Disappearance of ensemble-averaged Josephson current in dirty superconductor-normal-superconductor junctions of *d*-wave superconductors

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(Received 12 January 2001; revised manuscript received 12 March 2001; published 12 June 2001)

We discuss the Josephson current in superconductor/dirty normal conductor/superconductor junctions, where the superconductors have $d_{x^2-y^2}$ pairing symmetry. The low-temperature behavior of the Josephson current depends on the orientation angle between the crystalline axis and the normal of the junction interface. We show that the ensemble-averaged Josephson current vanishes when the orientation angle is $\pi/4$ and the normal conductor is in the diffusive transport regime. The $d_{x^2-y^2}$-wave pairing symmetry is responsible for this fact.

DOI: 10.1103/PhysRevB.64.014511 PACS number(s): 74.80.Fp, 74.25.Fy, 74.50.+r

I. INTRODUCTION

In recent years, the Josephson effect between the anisotropic superconductors has attracted much attention because the high-T_c superconductors might have the $d_{x^2-y^2}$ pairing symmetry.[1,2] In anisotropic superconductors, the sign of the pair potential depends on the direction of a quasiparticle's motion. As a consequence, the zero-energy states[3] (ZES) are formed at the normal-metal/superconductor (NS) interface when the potential barrier at the interface is large enough. The ZES are sensitive to the orientation angle between the crystalline axis of the high-T_c superconductors and the normal of the junction interface. The zero-bias conductance peak, which is due to the ZES, is observed in conductance spectra of N/I/*d*-wave superconductor junctions and the peak height is maximum when the orientation angle is $\pi/4$.[4] Here I is the insulator. In superconductor-insulator-superconductor (SIS) junctions, the ZES dominate the dc Josephson effect and the low-temperature anomaly in the Josephson current has been discussed in a number of theoretical works.[5–12] In experiments, it seems to be difficult to fabricate clean SIS and superconductor-normal-superconductor (SNS) junctions. Thus it is important to understand the effects of disorder on the Josephson current. So far it has been pointed out that the roughness at the interface suppresses the low-temperature anomaly of the Josephson current in SIS junctions.[5,8,9]

In this paper, we study the dc Josephson effect in superconductor/normal conductor/superconductor junctions, where the superconductors have the $d_{x^2-y^2}$-wave pairing symmetry and the normal metal is in the diffusive transport regime. We analytically derive the Josephson current in dirty SNS junctions based on a formula[13] in which the Josephson current is calculated from the two Andreev reflection coefficients.[14] We show that the ensemble-averaged Josephson current vanishes when the orientation angle is $\pi/4$. The analytic results are confirmed by a numerical simulation by using the recursive Green-function method.[15] Throughout this paper we take units of $k_B=\hbar=1$, where k_B is the Boltzmann constant.

This paper is organized as follows. In Sec. II, we derive the general expression of the Josephson current in SNS junctions. In Sec. III, we discuss the Josephson current in dirty SNS junctions. The discussion is given in Sec. IV. We summarize this paper in Sec. V.

II. JOSEPHSON CURRENT FORMULA

Let us consider the two-dimensional SNS junction as shown in Fig. 1, where the length of the normal metal is L_N and the width of the junction is W. The pair potential in the momentum space is schematically depicted in the superconductors. In the $d_{x^2-y^2}$-wave superconductor, the a axis points in the direction in which the amplitude of the pair potential takes its maximum. The angle between the x direction and the a axis is α_L (α_R) in the superconductor on the left- (right-) hand side. We assume that the effective mass (m) of an electron and the Fermi energy ($\mu_F=k_F^2/2m$) are common in the superconductors and the normal conductor, where k_F is the Fermi wave number. We describe the SNS junction by using the Bogoliubov-de Gennes equation[16]

$$\int d\mathbf{r}'\begin{bmatrix}\delta(\mathbf{r}-\mathbf{r}')h_0(\mathbf{r}') & \Delta(\mathbf{r},\mathbf{r}')\\ \Delta^*(\mathbf{r},\mathbf{r}') & -\delta(\mathbf{r}-\mathbf{r}')h_0(\mathbf{r}')\end{bmatrix}\begin{bmatrix}u(\mathbf{r}')\\ v(\mathbf{r}')\end{bmatrix} =E\begin{bmatrix}u(\mathbf{r})\\ v(\mathbf{r})\end{bmatrix}, \tag{1}$$

$$h_0(\mathbf{r})=-\frac{\nabla^2}{2m}+U(\mathbf{r})-\mu_F, \tag{2}$$

$$U(\mathbf{r})=V_b[\delta(x)+\delta(x-L_N)]+\sum_{i=1}^{N_i} v_I\delta(\mathbf{r}-\mathbf{r}_i), \tag{3}$$

where the first term in Eq. (3) is the barrier potential at the NS interface and the second term is the impurity potential in the normal metal. The energy of a quasiparticle (E) is measured from the chemical potential of the junction. The pair potential can be described by the Fourier representation

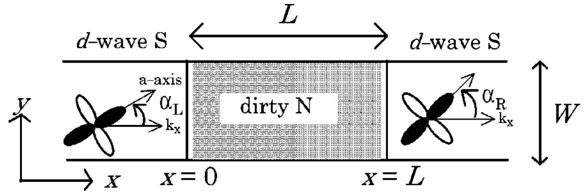

FIG. 1. The SNS junction of the *d*-wave superconductor is illustrated. The orientation angle in the left (right) superconductor is α_L (α_R).

