}}{2}\psi_\beta(x_j) = [\mathbf{J}_{sR}(x) + \mathbf{J}_{sL}(x)] + \mathbf{n}_s(x), \tag{3}$$

where $\mathbf{J}^s_R = (1/2)R^+_\sigma \vec{\sigma}_{\sigma\sigma'} R_{\sigma'}$; $\mathbf{J}^s_L = (1/2)L^+_\sigma \vec{\sigma}_{\sigma\sigma'} L_{\sigma'}$ are the ferromagnetic $(q=0)$ spin currents of right- and left-moving electrons respectively, and

$$\mathbf{n}_s(x) = e^{-i2k_F x_j}\mathbf{n}_R(x) + e^{+i2k_F x_j}\mathbf{n}_L(x), \tag{4}$$

where $\mathbf{n}_R = R^+_\sigma(\vec{\sigma}_{\sigma,\sigma'}/2)L_{\sigma'}$; $\mathbf{n}_L = L^+_\sigma(\vec{\sigma}_{\sigma,\sigma'}/2)R_{\sigma'}$ are the staggered magnetization $(q=2k_F)$ components of the 1DEG.

We work in the *weak interchain coupling limit*

$$J_K \ll J_H, E_F. \tag{5}$$

In this case, one is allowed to make *further approximation* by taking the continuum limit also for the Heisenberg spin chain[2] (such approximation is not valid in the opposite limit $J_K \gg J_H$, which is discussed elsewhere[3,6]). The local spin-chain field is then also decomposed into the smooth (ferromagnetic) and staggered (antiferromagnetic) components;

$$\vec{\boldsymbol{\tau}}_j=[\mathbf{J}_R^\tau(x_j)+\mathbf{J}_L^\tau(x_j)]+(-1)^j\mathbf{n}_\tau(x_j). \tag{6}$$

(Note: we will consistently use the subscripts ''τ,s'' to distinguish the spin-chain fields from the 1DEG fields).

The effective Fermi wave numbers (in the sense of the generalized Luttinger's theorem[1]) for the 1DEG and the spin chain are $2k_F$ and $2k_F^{Heis}=\pi/b$ respectively (where $b=x_{j+1}-x_j$ is the distance between the local spins of the Heisenberg chain). It is assumed that the two systems are relatively incommensurate, and that $2k_F$ is incommensurate with any underlying ionic lattice. In order to distinguish contributions coming from various interaction terms, we introduce distinct Kondo coupling coefficients for forward scattering (J_f), back scattering (J_b), and mixed interactions (J_m);

$$H_K=J_f(\mathbf{J}_R^\tau+\mathbf{J}_L^\tau)\cdot(\mathbf{J}_R^s+\mathbf{J}_L^s)+J_m(-1)^j\mathbf{n}_\tau\cdot(\mathbf{J}_R^s+\mathbf{J}_L^s)$$
$$+J_b\vec{\boldsymbol{\tau}}(x_j)\cdot[e^{-i2k_Fx_j}\mathbf{n}_{sR}(x_j)+e^{+i2k_Fx_j}\mathbf{n}_{sL}(x)]. \tag{7}$$

The back-scattering term J_b and the mixed interaction J_m are made irrelevant by the oscillatory factors $e^{\pm i2k_Fx_j}$ and $(-1)^j$ respectively. Therefore, at incommensurate filling in the weak coupling limit, the K-H Hamiltonian (1) reduces to

$$H=H_0+J_f\int \mathrm{d}x(\mathbf{J}_R^\tau+\mathbf{J}_L^\tau)\cdot(\mathbf{J}_R^s+\mathbf{J}_L^s), \tag{8}$$

where $H_0=H^{1DEG}+H^{Heis}$. Due to the incommensurate electron filling, after dropping terms that are irrelevant in the RG sense, the spin- and charge-sectors decouple, $H=\int dx[\mathcal{H}_c+\mathcal{H}_{spin}]$. The charge sector is described by a Gaussian model[5]

$$\mathcal{H}_c=\frac{v_c}{2}\left[K_c\Pi_c^2(x)+\frac{1}{K_c}(\partial_x\phi_c)^2\right]. \tag{9}$$

