Quantum force in a superconductor

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(Received 14 June 2000; revised manuscript received 31 October 2000; published 8 June 2001)

In order to account for a contradiction of the Little-Parks experiment with Ohm's law and other fundamental laws, the thermal fluctuation is considered as a dynamic phenomenon and an extra force, called the quantum force, is introduced. A persistent current can exist at zero voltage and nonzero resistance because of the quantum force induced by the thermal fluctuation. Not only the persistent current but also a persistent voltage (a direct voltage in the equilibrium state) can exist in an inhomogeneous superconducting ring. The directions of the persistent current and the persistent voltage coincide in a ring segment with lower critical temperature and are opposite in other ring segments with higher T_c. Consideration of a superconducting ring interrupted by Josephson junction shows a connection of the quantum force with a real mechanical force.

DOI: 10.1103/PhysRevB.64.012505 PACS number(s): 74.20.De, 64.70.−p, 73.23.Ra

Superconductivity is a macroscopic quantum phenomenon: some macroscopic effects observed in superconductors cannot be described by classical mechanics. One of them is the Little-Parks (LP) experiment[1] repeated in numerous works (see, for example, Ref. 2). It is considered[3,2] that the LP experiment was explained as long ago, as 1963.[4] But this explanation is not perfect. More perfect consideration shows a contradiction of the LP experiment in a loop[2] with some habitual knowledge. This contradiction is explained in the present work.

The resistance oscillations observed at the LP experiment is interpreted as a consequence of oscillations of the superconducting transition temperature T_c.[1,3,2] It is assumed[3] that $R_l(T)=R_l(T-T_c)$ in the resistive transition region where the resistance $R_l=\int_l dl\rho/s^2$ changes from $R_l=R_{ln}$ to $R_l=0$. Here R_{ln} is the resistance along the loop in the normal state; s is the area of the cross-sectional of the wire defining the loop.

The T_c oscillations is explained by the fluxoid quantization.[4,3,2] Because of the quantization the velocity circulation

$$\int_l dl v_s = \frac{\pi\hbar}{m}\left(n-\frac{\Phi}{\Phi_0}\right) \tag{1}$$

of superconducting pairs can not be equal zero when the magnetic flux Φ contained within a loop is not divisible by the flux quantum $\Phi_0=\pi\hbar c/e$. Therefore the energy of superconducting state increases and as consequence the T_c decreases when $\Phi\neq n\Phi_0$, $\Delta T_c \propto -v_s^2 \propto -(n-\Phi/\Phi_0)^2$.[3] The magnetic flux LI_s induced by the screening current $I_s=sj_s=s2en_s v_s$ is small $LI_s \ll \Phi_0$ at $T\simeq T_c$ (when the density of superconducting pairs n_s is close to zero) and therefore $\Phi=BS+LI_s\simeq BS$.[3] Here B is the magnetic induction induced by an external magnet; S is the area of the loop.

It is important that the theoretical dependence $\Delta T_c \propto -(n-\Phi/\Phi_0)^2$, where $v_s^2\propto(n-\Phi/\Phi_0)^2$ has minimum possible value[3] describes enough well the experimental data (see, for example, Fig. 4 in Ref. 2). Consequently, superconducting states with minimum $(n-\Phi/\Phi_0)^2$ value give the main contribution. This means that not only the average $\overline{v_s^2}=t_{\rm long}^{-1}\int_{t_{\rm long}} dt v_s^2$ but also $\overline{v_s}=t_{\rm long}^{-1}\int_{t_{\rm long}} dt v_s$ is not equal zero at $\Phi\neq n\Phi_0$ and $\Phi\neq(n+0.5)\Phi_0$. $\overline{v_s}=0$ at $\Phi=(n+0.5)\Phi_0$ because the permitted states with opposite direction of the velocity have the same v_s^2 value. $\overline{v_s}\approx(\pi\hbar/ml)(n-\Phi/\Phi_0)$ in a homogeneous loop when Φ is not close to $(n+0.5)\Phi_0$.

