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Hybrid resonances in the optical absorption of a three-dimensional anisotropic quantum well

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The optical absorption of electromagnetic radiation by an electron gas in a three-dimensional anisotropic quantum well subjected to a uniform magnetic field arbitrarily directed with respect to the potential symmetry axes is investigated. The dependence of the absorption coefficient on the radiation frequency, on the direction of the polarization vector, and on the magnitude and direction of the magnetic field is studied. It is shown that in the general case there are three resonance frequencies on the absorption curve; the amplitude and the position of the resonance peaks is investigated. In the limit $T\to 0$ (the case of a degenerate gas), the absorption curve as a function of the radiation frequency contains kinks.

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I. INTRODUCTION

The modern development of nanotechnology has enabled the confinement of a finite number of electrons in a localized space of a few hundred angstroms (quantum wells and dots). In particular, it has become possible to experimentally investigate wells in heterostructures with widths comparable to the cyclotron radius. Note that the physical properties of nanostructures cannot be extrapolated from behavior at larger sizes because of the prevalence of quantum effects. Hybridization of the size quantization and the magnetic one promises discoveries of new effects in such systems. Therefore, quantum wells and dots are attracting a great deal of experimental and theoretical interest.[1–4] For example, the magnetization of two-dimensional (2D) isotropic quantum dots in perpendicular and parallel magnetic fields was studied by Meir *et al.*[1] and by Ihm *et al.*,[2] respectively. The case of an oblique magnetic field is investigated in Ref. 3. The spectrum and magnetic moment of an electron gas in a 3D quantum well is studied in Ref. 4.

It is especially important to investigate the optical properties of nanostructures because the study of intraband optical transitions yields important information about the energy spectrum, the Fermi surface of electrons, and the value of the electronic effective mass. The effects of frequency hybridization cause the resonance absorption of electromagnetic radiation in such structures at the hybrid frequencies but not at the bare cyclotron frequencies. For example, in Ref. 5, far-infrared magnetotransmission measurements on a quasi-three-dimensional semiconductor structure were reported. In particular, when the sample is tilted to 13° and 30° with respect to the field direction, the cyclotron resonance splits into two branches. This splitting is explained just by the hybridization effect. In Ref. 6, the optical transitions of an electron gas in a two-dimensional symmetrical quantum well are investigated. The resonance absorption here was observed at the two hybrid frequencies. For arrays of quantum dots, the resonance absorption at the same frequencies was experimentally investigated by Meurer *et al.*[7] The electrodynamic response of quantum-dot structures containing 25–210 electrons per dot was observed by Demel *et al.*[8] In particular, in a magnetic field the resonance splits into two branches. However, for a strong magnetic field $B>4$ T, an additional resonance was observed. The optical absorption of two-dimensional quantum dots with a few electrons was experimentally investigated by Sikorski and Merkt.[9] The electronic energy levels of an anisotropic quantum well in a perpendicular magnetic field and the selection rules for dipole transitions were investigated by Madhav and Chakraborty.[10] The hybrid-phonon resonance in a 2D quantum well was studied in Ref. 11. We stress that the above investigations consist of a model of 2D parabolic quantum dots. The effects of electron-electron interactions in quantum wells are considered in Refs. 12 and 13. However, as was shown in Ref. 14, the electron-electron interaction in the case of a three-dimensional harmonic quantum well has no effect on the electron transitions.

In this paper, we present a theoretical study of the optical absorption of an electron gas in a three-dimensional quantum well subjected to a uniform magnetic field arbitrarily directed with respect to the potential symmetry axes. In the above-mentioned papers, the parabolic potential is used as the confinement potential of quantum wells. The choice of this potential is justified by a number of factors. On the one hand, it has been proved rigorously that for high-energy levels, any confinement potential is well approximated by a parabolic one.[15] On the other hand, in accordance with the generalized Kohn theorem,[14] electron-electron interaction in this case has no effect on the electron transitions. Finally, even in the presence of an external magnetic field, a parabolic potential gives a quadratic Hamiltonian whose spectrum can be obtained by a purely algebraic method, i.e., with the help of an appropriate linear canonical transformation of the phase space. This spectrum reduces to the algebraic sum of the spectra of three harmonic oscillators with some hybrid frequencies.[4]

To find an electrodynamic response, one needs the matrix elements of the momentum operators and the position ones. However, a direct calculation of wave functions and corresponding matrix elements is a complicated computational problem. To simplify these calculations, we use linear canonical transformations of the phase space. By means of these transformations, we find a new coordinate system such that in the new phase coordinates the wave functions have the simplest form.

