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Critical depinning force and vortex lattice order in disordered superconductors

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We simulate the ordering of vortices and its effects on the critical current in superconductors with varied vortex-vortex interaction strength and varied pinning strengths for a two-dimensional system. For strong pinning the vortex lattice is always disordered and the critical depinning force only weakly increases with decreasing vortex-vortex interactions. For weak pinning the vortex lattice is defect-free until the vortex-vortex interactions have been reduced to a low value, when defects begin to appear with a simultaneous rapid increase in the critical depinning force. In each case the depinning force shows a maximum for noninteracting vortices. The relative height of the peak increases and the peak width decreases for decreasing pinning strength in agreement with experimental trends associated with the peak effect. We show that scaling relations exist between the distance between defects in the vortex lattice and the critical depinning force.

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I. INTRODUCTION

A large class of condensed matter systems can be represented as an elastic lattice interacting with a quenched random substrate. Vortices in type II superconductors, which can be pinned by random defect sites in the material, are a particularly ideal example of such a system since the substrate and lattice interactions can be readily tuned experimentally. In this case, a rich variety of static[1–6] and dynamical phases[2,7,8] can occur due to the competition between the repulsive vortex-vortex interactions, which tend to order the vortex lattice, and the attractive vortex-pin interactions, which tend to disorder the vortex lattice. Unlike the early studies of collective pinning of a defect-free elastic lattice, recent experimental and theoretical work points to the importance of topological defects in pinned vortex matter. If the pinning in the sample is sufficiently strong, the vortex lattice will be highly defected; conversely, in a sample with weak pinning the vortex lattice can be relatively ordered and contain only a few defects. In contrast with the purely elastic model, however, the relationship between the critical force and the density of flux line defects is not known. Moreover, applications of superconducting materials require that the critical current be high, and so a key question is how the number of defects in the vortex lattice can affect the critical current.

In a given sample, i.e., with a given realization of quenched disorder, the relative strength of the pinning can be increased by lowering the vortex-vortex interaction strength, i.e., by softening the lattice. Vortex-vortex interactions become weak at low applied fields, when vortices are dilute, as well as very near H_{c2} and T_c, where the induction of the vortices overlaps significantly. In the latter regime one encounters the peak effect: a rapid increase in the critical current before it collapses to zero at the superconducting-normal phase boundary. At these two extreme field values, the pinning interaction dominates and the vortex lattice is expected to be highly disordered. Indeed, direct evidence for vortex lattice disordering through the peak effect regime has been observed in neutron scattering measurements.[9] A reentrant disordering has been found in the low field regime where the vortex lattice again softens.[4,10] Bitter decoration experiments have also provided evidence for the disordering of the vortex lattice at low fields.[11] In each of these cases the critical current J_c is high due to the dominance of pinning.

Experiments on superconducting samples with different amounts of pinning have revealed some systematic trends. The peak effect is much sharper and more pronounced in cleaner samples,[6,5,12] while in samples with stronger pinning the width of the peak region increases and *the relative valley-to-peak height decreases*, although the overall critical current is higher.[6,12] In samples with the strongest pinning, J_c is high through most of the applied field range, and the peak is completely absent. A possible interpretation of this behavior connects vortex lattice defects with high critical currents. A detailed understanding of the observed effects as a possible transition/crossover between regimes with low and high critical currents, i.e., between, say, a Bragg-glass-like ordered phase with few defects and a vortex-glass/pinned-liquid-like disordered vortex phase is still lacking. A key reason for this uncertainty is the absence of a systematic connection between the number of defects in the vortex lattice, the softness of the lattice, and the critical current.

