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Observation of progressive motion of ac-driven solitons

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We report the experimental observation of ac-driven phase-locked motion of a topological soliton at a nonzero average velocity in a periodically modulated lossy medium. The velocity is related by a resonant condition to the driving frequency. The observation is made in terms of the current-voltage, $I(V)$, characteristics for a fluxon trapped in an annular Josephson junction placed into dc magnetic field. Large zero-crossing constant-voltage steps, exactly corresponding to the resonantly locked soliton motion at different orders of the resonance, are found on the experimental $I(V)$ curves. A measured dependence of the size of the steps versus the external magnetic field is in good agreement with predictions of an analytical model based on the balance equation for the fluxon's energy. The effect has a potential application as a low-frequency voltage standard.

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The important role played by nonlinear excitations in the form of solitons in various physical systems is commonly known. However, experimental observation of dynamical effects produced by solitons is often difficult because real systems may be far from their idealized mathematical models which give rise to soliton solutions. Among perturbations that destroy soliton effects, dissipation is the most important one. To compensate dissipative losses and thus make the soliton dynamics visible, one must apply an external force that supports (in particular, *drives*) a soliton.[1]

Solitons of the simplest type are topological *kinks*, a well-known example being a magnetic flux quantum (fluxon) in long Josephson junctions (LJJ's).[2,3] A fluxon in LJJ can be easily driven by bias current applied to the junction. The motion of a fluxon gives rise to dc voltage V across the junction, which is proportional to the fluxon's mean velocity. Varying the dc bias current I, one can produce a dependence $V(I)$, which is the main dynamical characteristic of LJJ. An experimentally obtained $I(V)$ curve easily allows one to identify the presence of one or more fluxons trapped in LJJ.[2]

Microwave field irradiating LJJ gives rise to an ac drive acting on the fluxon. In a spatially homogeneous lossy system, an ac drive may only support an oscillatory motion of a kink, which is hard to observe in LJJ due to the absence of dc voltage. However, it was predicted[4] that an ac drive can support motion of a kink with a *nonzero* average velocity u in a system with a periodic spatial modulation. Indeed, a moving kink passes the modulation length (period) L during the time L/u. If this time is commensurate with the period $2\pi/\omega$ of the ac drive, i.e., $m(L/u)=2\pi/\omega$ with an integer m, or

$$u=m(L\omega/2\pi), \tag{1}$$

one may expect a resonance (of order m) between the two periodicities. In other words, a moving kink may be phase-locked to the ac drive. This provides for permanent transfer of energy from the drive to the kink, making it possible to compensate dissipative losses. The energy balance gives rise to a minimum (*threshold*) value $\Gamma_{\rm thr}$ of the ac drive's amplitude Γ, which can compensate the losses and support the motion of the ac-driven kink.

A more general case, when the system is simultaneously driven by the ac field and dc bias current I, was considered in Ref. 5. It was predicted that the corresponding $V(I)$ characteristic has *steps* (constant-voltage segments) at the resonant velocities (1). The motion of the fluxon under the action of pure ac drive then corresponds to *zero crossings*, when the steps cross the axis $I=0$. In fact, the most straightforward way to observe the ac-driven motion of the soliton is through zero-crossings on the $V(I)$ characteristic.

A formally similar feature is known in *small* Josephson junctions as Shapiro steps:[2] ac drive applied to the junction gives rise to dc voltage across it (an inverse ac Josephson effect). However, a drastic difference of the effect sought for in this work from the Shapiro steps is that they are only possible at high frequencies exceeding the junctions's plasma frequency, while the ac-driven motion of the fluxon can be supported by the ac drive with an arbitrarily low frequency. This circumstance also opens way for application to the design of voltage standards using easily accessible sources of low-frequency radiation, which are not usable with the usual voltage standards based on small Josephson junctions. Moreover, because the single-valued dc voltage in LJJ is controlled not only by the frequency and amplitude of the ac drive, but also by locally applied magnetic field (see below), the system studied here may be useful for designing an ac voltage standard.

The inverse Josephson effect was also studied in detail in ac-driven finite-length linear junctions with reflecting edges, corresponding to zero external magnetic field.[6] A fluxon turns into an antifluxon while bouncing from the edge. In that case, the system of constant-voltage steps is complicated, on the contrary to a very simple set of two symmetric steps in the circular junction, see below. This is explained by the fact that the *shuttle motion* of the polarity-reversing fluxon/antifluxon in the linear junction is different from the progressive motion of the fluxon, without any polarity reversal, in the circular LJJ.