Using this method, we calculate the absorption coefficient of the electromagnetic radiation of electrons in a 3D anisotropic quantum well for the general case of an arbitrary di-

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rection of the magnetic field as well as the polarization vector.

To calculate the absorption coefficient $\Gamma(\omega)$, we use an approach based on the method suggested in Refs. 16 and 17. The cases of both a nondegenerate and a degenerate gas are studied. We consider two important cases in detail, namely the Faraday geometry (the polarization vector is perpendicular to the magnetic field) and the Feucht geometry (the polarization vector is parallel to the magnetic field).

II. DIAGONALIZATION OF THE HAMILTONIAN

The spinless one-particle Hamiltonian of an electron in a 3D anisotropic parabolic quantum well has the form

$$H=\frac{1}{2m^*}\left(\mathbf{p}-\frac{e}{c}\mathbf{A}\right)^2+\frac{m^*}{2}(\Omega_x^2x^2+\Omega_y^2y^2+\Omega_z^2z^2), \quad (1)$$

where $\mathbf{A}$ is the vector potential of a magnetic field $\mathbf{B}$, Ω_i $(i=x,y,z)$ are the characteristic frequencies of the parabolic potential, and m^* is the effective electron mass.

It is convenient to choose the following gauge for the vector potential:

$$\mathbf{A}=(\tfrac{1}{2}B_yz-B_zy,0,B_xy-\tfrac{1}{2}B_yx).$$

In the phase space (p_x,p_y,p_z,x,y,z), the Hamiltonian (1) defines a quadratic form with the sixth-order symmetric matrix.[4] By means of a linear canonical transformation of the phase space, we find new phase coordinates $(\mathbf{P},\mathbf{Q})$ such that H has the canonical form

$$H(\mathbf{P},\mathbf{Q})=\frac{1}{2m^*}(P_1^2+P_2^2+P_3^2)+\frac{m^*}{2}(\omega_1^2Q_1^2+\omega_2^2Q_2^2+\omega_3^2Q_3^2), \quad (2)$$

where ω_i $(i=1,2,3)$ are the hybrid frequencies (they are functions of the magnitude and direction of the magnetic field $\mathbf{B}$). These frequencies are obtained from the following sixth-order algebraic equation:[4]

$$(\Omega_1^2-\lambda^2)(\Omega_2^2-\lambda^2)(\Omega_3^2-\lambda^2)-\omega_{xc}^2(\Omega_1^2-\lambda^2)\lambda^2-\omega_{yc}^2(\Omega_2^2-\lambda^2)\lambda^2-\omega_{zc}^2(\Omega_3^2-\lambda^2)\lambda^2=0, \quad (3)$$

where $\omega_i=\sqrt{|\lambda|}$ $(i=1,2,3)$ and $\omega_{jc}=eB_j/m^*c$ $(j=x,y,z)$ are the components of the cyclotron frequency.

Hence, the spectrum of the Hamiltonian (2) has the form

$$E_{nml}=\hbar\omega_1(n+\tfrac{1}{2})+\hbar\omega_2(m+\tfrac{1}{2})+\hbar\omega_3(l+\tfrac{1}{2}) \quad (n,m,l=0,1,\ldots), \quad (4)$$

and the corresponding wave functions in the new phase coordinates have the form

$$\Phi_{nml}(Q_1,Q_2,Q_3)=\Phi_n(Q_1)\Phi_m(Q_2)\Phi_l(Q_3), \quad (5)$$

where $\Phi_k(x)$ are the oscillator functions.

Further, we need the transition matrix $\Lambda=(a_{ji})$ $(i,j=1,\ldots,6)$ from the initial phase coordinates (p_1,p_2,p_3,x,y,z) to the new ones $(P_1,P_2,P_3,Q_1,Q_2,Q_3)$:

$$\begin{pmatrix} p_1\\ p_2\\ p_3\\ x\\ y\\ z \end{pmatrix}=\Lambda\begin{pmatrix} P_1\\ P_2\\ P_3\\ Q_1\\ Q_2\\ Q_3 \end{pmatrix}. \quad (6)$$

The matrix elements a_{ji} are given in the Appendix.