In this work, we directly examine the effect of vortex lattice softness on the critical current through a series of numerical simulations of samples with different pinning strengths. For strong pinning the vortex lattice is highly defective for the entire range of vortex-vortex interactions investigated. The critical depinning force, which is proportional to the critical current, only weakly increases with decreasing vortex-vortex interaction strength, reaching a maximum for noninteracting vortices. For the weakest pinning strengths, on the other hand, the lattice is almost defect-free over a large range of vortex-vortex interaction strengths. When defects begin to appear in the softest vortex lattices, the critical depinning force increases sharply from its low value to a peak at zero vortex-vortex interaction. The behavior for intermediate pinning is intermediate between these extremes. We find that the relative height of the critical current peak increases for decreasing pinning while the width of the peak decreases. Thus, the most pronounced peaks occur for the weakest pinning. These results are in agreement with the experimental trends described above. We discuss how these results can be connected to the behavior of the peak effect in superconductors with varying pinning strength.

II. SIMULATION

We consider a two-dimensional (2D) slice of a system of superconducting vortices interacting with a random pinning background. The applied magnetic field $\mathbf{H}=H\hat{\mathbf{z}}$ is perpendicular to our sample, and we use periodic boundary conditions in x and y. The $T=0$ overdamped equation of motion for a vortex i is

$$\mathbf{f}_i=\eta\frac{d\mathbf{r}_i}{dt}=\mathbf{f}_i^{vv}+\mathbf{f}_i^{vp}+\mathbf{f}_d=\eta\mathbf{v}_i\,, \tag{1}$$

where $\mathbf{v}_i$ is the velocity of vortex i and $\eta=1$ is the damping coefficient. The total force on vortex i from the other vortices is $f_i^{vv}=\Sigma_{j=1}^{N_v}A_v f_0 K_1(|\mathbf{r}_i-\mathbf{r}_j|/\lambda)\hat{\mathbf{r}}_{ij}$, where $\mathbf{r}_i$ is the position of vortex i, λ is the penetration depth, $f_0=\Phi_0^2/8\pi^2\lambda^3$, the prefactor A_v is used to vary the vortex lattice softness, and K_1 is the modified Bessel function. The pinning $\mathbf{f}_i^{vp}$ is modeled as randomly placed attractive parabolic wells of radius $r_p=0.15\lambda$ with $f_i^{vp}=(f_p/r_p)(|\mathbf{r}_i-\mathbf{r}_k^{(p)}|)\Theta(r_p-|\mathbf{r}_i-\mathbf{r}_k^{(p)}|)\hat{\mathbf{r}}_{ik}^{(p)}$, where $\mathbf{r}_k^{(p)}$ is the location of pin k, f_p is the maximum pinning force, which is varied from $0.1f_0$ to $3.0f_0$, Θ is the Heavside step function, and $\hat{\mathbf{r}}_{ik}^{(p)}=(\mathbf{r}_i-\mathbf{r}_k^{(p)})/|\mathbf{r}_i-\mathbf{r}_k^{(p)}|$. The pin density is $n_p=3.0/\lambda^2$ and the vortex density is $n_v=0.75/\lambda^2$. We simulate a $36\lambda\times36\lambda$ system containing $N_v=864$ vortices and $N_p=3887$ pins. We initialize the vortex positions by performing simulated annealing, starting from a high temperature and slowly cooling to $T=0$. This method of preparing the lattice is similar to field cooled experiments. To identify the depinning force f_c we apply a slowly increasing uniform driving force $\mathbf{f}_d$ on the vortices in the x direction, which would correspond to a Lorentz force from an applied current $\mathbf{J}=J\hat{\mathbf{y}}$. We use a time step of $dt=0.02$ and spend 10^4 time steps at each current value. For each drive increment we measure the average vortex velocity in the direction of drive, $V_x=(1/N_v)\Sigma_1^{N_v}v_x$. The f_d versus V_x curve corresponds experimentally to a $V(I)$ curve. The depinning force f_c is defined as the drive at which $V_x>0.03$.