Thus, according to the LP experiment and in spite of the Ohm's law $R_l I_{sc}=\int_l dlE=-(1/c)d\Phi/dt$ a direct screening current $\overline{I_{sc}}\approx s2e\overline{n_s}(\pi\hbar/ml)(n-\Phi/\Phi_0)$ flows along the loop at a constant magnetic flux $\Phi\neq n\Phi_0$ and $\Phi\neq(n+0.5)\Phi_0$, and $R_l\neq 0$. The latter is evident from the experiment.[2] The measured resistance[2] $R_m\approx R_l/4$ in a homogeneous loop. The LP experiment contradicts not only to the Ohm's law but also some more fundamental laws because a dissipation (friction) force $F_{\rm dis}$ should act at $I_{sc}=sj_{sc}\neq 0$ and $R_l\neq 0$, and an energy dissipation with power $R_l I_{sc}^2$ should take place.

This contradiction has a explanation having a single meaning. It is obvious that in a stationary state the screening current is equal superconducting current

$$j_s=\frac{2e\pi\hbar}{lm\langle n_s^{-1}\rangle}\left(n-\frac{\Phi}{\Phi_0}\right). \tag{2}$$

It can be nonzero when the whole of loop in superconducting state, i.e., $\langle n_s^{-1}\rangle^{-1}\neq 0$, and $R_l=0$. $\langle n_s^{-1}\rangle=l^{-1}\int_l dl n_s^{-1}$ is used because the j_s value should be constant along the loop in the stationary state. (n_s ought be considered as an effective density in order to take into account the Josephson current through segments with $n_s=0$.) Therefore the LP oscillations are observed only in the region of the resistive transition where loop segments are switched by the thermal fluctuation between superconducting state (when $\langle n_s^{-1}\rangle^{-1}\neq 0$, $j_s\neq 0$ but $R_l=0$) and normal state (when $\langle n_s^{-1}\rangle^{-1}=0$, $R_l\neq 0$ but $j_s=0$). These oscillations cannot be observed below the resistive transition where $j_s\neq 0$ but $R_l=0$ all time and above this transition where $R_l=R_{ln}$ but $j_s=0$ all time.

Thus, the LP experiment is evidence of a motion induced by fluctuation in the thermodynamic equilibrium state at nonzero dissipation. Such phenomena are called Brownian motion.[5] There is an important difference from the classical

Brownian motion. According to the classical mechanics the average velocity of any Brownian motion should be equal zero whereas the LP experiment is evidence of the persistent current (i.e., a direct current in the equilibrium state) $j_{\mathrm{PC}} = \overline{j_{sc}} \neq 0$ at $\Phi \neq n\Phi_0$ and $\Phi \neq (n+0.5)\Phi_0$.

The persistent current

$$j_{\mathrm{PC}} = q\sum_p v f_0\left(\frac{E(p)}{k_B T}\right) = \frac{q}{m}\sum_p \left(p - \frac{q}{c}A\right) f_0\left(\frac{E(p)}{k_B T}\right) \quad (3)$$

is a quantum phenomenon. It can exist in states with discrete spectrum $\int_l dlp = n2\pi\hbar$, at the energy difference between adjacent permitted states $E(n+1)-E(n) \gg k_B T$, when the summation Σ_p can not be replaced by integration. Here $p = mv + (q/c)A$ is the generalized momentum of a particle with a charge q; A is the vector potential. At continuous spectrum [at $E(n+1)-E(n) \ll k_B T$] $j_{\mathrm{PC}} = q\Sigma_p v f_0 = q\int dv v f_0 = 0$ because the distribution function in the equilibrium state f_0 depends on v only through $E(p)/k_B T$ and the kinetic energy is proportional to v^2 in a consequence of the space symmetry. Therefore, according to the classical mechanics any direct (nonchaotic) current can only be in a nonequilibrium state and it is postulated that the average value of the fluctuation force introduced by Langevin for description of the classical Brownian motion is equal zero $\overline{F_{\mathrm{Lan}}} = 0$.

