

Vortex structure in underdoped cuprates

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In underdoped cuprates the normal state is highly anomalous and is characterized as a pseudogap phase. The question of how to describe the “normal” core of a superconducting vortex is an outstanding open problem. We show that the SU(2) formulation of the t - J model provides a description of the normal state as well as the vortex core. Interestingly, the pseudogap persists inside the core. We also found that it is likely that the core consists of a state that breaks translational symmetry due to the existence of a staggered current that generates a staggered magnetic field with very slow dynamics. This staggered-flux state is likely to be the ground state for magnetic fields higher than H_{c2} . Experiments to test this picture are proposed.

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I. INTRODUCTION

It is now widely appreciated that high T_c superconductors are fundamentally different from conventional superconductors in that they emerge by introducing doped holes into a Mott insulator. This contrast is most apparent in the underdoped region where the density x of doped holes is small. Experimentally, this is also the regime where the physical properties are most anomalous.¹ Much attention has been focused on the normal state, which is characterized by a pseudogap regime below a relatively high temperature $T^* \approx 300$ K. The pseudogap appears in spin excitations and in tunneling and angle resolved photoelectron spectroscopy experiments. The superconducting state is anomalous as well in that the superfluid density is proportional to the hole density x and not the electron density (Fermi surface area) $1-x$ as in conventional superconductors. Recently, it has become possible to perform scanning tunneling microscope (STM) tunneling in the superconducting state and probe the electronic structure of the vortex core.^{2,3} This raises the following interesting question. Common sense would indicate that the vortex core should be made up of the normal state and one would expect the pseudogap, i.e., a dip in the tunneling density of states, to persist in the core region. This is in fact what is seen experimentally. Yet a conventional description of a vortex core requires that the order parameter vanishes inside the core, which is usually accompanied by the vanishing of the energy gap. Thus it is clear that the electronic structure of the vortex core in the underdoped region is qualitatively different from that given by conventional theory. This point was made eloquently in a recent paper by Franz and Tešanović (FT).⁴ Recently this problem is also addressed numerically using unrestricted mean-field theory by Han and Lee.⁵

It is clear that any attempt to model the underdoped vortex core must include the physics of the proximity to the Mott insulator, i.e., the strong correlation physics. One of the few analytic tools available for this purpose is the slave boson method used to treat the constraint of no double occupation in a strong correlation model such as the t - J model. FT employed the U(1) formulation of this theory where the electron operator $c_{i\sigma}$ is written as $c_{i\sigma} = f_{i\sigma} b_i^\dagger$ and the no double

occupation constraint is replaced by $f_{i\sigma}^\dagger f_{i\sigma} + b_i^\dagger b_i = 1$, which is in turn accomplished by the introduction of a U(1) gauge field \mathbf{a} . In the mean-field theory the pseudogap state is described by a pairing of the fermions $\Delta^f(\hat{\eta}) = \langle f_{\uparrow, i} f_{\downarrow, i+\hat{\eta}} \rangle$ where $\hat{\eta}$ is the nearest-neighbor vector and $\Delta^f(\hat{\eta})$ has d -wave symmetry.^{6,7} The superconducting state is described by Bose condensation of the bosons $\langle b \rangle \neq 0$. Δ^f is not gauge invariant and the onset of the pseudogap is merely a crossover, but the appearance of $\langle b \rangle = b_0 \neq 0$ triggers the appearance of the superconducting pairing amplitude $\langle c_{\uparrow, i} c_{\downarrow, i+\hat{\eta}} \rangle = b_0^2 \Delta^f$ that is gauge invariant and physical. Within this theory, FT proposed a description of the vortex state where the bosonic amplitude $\langle b \rangle$ vanishes inside the core but the fermion-pairing amplitude $|\Delta^f|$ remains finite. Since the electronic spectrum is given by the fermion dispersion, the core will retain the energy gap just as in the pseudogap state.

Upon closer examination, FT pointed out that this solution requires that the gauge field has negligible restoring force, i.e., a “Maxwell” term of the form $\sigma(\nabla \times \mathbf{a})^2$ must have a very small coefficient σ . This requirement is in fact related to a problem discussed by Sachdev⁸ and by Nagaosa and Lee⁹ some time ago. Due to the existence of the two fields Δ^f and $\langle b \rangle$, it is possible to construct several kinds of vortices. The field Δ^f is minimally coupled to \mathbf{a} in the form $|(\nabla/i + 2\mathbf{a})\Delta^f|^2$ whereas the field $\langle b \rangle$ is coupled to a combination of \mathbf{a} and the electromagnetic field \mathbf{A} in the form $|[(\nabla/i) + \mathbf{a} - (e/c)\mathbf{A}]\langle b \rangle|^2$.