III. VORTEX ORDER AND PINNING FOR VARIED VORTEX-VORTEX INTERACTION STRENGTH

We first consider the effect of the vortex lattice softness on the stationary vortex lattice. In Fig. 1 we show the Delaunay triangulation for a system with $f_p=0.25f_0$ for decreasing vortex-vortex interaction A_v, after the lattice has been annealed and with no driving force applied. Defect sites in the vortex lattice are indicated by circles. In Fig. 1(a) for $A_v=4.0$, the vortex lattice contains no defects. In Fig. 1(b) for $A_v=3.0$ a small number of 5–7 defect pairs appear. In Fig. 1(c), at $A_v=2.0$ the vortex lattice is considerably disordered but some domains of order are still present. For softer vortex lattices in Figs. 1(d)–1(f), the vortex lattice becomes progressively more disordered by the underlying random pinning. Since the simulation is for a two-dimensional system, we expect the system to be defective for any pinning strength.[13] The defect-free situation likely results from the finite size of the sample.

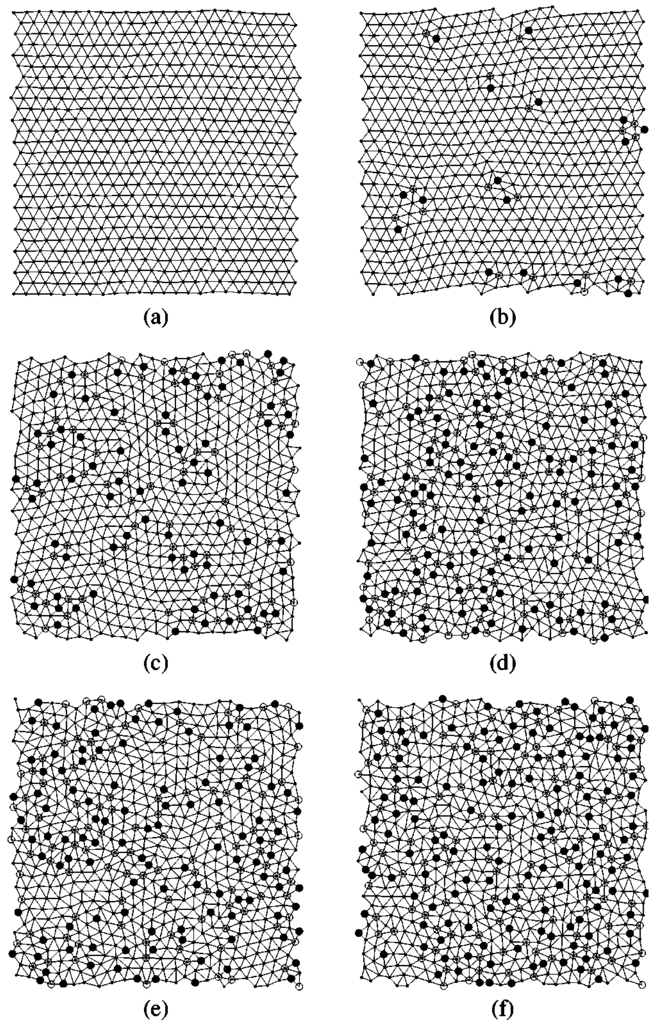

FIG. 1. The Delaunay triangulation of the vortex lattice for decreasing vortex-vortex interaction strength A_v in a sample with $f_p=0.25f_0$. Dark circles indicate fivefold coordinated vortices; open circles indicate sevenfold coordinated vortices. $A_v=$ (a) 4.0, (b) 3.0, (c) 2.0, (d) 1.0, (e) 0.75, and (f) 0.50. The vortex lattice is relatively ordered in (a). In (b) more defects appear. (c) and (d) show different domains of vortex orientation. In (e) and (f) the vortex lattice is highly disordered.