III. ABSORPTION COEFFICIENT

Using the method suggested by Bass and Levinson,[16] we find the absorption coefficient $\Gamma(\omega)$ by applying the perturbation theory for the interaction of electrons with a high-frequency field,

$$\Gamma(\omega)=\frac{2\pi\sqrt{\varepsilon(\omega)}}{c\hbar N_{\mathbf{f}}}\left[1-\exp\left(-\frac{\hbar\omega}{T}\right)\right]\times\sum_{nml}\sum_{n'm'l'}f_0(E_{nml})|\langle n,m,l|H_R|n',m',l'\rangle|^2\times\delta(E_{nml}-E_{n'm'l'}+\hbar\omega), \quad (7)$$

where $\varepsilon(\omega)$ is the real part of the dielectric constant (we suppose there is no dispersion in the frequency range considered here), $\mathbf{f}$ is the wave vector of photons, $f_0(E_{nml})$ is the electron distribution function for the nondegenerate gas, $N_{\mathbf{f}}$ is the number of initial-state photons with the frequency ω, and the factor $1-\exp(-\hbar\omega/T)$ takes into account spontaneous transitions.

The Hamiltonian H_R representing the interaction with the high-frequency electromagnetic field has the form

$$H_R=\frac{e}{m^*}\sqrt{\frac{2\pi\hbar N_{\mathbf{f}}}{\varepsilon(\omega)\omega}}\,\mathbf{e_f}\mathbf{p}', \quad (8)$$

where $\mathbf{e_f}$ is the polarization vector and $\mathbf{p}'=\mathbf{p}-e\mathbf{A}/c$ is the generalized momentum.

We calculate the matrix elements of H_R in the new phase coordinates because in this case the wave function of electrons has the simplest form (5). Note that it is easy to find the momentum operator $\mathbf{p}'$ $(p_1'=p_x-eA_x/c,\ p_2'=p_y,\ p_3'=p_z-eA_z/c)$ from Eq. (6). Calculation of the matrix elements of this operator yields

$$\langle nml|p_i'|n'm'l'\rangle=\alpha_{i1}\sqrt{\frac{n+1}{2}}\delta_{n,n'-1}\delta_{m,m'}\delta_{l,l'}+\alpha_{i2}\sqrt{\frac{m+1}{2}}\delta_{m,m'-1}\delta_{n,n'}\delta_{l,l'}+\alpha_{i3}\sqrt{\frac{l+1}{2}}\delta_{l,l'-1}\delta_{m,m'}\delta_{n,n'}, \quad (9)$$

where

$$\alpha_{1j}=\frac{\hbar}{il_j}\left(a_{1j}-\frac{m^*}{2}\omega_{cy}a_{6j}+m^*\omega_{cz}a_{5j}\right)+l_j\left(a_{1,j+3}-\frac{m^*}{2}\omega_{cy}a_{6,j+3}+m^*\omega_{cz}a_{5,j+3}\right), \tag{10}$$

$$\alpha_{2j}=\frac{\hbar}{il_j}a_{2j}+l_ja_{2,j+3}, \tag{11}$$

$$\alpha_{3j}=\frac{\hbar}{il_j}\left(a_{3j}-m^*\omega_{cx}a_{5j}+\frac{m^*}{2}\omega_{cy}a_{4j}\right)+l_j\left(a_{3,j+3}-m^*\omega_{cx}a_{5,j+3}+\frac{m^*}{2}\omega_{cy}a_{4,j+3}\right) \tag{12}$$

(here $i,j=1,2,3$).

In the general case, the direction of the polarization vector with respect to the potential symmetry axes x,y,z depends on the azimuth angle θ and the polar angle φ, respectively:

$$\mathbf{e_f}=\mathbf{i}\sin\theta\cos\varphi+\mathbf{j}\sin\theta\sin\varphi+\mathbf{k}\cos\theta. \tag{13}$$

Let us introduce the matrix $A=(\alpha_{ij})$ $(i,j=1,2,3)$ with the matrix elements α_{ij} from Eqs. (10)–(12) and consider the vector

$$\mathbf{X}=\frac{1}{\sqrt{m^*\hbar}}A\mathbf{e_f} \tag{14}$$

with coordinates (X_1,X_2,X_3).