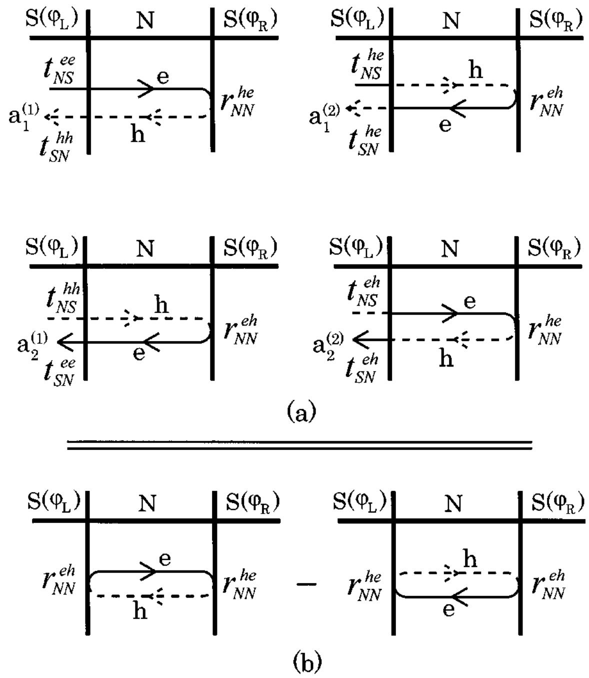

FIG. 2. The reflection coefficients in (a) contribute to the Josephson current proportional to $\sin(\varphi_L-\varphi_R)$. The Josephson current calculated from the four reflection coefficients in (a) is summarized in a reflection process in (b).

$$\Delta(\mathbf{R},\mathbf{r}-\mathbf{r}')=\frac{1}{(2\pi)^2}\int\int d\mathbf{k}\Delta(\mathbf{k})e^{i\mathbf{k}(\mathbf{r}-\mathbf{r}')}, \tag{4}$$

where we assume that the pair potential is uniform in the superconductors and neglect the dependence of Δ on $\mathbf{R}=(\mathbf{r}+\mathbf{r}')/2$. The s-wave superconductor is characterized by $\Delta(\mathbf{k})=\Delta e^{i\varphi_j}$. The $d_{x^2-y^2}$-wave superconductor is characterized by

$$\Delta(\mathbf{k})=\Delta e^{i\varphi_j}\cos(2\alpha_j-2\gamma), \tag{5}$$

$$e^{i\gamma}=\cos\gamma+i\sin\gamma=(k_x+ik_y)/k_F, \tag{6}$$

where Δ is the uniform amplitude of the pair potential, and φ_j and α_j with ($j=L$ or R) are the phase of the pair potential and the orientation angle, respectively.[6] At the Fermi surface, the wave number in the x direction is k_x and the one in the y direction is k_y. In the normal conductor, the pair potential is set to be zero.

We calculate the Josephson current by using the Furusaki-Tsukada formula[6,13]

$$J=eT\sum_{\omega_n}\sum_{k_y}\left[\frac{|\Delta_L^+|a_1(k_y,\omega_n)}{\Omega_L^+}-\frac{|\Delta_L^-|a_2(k_y,\omega_n)}{\Omega_L^-}\right], \tag{7}$$

$$\Delta_j^{\pm}=\Delta\cos 2(\alpha_j\mp\gamma), \tag{8}$$

$$\Omega_j^{\pm}=\sqrt{\omega_n^2+(\Delta_j^{\pm})^2}, \tag{9}$$

with $j=L$ or R. Here T and $\omega_n=(2n+1)\pi T$ are the temperature and the Matsubara frequency, respectively. The wave number in the y direction k_y specifies the propagating channel and the number of propagation channels for each spin is $N_c\equiv Wk_F/\pi$. The coefficients $a_1(k_y,\omega_n)$ and $a_2(k_y,\omega_n)$ are the analytic continuation ($E\to i\omega_n$) of the reflection coefficients of a quasiparticle from the left superconductor to the left superconductor. The Andreev reflection coefficient from the electron (hole) channels to the hole (electron) channels is denoted by a_1 (a_2). In the presence of the time-reversal symmetry, the Josephson current can be decomposed into the series of

$$J=\sum_{m=1}^{\infty}J_m\sin(m\varphi), \tag{10}$$

where $\varphi=\varphi_L-\varphi_R$. In this paper, we neglect the series of J_m for $m\geqslant 2$. This approximation is justified in the presence of the high potential barrier at the NS interface. In Fig. 2(a), we show the four reflection processes of a_1 and a_2 that contribute to J_1. In order to estimate a_1 and a_2, we calculate the transmission and the reflection coefficients of the single NS interface for fixed k_y. The sixteen coefficients are obtained from the continuity condition of the wave function at the NS interface since there are four incoming and four outgoing channels for each k_y. In Appendix A, we show eight transmission and four reflection coefficients, which are required in the following calculation. The Andreev reflection coefficients in Fig. 2(a) are estimated as

$$a_1^{(1)}(k_y,\omega_n)=\sum_{k_y'}t_{SN}^{hh}(k_y,\alpha_L,\varphi_L)t^h_{k_y,k_y'}\times r_{NN}^{he}(k_y',\alpha_R,\varphi_R)t^e_{k_y',k_y}t_{NS}^{ee}(k_y,\alpha_L,\varphi_L), \tag{11}$$

$$a_1^{(2)}(k_y,\omega_n)=\sum_{k_y'}t_{SN}^{he}(k_y,\alpha_L,\varphi_L)t^e_{k_y,k_y'}\times r_{NN}^{eh}(k_y',\alpha_R,\varphi_R)t^h_{k_y',k_y}t_{NS}^{he}(k_y,\alpha_L,\varphi_L), \tag{12}$$

$$a_2^{(1)}(k_y,\omega_n)=\sum_{k_y'}t_{SN}^{ee}(k_y,\alpha_L,\varphi_L)t^e_{k_y,k_y'}\times r_{NN}^{eh}(k_y',\alpha_R,\varphi_R)t^h_{k_y',k_y}t_{NS}^{hh}(k_y,\alpha_L,\varphi_L), \tag{13}$$

$$a_2^{(2)}(k_y,\omega_n)=\sum_{k_y'}t_{SN}^{eh}(k_y,\alpha_L,\varphi_L)t^h_{k_y,k_y'}\times r_{NN}^{he}(k_y',\alpha_R,\varphi_R)t^e_{k_y',k_y}t_{NS}^{eh}(k_y,\alpha_L,\varphi_L), \tag{14}$$