In a superconducting loop the difference between adjacent permitted states of the kinetic energy

$$E_p = s\int_l dl n_s \frac{2mv_s^2}{2} = \frac{s\pi^2\hbar^2}{lm\langle n_s^{-1}\rangle}\left(n - \frac{\Phi}{\Phi_0}\right)^2 \quad (4)$$

is proportional to $\langle n_s^{-1}\rangle^{-1}$ and of the energy of the magnetic flux induced by the superconducting current $E_L = LI_s^2/2c^2 = (Ls^2e^2 2\pi^2\hbar^2/c^2l^2m^2\langle n_s^{-1}\rangle^2)(n-\Phi/\Phi_0)^2 = (Ls/l\lambda_0^2)n_s' E_p$ is proportional to $\langle n_s^{-1}\rangle^{-2}$. Here $\lambda_0 = [c^2 2m/4e^2 n_s(0)]^{1/2}$ is the London penetration depth at $T = 0$; $n_s' = [n_s(0)\langle n_s^{-1}\rangle]^{-1}$; $n_s(0)$ is the density of superconducting pairs at $T=0$. At weak screening, when the LP oscillations are observed, $(Ls/l\lambda_0^2)n_s' < 1$ and consequently $E_L < E_p$.

Superconducting pairs, as condensed bosons, have the same value of the momentum circulation $\int_l dlp = n2\pi\hbar$. Therefore the $E(n+1)-E(n)$ value for superconducting pairs in a loop l at $\langle n_s^{-1}\rangle^{-1} \approx \langle n_s\rangle \neq 0$ is much more than the one for electron $E_p(n+1)-E_p(n) = (2\pi^2\hbar^2/l^2m)[(n+1)^2 - n^2] \approx 2\pi^2\hbar^2/l^2m$ because the average number of superconducting pairs $sl\langle n_s\rangle$ is very big in a real case. For a real length $l \simeq 4$ μm of the wire defining the loop[2] $2\pi^2\hbar^2/l^2m \simeq k_B 1K$. Therefore the persistent current in normal metal mesoscopic systems[6] is observed only at very low temperature.[7] In superconductor the screening persistent current $j_{\mathrm{PC}} = q\Sigma_p v f_{qu} = j_s$ is observed even in macroscopic samples (for example at the Meissner effect) because $E_p(n+1) - E_p(n) \approx sl\langle n_s\rangle(\pi^2\hbar^2/l^2m) \gg k_B T$ even near T_c.

Consequently, in the region of the resistive transition the fluctuations switch the loop between qualitatively different states: the superconducting state $\langle n_s^{-1}\rangle^{-1} \neq 0$ with strongly discrete spectrum $|E_p(n+1)-E_p(n)| \gg k_B T$, in which the circulation of the phase gradient $\nabla\varphi = p/\hbar$ of the wave function of superconducting pairs has a definite value $\int_l dl\nabla\varphi = 2\pi n$, and the state with continuous p spectrum, in which $\langle n_s^{-1}\rangle^{-1} = 0$, $R_l \neq 0$, the energy $E_p(n) = 0$ for any n value and therefore $\int_l dl\nabla\varphi$ is ''bad'' (vague) number. The later means that the ''random phase'' assumption is valid at $\langle n_s^{-1}\rangle^{-1} = 0$ and therefore the average velocity should be equal zero in the equilibrium state.[8]

Thus, the average value of the momentum circulation of superconducting pairs changes between $\int_l dlp = \int_l dl[2mv_s + (2e/c)A] = (2e/c)\Phi$ and $\int_l dlp = n2\pi\hbar$ at the switching between $\langle n_s^{-1}\rangle^{-1} = 0$ and $\langle n_s^{-1}\rangle^{-1} \neq 0$. At $(Ls/l\lambda_0^2)n_s' \ll 1$, when the A change is small, the momentum change on the unit volume $\Delta P \simeq (m/e)j_s$. These momentum changes induced by fluctuations explain the contradiction of the LP experiment with habitual laws. The persistent current $j_{\mathrm{PC}} = \overline{j_{sc}} \neq 0$ can exist at nonzero dissipation $\overline{F_{\mathrm{dis}}} \neq 0$ because the momentum circulation should return to the quantum value $n2\pi\hbar$ at switching to the state with $\langle n_s^{-1}\rangle^{-1} \neq 0$. The momentum circulation does not change systematically during a long time t_{long} at $\int_{t_{\mathrm{long}}} dt F_{\mathrm{dis}} = t_{\mathrm{long}}\overline{F_{\mathrm{dis}}} \neq 0$ because at reiterated switching $\int_l dl\overline{F_{\mathrm{dis}}} + \int_l dl\overline{\Delta P}\omega = 0$. $\overline{\Delta P} = N_{\mathrm{sw}}^{-1}\Sigma_k \Delta P(k)$; $\Delta P(k)$ is the momentum change at k switching in the state with $\langle n_s^{-1}\rangle^{-1} \neq 0$; $\omega = N_{\mathrm{sw}}/t_{\mathrm{long}}$; N_{sw} is the number of switching for t_{long}.