An objective of this paper is to report direct experimental observation of the ac-driven fluxon motion in periodically modulated LJJ's. Frequently, it is assumed that the necessary periodic spatial modulation along the junction can be in-

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duced by periodically changing the thickness of the dielectric barrier separating two superconductors. In the presence of the losses and drive, the modulated LJJ is described by the perturbed sine-Gordon (sG) equation,

$$\phi_{tt}-\phi_{xx}+\left(1+\varepsilon\sin\frac{2\pi x}{L}\right)\sin\phi=-\alpha\phi_t-\gamma-\Gamma\sin\omega t, \tag{2}$$

where x and t are the length along the junction and time, measured, respectively, in units of the Josephson length and inverse plasma frequency, ε is the normalized modulation amplitude, while γ and Γ are the dc and ac bias current densities, both normalized to the junction's critical current density.

The harmonic modulation of the local magnitude of the maximum Josephson current, assumed by this model, is very hard to realize in an experiment due to the exponential dependence of the critical current on the thickness of the dielectric barrier. A much easier and fully controllable way to induce a harmonic periodic modulation is to use an *annular* (ring- shaped) LJJ, to which uniform dc magnetic field is applied in its plane.[7] As it was demonstrated experimentally, fluxons can be readily trapped in annular LJJs.[8] In this case, the sG model takes the form

$$\phi_{tt}-\phi_{xx}+\sin\phi+h\sin\frac{x}{R}=-\alpha\phi_t-\gamma-\Gamma\sin\omega t, \tag{3}$$

where h is (renormalized) magnetic field, and R is the radius of the ring. In the case of the annular junction, solutions are subject to periodic boundary conditions, $\phi_x(x+2\pi R)\equiv\phi_x(x)$ and $\phi(x+2\pi R)\equiv\phi(x)+2\pi N$, N being the number of the trapped fluxons (in this work, $N=1$). Comparison with the experiment shows that, unlike the model (2), the one (3) is, virtually, exact.[9]

We assume the spatial size of the fluxon, which is ~ 1 in the present notation, to be much smaller than the circumference $L\equiv 2\pi R$ of the annular junction. Large L imposes an upper limit on the driving frequency ω which can support the ac-driven motion: as the fluxon's velocity cannot exceed the maximum (Swihart) group velocity of electromagnetic waves in LJJ, which is 1 in our notation, Eq. (1) implies that $\omega \lesssim 1/L$.

A different type of ac drive for fluxons in circular LJJs was proposed in Ref. 7, viz., ac magnetic field. In this case, the terms $h\sin(x/R)$ and $\Gamma\sin(\omega t)$ in Eq. (3) are replaced by a single one, $h\sin(x/R)\sin(\omega t)$, which may be decomposed into two waves traveling in opposite directions, $(1/2)\times[\cos(x/R-\omega t)-\cos(x/R+\omega t)]$. As it was shown,[7] either traveling wave may capture a fluxon, dragging it at the wave's phase velocity $\pm\omega R$. A similar model was proposed in Ref. 10, in which the fluxon is dragged by *rotating* magnetic field. A difference of our model (which corresponds to the real experiment reported below) is the separation between the fields that induce the spatial modulation and ac force. The separation makes it possible to control the dynamics in a more flexible way.

It is straightforward to derive an equation of motion for the fluxon in the adiabatic approximation, following the lines of Refs. 4 and 5 ($\dot{\xi}\equiv d\xi/dt$):

$$\frac{d}{dt}\left(\frac{\dot{\xi}}{\sqrt{1-\dot{\xi}^2}}\right)=\frac{\pi h}{4\sqrt{1-\dot{\xi}^2}}\cos\frac{\xi}{R}-\frac{\alpha\dot{\xi}}{\sqrt{1-\dot{\xi}^2}}+\frac{\pi}{4}[\gamma+\Gamma\sin(\omega t)]. \tag{4}$$

For further analysis, one may assume, following Refs. 7 and 10, that, in the lowest approximation, the fluxon is moving at a constant velocity $\dot{\xi}_0\equiv u$ belonging to the resonant spectrum (1), so that $\xi(t)=ut+R\delta$, where δ is a phase-locking constant. Then, the first correction to the instantaneous fluxon's velocity, generated by the spatial modulation, can be easily found from Eq. (4),

$$\dot{\xi}_1=(\pi Rh/4u)(1-u^2)\sin[(u/R)t+\delta]. \tag{5}$$

The approximation applies provided that the correction (5) is much smaller than the unperturbed velocity u, which amounts to $Rh\ll u^2/(1-u^2)$.

