Transverse-optical Josephson plasmons: Equations of motion

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(Received 17 November 2000; published 22 June 2001)

A detailed calculation is presented of the dielectric function in superconductors consisting of two Josephson coupled superconducting layers per unit cell, taking into account the effect of finite compressibility of the electron fluid. From the model, it follows that two longitudinal and one transverse-optical Josephson plasma resonance exist in these materials for electric-field polarization perpendicular to the planes. The latter mode appears as a resonance in the transverse dielectric function, and it couples directly to the electrical-field vector of infrared radiation. A shift of all plasma frequencies and a reduction of the intensity of the transverse-optical Josephson plasmon is shown to result from the finite compressibility of the electron fluid.

DOI: 10.1103/PhysRevB.64.024530 PACS number(s): 74.25.Gz, 74.25.Nf, 74.20.−z, 74.80.Dm

I. INTRODUCTION

In recent years the ‘‘second’’ Josephson plasma-resonance (JPR) phenomenon has been studied theoretically and has been observed experimentally[1–8] in double layer high-T_c cuprates. Originally the prediction of a transverse-optical Josephson resonance and the associated second JPR in layered superconductors with two layers per unit cell appeared in a short conference paper,[2] explaining briefly the main theoretical ingredients and results. One of the reasons for revisiting this problem is that experimentally the transverse-optical JPR was observed[7,8] in $SmLa_{0.8}Sr_{0.2}CuO_{4-\delta}$ with an oscillator strength much smaller than expected on the basis of the simple expression derived in Ref. 2. Following a suggestion by Bulaevskii, the expressions for the dielectric function will be rederived in this manuscript, while taking into account an extra term representing the finite compressibility of the electron fluid. The central result is that the dielectric function is

$$\frac{1}{\epsilon(\omega)}=\frac{\tilde{z}_I\omega^2}{\omega^2-\tilde{\omega}_I^2}+\frac{\tilde{z}_K\omega^2}{\omega^2-\tilde{\omega}_K^2},$$

which is the same expression as in Ref. 2, except that the volume fractions x_I and x_K have been replaced with *effective* weight factors $\tilde{z}_I$ and $\tilde{z}_K=1-\tilde{z}_I$, which depend on the volume fractions, the plasma frequencies ω_I and ω_K, and the compressibility.

Let us consider the optical response function for a material with two superconducting layers per unit cell. The lattice constant along the c direction, perpendicular to the planes, is d. The layers are grouped in pairs, with interlayer distances x_Id and x_Kd alternating $(x_I+x_K=1)$. Hence the z coordinate of the mth plane normalized by the lattice constant is $x_m=m/2$ if m is even, and $x_m=(m-1)/2+x_I$ if m is odd.

The charge fluctuation of each plane is characterized by a charge amplitude Q_m and a phase ϕ_m, where m is the layer index, and d is the length of the unit cell along the c direction.[9] The discussion in this paper will be restricted to the charge fluctuations perpendicular to the planes, corresponding to a homogeneous charge distribution within each plane. In this case the electric fields are perpendicular to the planes. The electric field of a single plane with an area A and charged with a positive charge Q is constant in space, and the field lines are directed away from the plane. As a result the potential energy of a positive test charge with charge q in this field *decreases* linearly as a function of the distance d from the plane: $E_{pot}(z)=-4\pi qQz/A$. The potential energy stored in the field of the charge fluctuations Q_m and Q_n of a pair of planes at a distance $d|x_m-x_n|$ is half this amount,

$$V_C(Q_m,Q_n)=-\frac{|x_m-x_n|Q_mQ_n}{2C_0}, \tag{1}$$

where $C_0=A/(4\pi d)$ is the capacitance of two planes at a distance corresponding to the lattice parameter d. The total potential energy stored in the fields of the charge fluctuations is just the linear superposition of the contributions from all pairs of planes in the crystal. In this context it is important to point out that the planes considered here are truly two-dimensional (2D) in the electrodynamical sense: Because the charge has no spatial degrees of freedom perpendicular to the planes (other than tunneling between planes), the individual planes provide no channel for metallic screening for fields along the c axis. This is quite different from the situation encountered in classical Josephson junctions between thick metallic layers. In the latter case individual planes *do* screen the electric fields polarized along the c direction.

The second source of potential energy is the electronic compressibility. This is due to the fact that if δN electrons are added to a plane, the free energy increases with an amount $\delta F=\mu\delta N+\delta N^2/(2K_0n^2)$, where μ, K_0, and n are the chemical potential, the electronic compressibility, and the electron density, respectively. For a Fermi gas, $K_0n^2=\partial n/\partial\mu$ corresponds to the density of states at the Fermi level. In the context of ‘‘excitons’’ in two-band superconductors, the compressibility term was first considered in 1966 by Leggett.[10] In neutral fluids the compressibility causes propagation of sound, whereas for electrons it causes the dispersion of plasmons. Plasma dispersion of the JPR in the cuprates has been described by Koyama and Tachiki.[11] In part the compressibility can be motivated by calculations based on the random-phase approximation, showing a finite

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dispersion of the charge-density fluctuations in single layer[12] and bilayer[13] cuprates and of spin, amplitude, and phase collective modes.[14] It has been shown by Artemenko and Kobel'kov that the frequency of the resonance, its dispersion, and its damping are strongly influenced by the presence of quasiparticles at finite temperature.[15] Although this type of calculation demonstrates that the Pauli exclusion principle causes a finite compressibility of the electron fluid leading to a finite dispersion of the collective modes, weak-coupling approaches are not well supported due to the strong electronic correlations in these materials. We therefore treat the electronic compressibility as a phenomenological parameter in this paper.