By applying a transport current to the annealed lattices, we determine the critical depinning force f_c that must be applied before the vortices begin to move. In Fig. 2(a) we plot f_c versus A_v for a sample with $f_p=0.25f_0$. For comparison, we measure the amount of order in the lattice before depinning using the Delaunay triangulation, and in Fig. 2(b) we plot P_6, the fraction of sixfold coordinated vortices, ver-

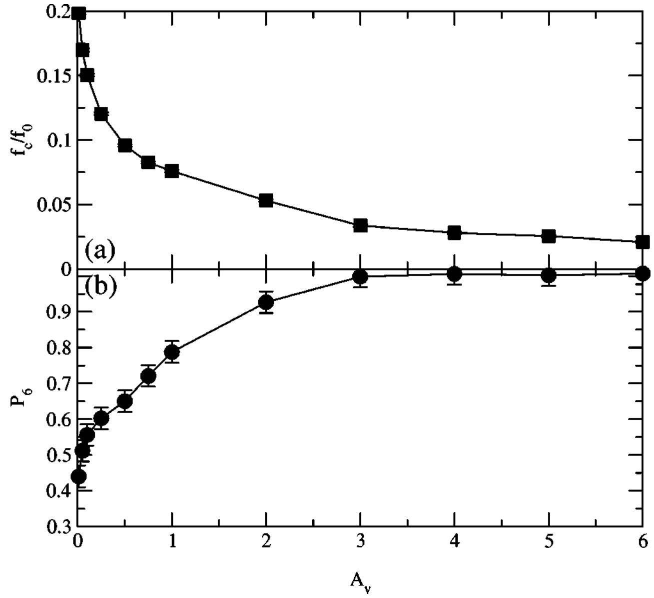

FIG. 2. (a) The critical depinning force f_c versus A_v for a sample with $f_p = 0.25 f_0$. (b) The fraction of sixfold coordinated vortices P_6 versus A_v. As the vortex-vortex interaction is lowered f_c increases while P_6 decreases.

sus A_v. Here $P_6 = 1.0$ indicates a perfect triangular lattice. As A_v is lowered f_c increases and simultaneously the order in the lattice, P_6, decreases. The maximum value of f_c occurs at $A_v \sim 0$ which coincides with the minimum value of P_6. Thus the softer lattices with low values of A_v and large amounts of disorder are more strongly pinned than stiffer, more ordered lattices.

IV. HEIGHT AND WIDTH OF CRITICAL DEPINNING FORCE PEAK

The shape and magnitude of the critical current peak at $A_v = 0$ are affected by the strength of the pinning in the sample. To demonstrate this, in Fig. 3(a) we plot f_c/f_p versus A_v. Scaling the curves with f_p in this way causes all of the curves to approach $f_c/f_p = 1$ at $A_v = 0$. We find that the relative height of the peak increases as the pinning becomes weaker. For example, comparing $f_p = 0.25$ (bottom curve) with $f_p = 3.0$ (top curve), the critical current f_c increases by a factor of 8 from $f_c = 0.1$ at $A_v = 6.0$ to $f_c = 0.8$ at $A_v = 0.01$ in the weakly pinned sample, whereas in the strongly pinned sample f_c increases only by a factor of 2 from $f_c = 0.38$ at $A_v = 6.0$ to $f_c = 0.8$ at $A_v = 0.01$. The peak not only becomes higher for weaker pinning, but also it becomes much narrower as can be seen by comparing the widths of the peaks in Fig. 3(a). As shown in Fig. 3(b), in the strongly pinned samples such as $f_p = 3.0 f_0$ with broad low peaks, P_6 is low over the entire range of A_v, indicating that the vortex lattice is always strongly disordered by the pinning. In contrast, for weakly pinned samples such as $f_p = 0.25 f_0$, P_6 drops a large amount from $P_6 \approx 1$ at large A_v, indicating a nearly perfect lattice, to $P_6 \approx 0.4$ at the lowest A_v, indicating a large amount of disorder in the lattice.