A key ingredient of the dynamical analysis is the *energy-balance* equation.[7] In the model (3) it is based on the correction (5) to the velocity[4,5] (while in the above-mentioned models with ac magnetic fields[7,10] the approximation $\dot{\xi}=u$ was sufficient). In the case of the fundamental resonance, with $m=1$ in Eq. (1), i.e., $u=R\omega$, the energy balance yields, after a straightforward algebra,

$$\gamma=\frac{4\alpha R\omega}{\pi\sqrt{1-(R\omega)^2}}-\frac{\pi h\Gamma}{8R\omega^2}[1-(R\omega)^2]\cos\delta. \tag{6}$$

Setting $|\cos\delta|=1$ and $\gamma=0$ in Eq. (6) gives a minimum (*threshold*) amplitude of the ac drive which can support the fluxon's motion in the absence of the dc bias current, $\Gamma_{\rm thr}=(32/\pi^2h)\alpha R^2\omega^3[1-(R\omega)^2]^{-3/2}$. For the comparison with experimental results, the most important consequence of Eq. (6) is an interval of dc bias current density γ in which the phase-locked ac-driven motion of the fluxon is expected. It is produced by varying $\cos\delta$ in Eq. (6) between -1 and $+1$:

$$\gamma_1^-<\gamma<\gamma_1^+;\quad \gamma_1^\pm\equiv\frac{4\alpha R\omega}{\pi\sqrt{1-(R\omega)^2}}\pm\frac{\pi R\Gamma[1-(R\omega)^2]}{8(R\omega)^2}h. \tag{7}$$

Note that the size of the interval strongly depends on the driving frequency, while in the model with the ac magnetic field[7] it does not depend on ω at all, provided that $2\pi R\gg 1$.

Experiments have been performed with Nb/Al-AlO$_x$/Nb Josephson annular junction[11] with the mean diameter $2R=95$ μm and the annulus width 5 μm, applying the bias current I and measuring the dc voltage V across the junction. The distribution of the bias current was uniform, which was concluded from measurement of the critical current I_c in the state without trapped fluxons at $H=0$. I_c was found to be

about 0.9 of its value for the small junction. The annular LJJ had the Josephson length $\lambda_J \approx 30$ μm and plasma frequency 50 GHz. Note that these parameters imply the ratio ~ 10 of the junction's length $2\pi R$ to the fluxon's size, which is $\sim \lambda_J$, i.e., the junction may indeed be regarded as a long one. The measurements were done at the temperature 4.2 K, using a shielded low-noise measurement setup. The ac driving current with the frequency $f = \omega/2\pi$ between 5 and 26 GHz was supplied by means of a coaxial cable ending with a small antenna inductively coupled to the junction. The antenna was oriented coaxial to the dc bias current supplied through the electrodes, therefore the ac magnetic field was perpendicular to the dc magnetic field. Thus, the magnetic component of the ac signal induced the driving force of the same type as the bias current. The ac power levels mentioned below pertain to the input at the top of the cryostat.

Following Ref. 9, trapping of a fluxon in the junction was achieved by cooling the sample below the critical temperature $T_c \approx 9.2$ K for the transition of Nb into the superconductive state, with a small dc bias current applied to the junction. At $H=0$, the fluxon depinning current $I_{\rm dep}$ was found to be very small, less than 1% of the Josephson critical current $I_{\rm c}$ measured without the trapped fluxon. As a fluxon can only be trapped by junction's local inhomogeneities in the absence of the magnetic field, this indicates at fairly high uniformity of the junction. At low values of the field H, linear increase of $I_{\rm dep}$ with H was observed, which is well described by the theoretical model based on Eqs. (3) and (4): the zero-voltage state exists as long as the maximum fluxon's pinning force exerted by the field-induced potential remains larger than the driving force induced by dc bias current, which is satisfied at $|\gamma| < h$. Note that the fluxon depinning and retrapping in weak external magnetic fields were studied experimentally and analytically in Ref. 9.

An evidence for the progressive ac-driven motion of the fluxon is presented in Fig. 1(a). This $I(V)$ characteristic was measured at $H=0.35$ Oe and $f=18.1$ GHz. Its salient feature is two large symmetric constant-voltage steps. The points where they intersect the zero-current axis correspond to the fluxon moving around the junction with a nonzero average velocity at zero dc driving force. Another remarkable feature is the absence of any step at the zero voltage, i.e., in the present case the fluxon cannot be trapped by the effective potential, even when the dc bias current is small. For comparison, in Fig. 1(b) we show the $I(V)$ curve measured at the same power and frequency of the drive, but with $H=0$. In this case, a substantial zero voltage step is seen, extending up to the current $I_0 \approx \pm 0.1$ mA. In the absence of the ac drive, the critical current I_0 is much smaller, less than 20 μA (this residual I_0 may be explained by small inhomogeneities of LJJ, see above).