where $t^{e(h)}_{k_y',k_y}$ is the transmission coefficient of the electronlike (holelike) quasiparticle in the normal conductor, and k_y' indicates the propagating channel at the right NS interface. The transmission coefficients in the normal metal are described by

$$t^{e}_{k_y',k_y}=\mathrm{i}v_F\cos\gamma e^{-ik_F\cos\gamma' L_N}\int_0^W dy\int_0^W dy'$$
$$\times\mathcal{G}_{\omega_n}(L_N,y';0,y)\chi^{*}_{k_y'}(y')\chi_{k_y}(y), \quad (15)$$

$$t^{h}_{k_y,k_y'}=-iv_F\cos\gamma' e^{ik_F\cos\gamma' L_N}\int_0^W dy\int_0^W dy'$$
$$\times\mathcal{G}_{-\omega_n}(L_N,y';0,y)\chi^{*}_{k_y}(y)\chi_{k_y'}(y'), \quad (16)$$

where $\mathcal{G}_{\omega_n}(\mathbf{r},\mathbf{r}')$ is the Green function of the normal conductor, v_F is the Fermi velocity, and $\chi_{k_y}(y)$ is the wave function in the y direction belonging to the channel specified by k_y.[17] In this paper, we assume that the NS interface is sufficiently clean so that k_y is conserved while the transmission and the reflection are at the interface. In $a_1^{(1)}$ in Eq. (11), a quasiparticle wave is initially incident into the normal part from the left superconductor through the channel specified by k_y. After the Andreev reflection at the right NS interface, we assume that the reflected wave transmits to the left superconductor through the initial channel of k_y because of the retro property of a quasiparticle under the time-reversal symmetry.[18] The two Andreev reflection coefficients in Eq. (7) are given by $a_1=a_1^{(1)}+a_1^{(2)}$ and $a_2=a_2^{(1)}+a_2^{(2)}$, respectively. By using the transmission and the reflection coefficients in Appendix A, the following equations can be derived,

$$\sum_{k_y}\left[\frac{|\Delta_L^+|}{\Omega_L^+}a_1^{(1)}-\frac{|\Delta_L^-|}{\Omega_L^-}a_2^{(2)}\right]=2i\sum_{k_y,k_y'}P_1, \quad (17)$$

$$\sum_{k_y}\left[\frac{|\Delta_L^+|}{\Omega_L^+}a_1^{(2)}-\frac{|\Delta_L^-|}{\Omega_L^-}a_2^{(1)}\right]=-2i\sum_{k_y,k_y'}P_2, \quad (18)$$

with

$$P_1\equiv r_{NN}^{eh}(k_y,\alpha_L,\varphi_L)t^{h}_{k_y,k_y'}r_{NN}^{he}(k_y',\alpha_R,\varphi_R)t^{e}_{k_y',k_y}, \quad (19)$$

$$P_2\equiv r_{NN}^{he}(k_y,\alpha_L,\varphi_L)t^{e}_{k_y,k_y'}r_{NN}^{eh}(k_y',\alpha_R,\varphi_R)t^{h}_{k_y',k_y}. \quad (20)$$

The Josephson current in Eq. (7) can be written as

$$J=2ieT\sum_{\omega_n}\sum_{k_y,k_y'}(P_1-P_2). \quad (21)$$

Equation (21) corresponds to the reflection processes shown in Fig. 2(b). After a little algebra, we find the final expression of the Josephson current,

$$J=4e\sin\varphi T\sum_{\omega_n}\sum_{k_y,k_y'}Q(k_y,\alpha_L)Q(k_y',\alpha_R)\mathcal{T}(k_y,k_y'), \quad (22)$$

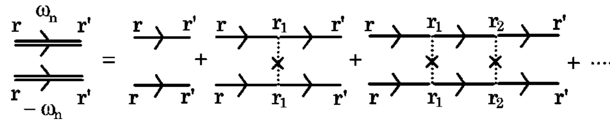

FIG. 3. The propagation process in the diffusive normal conductor is diagrammatically illustrated. The cross represents the impurity scattering.

with

$$\mathcal{T}(k_y,k_y')\equiv t^{h}_{k_y,k_y'}t^{e}_{k_y',k_y}=t^{e}_{-k_y,-k_y'}t^{h}_{-k_y',-k_y}, \quad (23)$$

$$Q(k_y,\alpha_j)\equiv\frac{\cos^2\gamma\Delta_j^-K_j^+}{\Xi_j}, \quad (24)$$

$$\Xi_j\equiv Z^2(\Delta_j^+\Delta_j^-+K_j^+K_j^-)+\cos^2\gamma\Delta_j^+\Delta_j^-, \quad (25)$$

$$K_j^{\pm}\equiv\Omega_j^{\pm}-|\omega_n|, \quad (26)$$

where $Z=mV_b/k_F$ represents the strength of the barrier potential. In what follows, we consider the high potential barrier at the NS interface (i.e., $Z\gg 1$). We note that $Q(k_y,\alpha_j)$ is proportional to the Andreev reflection coefficient at the NS interface. Equations (21) and (22) are the general expressions of the Josephson current proportional to $\sin\varphi$. It is possible to apply these expressions to various junctions if the transmission coefficients in the normal part can be calculated.

III. DIRTY SNS JUNCTIONS

When the normal conductor is in the diffusive transport regime, $t^{e(h)}_{k_y',k_y}$ is almost independent of the propagating channels,

$$\langle\mathcal{T}(k_y,k_y')\rangle\rightarrow\overline{\langle t^e t^h\rangle}, \quad (27)$$

where $\langle\cdots\rangle$ means the ensemble average and $\overline{\cdots}$ is the average over the propagation channels. The transmission coefficients are approximately given by

$$\overline{\langle t^e t^h\rangle}=\frac{v_F^2}{2N_c^2}\int_0^W dy\int_0^W dy' X(L_N,y;0,y'), \quad (28)$$

$$X(\mathbf{r},\mathbf{r}')\equiv\langle\mathcal{G}_{\omega_n}(\mathbf{r},\mathbf{r}')\mathcal{G}_{-\omega_n}(\mathbf{r},\mathbf{r}')\rangle. \quad (29)$$