The subsequent analysis and manipulations deal only with the spin-sector fields. The spin part of H_0 can be written as the sum of two level $k=1$ SU(2) Wess-Zumino-Novikov-Witten models. The corresponding Hamiltonian density is

$$\mathcal{H}_0^{spin}=\sum_{\mu=s,\tau}\frac{2\pi v_\mu}{3}(:\mathbf{J}_R^\mu\mathbf{J}_R^\mu:+:\mathbf{J}_L^\mu\mathbf{J}_L^\mu:) \tag{10}$$

where v_τ,v_s are the spin-wave velocities of the Heisenberg chain and 1DEG respectively ($v_\tau=\pi J_{\mathrm{H}}/2$).

For $J_f>0$, the noninteracting fixed point (10) is unstable,[2] and the low-energy physics is governed by some ''strong-coupling'' fixed point. Nothing more can be deduced from perturbative RG analysis, and the character of the strong-coupling fixed point should be studied by means of nonperturbative methods. Sikkema-Affleck-White[2] noticed that the relevant spin sector of the K-H Hamiltonian at incommensurate filling is equivalent to that of the two-leg zigzag spin ladder. In turn, the zigzag ladder was shown to possess a spin gap by means of exact numerical simulations.

We use the bosonized representation of the spin-1/2 fermionic fields;[5] $L_\sigma(x)=(F_\sigma/\sqrt{2\pi a})e^{-i\sqrt{\pi}[\theta_\sigma(x)+\phi_\sigma(x)]}$, $R_\sigma(x)=(F_\sigma/\sqrt{2\pi a})e^{-i\sqrt{\pi}[\theta_\sigma(x)-\phi_\sigma(x)]}$, where $\theta_\sigma(x)=\int_{-\infty}^x dx'\Pi_\sigma(x')$, and $[\Pi_\sigma(x'),\phi_\sigma(x)]=-i\delta(x'-x)$, $\sigma=\uparrow,\downarrow$. The Klein factors, $\{F_\sigma,F_{\sigma'}\}=\delta_{\sigma,\sigma'}$, enforce proper anticommutation of fermions with different spin. As commonly done, we re-express the operators in terms of bosonic spin fields $\phi_s(x)=1/\sqrt{2}[\phi_\uparrow-\phi_\downarrow]$, and charge fields $\phi_c(x)=1/\sqrt{2}[\phi_\uparrow+\phi_\downarrow]$, and correspondingly defined momenta Π_s and Π_c. Similarly, the spin-chain fields are bosonized. In particular, the bosonized expression for the staggered magnetization is

$$\mathbf{n}_\tau\sim[\sin(\sqrt{2\pi}\theta_\tau),-\cos(\sqrt{2\pi}\theta_\tau),\sin(\sqrt{2\pi}\phi_\tau)]. \tag{11}$$

In what follows we shall also need the bosonized expression for the triplet-superconducting order parameter:

$$(-i/2)[R_\alpha^\dagger(\vec{\sigma}\sigma_2)_{\alpha\beta}L_\beta^\dagger]\sim \mathrm{e}^{\mathrm{i}\sqrt{2\pi}\theta_c}[\sin(\sqrt{2\pi}\theta_s),$$
$$-\cos(\sqrt{2\pi}\theta_s),-\sin(\sqrt{2\pi}\phi_s)]. \tag{12}$$

The model (8) is exactly solvable by the Bethe ansatz[7,8] for general v_s,v_τ. However, for the purpose of calculating correlation functions, we take advantage of a *special point* in parameter space where the spin velocities are equal, i.e., $\Delta v_s=v_s-v_\tau=0$.