The terms in the free energy proportional to δN only shift the equilibrium density. In harmonic approximation, the charge fluctuations around equilibrium follow from the quadratic terms

$$V_K(Q_m)=\frac{(Q_m/e)^2}{2K_0 n^2 A}=\frac{\gamma_0}{2C_0}Q_m^2. \tag{2}$$

For later convenience the dimensionless constant $\gamma_0 = 1/(4\pi d e^2 K_0 n^2)$, proportional to the 2D bulk modulus, is introduced here to characterize the compressibility. In Ref. 2 we left these terms out of consideration.

We calculate the longitudinal dielectric function following the usual procedure of adding external charges Q_m^e to each layer, distributed such as to provide an external electric field D (the displacement field) of the plane-wave form with wave vector k. The *definition* of the dielectric constant[16] implies that the internal and external charge distributions interact only via the electromagnetic field. Hence, the interaction between internal and external charge is described by the Coulomb term, Eq. (1), but the external charge does not enter the compressibility term, Eq. (2). The charge dynamics enters via the Josephson coupling J_m^{m+1} between each set of nearest-neighbor planes

$$H_{kin}=-J_m^{m+1}\cos(\phi_m-\phi_{m+1}). \tag{3}$$

Our aim is to determine the dielectric constant and collective modes in the absence of external dc magnetic fields. For this purpose we will need the equations of motion for the *internal* charge acceleration $d^2Q_m^i/dt^2$ subject to the fields of the internal *and* external charges $Q_m^t=Q_m^i+Q_m^e$. These follow from the Hamiltonian

$$H=-\sum_{m>n}\frac{|x_m-x_n|}{2C_0}(Q_m^i+Q_m^e)(Q_n^i+Q_n^e)+\sum_m\frac{\gamma_0}{2C_0}(Q_m^i)^2 -\sum_m J_m^{m+1}\cos(\phi_m-\phi_{m+1}). \tag{4}$$

Here the phases ϕ_m and the internal charges Q_m^i are conjugate variables, which are subject to the Hamilton-Josephson equations of motion: $(\hbar/e^*)d\phi_m/dt=\partial H/\partial Q_m^i$, and $(\hbar/e^*)dQ_m^i/dt=-\partial H/\partial\phi_m$, where $e^*=2e$ is the charge of a Cooper pair.

II. EQUATIONS OF MOTION

Working in the linear-response regime, we obtain the equations of motion for the *internal* charge acceleration $d^2Q_m^i/dt^2$ subject to the fields of the internal *and* external charges

$$-2C_0\left\{\frac{\hbar}{e^*}\right\}^2\frac{d^2Q_m^i}{dt^2}=-\sum_n\{[J_m^{m+1}+J_{m-1}^m]|x_n-x_m| -J_m^{m+1}|x_n-x_{m+1}|-J_{m-1}^m|x_n-x_{m-1}|\}Q_n^t+2\gamma_0(J_m^{m+1} +J_{m-1}^m)Q_m^i-2\gamma_0J_m^{m+1}Q_{m+1}^i-2\gamma_0J_{m-1}^mQ_{m-1}^i. \tag{5}$$

Due to the lattice periodicity, the solutions have to be plane waves with a wave vector $k=\phi/d$. Therefore we can use generalized charge coordinates R and S defined as $Q_{2m}=Se^{im\phi}$, $Q_{2m+1}=Re^{im\phi}$ to describe the charge fluctuations in the even and odd planes. The Josephson coupling energies $J_{2m}^{2m+1}=I$ and $J_{2m-1}^{2m}=K$ characterize the two types of junctions.

It is quite easy to extend this to the situation where we have a lattice polarizability characterized by dielectric constants ϵ_I^s and ϵ_K^s for each type of Josephson junction. This corresponds to the transformation $C_0\to C_{av}$, $\gamma_0\to\gamma$, $x_I\to z_I$, and $x_K\to z_K$ in Eqs. (4) and (5). Here the following definitions have been used: $C_{av}=A\epsilon_{av}^s/4\pi d$ for the average capacitance, $\gamma=\epsilon_{av}/(4\pi d e^2K_0n^2)$ for the compressibility, $z_I=x_I\epsilon_{av}^s/\epsilon_I^s$ and $z_K=x_K\epsilon_{av}^s/\epsilon_K^s$ (together satisfying $z_K+z_I=1$) for the weight factors, and $1/\epsilon_{av}^s=x_K/\epsilon_K^s+x_I/\epsilon_I^s$ for the average dielectric constant. We are now ready to formulate the equations of motion for the generalized coordinates S and R for each wave number $k=\phi/d$.