At the closing of the superconducting state in the loop, as well as at the Meissner effect, superconducting pairs are accelerated against the force of the electric field $\int$ldlE $= -(1/c)d\Phi/dt$. *In order to eliminate the contradiction with the Newton's law a force* F_q *may be introduced,* $\overline{F_q} = \overline{\Delta P}\omega$. *Because the* ΔP *is induced by quantization it is natural to call* F_q *as quantum force. The necessity to introduce the* F_q *is conditioned by the well known difference between superconductor and a classical conductor with infinite conductivity. It is important that the quantum force can not be localized in any segment of the loop in principle because of the uncertainty relation* $\Delta p \Delta l > \hbar$. *The* v_s *becomes nonzero when the momentum takes a certain value* $\Delta p \ll p_{n+1} - p_n = 2\pi\hbar/l$, *i.e., when superconducting pairs cannot be localized in any segment of the loop.* $\overline{F_q}$ *should be uniform along the loop because* $\Delta P \propto j_s$.

The quantum force $\overline{F_q}$ takes the place of the Faraday's voltage $-(1/c)d\Phi/dt$ which maintains the screening current in a conventional loop with $R_l \neq 0$. Therefore the $j_{\mathrm{PC}} = \overline{j_{sc}} \neq 0$ is observed at $\overline{R_l} \neq 0$ and $\overline{d\Phi/dt}$ in the LP experiment. The periodic variation of the resistance with magnetic field $R_l(\Phi/\Phi_0)$ is observed in the LP experiment[2] because the probability of superconducting state $P(\langle n_s^{-1}\rangle^{-1} \neq 0) \propto \exp -(E_p + E_L)/k_B T$ decreases at $\Phi \neq n\Phi_0$. The approximation,[3] in which only state with minimum $|n - \Phi/\Phi_0|$ is taken into account, describes enough well the experimental data[2] because in the superconducting state $|E_p(n+1) - E_p(n)| \gg k_B T$ even in the fluctuation region near T_c.

Thus, the LP experiment is evidence of a direct (nonchaotic) one-dimensional Brownian motion. The Brownian particle in this case is the superconducting condensate. Its kinetic energy changes randomly in time: the E_p (and also E_L) is increased by the quantum force and dissipates after the switching a loop segment in the normal state. The quantum force induced by the fluctuations is the Langevin force F_{Lan}. Contrary to the classical Brownian motion $\overline{F_{\mathrm{Lan}}}=\overline{F_q}=\overline{\Delta P}\omega\neq 0$ at $j_{\mathrm{PC}}\neq 0$ and $\overline{R_l}\neq 0$.

Because the LP experiment is explained by the fluctuation switching between $j_{sc}=q\Sigma_p v f_{cl}=0$, where the distribution function f_{cl} is in the equilibrium $f_0=f_{cl}$ above T_c, and $j_{sc}=q\Sigma_p v f_{qu}\neq 0$ where f_{qu} is in the equilibrium $f_0=f_{qu}$ below T_c, it is useful to consider the motion along the loop both superconducting pairs and electrons at the transition between f_{cl} and f_{qu}. The reduction of j_{sc} at $R_l\neq 0$ can be described by the classical Boltzmann transport equation[9] because the ''random phase'' assumption is valid at $\langle n_s^{-1}\rangle^{-1}=0$. But the j_{sc} appearance contradicts classical mechanics. For a phenomenological description of the transition $f_{cl}\to f_{qu}$, a new term $\mathcal{N}$ may be added to the Boltzmann equation

$$\frac{df}{dt}=\frac{\partial f}{\partial t}+v\frac{\partial f}{\partial l}+qE_V\frac{\partial f}{\partial p}=\mathcal{N}-\frac{f_1}{\tau} \tag{5}$$