Using a *transformation to composite spin fields*; $\theta_\pm=1/\sqrt{2}(\theta_s\pm\theta_\tau)$ and $\phi_\pm=1/\sqrt{2}(\phi_s\pm\phi_\tau)$, the spin sector of the Kondo-Heisenberg Hamiltonian is simplified to the form; $H=H_0^{ZZ}+\Delta H_0^{ZZ}+H_\perp$;

$$H_0^{ZZ}=\frac{\overline{v_s}}{2}\int dx\left[\Pi_+^2+\left(1+\frac{J_z^f}{2\pi\overline{v_s}}\right)(\partial_x\phi_+)^2\right]$$
$$+\frac{\overline{v_s}}{2}\int dx\left[\Pi_-^2+\left(1-\frac{J_z^f}{2\pi\overline{v_s}}\right)(\partial_x\phi_-)^2\right], \tag{13}$$

$$\Delta H_0^{ZZ}=\frac{\Delta v_s}{4}\int dx\{\Pi_+\Pi_-+(\partial_x\phi_+)(\partial_x\phi_-)\}, \tag{14}$$

$$H_\perp=\frac{J_\perp^f}{(\pi a)^2}\int dx\cos(\sqrt{4\pi}\theta_-)$$
$$\times[\cos(\sqrt{4\pi}\phi_+)+\cos(\sqrt{4\pi}\phi_-)], \tag{15}$$

where $\overline{v}=\frac{1}{2}(v_s+v_\tau)$. Note that the J_z part of the interaction has been completely absorbed into the kinetic-energy part (13) of the Hamiltonian in terms of the new fields, $\phi_\pm$. Intuitively, a spin gap can be established due to the $J_\perp^f$ interaction term $\cos(\sqrt{4\pi}\theta_-)\cos(\sqrt{4\pi}\phi_+)$ in Eq. (15), where a self-consistent expectation value can be obtained for the composite fields $\langle\cos(\sqrt{4\pi}\theta_-)\rangle\neq 0$ and $\langle\cos(\sqrt{4\pi}\phi_+)\rangle\neq 0$. In contrast to $\langle\cos(\sqrt{4\pi}\theta_-)\cos(\sqrt{4\pi}\phi_-)\rangle=0$ (since $e^{i\sqrt{4\pi}\theta_-}$ and $e^{i\sqrt{4\pi}\phi_-}$ are respective disorder/order parameters).[9] Therefore, for determining spin-gap physics of the fixed point, we rigorously need to keep only the

$\cos(\sqrt{4\pi}\theta_-)\cos(\sqrt{4\pi}\phi_+)$ interaction term. An additional simplification (which we justify later on) is obtained if we neglect the velocity renormalization in Eq. (13) (i.e., equivalent to the anisotropic $J_z=0$ limit). Thus, we obtain a decoupling of the spin sector into two commuting sine-Gordon-type Hamiltonians,

$$H=\int dx\sum_{i=1,2}\left\{\frac{v_s}{2}[(\partial_x\Theta_i)^2+(\partial_x\Phi_i)^2]+(-1)^i\frac{\Delta v_s}{4}(\partial_x\Phi_i)\times(\partial_x\Theta_i)+\frac{J_\perp^f}{2(\pi a)^2}.\cos(\sqrt{8\pi}\Phi_i)\right\}, \tag{16}$$

where, Φ_i are new *nonchiral* fields combining the chiral components of ϕ_s and ϕ_τ as follows:

$$\Phi_1=\frac{\phi_+ +\theta_-}{\sqrt{2}};\quad \Phi_2=\frac{\phi_+ -\theta_-}{\sqrt{2}};$$

$$\Theta_1=\frac{\theta_+ +\phi_-}{\sqrt{2}};\quad \Theta_2=\frac{\theta_+ -\phi_-}{\sqrt{2}}.$$

In the limit $\Delta v_s=0$, the Hamiltonian (16) is equivalent to the spin sector of the SU(2) Thirring model that is known to have an exponentially small gap,[5] as anticipated by the RG arguments.[2] *The spin-gap fixed point is perturbatively stable with respect to all the interactions that were neglected for arriving at Eq. (16).* In the ground state $\sqrt{2\pi}\Phi_j=\pi n$, where n is an integer. Thus

$$\langle\cos(\sqrt{2\pi}\Phi_j)\rangle\neq 0,\quad \langle\sin(\sqrt{2\pi}\Phi_j)\rangle=0 \tag{17}$$

and there is an additional discrete $Z_2\times Z_2$ symmetry corresponding to the signs of $\langle\cos(\sqrt{2\pi}\Phi_j)\rangle$ that is spontaneously broken in the ground state, and to the (Φ_1,Φ_2) separation (the later is only an approximate symmetry that is broken by $J_z\neq 0$ coupling terms).