$$2C_{av}\left\{\frac{\hbar\omega}{e^*}\right\}^2S^i=-\sum_n e^{i\phi n}\{|n|(I+K)-|n-z_I|I -|n+z_K|K\}S^t-\sum_n e^{i\phi n}\{|n+z_I|(I+K)-|n|I -|n+1|K\}R^t+2\gamma(I+K)S^i-2\gamma(I+e^{-i\phi}K)R^i,$$

$$2C_{av}\left\{\frac{\hbar\omega}{e^*}\right\}^2R^i=-\sum_n e^{i\phi n}\{|n|(I+K)-|n+z_I|I -|n-z_K|K\}R^t-\sum_n e^{i\phi n}\{|n-z_I|(I+K)-|n|I -|n-1|K\}S^t+2\gamma(I+K)R^i-2\gamma(I+e^{i\phi}K)S^i. \tag{6}$$

The convergent lattice sums over n can be replaced with the identities

$$\sum_n |n+p\pm x|e^{in\phi}=e^{-ip\phi}\frac{(1-e^{\mp i\phi})x-1}{1-\cos\phi}. \qquad (7)$$

Before we continue it is convenient to partly diagonalize Eq. (6), by transforming the expressions to new generalized charge coordinates Q and P, defined as $Q=S+R$ and $P=e^{i\phi}S+R$, with identical transformations for the internal and external charges. We will see later that Q and P correspond to charge fluctuations across the barriers of type K and I, respectively. The reverse transformations are $S=(Q-P)/(1-e^{i\phi})$, and $R=(P-e^{i\phi}Q)/(1-e^{i\phi})$. In addition it will turn out to be convenient to introduce the Josephson plasma-resonance frequencies characteristic of the two types of junctions $\omega_K^2=z_K K(e^*/\hbar)^2C_{av}^{-1}$, and $\omega_I^2=z_I I(e^*/\hbar)^2C_{av}^{-1}$ and the corresponding local charge response functions $\epsilon_K=1-\omega_K^2/\omega^2$, and $\epsilon_I=1-\omega_I^2/\omega^2$. With the help of the identities, Eq. (7), the equations of motion of the external and total generalized charge coordinates become

$$Q^e=\epsilon_K Q^t-2\gamma z_K^{-1}\omega_K^2\omega^{-2}[Q^i-P^ie^{-i\phi/2}\cos(\phi/2)],$$

$$P^e=\epsilon_I P^t-2\gamma z_I^{-1}\omega_I^2\omega^{-2}[P^i-Q^ie^{i\phi/2}\cos(\phi/2)]. \qquad (8)$$

We see that if the compressibility term $\gamma=0$, these equations of motion are already diagonal, corresponding to two nondispersing longitudinal plasmons at frequencies ω_K and ω_I. It is also immediately clear from this that Q and P correspond to the charge fluctuations across junctions of type K and I, respectively. If γ is finite, the equations of motion are coupled, and the plasma frequencies will have a finite dispersion as a function of $k=\phi/d$.

III. CALCULATION OF THE DIELECTRIC FUNCTION

We are interested in the response of the total electric field E to an external field D, which is polarized along the c axis, and which varies harmonically in time and space along the c direction, i.e.,

$$\vec{D}(\vec{r},t)=D_0\hat{z}e^{i(kz-\omega t)}. \qquad (9)$$

The dielectric function $\epsilon(k,\omega)$ is calculated from the definition $\vec{D}=\epsilon\vec{E}$. We will employ the fact that $\vec{D}=\vec{\nabla}V^e$ and $\vec{E}=\vec{\nabla}V$. V, V^i, and V^e are the total, internal, and the external voltages, respectively. Thus we need to arrange the external charges in a such a way as to guarantee that the external field $D(z)$ has a plane-wave form. For z coinciding with the coordinates of the conducting planes, this requires that $V^e_{2m+1}/V^e_{2m}=e^{iz_I\phi}$. We satisfy this requirement by giving the external charges the ratio $R^e/S^e=\{e^{iz_I\phi}-z_K-z_Ie^{i\phi}\}/\{1-z_Ke^{iz_I\phi}-z_Ie^{-iz_K\phi}\}$. For the generalized coordinates this implies

$$\frac{P^e}{Q^e}=\frac{z_K\sin(z_I\phi/2)}{z_I\sin(z_K\phi/2)}e^{i\phi/2}. \qquad (10)$$

For the calculation of ϵ we need to calculate the macroscopic average of E and D, corresponding to the macroscopic electric and displacement fields. For this it is sufficient to know the voltages at the positions of the planes. Now, using Eq. (1) we observe that the voltages in each plane are

$$V_{2m}=\frac{-1}{C_{av}}\sum_n (Q_{2n}|n-m|+Q_{2n+1}|n-m+z_I|),$$

$$V_{2m+1}=\frac{-1}{C_{av}}\sum_n (Q_{2n}|n-m-z_I|+Q_{2n+1}|n-m|). \qquad (11)$$