Now we write the squares of the matrix elements of the perturbation operator H_R in terms of $\mathbf{X}$:

$$|\langle n,m,l|H_R|n',m',l'\rangle|^2=\frac{e^2}{m^*}\frac{\pi\hbar^2N_{\mathbf{f}}}{\varepsilon(\omega)\omega}\{X_1^2(n+1)\delta_{n,n'-1}\delta_{m,m'}\delta_{l,l'}+X_2^2(m+1)\delta_{m,m'-1}\delta_{n,n'}\delta_{l,l'}+X_3^2(l+1)\delta_{l,l'-1}\delta_{n,n'}\delta_{m,m'}\}. \tag{15}$$

Substituting Eq. (15) into Eq. (7), we get

$$\Gamma(\omega)=\frac{2\pi^2e^2}{cm^*\sqrt{\varepsilon}\omega}\sum_{n,m,l=0}^{\infty}f_0(E_{nml})\{X_1^2(n+1)\delta(\omega-\omega_1)+X_2^2(m+1)\delta(\omega-\omega_2)+X_3^2(l+1)\delta(\omega-\omega_3)\}. \tag{16}$$

Equation (16) shows that there are three resonance frequencies in the general case.

The normalization constant in the distribution function for a nondegenerate gas is determined from the equation

$$\sum_{n,m,l=0}^{\infty}f_0(E_{mnl})=N, \tag{17}$$

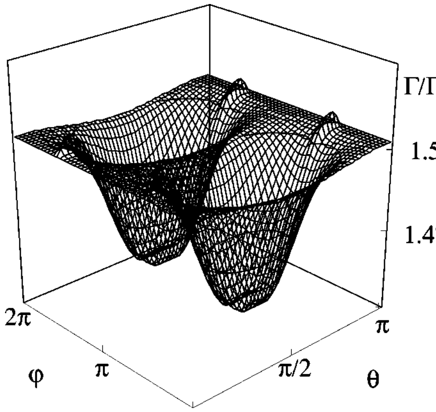

FIG. 1. The absorption coefficient Γ as a function of the orientation of the magnetic field **B** with respect to the symmetry axes of the well (the angles θ and φ) at $B=1.5$ T, $\omega=1.25\times10^{13}$ c^{-1}, $\Omega_x=1.9\times10^{13}$ c^{-1}, $\Omega_y=1.1\times10^{13}$ c^{-1}, $\Omega_x=1.5\times10^{13}$ c^{-1}.

where N is the electron density.

To take into account the smearing of the energy levels caused by collisions, we replace the Dirac δ function by the Lorentzian function,

$$\delta_\tau(x)=\frac{(\pi\tau)^{-1}}{\tau^{-2}+x^2} \tag{18}$$

(here τ is the phenomenological relaxation time).

Substituting the function (18) into Eq. (16), we obtain the following expression for the absorption coefficient

$$\frac{\Gamma(\omega)}{\Gamma_0}=\frac{X_1^2/\omega}{1+\tau^2(\omega-\omega_1)^2}+\frac{X_2^2/\omega}{1+\tau^2(\omega-\omega_2)^2}+\frac{X_3^2/\omega}{1+\tau^2(\omega-\omega_3)^2}, \tag{19}$$

where $\Gamma_0=\pi e^2N\tau/\sqrt{\varepsilon}cm^*$.

It is interesting to consider the two most important cases: the polarization vector is perpendicular to the magnetic field (the Feucht geometry) and the polarization vector is parallel to the magnetic field (the Faraday geometry). Without loss of generality, we fix the y direction as the direction of the polarization vector and vary the direction of the magnetic field.

The graphs plotted in Fig. 1 show that the absorption coefficient Γ depends on the orientation of the field **B** with respect to the symmetry axes of the well (the angles θ and φ). The dependence Γ as a function of the direction of the polarization vector is depicted in Fig. 2.

A. Feucht geometry

In this case, the magnetic field is applied in the z direction. The hybrid frequencies $\omega_{1,2}$ are determined by the formula

$$\omega_{1,2}=\tfrac{1}{2}\left[\sqrt{(\Omega_x+\Omega_y)^2+\omega_c^2}\pm\sqrt{(\Omega_x-\Omega_y)^2+\omega_c^2}\right],$$

$$\omega_3=\Omega_z. \tag{20}$$

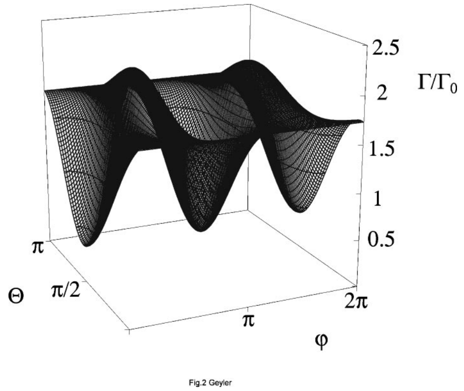

FIG. 2. The absorption coefficient Γ as a function of the direction of the polarization vector with respect to the symmetry axes of the well (the angles θ and φ) at $B=1.5$ T, $\omega=1.25\times10^{13}$ c^{-1}, $\Omega_x=1.9\times10^{13}$ c^{-1}, $\Omega_y=1.1\times10^{13}$ c^{-1}, $\Omega_x=1.5\times10^{13}$ c^{-1}.