A spin-gapped one-dimensional system is expected to manifest enhanced pairing and CDW correlations. Furthermore, the generalized Luttinger's theorem[1] mandates the existence of a gapless CDW mode at wave vector $2k_F^*=2k_F+(\pi/b)$. As we shall see, these intuitive expectations are satisfied in a rather nontrivial manner.

The rigidity of the *composite* bosonic fields, enforced in Eq. (17), implies that the correlation function of any order parameter for which the spin part cannot be written purely in terms of $\cos(\sqrt{2\pi}\Phi_j)$ is exponentially decaying, i.e., is incoherent. In particular, the usual one-dimensional electron-gas singlet charge-$2e$ pairing $\Delta=(1/\sqrt{2})(R_\uparrow^\dagger L_\downarrow^\dagger+L_\uparrow^\dagger R_\downarrow^\dagger)$, and the $2k_F$ CDW $\hat{O}_{CDW}=[(1/\sqrt{2})(R_\uparrow^\dagger L_\uparrow+R_\downarrow^\dagger L_\downarrow)+h.c.]$ are incoherent.

Instead, there are gapless modes of a *composite* nature: A composite odd-parity/odd-frequency singlet

$$\hat{O}_{c-SP}(x)=\frac{-i}{2}[R_\alpha^\dagger(\vec{\sigma}\sigma_2)_{\alpha\beta}L_\beta^\dagger]\cdot\vec{\tau},\quad \sim e^{+i\sqrt{2\pi}\theta_c}\langle\cos(\sqrt{2\pi}\Phi_1)\cos(\sqrt{2\pi}\Phi_2)\rangle(-1)^j \tag{18}$$

and a composite CDW (a charge-0 spin-0 operator)

$$\hat{O}_{c-CDW}(x)=\vec{n}_{1DEG}\cdot\vec{\tau}\sim e^{+i\sqrt{2\pi}\phi_c}\langle\cos(\sqrt{2\pi}\Phi_1)\cos(\sqrt{2\pi}\Phi_2)\rangle\times e^{+i(2k_Fx+\pi j)}, \tag{19}$$

where it is the staggered component of the impurity spin chain, $\vec{\tau}\to(-1)^j\vec{\mathbf{n}}_\tau$, which is contributing the gapless modes with power-law correlations.

The staggering factor $(-1)^j$ in the corresponding correlation functions is effectively modulating the usual correlations by the reciprocal lattice vector π/b of the spin chain. As a result, the composite gapless modes are found at unusual finite momentum values: the composite singlet pairs with momentum π/b (and there is no $k=0$ singlet pairing with charge $2e$), and the gapless composite CDW mode at momentum $2k_F^*=2k_F+(\pi/b)$ (and not at $2k_F$ of the bare 1DEG). The pure charge sector is not affected, as is evidenced by the fact that the gapless $\eta-pairing$ mode ($\eta_R=R_\uparrow^\dagger R_\downarrow^\dagger$ and $\eta_L=L_\uparrow^\dagger L_\downarrow^\dagger$) remains at momentum $2k_F$. The gapless modes are inter-related by the commutation relation $[\hat{O}_{c-CDW},\eta_R-\eta_L/\sqrt{2}]=2\hat{O}_{c-SP}$. An extended discussion of these order parameters can be found in Ref. 6.

To summarize, we developed a solution of the one-dimensional Kondo-Heisenberg model at weak exchange coupling $J_K\ll J_H,E_F$, for a special value of parameters $v_s=v_\tau$. We were able to explicitly demonstrate the spin gap and characterized the gapless modes properties of the fixed point previously alluded to by perturbative RG arguments.

The spin-wave velocities v_s and v_τ in general are not identical. What then are the limits of validity (and hence the significance) of our solution? First, the spin gap guarantiees that our fixed-point solution is perturbatively stable for $\Delta v_s=v_s-v_\tau\neq 0$. Moreover, unless there is a phase transition driven by large Δv_s anisotropy (to a yet another unknown phase), our results (gapless modes identification) in the limit $\Delta v_s=0$ are *universal* for all Δv_s so long as the weak coupling condition $J_K\ll v_\tau,v_s$ is maintained. The same argument applies to all other approximations we undertook for arriving at Eq. (16).