We consider a charge oscillation with wave vector $k=\phi/d$. The charges in the alternating layers are $Q^t_{2n}=e^{in\phi}S^t$ and $Q^t_{2n+1}=e^{in\phi}R^t$. After summation over the index n, the voltages in the zeroth and first plane are

$$V_0=\frac{-1}{C_{av}(\cos\phi-1)}\{S^t+R^t(z_K+z_Ie^{-i\phi})\},$$

$$V_1=\frac{-1}{C_{av}(\cos\phi-1)}\{S^t(z_K+z_Ie^{i\phi})+R^t\}, \qquad (12)$$

with similar expressions for the external charges for which C_{av} must be replaced with C_0. The electric fields integrated between the nearest-neighbor planes are

$$\int_{(m-z_K)d}^{md}E(z)dz=V_{2m}-V_{2m-1}=C_{av}^{-1}e^{im\phi}(e^{-i\phi}-1)z_KQ^t,$$

$$\int_{md}^{(m+z_I)d}E(z)dz=V_{2m+1}-V_{2m}=C_{av}^{-1}e^{im\phi}(e^{-i\phi}-1)z_IP^t, \qquad (13)$$

with similar expressions for D, for which C_{av} must be replaced with C_0. In the limit $k\rightarrow 0$ the macroscopic electric field is just the sum of the two integrals divided by the lattice parameter d. We conclude from this that for $k\rightarrow 0$ the dielectric function is

$$\epsilon(\omega)=\frac{\int_0^d D(z)dz}{\int_0^d E(z)dz}=\epsilon_{av}\frac{z_KQ^e+z_IP^e}{z_KQ^t+z_IP^t}. \qquad (14)$$

From Eq. (10) we see that $P^e=Q^e$ in the limit $k\rightarrow 0$. We can combine this identity with the equations of motion, Eq. (8), to prove that $P^t/Q^t=\{\epsilon_K-2\gamma(\omega_K^2/z_K+\omega_I^2/z_I)\omega^{-2}\}/\{\epsilon_I-2\gamma(\omega_K^2/z_K+\omega_I^2/z_I)\omega^{-2}\}$, that $P^e/P^t=\epsilon_I-\{(\epsilon_K-\epsilon_I)2\gamma\omega_I^2/z_I\}/\{\epsilon_K\omega^2-2\gamma(\omega_K^2/z_K+\omega_I^2/z_I)\}$, and $Q^e/Q^t=\epsilon_K-\{(\epsilon_I-\epsilon_K)2\gamma\omega_K^2/z_K\}/\{\epsilon_I\omega^2-2\gamma(\omega_K^2/z_K+\omega_I^2/z_I)\}$. The dielectric function is now easily obtained,

$$\frac{1}{\epsilon(\omega)}=\frac{1}{\epsilon_{av}}\frac{\omega^2(\omega^2-\tilde{\omega}_T^2)}{(\omega^2-\tilde{\omega}_I^2)(\omega^2-\tilde{\omega}_K^2)} \qquad (15)$$

with the definitions

$$\tilde{\omega}_K^2=\left(\frac{1}{2}+\frac{\gamma}{z_K}\right)\omega_K^2+\left(\frac{1}{2}+\frac{\gamma}{z_I}\right)\omega_I^2+\sqrt{\left[\left(\frac{1}{2}+\frac{\gamma}{z_K}\right)\omega_K^2-\left(\frac{1}{2}+\frac{\gamma}{z_I}\right)\omega_I^2\right]^2+\frac{(2\gamma\omega_K\omega_I)^2}{z_Kz_I}},$$

$$\tilde{\omega}_I^2=\left(\frac{1}{2}+\frac{\gamma}{z_K}\right)\omega_K^2+\left(\frac{1}{2}+\frac{\gamma}{z_I}\right)\omega_I^2-\sqrt{\left[\left(\frac{1}{2}+\frac{\gamma}{z_K}\right)\omega_K^2-\left(\frac{1}{2}+\frac{\gamma}{z_I}\right)\omega_I^2\right]^2+\frac{(2\gamma\omega_K\omega_I)^2}{z_Kz_I}},$$

$$\tilde{\omega}_T^2=\left(z_I+\frac{2\gamma}{z_K}\right)\omega_K^2+\left(z_K+\frac{2\gamma}{z_I}\right)\omega_I^2. \tag{16}$$

Here we have adopted the convention for labeling the two plasma resonances, such that K always refers to the highest plasma-resonance frequency.