The values of X_i^2 have a simple form,

$$X_i^2=\frac{\omega_i(\Omega_x^2-\omega_i^2)^2}{\omega_c^2\Omega_x^2+(\Omega_x^2-\omega_i^2)^2},\tag{21}$$

where $\omega_c=\omega_{cz}$.

Let us consider the important case of a symmetric well: $\Omega_x=\Omega_y\equiv\Omega$. In this case, the hybrid frequencies $\omega_{1,2}$ have the form

$$\omega_{1,2}=\tfrac{1}{2}\left[\sqrt{4\Omega^2+\omega_c^2}\pm\omega_c\right],\qquad \omega_3=\Omega_z.\tag{22}$$

Note that the absorption coefficient depends strongly on the correlation between the size quantization and the magnetic one.

In the case in which the magnetic field is weak ($\omega_c\ll\Omega$), the frequencies $\omega_{1,2}$ can be represented in the form $\omega_{1,2}\sim\Omega\pm\omega_c/2$ up to terms of order ω_c^2/Ω.

In this case, $X_1^2=X_2^2\sim\Omega/2$, and the absorption coefficient has the form

$$\frac{\Gamma(\omega)}{\Gamma_0}=\frac{\Omega}{2\omega}\left[\frac{1}{1+\tau^2(\omega-\Omega-\omega_c/2)^2}+\frac{1}{1+\tau^2(\omega-\Omega+\omega_c/2)^2}+\frac{\Omega_z}{\Omega}\frac{(\Omega^2-\Omega_z^2)^2}{[\omega_c^2\Omega^2+(\Omega^2-\Omega_z^2)^2][1+\tau^2(\omega-\Omega)^2]}\right].\tag{23}$$

For the case of a strong magnetic field ($\omega_c\gg\Omega_x,\Omega_y$) and a symmetric structure ($\Omega_x=\Omega_y\equiv\Omega$), Eq. (22) yields the estimates $\omega_1\sim\omega_c$, $\omega_2\sim\Omega^2/\omega_c$. In this case, the resonance frequency ω_2 becomes appreciably smaller than ω_1. The same statement is true for the quantities $X_{1,2}^2$: $X_1^2\sim\omega_c$, $X_2^2\sim\Omega^4/\omega_c^3$.

Thus, in the case of a strong magnetic field, the absorption coefficient is given by

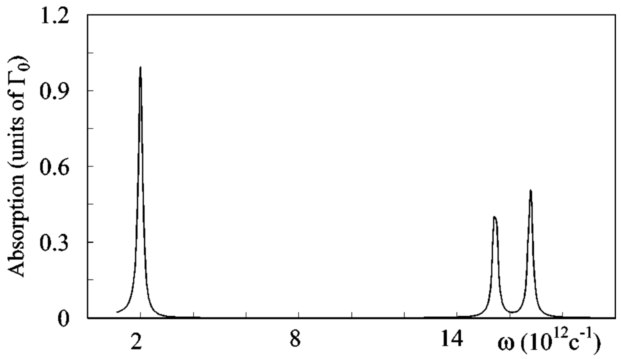

FIG. 3. The absorption coefficient $\Gamma(\omega)$ in the case of the Feucht geometry at $\Omega=1.6\times10^{13}$ c^{-1}, $\Omega_z=2\times10^{12}$ c^{-1}, $B=0.5$ T.

$$\frac{\Gamma(\omega)}{\Gamma_0}=\frac{\omega_c}{\omega}\frac{1}{1+\tau^2(\omega-\omega_c^2)^2}+\frac{\Omega^4}{\omega_c^3\omega}\frac{1}{1+\tau^2(\omega-\Omega^2/\omega_c)^2}+\frac{\Omega_z}{\omega}\frac{(\Omega^2-\Omega_z^2)^2}{[\omega_c^2\Omega^2+(\Omega^2-\Omega_z^2)^2][1+\tau^2(\omega-\Omega_z)^2]}.\tag{24}$$

B. Faraday geometry

In this case, the magnetic field is applied in the y direction (the polarization vector is parallel to the magnetic field). The hybrid frequencies are found similarly with the previous case,

$$\omega_{1,3}=\tfrac{1}{2}\left[\sqrt{(\Omega_x+\Omega_z)^2+\omega_c^2}\pm\sqrt{(\Omega_x-\Omega_z)^2+\omega_c^2}\right],$$
$$\omega_2=\Omega_y.\tag{25}$$

The quantities X_i^2 can be written as

$$X_i^2=\frac{\omega_i[(\Omega_x^2-\omega_i^2)(\Omega_z^2-\omega_i^2)-\omega_c^2\omega_i^2]^2}{\omega_c^4\Omega_x^4+[(\Omega_x^2-\omega_i^2)(\Omega_z^2-\omega_i^2)-\omega_c^2\omega_i^2]^2},\tag{26}$$

where $\omega_c=\omega_{cy}$.