$\mathcal{N}=df/dt+f_1/\tau$ during a time Δt_{qu} of the transition $f_{cl}\to f_{qu}$ and $\mathcal{N}=0$ during any other time, $\int_{\Delta t_{qu}}dt\mathcal{N}=f_{qu}-f_{cl}+\int_{\Delta t_{qu}}dt f_1/\tau$. p is the generalized momentum. Therefore $E_V=-\nabla V$ is the potential part of the electric field $E=-\nabla V-(1/c)\partial A/\partial t$: $qE_V=\partial p/\partial t=m\partial v/\partial t+(q/c)\partial A/\partial t=qE+(q/c)\partial A/\partial t=-q\nabla V$. The distribution function $f=n_s+f_e$ describes both superconducting pairs and electrons. $q=e$ for electron and $q=2e$ for superconducting pair. $f_1=f-f_0$ is the deviation of the distribution function f from the one f_0 in the equilibrium state. It is assumed that the equilibrium distribution $f_0=f_{cl}$ at $\langle n_s^{-1}\rangle^{-1}=0$ and $f_0=f_{qu}$ at $\langle n_s^{-1}\rangle^{-1}\neq 0$. The difference between $f_1=f-f_{cl}$ and $f_1=f-f_{qu}$ is not important in our consideration because the mean time between collisions τ is infinite for superconducting pairs and the equilibrium distributions for electrons f_e are approximately the same at $\langle n_s^{-1}\rangle^{-1}=0$ and $\langle n_s^{-1}\rangle^{-1}\neq 0$.

The balance on the average forces

$$\frac{\partial P}{\partial t}-F_p-F_e=F_q-F_{\mathrm{dis}} \tag{6}$$

is obtained by multiplication of the transport equation (5) by the momentum and summing over the p states. Here $P=\Sigma_p pf=p_s n_s+\Sigma_p pf_e=P_s+P_e$; $P_s=p_s n_s$ is the momentum per unit volume of superconducting pairs; $P_e=\Sigma_p pf_e$ is the momentum per unit volume of normal electrons; $F_p=-\partial(\Sigma_p pvf)/\partial l=-\partial(n_q\langle pv\rangle)/\partial l$ is the force of the pressure; $F_e=-eE_V\Sigma_p p\partial f/\partial p=eE_V n_q=2eE_V n_s+eE_V n_e$ is the force of the electric field; n_e is the density of normal electrons; $n_q=n_e+2n_s$ is the total density of electrons; $F_{dis}=\Sigma_p pf_1/\tau$ is the dissipation force; and $F_q=\Sigma_p p\mathcal{N}$ is the quantum force. $F_q=\Sigma_p pdf/dt+\Sigma_p pf_1/\tau$ during Δt_{qu}.

The quantum force $F_{qs}=\Sigma_p pdf/dt$ acts directly on superconducting pairs $\int_{\Delta t_{qu}}dtF_{qs}=\Sigma_p pf_{qu}-\Sigma_p pf_{cl}=\Delta P_s=(m/e)j_{\mathrm{PC}}[1+(Ls/l\lambda_0^2)n_s']$ and $F_{qe}=\Sigma_p pf_1/\tau$ acts on normal electrons through the Faraday's voltage $\int_l dlE=-(L/c)dI/dt$. The dissipation force F_{dis} strives to retain zero average velocity. Therefore $\Delta P_e=\int_{\Delta t_{qu}}dtF_{qe}=\int_{\Delta t_{qu}}dtF_{dis}=n_e(e/c)Lsj_s/l$.

Both P_s and P_n return to initial values after the transition $f_{qu}\to f_{cl}$ because of the dissipation force. After the switching of a l_b segment in the normal state with $R_{bn}=\rho_n l_b/s\neq 0$, when the resistance of other l_a segment $R_a=0$, a potential difference V and a pressure difference is induced by the deviation Δn_q of the electron density from its equilibrium value ($\Delta n_q\ll n_q$). But $\int_l dlF_p=-\int_l dl\partial(n_q\langle pv\rangle)/\partial l=0$ and $\int_l dlF_e=en_q\int_l dlE_V=-en_q\int_l dl\nabla V=0$.