IV. CENTRAL RESULT

Provided that $\tilde{\omega}_I<\tilde{\omega}_T<\tilde{\omega}_K$, it is always possible to express $\tilde{\omega}_T$ as a weighted average of the two longitudinal frequencies

$$\tilde{\omega}_T^2=\tilde{z}_K\tilde{\omega}_I^2+\tilde{z}_I\tilde{\omega}_K^2, \tag{17}$$

with weight factors satisfying $\tilde{z}_K+\tilde{z}_I=1$. The latter are no longer the volume fractions z_I and z_K, as in Ref. 2. Instead they depend on the volume fractions *and* on the microscopic electronic parameters characterizing the two types of junctions. The effective fractions can be calculated by inverting the above relation, i.e., using

$$\tilde{z}_K=(\tilde{\omega}_K^2-\tilde{\omega}_T^2)/(\tilde{\omega}_K^2-\tilde{\omega}_I^2). \tag{18}$$

As a result[17] we can write the inverse dielectric function, Eq. (15), as a linear superposition of two plasma resonances

$$\frac{\epsilon_{av}}{\epsilon(\omega)}=\frac{\tilde{z}_I\omega^2}{\omega^2-\tilde{\omega}_I^2}+\frac{\tilde{z}_K\omega^2}{\omega^2-\tilde{\omega}_K^2}, \tag{19}$$

which is the same expression as in Ref. 2, except that z_I and z_K have been replaced with *effective* volume fractions

$$\tilde{z}_K=\frac{1}{2}\pm\frac{(z_K-z_I)(z_Kz_I+2\gamma)}{2(z_Kz_I+2\gamma+4\gamma^2)}\sqrt{1-\frac{4(2\gamma)^2\tilde{\omega}_K^2\tilde{\omega}_I^2}{(z_Kz_I+2\gamma)(\tilde{\omega}_K^2-\tilde{\omega}_I^2)^2}}-\frac{2\gamma(z_Kz_I+\gamma)}{z_Kz_I+2\gamma+4\gamma^2}\frac{\tilde{\omega}_K^2+\tilde{\omega}_I^2}{\tilde{\omega}_K^2-\tilde{\omega}_I^2}. \tag{20}$$

Together, Eqs. (19) and (20) form the central result of this paper. From the intensities of the experimental loss functions the effective volume fractions $\tilde{z}_K$ and $\tilde{z}_I$ can be extracted. These can be used to calculate γ, and this in turn can be used to determine the density of states

$$\frac{\partial n}{\partial\mu}=\frac{1}{4\pi de^2}\frac{1}{\gamma} \tag{21}$$

or, using $K_0n^2=\partial n/\partial\mu$, the compressibility. Once $\tilde{\omega}_I$ and $\tilde{\omega}_K$ have been measured, and γ has been calculated from the weight factors $\tilde{z}_K$ and $\tilde{z}_I$ using Eq. (20), it becomes possible to make two further deductions, namely, the determination of ω_K and ω_I using

$$\omega_K^2=\frac{z_K}{z_K+2\gamma}\left\{\frac{\tilde{\omega}_K^2+\tilde{\omega}_I^2}{2}\pm\frac{\tilde{\omega}_K^2-\tilde{\omega}_I^2}{2}\sqrt{1-\frac{4(2\gamma)^2\tilde{\omega}_K^2\tilde{\omega}_I^2}{(z_Kz_I+2\gamma)(\tilde{\omega}_K^2-\tilde{\omega}_I^2)^2}}\right\},$$

$$\omega_I^2=\frac{z_I}{z_I+2\gamma}\left\{\frac{\tilde{\omega}_K^2+\tilde{\omega}_I^2}{2}\mp\frac{\tilde{\omega}_K^2-\tilde{\omega}_I^2}{2}\sqrt{1-\frac{4(2\gamma)^2\tilde{\omega}_K^2\tilde{\omega}_I^2}{(z_Kz_I+2\gamma)(\tilde{\omega}_K^2-\tilde{\omega}_I^2)^2}}\right\}, \tag{22}$$

from which we can calculate directly the Josephson coupling energies

$$K=\frac{\epsilon_{av}}{4\pi d}\left\{\frac{\hbar\omega_K}{e^*}\right\}^2 \quad\text{and}\quad I=\frac{\epsilon_{av}}{4\pi d}\left\{\frac{\hbar\omega_I}{e^*}\right\}^2. \tag{23}$$

V. EVOLUTION OF THE OSCILLATOR STRENGTH AS A FUNCTION OF γ

For the analysis of experimental data, Eqs. (19) and (20) suffice to deduce the microscopic parameters I, K, and K_0n^2, i.e., the two Josephson energies and the compressibility factor. To predict the plasma-resonance frequencies $\tilde{\omega}_T$, $\tilde{\omega}_I$, and $\tilde{\omega}_K$, we can use Eq. (16). The intensities of the two peaks in the energy-loss function $\mathrm{Im}-1/\epsilon(\omega)$ are just the weight factors $\tilde{z}_K$ and $\tilde{z}_I$. Their dependence on the microscopic parameters ω_I, ω_K, and K_0n^2 is given by the expressions

$$\tilde{z}_K=1-\tilde{z}_I=\frac{1}{2}+\frac{(z_K-z_I)(\omega_K^2-\omega_I^2)-2\gamma(\omega_K^2/z_K+\omega_I^2/z_I)}{2\sqrt{\left[\left(1+\frac{2\gamma}{z_K}\right)\omega_K^2-\left(1+\frac{2\gamma}{z_I}\right)\omega_I^2\right]^2+4\frac{(2\gamma\omega_K\omega_I)^2}{z_Kz_I}}}. \tag{24}$$