Let us consider the important case in which the well is symmetric $\Omega_x=\Omega_z\equiv\Omega$. In this case, the frequencies $\omega_{1,3}$ have the form

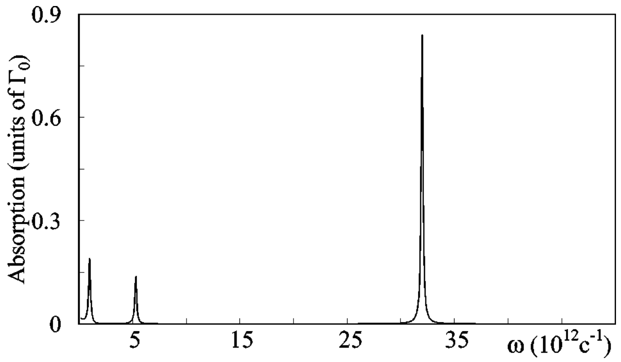

FIG. 4. The absorption coefficient $\Gamma(\omega)$ in the case of the Feucht geometry at $\Omega=1.3\times10^{13}$ c^{-1}, $\Omega_z=10^{12}$ c^{-1}, $B=10$ T.

$$\omega_{1,3}=\tfrac{1}{2}\left[\sqrt{4\Omega^2+\omega_c^2}\pm\omega_c\right]. \tag{27}$$

It follows from Eqs. (26) and (27) that the values of $X_{1,3}^2$ are equal to zero. Hence, we have only one resonance peak at the frequency $\omega=\Omega_y$. It is interesting to note that the resonance frequency is independent of the magnetic field. Therefore, in the case of the Faraday geometry, the absorption coefficient has the following form:

$$\frac{\Gamma(\omega)}{\Gamma_0}=\frac{\Omega_y}{\omega}\,\frac{[(\Omega^2-\Omega_y^2)^2-\omega_c^2\Omega_y^2]^2}{\{\omega_c^4\Omega^4+[(\Omega^2-\Omega_y^2)^2-\omega_c^2\Omega_y^2]^2\}[1+\tau^2(\omega-\Omega_y)^2]}. \tag{28}$$

IV. DISCUSSION

In this paper, we have investigated the optical absorption of a three-dimensional anisotropic quantum well subjected to a uniform magnetic field arbitrarily directed with respect to the potential symmetry axes. Using the method of linear canonical transformations of the phase space, we have found the matrix elements of the momentum operators and the position ones. The dependence of the absorption coefficient on the radiation frequency, on the direction of the polarization vector, and on the magnitude and direction of the magnetic field has been studied. It has been shown that in the general case there are three resonance frequencies. We stress that the amplitude and the position of the resonance peaks depend strongly on the correlations between the size quantization and the magnetic one. However, in some cases the disappearance of one or two peaks is possible. Note that the resonance peaks have a similar symmetric shape. Let us consider these cases in detail.

First of all, we consider the case of the Feucht geometry. Equation (23) shows that the absorption peaks have a doublet structure in the case of a weak magnetic field. Note that the amplitudes of the peaks are approximately equal. The distance between the doublet components is equal to the cyclotron frequency ω_c. The amplitude of the peak at the frequency $\omega_3=\Omega_z$ depends strongly on the correlation between the characteristic frequencies of the parabolic potential. For example, if $\Omega_z\ll\Omega$ or $\Omega_z\gg\Omega$, then the amplitude of this peak is nearly equal to 1, whereas the amplitude of the doublet peaks is nearly equal to $\frac{1}{2}$ (Fig. 3). Note that when $\Omega_z=\Omega$, there are only two resonance peaks. The amplitude of the peak at the frequency $\omega_3=\Omega_z$ is equal to zero [see Eq. (23)].