The different kinds of vortices are described below.

(i) A vortex carrying the conventional $hc/2e$ flux quantum. A gauge vortex carrying half a flux quantum $\frac{1}{2}h$ is generated so that $\langle b \rangle$ has no singularity. The phase of Δ^f winds by 2π and its amplitude vanishes in the core. This is just like the conventional vortex in that the energy gap vanishes inside the core. This describes the optimal or overdoped region.

(ii) An hc/e vortex. This involves no winding of Δ^f and no gauge flux. The advantage is that $|\Delta^f|$ is finite in the core and the pseudogap is preserved. This state is energetically favorable because the cost of the boson vortex is small for small x . The price one pays is that because the boson carries a charge e , this vortex carries a double superconducting flux quantum hc/e . This has so far not been observed.

(iii) The FT vortex. A third possibility proposed by FT is

that a flux tube for the gauge field \mathbf{a} carrying $-\frac{1}{2}h$ gauge flux is attached to the core. Now the \mathbf{A} flux can be a conventional flux quantum $hc/2e$ and the phase of $\langle b \rangle$ winds by 2π with $\langle b \rangle = 0$ in the core. On the other hand, Δ^f sees only a flux tube and remains nonzero in the core. It is this latter requirement that forces the \mathbf{a} flux to be a flux tube, i.e., confined to a lattice plaquette.

Actually this possibility was considered by Nagaosa and Lee and dismissed because the energy cost of a flux tube is large in the presence of a Maxwell term. The point is that the theory for Δ^f and $\langle b \rangle$ must be considered as a low-energy effective action and the terms allowed by symmetry such as the Maxwell term will be generated by eliminating the high-energy degrees of freedom. We expect the energy of the flux tube will be of the order of the cut-off scale, i.e., the fermion band width J . This will make this kind of vortex very costly in energy compared with the hc/e vortex in the limit of small x .

FT appealed to the papers by Nayak¹⁰ and Lee¹¹ to justify setting $\sigma = 0$. Even assuming, for the sake of argument, that σ vanishes and the \mathbf{a} flux tube costs no energy, the FT vortex still had a core energy at least of order J . Although the pairing field Δ^f cannot see the $h/2$ flux tube of \mathbf{a} , the fermions see the flux tube. The mismatch of phase π at the lattice scale in the fermion wave function will cost an energy of order J . (Actually, this is why the $h/2$ flux tube costs an energy of order J as discussed above.) In order to reduce this energy cost, Δ^f likes to vanish in a region of size of coherent length $\xi_F \sim v_f/\Delta_0$, where Δ_0 is the spin gap. Such a vortex has a core very similar to the standard BCS vortex [case (i) mentioned above]. The fermion contribution to the core energy is reduced to a value of order Δ_0 .

We should add that recently Senthil and Fisher¹⁴ proposed a model of the vortex based on their $Z(2)$ gauge theory that carries $hc/2e$ flux quantum and contains a pseudogap in the core. This is accomplished by attaching a $Z(2)$ vortex to the core. Senthil and Fisher¹⁵ recently showed how the $Z(2)$ gauge theory can be placed in the context of the $U(1)$ theory and it becomes clear that their model of the vortex is intimately related to that of FT. Senthil and Fisher combined the phase of the boson and half that of Δ^f to form the phase of the ‘‘chargon’’ that Bose condenses. The $Z(2)$ vortex is then the residue of the half flux tube of FT. The $Z(2)$ vortex is also localized to a lattice plaquette and has an energy gap that Senthil and Fisher identify with the pseudogap scale. This also renders this vortex costly in comparison with the hc/e vortex (where the chargon winds by 4π) in the limit of small x . Thus we conclude that models based on $U(1)$ mean-field theory still have difficulties coming up with a $hc/2e$ vortex with a pseudogap core that is stable against merging into hc/e vortex in the limit of small doping.

Several years ago, we introduced an alternate formulation of the constraint in the t - J model called the $SU(2)$ theory.¹⁶ This model is designed to connect smoothly to the Mott insulator at half-filling in that the $SU(2)$ symmetry known to be present at half-filling is preserved for finite doping. The $SU(2)$ mean-field theory should have a better chance of describing the small doping limit. In this paper we show that

this theory leads naturally to a stable $hc/2e$ vortex in the underdoped limit. The spin gap is finite both inside and outside the vortex core. Possible experimental consequences are explored at the end of the paper.