The order of F_p and F_e magnitudes can be estimated by relations $F_p\approx-\langle pv\rangle\Delta n_q/\Delta l$ and $F_e\approx q^2 n_q\Delta n_q\Delta l=q^2/n_q^{-1/3}(\Delta l/n_q^{-1/3})^2\Delta n_q/\Delta l$. Because $\Delta n_q\ll n_q$ the characteristic length Δl over which n_q changes is much longer than the distance between electrons: $\Delta l\gg n_q^{-1/3}$. In any metal $\langle pv\rangle\approx q^2/n_q^{-1/3}$.[9] Consequently, the force of the pressure $F_p\ll F_e$ is not important in our consideration.

The time of the Δn_q appearance is very short because the capacitance is very small. After this short time the j_{sc} value is the same in the superconducting l_a, $j_{sc}=j_s+j_{na}$, and in the normal l_b, $j_{sc}=j_{nb}$, segments. The dissipation force acts on superconducting pairs through the electric force $\partial P_s/\partial t=F_e=-2en_s\nabla V$ and $dj_s/dt=(2e^2n_s/m)E_a=(2e^2n_s/m)[-\nabla V_a-(Ls/c^2l)dj_{sc}/dt]$. The current of normal electrons $j_{na}=\rho_n E_a$ in the l_a segment and $j_{nb}=\rho_n E_b$ in the l_b segment. Because $\int_l dl\nabla V=l_a\langle\nabla V_a\rangle+l_b\langle\nabla V_b\rangle=\langle V_a\rangle+\langle V_b\rangle=0$ the electric field $E_a=-\langle V_b\rangle/\mathrm{la}$ $[-(Ls/c2l)djsc/dt]$ *in the* l_a *segment and* $E_b=\langle V_b\rangle/l_b$ $[-(Ls/c^2l)dj_{sc}/dt]$ *in the* l_b *segment. At* $l_a\gg l_b$, *when* $j_{na}\ll j_{sc}$, $\langle V_b\rangle\simeq R_{bn}I_{sc}\simeq R_{bn}I_s\exp-t/\tau_{RL}$, *where* $\tau_{RL}=(l_a/l+l_a\lambda_0^2/Lsn_s')L/R_{bn}$ *is the decay time of the current.*

At $T\simeq T_{cb}\langle T_{ca}$ only l_b segment with lowest critical temperature T_{cb} is switched in the normal state by the fluctuation. In this case $\overline{R_b}\neq 0$, $\overline{R_a}=0$ and $-\overline{\langle V_a\rangle}=\overline{\langle V_b\rangle}=L\overline{I_s}\omega(l_a/l+l_a\lambda_0^2/Lsn_s')$. Thus, not only the persistent current $I_{p.c}$ but also the persistent voltage $V_{\mathrm{PV}}=\overline{\langle V_b\rangle}$ can be induced by fluctuations in an inhomogeneous loop. This result was published first in Ref. 10. The possibility of the persistent voltage is a direct consequence of the existence of the nonchaotic Brownian motion at which $\overline{F_{\mathrm{Lan}}}=\overline{F_q}\neq 0$. The average force of the electric field $\overline{F_e}=en_q\overline{E}$ should be not equal zero in an inhomogeneous loop, in which the dissipation force $\overline{F_{\mathrm{dis}}}$ has different value in segments, because $\overline{F_q}$ should be uniform along the loop and according to Eq. (6) $\overline{F_e}\simeq\overline{F_q}-\overline{F_{\mathrm{dis}}}$ (because $F_p\ll F_e$). In a homogeneous loop $\overline{F_e}=\overline{F_q}-\overline{F_{\mathrm{dis}}}=0$ because the switching probability of any segment is the same and $\overline{F_{\mathrm{dis}}}$ is uniform along the loop.