In Fig. 4 we show explicitly how the relative peak height

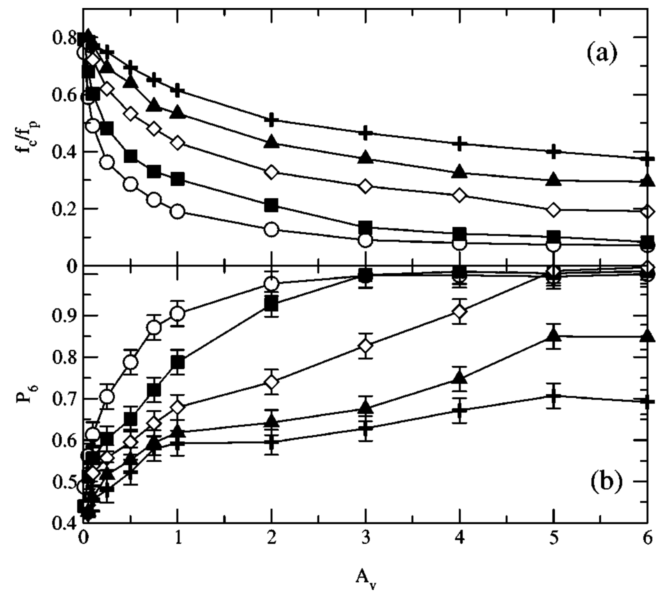

FIG. 3. Scaled critical depinning force and P_6 versus A_v for samples with different pinning strengths. Open circles, $f_p = 0.10 f_0$; filled squares, $f_p = 0.25 f_0$; open diamonds, $f_p = 0.75 f_0$; filled triangles, $f_p = 1.5 f_0$; plus signs, $f_p = 3.0 f_0$. (a) The plot of f_c/f_p versus A_v shows that the peak at low A_v becomes sharper for weakening pinning strength f_p. (b) The corresponding P_6 values show that for large $f_p = 3.0$ the vortex lattice is disordered over the entire range of A_v.

$H = f_c(A_v = 0.01)/f_c(A_v = 6.0)$ and the width at half maximum dW vary with pinning strength f_p. Here we see that the relative strength and sharpness of the peak increase with weaker pinning. The width dW depends linearly on f_p, while the relative height H shows a nonlinear increase that cannot be fitted by a simple functional form.

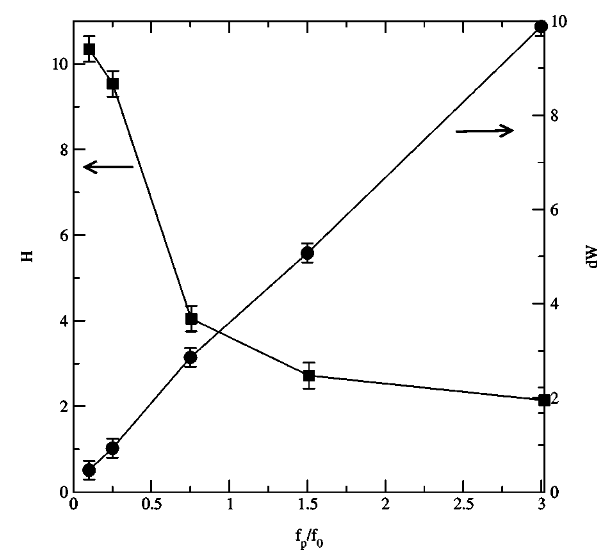

FIG. 4. Squares: the relative height $H = f_c(A_v = 0.01)/f_c(A_v = 6.0)$ of the peak in f_c for samples with different pinning strength f_p. Circles: The width of the peak dW at half maximum as a function of f_p. The peak becomes sharper and more pronounced for weaker pinning.

The results in Fig. 3 and Fig. 4 suggest that in clean superconductors, where the vortex lattice softness has a much more significant impact on the effectiveness of the pinning, the critical current enhancements should be both sharper and of larger relative height than in strongly pinned superconductors. This behavior agrees with the results of recent experiments[5,6] on the peak effect. Our results also suggest that the peak can be made arbitrarily sharp simply by adjusting the disorder strength, and that such large peaks in the critical current must be accompanied by the proliferation of defects.