The conspicuous zero-voltage step in Fig. 1(b) may be explained by the fact that the magnetic component of the ac drive creates its own modulated potential. This argument also helps to explain two symmetric constant-voltage steps at $V \approx 37$ μV in Fig. 1(b) as resonant steps supported by the ac-drive-induced modulation. Note, however, that the latter steps do not feature zero-crossing. All the data collected in the experiments show that the zero crossing is possible *solely*

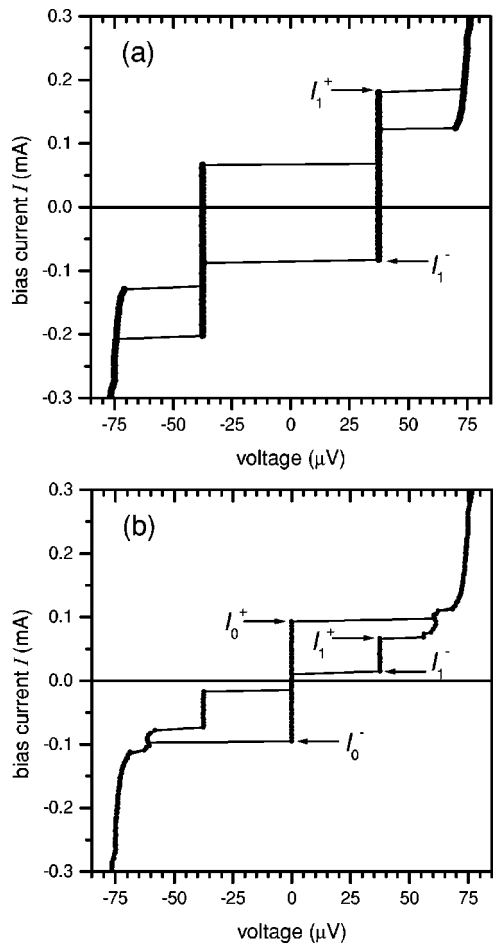

FIG. 1. Current-voltage characteristics for a fluxon trapped in the annular Josephson junction irradiated by the ac signal with the frequency 18.1 GHz and power $P_{\rm ac} = -8$ dBm. The dc magnetic field is (a) $H=0.35$ Oe and (b) $H=0$.

on the resonant steps that occur in the presence of dc magnetic field. In other words, the ac-driven motion of the fluxons is not possible without a stationary spatially periodic potential. This inference is in no contradiction with numerical results of Ref. 7, where the drive itself was spatially modulated.

Coming back to the resonant steps induced by the dc field H, which is the main subject of the work, we have also measured their size vs H, see Fig. 2. The result is that both edge values I_1^+ and I_1^- indicated in Fig. 1 vary nearly linearly with H, up to $H \approx 0.37$ Oe. At still larger fields, the phase-locked ac-driven motion of the fluxon gets interrupted in some current range (the perturbation theory does not apply to so strong fields). These findings are in reasonable agreement with the theoretical prediction (7), a fit to which is shown by the dashed lines. This pertains to both the upward shift of the lines $I_1^{\pm}(H)$ (recall that I and H are proportional to γ and h, respectively) and their linear change with the magnetic field. The residual nonzero value of $I_1^+ - I_1^-$ at $H=0$ matches the small non-zero-crossing step in Fig. 1(b). It is noteworthy too that the current range of the zero-voltage state, $I_0^- < I < I_0^+$ [see Fig. 1(b)], decreases nearly linearly with H, disappearing at $H \approx 0.09$ Oe.

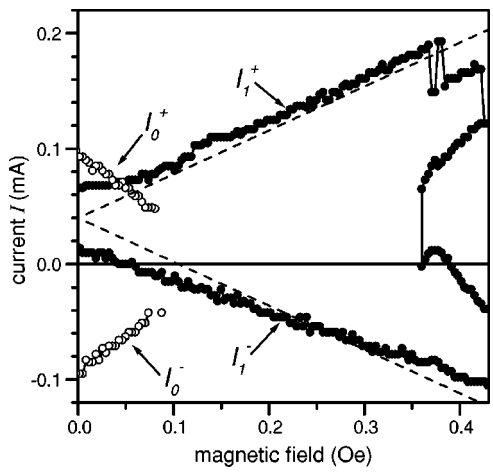

FIG. 2. The critical values $I_0^{\pm}$ and $I_1^{\pm}$ of the dc bias current, marked in Fig. 1, vs the external dc magnetic field, the dashed lines showing a fit to Eq. (7).