The inhomogeneous superconducting loop with $V_{\mathrm{PV}}\neq 0$ is an electric circuit in which the l_a segment with higher T_c is a power source $W_s=\overline{\langle V_a\rangle I_{sc}}<0$, and the l_b segment with lower T_c is a load, $W_l=\overline{\langle V_b>I_{sc}}0$. The power W_s induced by the thermal fluctuation cannot exceed $(k_BT)^2/\hbar$ because the energy of fluctuation is $k_B T$ and the frequency of switching $\omega < k_B T/\hbar$ in accordance with the uncertainty relation. Consequently $V_{PV} = (R_b W_l)^{0.5} < k_B T_c (R_b/\hbar)^{0.5}$ in any case. $(k_B T)^2/\hbar \simeq 10^{-10}$ Wt at $T = 10$ K and $(k_B T)^2/\hbar \simeq 10^{-8}$ Wt at $T = 100$ K. Therefore, at a real value $R_b = 1\ \Omega$, $V_{PV} < 10^{-5}$ V=10 μV for a low-T_c superconductor with $T_c \approx 10$ K and $V_{PV} < 10^{-4}$ V=100 μV for a high-T_c superconductor with $T_c \approx 100$ K. These voltage values are large enough to be measured experimentally.

The persistent voltage can be induced also in an inhomogeneous normal metal mesoscopic loop[11] in which the persistent current can exist.[6,7] The mesoscopic loop, in which electrons are scattered in only segment, is like the inhomogeneous superconducting loop considered above. Superconducting condensate can be considered as a big particle which is scattered on the normal loop segment similar to the way electrons are scattered on impurities. In details the problem of the persistent voltage in an inhomogeneous normal metal mesoscopic loop will be considered elsewhere.

The transition between f_{cl} and f_{qu} states can be induced not only by the fluctuation but also by temperature change and by mechanical interrupting and closing of the superconducting loop. In the first case the loop can be considered as dc generator in which heat energy is transformed in electric energy.[12] In the second case the mechanical energy is transformed to the electric energy. In order to close the loop interrupted by Josephson junction, an additional work $\int db F_q = \Delta b \langle F_q \rangle$ should be expended because the energy is increased on $E_p + E_L \approx E_p \approx (s/\lambda^2)(\Phi_0^2/4\pi R)(n - \Phi/\Phi_0)^2$ at the I_{sc} appearance. The Josephson current decreases exponentially with increasing of break width b and has a negligible value when b exceeds some nanometers.[13] Consequently in order to close the loop at $n - \Phi/\Phi_0 = 1/2$ the quantum force, the average value of which equals $F_q \approx (s/\lambda^2)(\Phi_0^2/2l\Delta b)0.25$, should be overcome, where $\Delta b \approx 10$ nm. At $l = 4$ μm, when $\Phi_0^2/2l \approx 3\times10^{-20}$ J, $\langle F_q \rangle \approx (s/\lambda^2)3\times10^{-12}$ N. This consideration shows that the wave function can have an elasticity and that the quantum force can be connected with a real classical force which can be measured.

[1] W.A. Little and R.D. Parks, Phys. Rev. Lett. **9**, 9 (1962); Phys. Rev. **133**, A97 (1964).

[2] H. Vloeberghs *et al.*, Phys. Rev. Lett. **69**, 1268 (1992).

[3] M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, New York, 1975).

[4] M. Tinkham, Phys. Rev. **129**, 2413 (1963).

[5] A. Isihara, *Statistical Physics* (Academic Press, New York, 1971).

[6] I.O. Kulik, Pisma Zh. Éksp. Teor. Fiz. **11**, 407 (1970) [JETP Lett. **11**, 275 (1970)]; M. Buttiker, Y. Imry, and R. Landauer, Phys. Lett. A **96**, 365 (1983).

[7] E.M.Q. Jariwala *et al.*, Phys. Rev. Lett. **86**, 1594 (2001); V. Chandrasekhar *et al.*, *ibid.* **67**, 3578 (1991); L.P. Levy *et al.*, *ibid.* **64**, 2074 (1990).

[8] W. Kohn and J.M. Luttinger, Phys. Rev. **108**, 590 (1957).

[9] A.A. Abrikosov, *Fundamentals of the Theory of Metals* (North Holland, Amsterdam, 1989).

[10] A.V. Nikulov and I.N. Zhilyaev, J. Low Temp. Phys. **112**, 227 (1998).

[11] A.V. Nikulov (unpublished).

[12] A.V. Nikulov, in *Symmetry and Pairing in Superconductors*, edited by M. Ausloos and S. Kruchinin (Kluwer Academic Publishers, Dordrecht, 1999), p. 373.

[13] A. Barone and G. Paterno, *Physics and Application of the Josephson Effect* (Wiley, New York, 1982).