In the strong-field limit, $\omega_c\gg\Omega$, the amplitude of the peak at the frequency Ω^2/ω_c is smaller than that at the frequency $\omega_1=\omega_2$ (the first amplitude is equal to Ω^2/ω_c^2 times the second one). As one would expect, a strong absorption is observed at the cyclotron frequency in the case of strong magnetic quantization.

In the case of a symmetrical well ($\Omega_x=\Omega_y=\Omega_z$), the amplitude of the peak at the frequency $\omega_3=\Omega_z$ is equal to zero. Note that this effect takes place in the case of an arbitrarily directed magnetic field. The amplitude of this peak depends on the relations between the characteristic frequencies. However, if $\Omega_z\ll\Omega$, then the amplitude of this peak is of the order of the amplitude of the peak at the resonance frequency $\omega_2=\Omega^2/\omega_c$ and appreciably smaller than the amplitude of the peak at the resonance frequency $\omega_1=\omega_c$ (Fig. 4). In the opposite case ($\Omega_z\gg\Omega$), if $\Omega_z\gg\omega_c$, then the amplitude of the peak at the resonance frequency $\omega_3=\Omega_z$ is of the order of the amplitude of the peak at the resonance frequency $\omega_1=\omega_c$ (Fig. 5). In the case $\Omega_z\ll\omega_c$, the amplitude of the peak at the resonance frequency $\omega_3=\Omega_z$ becomes appreciably smaller than that for the peak at the resonance frequency $\omega_1=\omega_c$ (Fig. 6).

Let us consider the case of the Faraday geometry. In general, there are three resonance frequencies here. However, when the well is symmetric ($\Omega_x=\Omega_z$), there is only one resonance peak at the frequency $\omega=\Omega_y$. We stress that if the characteristic frequencies obey the relation $\Omega^2=\Omega_y(\Omega_y\pm\omega_c)$, then we cannot employ the method considered here

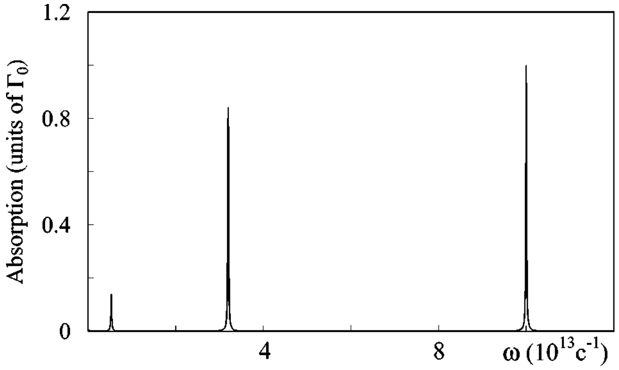

FIG. 5. The absorption coefficient $\Gamma(\omega)$ in the case of the Feucht geometry at $\Omega=1.3\times10^{13}$ c^{-1}, $\Omega_z=10^{14}$ c^{-1}, $B=10$ T.

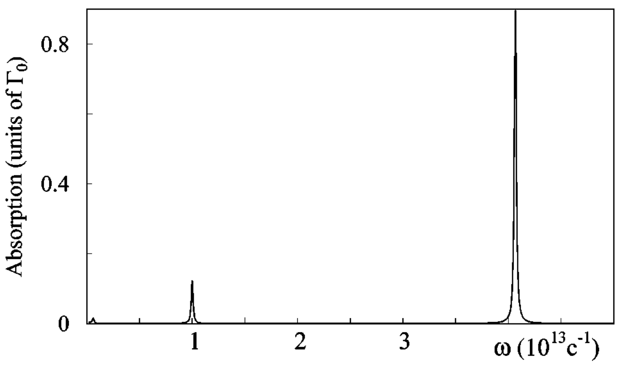

FIG. 6. The absorption coefficient $\Gamma(\omega)$ in the case of the Feucht geometry at $\Omega=5\times10^{12}$ c^{-1}, $\Omega_z=10^{13}$ c^{-1}, $B=15$ T.

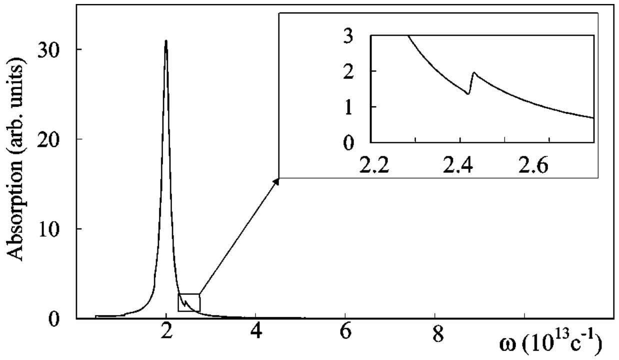

FIG. 7. The absorption coefficient $\Gamma(\omega)$ in the case of a degenerate gas at $\Omega=1.6\times10^{13}$ c^{-1}, $\Omega_z=2\times10^{12}$ c^{-1}, $B=0.5$ T; the neighborhood of a kink is shown in the inset.

because of the vanishing determinant of the transition matrix Λ.