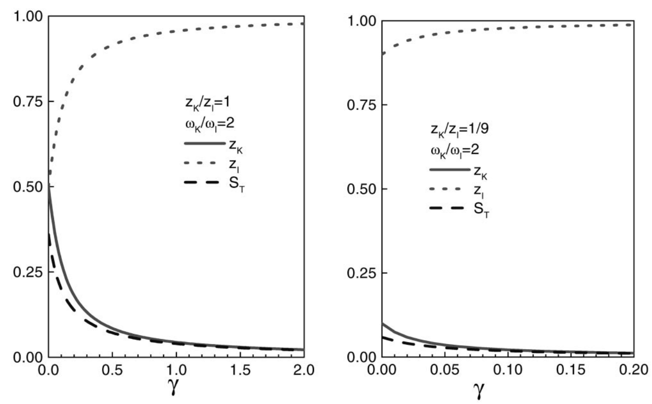

FIG. 1. Oscillator strengths as a function of γ.

The compressibility is characterized by the dimensionless parameter γ. The most important effect of this extra term is that the intensity of the highest plasma resonance $\tilde{\omega}_K$ is reduced compared to what it would have been if γ were zero: It is clear from Eq. (24) that the effective volume fraction $\tilde{z}_K$ is smaller than z_K.

The oscillator strength of the transverse-optical plasmon follows directly from the pole strength of the pole near $\tilde{\omega}_T$ in $\epsilon(\omega)$. From Eq. (15) it follows that $S_T=(\tilde{\omega}_K^2-\tilde{\omega}_T^2)(\tilde{\omega}_T^2-\tilde{\omega}_I^2)/\tilde{\omega}_T^4$. With the help of this, it follows that the dependence of S_T on the microscopic parameters ω_I, ω_K, and K_0n^2 is

$$S_T=\frac{(\omega_K^2-\omega_I^2)^2(z_Iz_K+2\gamma)}{\left(z_I+\dfrac{2\gamma}{z_K}\right)\omega_K^2+\left(z_K+\dfrac{2\gamma}{z_I}\right)\omega_I^2}. \qquad (25)$$

In Figs. 1 and 2 the plasmon strengths $\tilde{z}_I$ and $\tilde{z}_K$ and the pole strength S_T are displayed as a function of γ for a few different sets of ω_K, ω_I, and the volume fraction z_K.

VI. DISPERSION OF THE PLASMA MODES

In the previous section we found for the double layer cuprates *two* longitudinal plasma modes with the electric-field vector polarized along the *c* direction. The dispersion of the longitudinal modes can be calculated by realizing that for longitudinal modes $\vec{D}=0$ by virtue of the fact that longitudinally polarized free photons don't exist. Hence in Eq. (8) the external charge coordinates $Q^e=P^e=0$. The corresponding 2×2 matrix is easily solved, providing the two longitudinal branches

$$\tilde{\omega}_K^2=\left(\frac{1}{2}+\frac{\gamma}{z_K}\right)\omega_K^2+\left(\frac{1}{2}+\frac{\gamma}{z_I}\right)\omega_I^2+\sqrt{\left[\left(\frac{1}{2}+\frac{\gamma}{z_K}\right)\omega_K^2-\left(\frac{1}{2}+\frac{\gamma}{z_I}\right)\omega_I^2\right]^2+\frac{(2\gamma\omega_K\omega_I)^2}{z_Kz_I}\cos^2\frac{k_zd}{2}},$$

$$\tilde{\omega}_I^2=\left(\frac{1}{2}+\frac{\gamma}{z_K}\right)\omega_K^2+\left(\frac{1}{2}+\frac{\gamma}{z_I}\right)\omega_I^2-\sqrt{\left[\left(\frac{1}{2}+\frac{\gamma}{z_K}\right)\omega_K^2-\left(\frac{1}{2}+\frac{\gamma}{z_I}\right)\omega_I^2\right]^2+\frac{(2\gamma\omega_K\omega_I)^2}{z_Kz_I}\cos^2\frac{k_zd}{2}}. \qquad (26)$$

An example of this dispersion is given in Fig. 3. In Fig. 4 the oscillations of these modes are sketched. The longitudinal modes in the left-hand side of the figure have a finite dispersion, provided that $\gamma\neq0$, in other words, if the electron gas has finite compressibility.