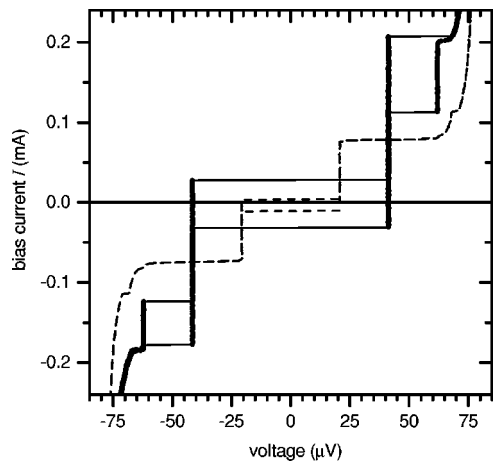

FIG. 3. Current-voltage characteristics for a fluxon in the annular Josephson junction at $H=0.40$ Oe, irradiated by the ac signal at the frequency 10.0 GHz. The signal's power P_{ac} is -3.4 dBm (solid line) and -12.4 dBm (dashed line). The constant-voltage steps on the two lines correspond, respectively, to the second-order and fundamental resonance in Eq. (1).

Equation (7) also predicts a linear dependence of the step's size on the ac-drive's amplitude Γ. Comparison with experimental data shows an agreement in a broad power range; we do not display detailed results of the comparison, as they display no interesting features, the linear dependence of the range of existence of the inverse Josephson effect vs the amplitude of the ac signal being a common feature of all the manifestations of this effect, including the Shapiro steps in small junctions.[2]

As for the dependence of the step's size on the ac-drive's frequency at a fixed value of its power, it is hard to measure it, as variation of the frequency inevitably entails a change in the ac power coupled to the junction. Nevertheless, basic features reported in this work, i.e., the zero crossing steps at finite voltages and disappearance of the zero-voltage state, have been observed in a broad range of the ac frequencies, starting from about 5 GHz and up. On the other hand, as it was mentioned above, the condition that the moving fluxon cannot exceed the Swihart velocity $\bar{c}$ sets an upper cutoff for the frequency that can support the phase-locked motion of fluxons. In our system, $\bar{c}$ corresponds to the dc voltage $V=\bar{c}\Phi_0/(2\pi R)\approx 80$ μV, which translates, via Eq. (1), into the cutoff frequency ~ 40 GHz for the case of the fundamental resonance.

All the above results pertained to the fundamental resonance, $m=1$ in Eq. (1). It is also easy to observe zero-crossings corresponding to higher-orders resonances. This is illustrated in Fig. 3, showing $V(I)$ curves with the resonant steps generated by both the fundamental and second-order (corresponding to $m=2$) resonances.

In conclusion, we have reported the observation of ac-driven motion of a topological soliton in a periodically modulated lossy medium. The observation was made in an annular uniform Josephson junction placed into constant magnetic field. Experimentally measured data, such as the size of the constant-voltage step, are in good agreement with predictions of the analytical model. The effect may take place in a broad class of nonlinear systems and, in terms of the Josephson junctions, it may find a potential application as a low-frequency voltage standard.

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[1] Y. S. Kivshar and B. A. Malomed, Rev. Mod. Phys. **61**, 763 (1989).
[2] A. Barone and G. Paternó, *Physics and Applications of the Josephson Effect* (Wiley, New York, 1982).
[3] A. V. Ustinov, Physica D **123**, 315 (1998).
[4] G. Filatrella, B. A. Malomed, and R. D. Parmentier, Phys. Lett. A **198**, 43 (1995).
[5] G. Filatrella *et al.*, Phys. Lett. A **228**, 250 (1997).
[6] G. Costabile *et al.*, Phys. Rev. B **42**, 2651 (1990); B. A. Malomed, *ibid.* **41**, 2037 (1990); M. Salerno *et al.*, *ibid.* **41**, 6641 (1990); G. Filatrella *et al.*, Phys. Lett. A **173**, 127 (1993).
[7] N. Grønbech-Jensen, P. S. Lomdahl, and M. R. Samuelsen, Phys. Lett. A **154**, 14 (1991).
[8] I. V. Vernik *et al.*, J. Appl. Phys. **81**, 1335 (1997).
[9] A. V. Ustinov, B. A. Malomed, and N. Thyssen, Phys. Lett. A **233**, 239 (1997).
[10] N. Grønbech-Jensen, B. A. Malomed, and M. R. Samuelsen, Phys. Rev. B **46**, 294 (1992).
[11] HYPRES Inc., Elmsford, NY 10523.