In the diffusive regime, the function $X(\mathbf{r},\mathbf{r}')$ for small ω_n satisfies the diffusion equation,

$$\tau_0(2|\omega_n|-D_0\nabla_{\mathbf{r}}^2)X(\mathbf{r},\mathbf{r}')\approx 2\pi N_0\tau_0\delta(\mathbf{r}-\mathbf{r}'), \quad (30)$$

where D_0, τ_0, and N_0 are the diffusion constant, the mean free time, and the density of states at the Fermi energy per unit area for each spin degree of freedom, respectively. The propagating process in the normal conductor is diagrammatically illustrated in Fig. 3, where the cross represents the impurity scattering. We solve Eq. (30) and show the results in Appendix B. The averaged transmission coefficients are

$$\overline{\langle t^e t^h \rangle} = \left(\frac{1}{N_c} \right)^2 g_N \frac{ln}{\sinh ln}, \tag{31}$$

where $ln = \sqrt{2n+1} L_N / \xi_D$, $\xi_D = \sqrt{D_0 / 2\pi T}$ is the coherence length, and $G_N \equiv (2e^2/h) g_N$ is the conductance in the normal metal. The ensemble-averaged Josephson current in dirty SNS junctions is calculated by using Eqs. (22) and (31),

$$\langle J(\alpha_L, \alpha_R) \rangle = eT \sum_{\omega_n} N(\alpha_L) N(\alpha_R) g_N \frac{ln}{\sinh ln}, \tag{32}$$

$$N(\alpha_j) \equiv \int_{-\pi/2}^{\pi/2} d\gamma \frac{\cos^3 \gamma \Delta_j^- K_j^+}{\Xi_j}, \tag{33}$$

where we replace the summation Σ_{k_y} by the integration $(N_c/2) \int_{-\pi/2}^{\pi/2} d\gamma \cos \gamma$. When the orientation angles are zero (i.e., $\alpha_{L,R} = 0$), the ensemble-averaged critical current in units of $\pi \Delta_0 / e R_J$ is given by,

$$\langle \overline{J}(0,0) \rangle = \frac{9}{25} \frac{\Delta}{\Delta_0} T \sum_{\omega_n} \frac{\Delta}{\omega_n^2 + \Delta^2} \frac{ln}{\sinh ln}, \tag{34}$$

where the resistance of the junction is described by

$$R_J = \left[\frac{2e^2}{h} \frac{4}{9} \frac{g_N}{Z^4} \right]^{-1}, \tag{35}$$

and Δ_0 is the amplitude of the pair potential at $T=0$. In the case of the *s*-wave junctions, the numerical factor $\frac{9}{25}$ in Eq. (34) is replaced by 1 and the expression is identical to that of the Usadel equation.[19] In recent studies (Refs. 20 and 21, and references therein), it is pointed out that the results in Ref. 19 are not correct in the low-temperature regime and the non-sinusoidal current-phase relation is observed. In these studies, the electron transmission at the NS interface is perfect and the higher harmonics contribute to the Josephson current. In our results, the amplitude of the Josephson current takes its maximum at $\varphi = \pi/2$ for all temperature regimes because of the potential barrier at the NS interface.

When $\alpha_{L,R} = \pi/4$, we find that $N(\pi/4) = 0$, therefore,

$$\langle \overline{J}(\pi/4, \pi/4) \rangle = 0. \tag{36}$$

This is because $K_{L,R}^+$ and $\Xi_{L,R}$ are even functions of γ, whereas $\Delta_{L,R}^- = -\Delta \sin 2\gamma$ is an odd function of γ in Eq. (33). The symmetry of the pair potential is responsible for the disappearance of the ensemble-averaged Josephson current.

When the orientation angle is $\pi/8$, the averaged critical current results in

$$\langle \overline{J}(\pi/8, \pi/8) \rangle = \frac{9\pi^2}{512} \frac{\Delta}{\Delta_0} T \sum_{\omega_n} \frac{\Delta}{\omega_n^2 + \Delta^2/2} \frac{ln}{\sinh ln}. \tag{37}$$

The condition for the ZES (i.e., $\Delta_j^+ \Delta_j^- < 0$) is satisfied in the range of $\pi/8 \leqslant |\gamma| \leqslant 3\pi/8$ in Eq. (33). For the same reason as with the case of $\alpha_{L,R} = \pi/4$, the contribution from this range becomes zero. In Eq. (37), the integration in the range of $0 \leqslant |\gamma| \leqslant \pi/8$ in Eq. (33) contributes to the averaged Josephson

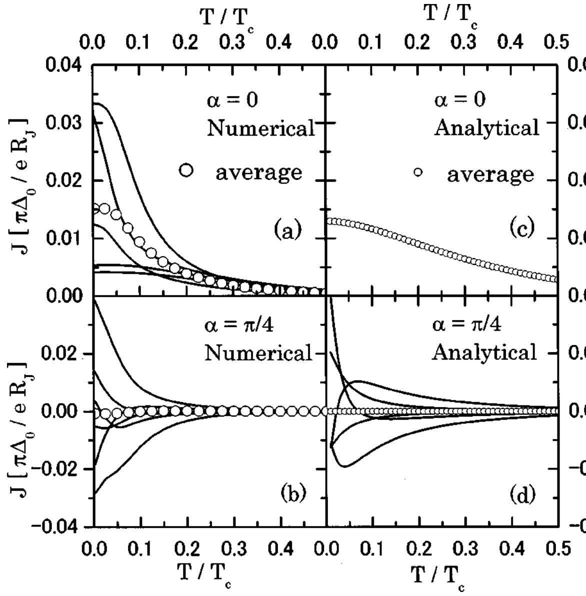

FIG. 4. The Josephson current in dirty SNS junctions is shown as a function of the temperatures. The numerical results for $\alpha_{L,R} = 0$ and $\pi/4$ are plotted in (a) and (b), respectively. The lines are the numerical results of several samples with different random configurations and the open circles are the average over a number of samples. For comparison, we show the analytical results for $\alpha_{L,R} = 0$ in (c) and for $\pi/4$ in (d).

current. We conclude that the ZES do not contribute to the averaged Josephson current in dirty SNS junctions of the *d*-wave superconductor.