Note that in the case of the degenerate gas, we obtain an expression for $\Gamma(\omega)$ if we replace in Eq. (16) $f_0(\varepsilon_{nml})$ by $f_0(\varepsilon_{nml})[1-f_0(\varepsilon_{nml}+\hbar\omega)]$ [here $f_0(\varepsilon_{mnl})$ is the Fermi distribution]. The existence of a resonance is independent of the electron statistics.

The singularities of the curve $\Gamma^{\mathrm{deg}}(\omega)$ incorporate all singularities of the curve $\Gamma(\omega)$. However, in the case of the degenerate gas, there exist additional singularities caused by the degeneracy. Namely, in this case there are kinks on the curve $\Gamma(\omega)$. Let us discuss the physical origin of this effect. These kinks are caused by the fact that the energy $\mu-\hbar\omega$ crosses the energy of the highest occupied level in the quantum well. A neighborhood of a kink is shown in Fig. 7 (inset).

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APPENDIX: THE MATRIX ELEMENTS OF THE MATRIX Λ

The matrix elements a_{ji} have the form

$$a_{ji}=\frac{f_{ji}}{L_i}(L_{i+3}=L_i), \qquad \text{(A1)}$$

where

$$\begin{aligned}L_i=\{&\omega_{yc}^2\omega_i^4(\omega_{xc}^2+\omega_{yc}^2)+\omega_{xc}^2\Omega_z^2(\Omega_x^2-\omega_i^2)^2\\&+\omega_{zc}^2\Omega_x^2(\Omega_z^2-\omega_i^2)^2\\&+[(\Omega_x^2-\omega_i^2)(\Omega_z^2-\omega_i^2)-\omega_{yc}^2\omega_i^2]^2\}^{1/2}.\end{aligned} \qquad \text{(A2)}$$

The values of f_{ji} are as follows. If $i=1,2,3$, then

$$f_{1i}=\frac{1}{\omega_i}[-\omega_{zc}\Omega_x^2(\Omega_z^2-\omega_i^2)+\tfrac{1}{2}\omega_{zc}\omega_{yc}^2\omega_i^2], \qquad \text{(A3)}$$

$$f_{2i}=0,$$

$$f_{3i}=\frac{1}{\omega_i}[\omega_{xc}\Omega_z^2(\Omega_x^2-\omega_i^2)-\tfrac{1}{2}\omega_{xc}\omega_{yc}^2\omega_i^2], \qquad \text{(A4)}$$

$$f_{4i}=-\frac{1}{m^*}\omega_{xc}\omega_{yc}\omega_i, \qquad \text{(A5)}$$

$$f_{5i}=\frac{1}{m^*\omega}[(\Omega_x^2-\omega_i^2)(\Omega_z^2-\omega_i^2)-\omega_{yc}^2\omega_i^2], \qquad \text{(A6)}$$

$$f_{6i}=-\frac{1}{m^*}\omega_{yc}\omega_{zc}\omega_i. \qquad \text{(A7)}$$

If $i=4,5,6$, then

$$f_{1i}=-\frac{m^*}{2}\omega_{xc}\omega_{yc}\omega_{i-3}(\Omega_x^2+\omega_{i-3}^2), \qquad \text{(A8)}$$

$$f_{2i}=m^*[\omega_{i-3}(\Omega_x^2-\omega_{i-3}^2)(\Omega_z^2-\omega_{i-3}^2)-\omega_{yc}^2\omega_{i-3}^3], \qquad \text{(A9)}$$

$$f_{3i}=-\frac{m^*}{2}\omega_{yc}\omega_{zc}\omega_{i-3}(\Omega_z^2+\omega_{i-3}^2), \qquad \text{(A10)}$$

$$f_{4i}=\omega_i\omega_{zc}(\Omega_z^2-\omega_{i-3}^2), \qquad \text{(A11)}$$

$$f_{5i}=0,$$

$$f_{6i}=-\omega_i\omega_{xc}(\Omega_x^2-\omega_{i-3}^2). \qquad \text{(A12)}$$

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