II. REVIEW OF THE $SU(2)$ FORMULATION

First we summarize some of the salient features of the $SU(2)$ formulation. This is well understood in the undoped case where $SU(2)$ doublets $\psi_{\uparrow j} = (f_{\uparrow j}, f_{\downarrow j}^\dagger)$ and $\psi_{\downarrow j} = (f_{\downarrow j}, -f_{\uparrow j}^\dagger)$ were introduced on each site j to represent the destruction of spin up and spin down in the subspace of one fermion per site.^{17,18} Wen and Lee extended the $SU(2)$ formulation away from half-filling by introducing a doublet of bosons $h_j = (b_{1j}, b_{2j})$. The physical electron is represented as an $SU(2)$ singlet formed out of the fermion and boson doublets: $c_{\sigma j} = (1/\sqrt{2})h_j^\dagger \psi_{\sigma j}$. The constraint of no double occupation is enforced by projecting onto the $SU(2)$ -singlet subspace of the extended $h_i, \psi_{\sigma i}$ Hilbert space. On each site there are three such singlets corresponding to $|\text{spin up}\rangle = f_{\uparrow}^\dagger|0\rangle$, $|\text{spin down}\rangle = f_{\downarrow}^\dagger|0\rangle$ and

$$|\text{hole}\rangle = \frac{1}{\sqrt{2}}(b_1^\dagger + b_2^\dagger f_{\uparrow}^\dagger)|0\rangle. \quad (1)$$

The role of the two bosons can be visualized as follows. In contrast to the $U(1)$ formulation, the fermions may remain at half-filling upon doping. Then a typical fermion configuration will contain spin up or spin down singly occupied sites as well as empty and doubly occupied sites. The latter sites are both spin singlets and have the correct spin quantum number for a vacancy. The b_1 boson is used to mark the empty site and the b_2 boson the doubly occupied site, and both b_1 and b_2 carry unit charge. This picture is a bit oversimplified in that it is a linear superposition given by Eq. (1) that correctly specifies a physical hole.

In order to perform the projection to $SU(2)$ singlet, three sets of gauge fields $a_{0j}^{(l)}$ associated with the three Pauli matrices τ^l , $l=1,2,3$ are needed. These are the generalization of the time component of the gauge field a_{0j} in the $U(1)$ formulation. The exchange and hopping terms are decoupled to give the mean-field Hamiltonian,

$$\begin{aligned} H = & \sum_{\langle jk \rangle} (J \psi_{\alpha j}^\dagger U_{jk} \psi_{\alpha k} + t h_j^\dagger U_{jk} h_k + \text{c.c.}) \\ & + \sum_j a_{0j}^{(l)} \left(\frac{1}{2} \psi_{\alpha j}^\dagger \tau^l \psi_{\alpha j} + h_j^\dagger \tau^l h_j \right) - \mu \sum_j h_j^\dagger h_j \\ & + \frac{J}{2} \sum_{\langle jk \rangle} \text{Tr}(U_{jk}^\dagger U_{jk}). \end{aligned} \quad (2)$$

The matrix U_{jk} is given by

$$U_{jk} = \begin{pmatrix} -\chi_{jk}^* \Delta_{jk}^f \\ \Delta_{jk}^{f*} \chi_{jk} \end{pmatrix} \quad (3)$$

where

$$\begin{aligned}\chi_{jk} &= \langle f_{\alpha j}^\dagger f_{\alpha k} \rangle, \\ \Delta_{jk}^f &= \langle \epsilon_{\alpha\beta\gamma} f_{\alpha j} f_{\beta k} \rangle.\end{aligned}\quad (4)$$

The hole density is $\langle b_1^\dagger b_1 + b_2^\dagger b_2 \rangle = x$ and is enforced by the chemical potential μ . The Lagrangian associated with Eq. (2) is invariant under the local SU(2) gauge transformation

$$\begin{aligned}\psi_{\alpha j} &\rightarrow g_j^\dagger \psi_{\alpha j}, \\ h_j &\rightarrow g_j^\dagger h_j, \\ U_{jk} &\rightarrow g_j^\dagger U_{jk} g_k, \\ a_{0j}^{(l)} \tau^l &\rightarrow g_j^\dagger a_{0j}^{(l)} \tau^l g_j - g_j \partial_\tau g_j^\dagger,\end{aligned}\quad (5)$$

where $g_j = \exp(i\mathbf{A}_j \cdot \boldsymbol{\tau})$ is a space and τ dependent 2×2 matrix that represents an SU(2) group element.