In Figs. 4(a) and 4(b), we show the numerical results of the Josephson current at $\varphi = \pi/2$ by using the recursive Green-function method on the two-dimensional tight-binding model,[15] where $\Delta_0 = 0.01\mu_F$, $L_N = 100a_0$, $W = 30a_0$, and a_0 is the lattice constant. The mean free path (ℓ) is about $6.2a_0$. The numerical results for $\alpha_{L,R} = 0$ and $\pi/4$ are shown in Fig. 4(a) and 4(b), respectively. The dependence of Δ on T is described by the BCS theory. The lines are the Josephson current for several samples with different random configurations and the open circles denote the ensemble average. In Fig. 4(a), the results for all samples increase with decreasing temperature and the averaged Josephson current agrees with the analytic results in Eq. (34) which are plotted in Fig. 4(c), where a parameter $L_N / \xi_D|_{T=T_c} = 7.6$ is estimated from the numerical simulation. In Fig. 4(b), the averaged Josephson current is almost zero for $\alpha_{L,R} = \pi/4$ as is predicted in Eq. (36). This fact, however, does not mean the absence of the Josephson current in experiments for a single sample. The sign of the Josephson current is either positive or negative depending on the random configuration and $|J|$ of a single sample increases rapidly with decreasing temperature as shown in Fig. 4(b). These results indicate the importance of the mesoscopic fluctuations in the Josephson current.[22,23] In Eq. (27), we assume that $\langle \mathcal{T}(k_y, k_y') \rangle$ is independent of k_y and k_y'. In other words, the transmission coefficients are isotropic in the momentum space. In order to explain the numerical results in Fig. 4(b), we have to consider a samplespecific anisotropy in the transmission coefficients. Here we introduce the anisotropy in a simple function,

$$\mathcal{T}(k_y,k_y')\rightarrow\overline{\langle t^e t^h\rangle}[1+f_1\delta_{k_y,k_1}\delta_{k_y',k_1}+f_2\delta_{k_y,k_2}\delta_{k_y',-k_2}], \tag{38}$$

where f_1 and f_2 are positive numerical factors much smaller than unity. In Eq. (38), we consider the situation where the two elements $\mathcal{T}(k_1,k_1)$ and $\mathcal{T}(k_2,-k_2)$ are slightly larger than the average. By using Eq. (38), the Josephson current in a single sample with $\alpha_{L,R}=\pi/4$ results in

$$\overline{J}(\pi/4,\pi/4)=\frac{\Delta}{\Delta_0}T\sum_{\omega_n}\frac{ln}{\sinh ln}[f_1Y(\gamma_1)-f_2Y(\gamma_2)], \tag{39}$$

$$Y(\gamma)\equiv\frac{3}{4}\frac{\Delta\cos^6\gamma\sin^2 2\gamma}{\left[|\omega_n|+\dfrac{\Delta}{2Z^2}\cos^2\gamma|\sin 2\gamma|\right]^2}. \tag{40}$$

The denominator of Eq. (40) approaches zero in the limit of $\omega_n\rightarrow 0$, and $Z\gg 1$, which reflects the ZES at the NS interface. In Fig. 4(d), we plot the analytical results in Eq. (39) for several choices of f_1, f_2, γ_1, and γ_2, where f_1 and f_2 are of the order of 10^{-2}. Some of the results show nonmonotonic temperature dependence, and others change the sign with decreasing temperature as shown in both Figs. 4(b) and (d). The analytical results explain the numerical results. These results indicate the enhancement of the mesoscopic fluctuations of the Josephson current in the limit of $T\rightarrow 0$.

IV. DISCUSSION

In Sec. III, we calculate $\mathcal{T}(k_y,k_y')$ by solving the diffusion equation, Eq. (30). The conclusion in Eq. (36) is independent of the detail of the approximation. In the diffusive conductors, $\langle\mathcal{T}(k_y,k_y')\rangle$ are independent of k_y and k_y'. The conclusion in Eq. (36) always holds when $\langle\mathcal{T}(k_y,k_y')\rangle$ is isotropic in the momentum space because the integration with respect to γ at the two NS interfaces can be carried out independently in Eq. (22). For this reason, we derive an equation for the dirty SNS junctions,

$$\langle J(\alpha_L,\pi/4)\rangle=\langle J(\pi/4,\alpha_R)\rangle=0 \tag{41}$$

for any α_L and α_R. We also derive relations

$$\begin{aligned}\langle J(\alpha_L,\alpha_R)\rangle&=\langle J(\alpha_L,-\alpha_R)\rangle\\&=\langle J(-\alpha_L,\alpha_R)\rangle=\langle J(-\alpha_L,-\alpha_R)\rangle\end{aligned} \tag{42}$$

because the zero-energy states do not contribute to the ensemble average of the Josephson current.

In the *s*-wave SNS junctions, the amplitude of the fluctuations is $\delta J_c=(eE_c/\hbar)C_d$, where $E_c=\hbar D_0/L_N^2$ is the Thouless energy of the normal conductor and C_d is a constant of the order of unity.[22] The amplitude of the fluctuations in the Josephson current for $\alpha_{L,R}=0$ at $T=0$ is expected to be proportional to the Thouless energy and depends on the sample size since the characteristic features of the Josephson current are essentially the same as those in the *s*-wave junctions. On the other hand, the fluctuations for $\alpha_{L,R}=\pi/4$ in Fig. 4(b) are mainly due to the ZES at the interfaces and strongly depend on Z and T as shown in Eq. (40) and Fig. 4(d). Though the average of the Josephson current is zero, the fluctuations for $\alpha_{L,R}=\pi/4$ can be larger than those for $\alpha_{L,R}=0$. Further theoretical investigations are necessary to understand the amplitude of the fluctuations.