The small J_K condition manifests itself through the coefficient $(1-J_z^f/2\pi v_s)$ of the $(\partial_x\phi_-)^2$ term in the Hamiltonian (13). It indicates that something may indeed breakdown when $J_z^f>2\pi v_s$ (Ref. 6). Indeed, we remark in this context that $J_z^f=2\pi v_s$ is the so-called "Toulouse-point" value on which we further comment below. Therefore, the perturbative RG flows to "strong coupling" found in Ref. 2 should be understood as being only to some intermediate coupling fixed point $J_z^f<2\pi v_s$ with a finite basin of attraction (the true strong J_K coupling limit of the K-H model is gapless[10]).

We now discuss the significance of our results in various context: In previous paper,[3] a spin-gap phase of the Kondo-Heisenberg model (1) was found in another region of the parameters space ($J_H \ll J_K \sim E_F$); the so-called "Toulouse-limit" solution. That solution has the same composite order parameters as the present one, but in addition it also possesses gapless modes of the conventional CDW and even-parity singlet-pairing Δ order parameters. Hence, we conclude that the "Toulouse point"[3] and the "zigzag ladder limit"[2] spin-gap phases of the Kondo-Heisenberg model are distinct. An elaborate comparison and implications for the general phase diagram of the Kondo-Heisenberg model is presented elsewhere.[6]

The two-leg ladder system consists of two *equivalent* Hubbard-model chains that are coupled by single-particle hopping with amplitude $t_\perp$. As is well known, the effect of $t_\perp$ interaction can be treated exactly by introducing "bonding" and "antibonding" bands described by fermion fields $\psi_{A,B} = 1/\sqrt{2}(\psi_1 \pm \psi_2)$, where $\psi_{1,2}(x)$ are fermion fields on the legs $\{1,2\}$ of the ladder. The bonding and antibonding bands are *inequivalent*. In particular, there is a chemical potential difference $\mu_A - \mu_B \sim t_\perp$. Consequently there could exists a range of doping for which, depending on model parameters, holes may enter only into the antibonding band (which becomes gapless), while the bonding band remains half filled and retains a Mott-Hubbard gap. In that range of doping, the Fermi energy lies in the gap of the bonding band and cuts only the antibonding band. As far as the low-energy physics of such a state is concerned, the half-filled band is equivalent to a Heisenberg chain of localized spins 1/2, $\{\vec{\tau}_j\}$, with effective antiferromagnetic coupling J_H. On the other hand, the gapless band represents a one-dimensional electron gas (described by the Hamiltonian H^{1DEG}),[5] with an incommensurate Fermi momentum k_F. Hence, the only relevant interaction between the two bands is spin-exchange interaction, $J_K > 0$. We conclude that in this particular doping range, the low-energy physics of the two-leg ladder is effectively captured by the K-H model (1). The conditions under which the above scenario is realized require an elaboration beyond the scope of this paper. The exact dependence of model parameters $\{J_H, J_K\}$ on the original ladder parameters $\{t, U, V, t_\perp\}$ is unimportant since, as noted in the introduction, our analysis addresses general fixed-point properties. Consequently, we suggest that in the general phase diagram of doped ladder,[4] the spin-gap region (labeled C1S0 in[4]) should be divided in two: A certain low-doping region with only odd-*w* pairing, and a higher-doping region with conventional pair states as discussed, for example, in Ref. 4. Consequently, the same would be true for the putative superconducting state in a system of coupled two-leg ladders.

A model of an incommensurate 1DEG coupled with a ladder environment[11] was proposed in the context of stripe phases in HTc cuprates. In the case of a gapless spin-ladder environment, if only spin-exchange interactions are considered[12] one arrives at the effective model given by Eq. (1). Combining our analysis (where only odd-ω pairing is coherent) with the experimental observation that superconductivity in high-T_c cuprates is due to *d*-wave BCS paired electron, we conclude that, *within a stripe-state scenario,*[12] spin-exchange interactions are ruled out as a possible source of the spin gap in high-T_c cuprates. While unrelated to cuprates, our solution does indicates the possibility of making pure odd-time composite pairing superconductors by constructing 2D or 3D weakly coupled arrays of the 1D chain model (1).

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