V. CRITICAL DEPINNING FORCE DEPENDENCE ON DEFECT DENSITY

Using the data obtained by varying A_v, we can determine the relationship between the critical current and the density of defects. In Fig. 5(a) we plot f_c/f_p versus P_6, showing that the curves for samples with different pinning strengths roughly collapse. At lower values of P_6 when the vortex lattice becomes defected, the critical current increases rapidly with the onset of defects. In Fig. 5(b) we plot f_c/f_p versus the distance between vortex lattice defects, $d_d = 1/\sqrt{1.0-P_6}$. In samples with weak pinning there is a range of high A_v values that produce well ordered vortex lattices. In this regime, when there are very few defects in the vortex lattice which are far apart, we find that the f_c/f_p values do not collapse, and that the rate at which f_c/f_p changes with d_d is slow. For $d_d<3.0$, or when the lattice contains approximately 10% or more defects, all the curves collapse and

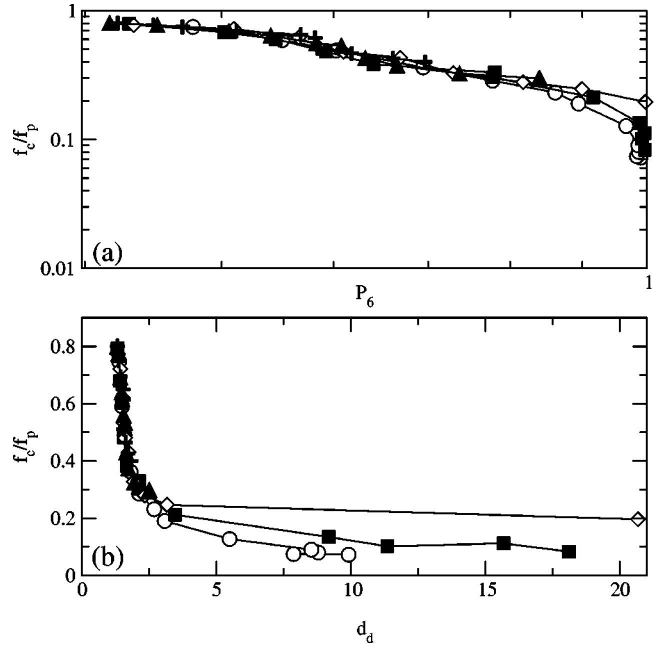

FIG. 5. (a) Scaled critical depinning force f_c/f_p versus P_6. (b) f_c/f_p versus the distance between defects d_d showing a collapse for $d_d<3.0$ coinciding with a rapid increase in f_c.

f_c/f_p increases rapidly with decreasing d_d. The behavior of f_c versus d_d suggests that there is a critical distance between defects below which J_c begins to rapidly increase.

We can compare the dependence of the critical depinning force on the defect density with the collective pinning results