The impurities in the normal metal may suppress the pair potentials near the NS interfaces because the normal impurities break a Cooper pair in the anisotropic superconductors. The pair potential should be determined self-consistently in such situations. So far in SIS junctions, Tanaka and Kashiwaya[24] calculated the Josephson current where the pair potentials were determined in a self-consistent way and compared it with the Josephson current obtained in a non-self-consistent manner. Their results show that there are no qualitative differences in the Josephson current between the two methods. In their study, the pair breaker is the insulator. The pair breaker in the present paper is the impurities in the normal metal. Though the origin of the pair break is different in the two systems, the suppression of the pair potentials, and therefore, the suppression of the Josephson current are considered to be the common consequence when we determine the pair potential self-consistently. Thus it may be possible to infer that the Josephson current in real SNS junctions would be smaller than that of the present paper. The self-consistent study has to be done to discuss the amplitude of the Josephson current quantitatively. However the main conclusion in Eq. (36) is not sensitive to the profile of the pair potential because the *d*-wave symmetry is responsible for the disappearance of the averaged Josephson current. The impurities may also modify the symmetry of the pair potentials. If the finite values of the averaged Josephson current are observed in experiments, such results might be evidence for the change of the pairing symmetry due to the impurities. This is because the *d*-wave component of the pair potentials does not contribute to the averaged current for $\alpha=\pi/4$.

The sign change of the pair potential on the Fermi surface, which is a characteristic feature of the anisotropic superconductors, leads to the disappearance of the Josephson current. Thus the conclusion in Eq. (36) may be applied to the SNS junctions of the superconductors with *p*-wave pairing symmetry such as UPt_3 for certain orientation angles between the junction interface normal and the node lines of the pair potentials. An investigation in this direction is now in progress.

V. CONCLUSION

In conclusion, we analytically derive the general expression of the Josephson current in SNS junctions of the *d*-wave superconductors. In dirty SNS junctions, we show that the ZES do not contribute to the ensemble-averaged Josephson current because of the symmetry of the pair potential. In particular, when the orientation angle of the *d*-wave superconductor is $\alpha=\pi/4$, the ensemble-averaged Josephson current vanishes. The critical current of a single sample, how-

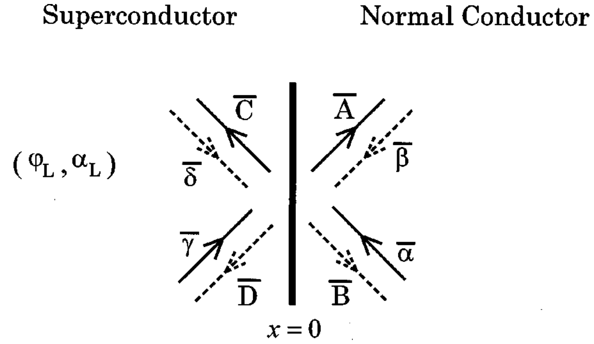

FIG. 5. The transmission and the reflection coefficients at the left NS interface. There are four incoming and outgoing channels.

ever, remains finite, which indicates an importance of the mesoscopic fluctuations of the Josephson current.

ACKNOWLEDGMENTS

The author is indebted to N. Tokuda, H. Akera, Y. Tanaka, and Y. Takane for useful discussions.

APPENDIX A: TRANSMISSION AND REFLECTION COEFFICIENTS

We derive the transmission and reflection coefficients of the NS interface at $x=0$ as shown in Fig. 5. In what follows, we calculate the coefficients after analytic continuation ($E \rightarrow i\omega_n$) for $\omega_n>0$ and we assume that $\Delta \ll \mu_F$. In the normal metal, the wave function of the quasiparticle with $k_y = k_F \sin\gamma$ can be described by

$$\hat{\Psi}^N_{k_y}(x,y)=\left[\bar{\alpha}\begin{pmatrix}1\\0\end{pmatrix}e^{-ip_+x}+\bar{\beta}\begin{pmatrix}0\\1\end{pmatrix}e^{ip_-x}+\bar{A}\begin{pmatrix}1\\0\end{pmatrix}e^{ip_+x}\right.$$
$$\left.+\bar{B}\begin{pmatrix}0\\1\end{pmatrix}e^{-ip_-x}\right]\chi_{k_y}(y), \tag{A1}$$

where $\bar{\alpha}$ and $\bar{\beta}$ ($\bar{A}$ and $\bar{B}$) are the amplitudes of the incoming (outgoing) waves in the electron and hole channels, respectively. The wave function in the y direction is $\chi_{k_y}(y)$, $p_\pm = k_F\cos\gamma \pm i\omega_n/v_F\cos\gamma$, and $v_F=k_F/m$ is the Fermi velocity.

In the same way, the wave function in the superconductor is given by

$$\hat{\Psi}^S_{k_y}(x,y)=\left[\bar{\gamma}\Phi^+_L\begin{pmatrix}u^+_L\\v^+_L\end{pmatrix}e^{ik_+x}+\bar{\delta}\Phi^-_L\begin{pmatrix}v^-_L\\u^-_L\end{pmatrix}e^{-ik^*_-x}\right.$$
$$\left.+\bar{C}\Phi^-_L\begin{pmatrix}u^-_L\\v^-_L\end{pmatrix}e^{-ik_-x}+\bar{D}\Phi^+_L\begin{pmatrix}v^+_L\\u^+_L\end{pmatrix}e^{ik^*_+x}\right]$$
$$\times\chi_{k_y}(y), \tag{A2}$$

where $\bar{\gamma}$ and $\bar{\delta}$ ($\bar{C}$ and $\bar{D}$) are the amplitudes of incoming (outgoing) waves in the electron and hole channels, respectively. We define that

$$\Delta^\pm_j=\Delta\cos 2(\gamma\mp\alpha_j), \tag{A3}$$

$$\Omega^\pm_j=\sqrt{\omega_n^2+\Delta^{\pm 2}_j}, \tag{A4}$$

$$u^\pm_j=\sqrt{(1+\Omega^\pm_j/\omega_n)/2}, \tag{A5}$$

$$v^\pm_j=\sqrt{(1-\Omega^\pm_j/\omega_n)/2}, \tag{A6}$$

$$k_\pm=k_F\cos\gamma+i\Omega^\pm_j/v_F\cos\gamma, \tag{A7}$$

where $j=L$ or R. The phase and the sign of the pair potential is considered in the matrix