In addition to the two longitudinal plasma modes there is *one* transverse-optical plasma mode and *one* transverse polarized pole of the dielectric function at $\omega=0$, representing the superconducting dielectric response for fields and currents polarized along the *c* axis and with the direction of propagation parallel to the planes. In the right-hand panel of Fig. 4 the oscillations of these modes are sketched. The transverse mode is coupled to electromagnetic radiation for long wavelengths, giving rise to coupled plasma-polariton

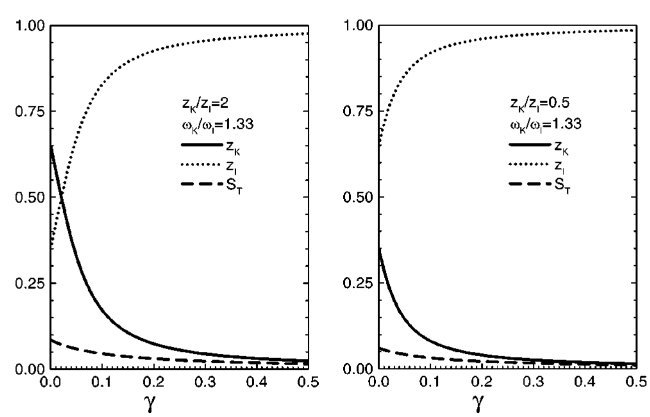

FIG. 2. Oscillator strengths as a function of γ.

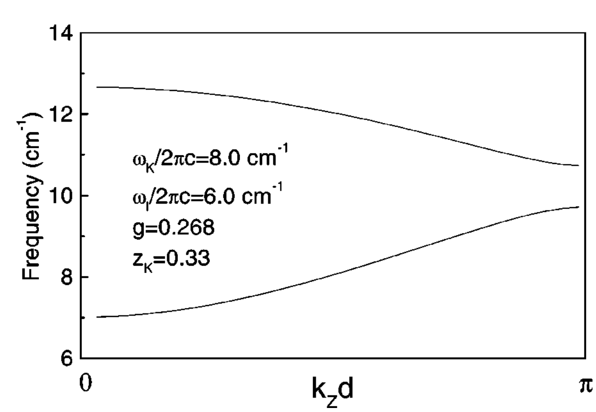

FIG. 3. Dispersion of the longitudinal JPR's.

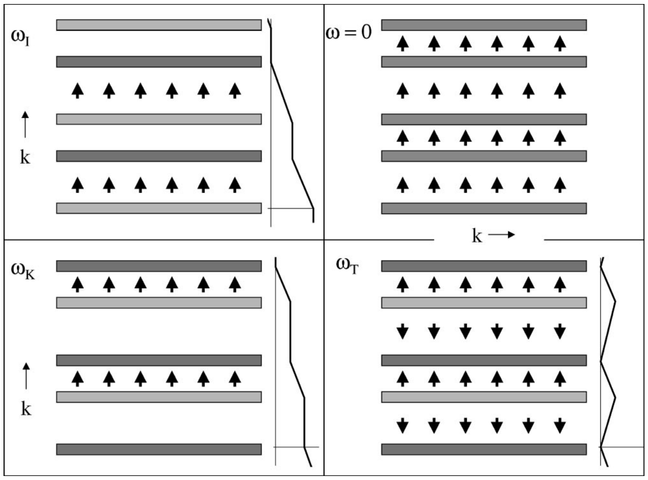

FIG. 4. Snapshot of the currents (arrows) and planar charge fluctuation amplitudes (indicated by gray scales) of the two sets of transverse and longitudinal modes with polarization along the c direction. On the right-hand side of each plot the voltage distribution is indicated.

modes instead of separate photons and transverse-optical plasma modes. The plasma-polariton dispersion follows from Maxwell's equations in dielectric media and is given by the relation between wave number k (in the solid) and frequency (ω)

$$k^2c^2=\epsilon(\omega)\omega^2. \tag{27}$$

With the dielectric constant given by Eq. (15) we get

$$\omega(k)=\left\{\left(\frac{\tilde{\omega}_I^2+\tilde{\omega}_K^2+k^2c^2}{2}\right)\right. \pm\left.\sqrt{\left(\frac{\tilde{\omega}_I^2+\tilde{\omega}_K^2+k^2c^2}{2}\right)^2-\tilde{\omega}_I^2\tilde{\omega}_K^2-k^2c^2\tilde{\omega}_T^2}\right\}^{1/2}. \tag{28}$$

This dispersion is sketched in Fig. 5. We see that there are two plasma-polariton branches. The lowest starts at frequency ω_I in the long-wavelength limit (small k) and quickly merges with the transverse-optical (TO) plasma frequency ω_T as the wavelength is reduced below c/ω_T, which is of the order of a millimeter if $\omega_T/2\pi$ is of order 300 GHz. At these shorter wavelengths the character is almost purely the TO JPR. The upper branch corresponds to a conventional transverse Josephson plasmon[18] (without the adjective ''optical'') as it merges with the light line in the short-wavelength limit. The lower branch is novel. It corresponds to a real polarization wave, and these modes can be used to convert electromagnetic radiation into microscopic currents, or vice versa.