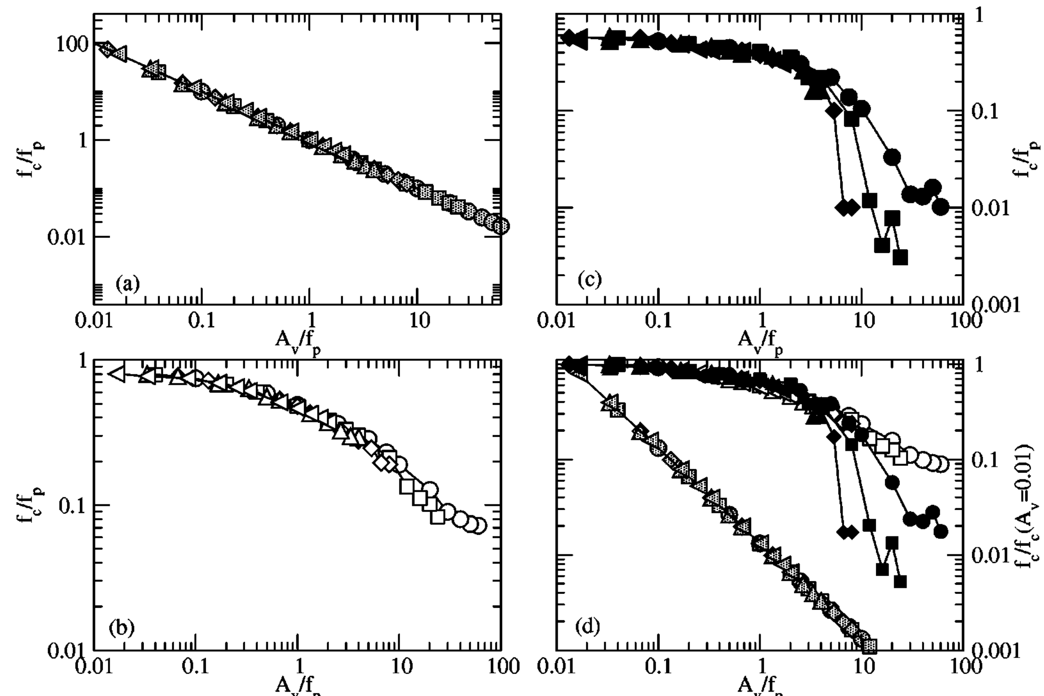

FIG. 6. (a) Shaded symbols: f_c/f_p as predicted by LO theory. (b) Open symbols: f_c/f_p measured in the simulations. (c) Filled symbols: f_c/f_p computed from the distance between dislocations. (d) All three methods of obtaining f_c are scaled by the value at $A_v=0.01$, and plotted together. For all panels, circles, $f_p=0.10f_0$; squares, $f_p=0.25f_0$; diamonds, $f_p=0.75f_0$; triangle up, $f_p=1.5f_0$; triangle left, $f_p=3.0f_0$.

of the Larkin-Ovchinnikov (LO) theory.[14] In Fig. 6(a) we plot f_c as predicted by LO, using $f_c \propto f_p^2/A_v$. Figure 6(b) shows f_c computed from the distance between dislocations, using $f_c \propto f_p/d_d$. The value of f_c measured in the simulations is shown in Fig. 6(c). We compare f_c obtained by the LO and dislocation distance methods with the actual f_c in Fig. 6(d). The agreement with the LO prediction is poor, but that with the dislocation distance is good out to values $A_v/f_p \sim 5$, beyond which d_d becomes of the order of the system size.

VI. CONCLUSION

We have investigated the dependence of the critical depinning force f_c and vortex lattice topological order P_6 on the vortex lattice rigidity for different values of pinning strength. The study is restricted to the two-dimensional case which is appropriate for vortex lattices where the longitudinal correlation length is large enough to be effectively given by the sample thickness. In all cases, f_c increases with decreasing vortex-vortex interaction strength, reaching a maximum for noninteracting vortices, as expected. Although for very weak pinning and strong vortex-vortex interaction the lattice is relatively defect-free, for strong pinning the lattice is defective for the entire vortex interaction range investigated. These results are in accord with previous studies. We have, in addition, studied the systematic variation of f_c with varying pinning and intervortex interaction. The results show that the pinning force variation departs significantly from the expectations of an elastic picture. The increase in f_c toward the maximum occurs very rapidly with decreasing interaction or increasing pinning. The relative height increases while the width decreases with decreasing pinning strength, strongly reminiscent of the peak effect phenomenon. Furthermore, a pseudo-Larkin picture where a plastic length, the distance between topological defects, replaces the elastic correlation length, provides a good account of the variation of f_c. We also find evidence that there is a critical value of the plastic length, typically spanning 3–5 vortices on each side, at which the rapid crossover occurs. These results should be relevant to experiments in very weak pinning (quasi-2D) flux lattices, as well as to general systems of two-dimensional lattices with quenched disorder where the lattice interaction can be tuned.

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