$$\Phi^\pm_j=\begin{pmatrix}e^{i(\varphi_j+\bar{\phi}^\pm_\gamma)/2} & 0\\ 0 & e^{-i(\varphi_j+\bar{\phi}^\pm_\gamma)/2}\end{pmatrix}, \tag{A8}$$

where $e^{i\bar{\phi}^\pm_\gamma}=\mathrm{sgn}(\Delta^\pm)$. The two wave functions satisfy the continuity condition at the left NS interface (i.e., $x=0$ and $\alpha_j=\alpha_L$),

$$\hat{\Psi}^N_{k_y}(0,y)=\hat{\Psi}^S_{k_y}(0,y), \tag{A9}$$

$$\left.\frac{\partial}{\partial x}\hat{\Psi}^N_{k_y}(x,y)\right|_{x=0}=\left.\frac{\partial}{\partial x}\hat{\Psi}^S_{k_y}(x,y)\right|_{x=0}+2mV_b\hat{\Psi}^S_{k_y}(0,y). \tag{A10}$$

From Eqs. (A9) and (A10), the amplitudes of the outgoing waves are connected with those of the incoming waves.

$$\begin{pmatrix}\bar{A}\\ \bar{B}\\ \bar{C}\\ \bar{D}\end{pmatrix}=\begin{pmatrix}r^{ee}_{NN} & r^{eh}_{NN} & t^{ee}_{NS} & t^{eh}_{NS}\\ r^{he}_{NN} & r^{hh}_{NN} & t^{he}_{NS} & t^{hh}_{NS}\\ t^{ee}_{SN} & t^{eh}_{SN} & r^{ee}_{SS} & r^{eh}_{SS}\\ t^{he}_{SN} & t^{hh}_{SN} & r^{he}_{SS} & r^{hh}_{SS}\end{pmatrix}\begin{pmatrix}\bar{\alpha}\\ \bar{\beta}\\ \bar{\gamma}\\ \bar{\delta}\end{pmatrix}. \tag{A11}$$

We explicitly show 10 coefficients that are required for calculating the Josephson current,

$$t^{ee}_{NS}=-iM^*\frac{\Delta^-_L K^+_L}{v^+_L}\frac{\Omega^+_L}{\omega_n}e^{-i(\bar{\phi}^+_\gamma)/2}e^{i(\varphi_L/2)}, \tag{A12}$$

$$t^{hh}_{SN}=-iM\frac{\Delta^-_L K^+_L}{v^+_L}e^{-i(\bar{\phi}^+_\gamma/2)}e^{i(\varphi_L/2)}, \tag{A13}$$

$$t^{he}_{NS}=-N\frac{\Delta^+_L K^-_L}{u^+_L}\frac{\Omega^+_L}{\omega_n}e^{i(\bar{\phi}^+_\gamma/2)}e^{-i(\varphi_L/2)}, \tag{A14}$$

$$t^{he}_{SN}=N\frac{\Delta^+_L K^-_L}{u^+_L}e^{i(\bar{\phi}^+_\gamma/2)}e^{-i(\varphi_L/2)}, \tag{A15}$$

$$t^{hh}_{NS}=-iM\frac{\Delta^+_L K^-_L}{v^-_L}\frac{\Omega^-_L}{\omega_n}e^{i(\bar{\phi}^-_\gamma/2)}e^{-i(\varphi_L/2)}, \tag{A16}$$

$$t_{SN}^{ee}=-iM^{*}\frac{\Delta_L^{+}K_L^{-}}{v_L^{-}}e^{i(\bar{\phi}_{\gamma}^{-}/2)}e^{-i(\varphi_L/2)}, \quad \text{(A17)}$$

$$t_{NS}^{eh}=N\frac{\Delta_L^{-}K_L^{+}}{u_L^{-}}\frac{\Omega_L^{-}}{\omega_n}e^{-i(\bar{\phi}_{\gamma}^{-}/2)}e^{i(\varphi_L/2)}, \quad \text{(A18)}$$

$$t_{SN}^{eh}=-N\frac{\Delta_L^{-}K_L^{+}}{u_L^{-}}e^{-i(\bar{\phi}_{\gamma}^{-}/2)}e^{i(\varphi_L/2)}, \quad \text{(A19)}$$

$$r_{NN}^{he}=-iQ(-k_y,\alpha_L)e^{-i\varphi_L}, \quad \text{(A20)}$$

$$r_{NN}^{eh}=-iQ(k_y,\alpha_L)e^{i\varphi_L}, \quad \text{(A21)}$$

$$M\equiv\frac{\cos\gamma(\cos\gamma+iZ)}{\Xi_L}, \quad \text{(A22)}$$

$$N\equiv\frac{Z\cos\gamma}{\Xi_L}, \quad \text{(A23)}$$

$$\Xi_j\equiv Z^2(\Delta_j^{+}\Delta_j^{-}+K_j^{+}K_j^{-})+\cos^2\gamma\Delta_j^{+}\Delta_j^{-}, \quad \text{(A24)}$$

$$Z\equiv\frac{mV_b}{k_F}, \quad \text{(A25)}$$

$$K_j^{\pm}\equiv\Omega_j^{\pm}-|\omega_n|, \quad \text{(A26)}$$

$$Q(k_y,\alpha_j)\equiv\frac{\cos^2\gamma\Delta_j^{-}K_j^{+}}{\Xi_j}, \quad \text{(A27)}$$

$$Q(-k_y,\alpha_j)\equiv\frac{\cos^2\gamma\Delta_j^{+}K_j^{-}}{\Xi_j}. \quad \text{(A28)}$$

These coefficients are a function of k_y, α_L, and φ_L. On the derivation, we use the approximation $k_{\pm}=p_{\pm}=k_F\cos\gamma$, where we assume that $\omega_n\ll\mu_F$.