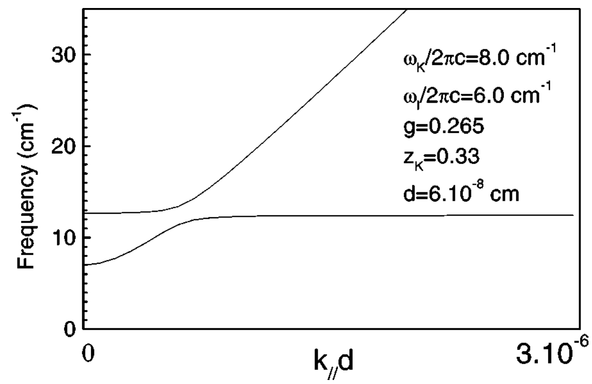

FIG. 5. Dispersion of the transverse JPR's for small values of $k_{\|}$ (about one millionth of the Brillouin zone).

Finally it should be added that in the short-wavelength limit, the presence of finite compressibility gives rise to nontrivial dependence of $\epsilon(k,\omega)$ on the wave vector k, which for the transverse modes are parallel to the planes. This effect becomes prominent on short wavelengths of the order of lattice spacings. Figure 5 was sketched on the scale of k of the order of a few inverse millimeter. On the scale of a few inverse angstrom the k dependence of $\epsilon(k,\omega)$ will give rise to finite k dispersion of $\tilde{\omega}_I(k)$, $\tilde{\omega}_T(k)$, and $\tilde{\omega}_K(k)$.

VII. CONCLUSIONS

An expression was derived for the dielectric function of superconductors consisting of two Josephson coupled superconducting layers per unit cell, taking into account the effect of finite compressibility of the electron fluid. In this model two longitudinal and one transverse-optical Josephson plasma resonance exist. The latter mode appears as a resonance in the transverse dielectric function, and it couples directly to the electric-field vector of infrared radiation. A shift of all plasma frequencies and a reduction of the intensity of the transverse-optical Josephson plasmon is shown to result from the finite compressibility of the electron fluid.

ACKNOWLEDGMENTS

We wish to thank L.N. Bulaevksii for drawing our attention to the dispersion term. This investigation was supported by the Netherlands Foundation for Fundamental Research on Matter (FOM) with financial aid from the Nederlandse Organisatie voor Wetenschappelijk Onderzoek (NWO).

[1] P.W. Anderson, Science **279**, 1196 (1998).
[2] D. van der Marel and A. Tsvetkov, Czech. J. Phys. **46**, 3165 (1996).
[3] H. Shibata and T. Yamada, Phys. Rev. Lett. **81**, 3519 (1998).
[4] D. Munzar, C. Bernhard, A. Golnik, J. Humlicek, and M. Cardona, Solid State Commun. **112**, 365 (1999).
[5] M. Grüninger, D. van der Marel, A.A. Tsvetkov, and A. Erb, Phys. Rev. Lett. **84**, 1575 (2000).
[6] V. Zelezny, S. Tajima, T. Motohashi, J. Shimoyama, K. Kishio, and D. van der Marel, J. Low Temp. Phys. **117**, 1019 (1999).

[7] D. Dulic, Ph.D. thesis, Rijks Universiteit Groningen, 2000; D. Dulić, A. Pimenov, D. van der Marel, D.M. Broun, S. Kamal, W.N. Hardy, A.A. Tsvetkov, I.M. Sutjaha, Ruixing Liang, A.A. Menovsky, A. Loidl, and S.S. Saxena, Phys. Rev. Lett. **86**, 4144 (2001).
[8] H. Shibata, Phys. Rev. Lett. **86**, 2122 (2001); T. Kakeshita, S. Uchida, K.M. Kojima, S. Adachi, S. Tajima, B. Gorshunov, and M. Dressel, Phys. Rev. Lett **86**, 4140 (2001).
[9] W.E. Lawrence and S. Doniach, in *Proceedings of the 12th International Conference on Low Temperature Physics, Kyoto, 1970*, edited by E. Kanda (Keigaku, Tokyo, 1970).
[10] A.J. Leggett, Prog. Theor. Phys. **36**, 901 (1966).
[11] T. Koyama and M. Tachiki, Phys. Rev. B **54**, 16 183 (1996).
[12] H.A. Fertig and S. Das Sarma, Phys. Rev. Lett. **65**, 1482 (1990).
[13] E.H. Hwang and S. Das Sarma, Int. J. Mod. Phys. B **12**, 2769 (1998).
[14] D. van der Marel, Phys. Rev. B **51**, 1147 (1995).
[15] S.N. Artemenko and A.G. Kobelkov, Physica C **253**, 373 (1995).
[16] See, e.g., G.D. Mahan, *Many Particle Physics*, 2nd ed. (Plenum, New York, 1990), Chap. 5.
[17] The order of presentation of the main results is not the order in which the expressions are derived at this point: To derive Eq. (19) we used Eqs. (22) and (15) to express $\tilde{\omega}_T$ in terms of γ, $\tilde{\omega}_I$, and $\tilde{\omega}_K$. This is inserted in Eq. (18), leading to Eq. (19).
[18] M. Tachiki, T. Koyama, and S. Takahashi, Phys. Rev. B **50**, 7065 (1994).