The reflection coefficients at the right NS interface are obtained in the same way. The two Andreev coefficients are given by

$$r_{NN}^{he}(k_y,\alpha_R,\varphi_R)=-iQ(k_y,\alpha_R)e^{-i\varphi_R}, \quad \text{(A29)}$$

$$r_{NN}^{eh}(k_y,\alpha_R,\varphi_R)=-iQ(-k_y,\alpha_R)e^{i\varphi_R}. \quad \text{(A30)}$$

APPENDIX B: SOLUTION OF THE DIFFUSION EQUATION

In this paper, we consider the low-temperature regime $T<T_c\sim10^{-3}-10^{-4}\mu_F$ and the diffusive transport regime $1/\mu_F\tau_0\sim10^{-1}-10^{-2}$. Thus a relation $2\pi T\tau_0<1$ is satisfied. In these regimes, we solve the diffusion equation

$$\tau_0(2|\omega_n|-D_0\nabla_{\mathbf{r}}^2)X(\mathbf{r},\mathbf{r}')\approx2\pi N_0\tau_0\delta(\mathbf{r}-\mathbf{r}'), \quad \text{(B1)}$$

under the appropriate boundary condition.[25,26] The right-hand side of Eq. (B1) corresponds to the first term in Fig. 3, which can be calculated to be

$$\frac{N_0}{v_F r}e^{-r/\ell}, \quad \text{(B2)}$$

where $\ell=v_F\tau_0$ is the mean free path and $r=|\mathbf{r}-\mathbf{r}'|$. In Eq. (B1), we replace this function by the δ function in two dimensions because the smallest length scale in the diffusion equation is ℓ. The function $X(\mathbf{r},\mathbf{r}')$ is the Green function of the Schrödinger-type equation

$$D_0\nabla^2 f(\mathbf{r})=\lambda f(\mathbf{r}). \quad \text{(B3)}$$

The normal conductor is separated by the high potential barrier at the two NS interfaces and there is no current flow in the y direction. In such situations, we solve Eq. (B3) under the boundary conditions,

$$\left.\frac{\partial f(\mathbf{r})}{\partial x}\right|_{x=0,L_N}=0, \quad \text{(B4)}$$

$$\left.\frac{\partial f(\mathbf{r})}{\partial y}\right|_{y=0,W}=0. \quad \text{(B5)}$$

The function $X(\mathbf{r},\mathbf{r}')$ is represented by

$$X(\mathbf{r},\mathbf{r}')=2\pi N_0\sum_{m=0}^{\infty}\sum_{m'=0}^{\infty}A_m^2B_{m'}^2\times\frac{\cos\left(\frac{m\pi x}{L_N}\right)\cos\left(\frac{m\pi x'}{L_N}\right)\cos\left(\frac{m'\pi y}{W}\right)\cos\left(\frac{m'\pi y'}{W}\right)}{2|\omega_n|+D_0[(m\pi/L_N)^2+(m'\pi/W)^2]}, \quad \text{(B6)}$$

$$A_m=\begin{cases}\sqrt{\frac{1}{L_N}}, & \text{for}\quad m=0,\\ \sqrt{\frac{2}{L_N}}, & \text{for}\quad m\neq0,\end{cases} \quad \text{(B7)}$$

$$B_m=\begin{cases}\sqrt{\frac{1}{W}}, & \text{for}\quad m=0,\\ \sqrt{\frac{2}{W}}, & \text{for}\quad m\neq0.\end{cases} \quad \text{(B8)}$$

Next we calculate the function

$$\int_0^W dy\int_0^W dy' X(\mathbf{r},\mathbf{r}')|_{x=L_N,x'=0}=2\pi N_0\tau_0\frac{W}{L_N}\frac{1}{2|\omega_n|\tau_0}\frac{ln}{\sinh ln}, \quad \text{(B9)}$$

where $ln=\sqrt{2n+1}L_N/\xi_D$ and $\xi_D=\sqrt{D_0/2\pi T}$ is the coherence length. Finally the averaged transmission coefficients are

$$\overline{\langle t^e t^h \rangle} = \left(\frac{1}{N_c}\right)^2 g_N \frac{ln}{\sinh ln} \frac{1}{2|\omega_n|\tau_0}, \tag{B10}$$

where $g_N = 2\pi N_0 D_0 W/L_N$ is the dimensionless conductance of the normal metal for each spin degree of freedom. In Eq. (B10), $\overline{\langle t^e t^h \rangle}$ seems to be singular in the limit of $\omega_n \to 0$. We note that the singularity is stemming from the boundary condition in Eq. (B4). When there is no potential barrier at the NS interface, we apply the boundary condition $f(\mathbf{r})|_{x=0,L_N} = 0$ instead of Eq. (B4) and obtain

$$\overline{\langle t^e t^h \rangle} = \left(\frac{1}{N_c}\right)^2 g_N \frac{ln}{\sinh ln} \frac{1}{2|\omega_n|\tau_0} \frac{\cosh(2\sqrt{2|\omega_n|\tau_0}) - 1}{2}, \tag{B11}$$

where the integration with respect to y and y' is carried out at $x = L_N + \ell/\sqrt{2}$ and $x' = -\ell/\sqrt{2}$. Since the propagation of a quasiparticle in the normal conductor is not sensitive to the boundary condition at the NS interface and $N_c^2 \overline{\langle t^e t^h \rangle}$ is close to g_N in the limit of $\omega_n \to 0$, we regularize Eq. (B10) by introducing the cutoff,

$$\overline{\langle t^e t^h \rangle} = \left(\frac{1}{N_c}\right)^2 g_N \frac{ln}{\sinh ln} F(2|\omega_n|\tau_0), \tag{B12}$$

$$F(x) = \Theta(1-x) + \Theta(x-1)\frac{1}{x}, \tag{B13}$$

where $\Theta(x)$ is the step function. Since $\overline{\langle t^e t^h \rangle}$ decays as $\exp(-\sqrt{2n+1} L_N/\xi_D)$, the contributions from small n are dominant in the summation with respect to ω_n in Eq. (32). Thus in most cases, $F(2|\omega_n|\tau_0)$ is unity.

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