In Ref. 16 the SU(2) mean-field theory was worked out by making the approximation that $a_{0j}^{(l)}$ is independent of space and τ . Of special interest is the pseudogap phase that occupies the low-doping part of the phase diagram. [Note that the pseudogap phase is called staggered flux or s -flux phase in Refs. 16 and 19. Despite its name, the s -flux phase in the SU(2) theory is translation invariant and has no staggered physical current or magnetic field. In this paper, we will reserve the name ‘‘staggered-flux phase’’ for the staggered-flux phase in the U(1) theory, which does have staggered physical orbital current and magnetic field.²⁰ We will use ‘‘spin-gap phase’’ to refer to what we previously called the staggered-flux phase in the SU(2) theory.) The spin-gap phase is described by the ansatz

$$a_{0j}^{(l)} = 0, \quad (6)$$

$$\chi_{i,i+\hat{x}} = \chi_{i,i+\hat{y}} = \chi,$$

$$\Delta_{i,i+\hat{x}}^f = -\Delta_{i,i+\hat{y}}^f = \Delta^f,$$

or

$$U_{j,j+\hat{x}} = -\chi\tau_3 + \Delta^f\tau_1,$$

$$U_{j,j+\hat{y}} = -\chi\tau_3 - \Delta^f\tau_1. \quad (7)$$

This resembles the d -wave fermion-pairing phase in the U(1) theory. However, according to Eq. (5), we can perform a SU(2) gauge transformation with

$$g_i = \exp[-i\pi(i_x + i_y)\tau_1/2] e^{i\pi\tau_1/4} (i\tau_3)^{i_x+i_y} \quad (8)$$

and the same mean-field state can be constructed with the choice

$$U_{j,j+\hat{x}} = -i\chi - (-1)^{j_x+j_y}\Delta^f\tau_3, \quad (9)$$

$$U_{j,j+\hat{y}} = -i\chi + (-1)^{j_x+j_y}\Delta^f\tau_3.$$

This resembles the staggered flux phase in U(1) mean-field theory²⁰ because the hopping matrix elements are complex and the sum of the phase angle around a plaquette gives a flux that alternates in sign as shown in Fig. 1. In the follow-

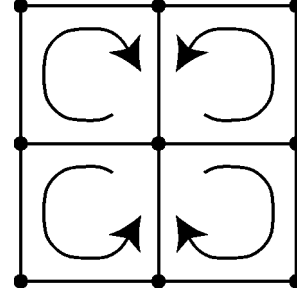


FIG. 1. Pictorial representation of the staggered flux state. The hopping integrals are complex in such a way that the sum of the phase angle around a plaquette yields a net flux that alternates in sign from plaquette to plaquette. This gives rise to circulating fermion currents on the bonds as indicated by the arrow. In the presence of hole doping and condensation of the bosons, circulating physical-hole currents appear. We refer to this state as the staggered-flux state.

ing, we will use the ansatz Eq. (9) to describe the spin-gap phase. In this ansatz the gauge fields $a_\mu^{1,2}$ have a finite energy gap and will be ignored. The gapless U(1) gauge fluctuations are described by a_μ^3 . After including the gapless gauge fluctuations a_μ^3 and the electromagnetic gauge field \mathbf{A} , the effective Hamiltonian has a form

$$\begin{aligned}H &= \sum_{\langle jk \rangle} (J\psi_{\alpha j}^\dagger U_{jk} e^{-ia_{jk}^3 \tau^{(3)}} \psi_{\alpha k} + t h_j^\dagger U_{jk} e^{-ia_{jk}^3 \tau^{(3)} + iA_{jk}} h_k \\ &+ \text{c.c.}) + \sum_j a_{0j}^{(l)} \left(\frac{1}{2} \psi_{\alpha j}^\dagger \tau^l \psi_{\alpha j} + h_j^\dagger \tau^l h_j \right) - \mu \sum_j h_j^\dagger h_j \\ &+ \frac{J}{2} \sum_{\langle jk \rangle} \text{Tr}(U_{jk}^\dagger U_{jk}),\end{aligned}\quad (10)$$

where $A_{jk} = (e/c) \int_k^j d\mathbf{x} \cdot \mathbf{A}$ and U_{jk} is given by Eq. (9).

The fermion dispersion for ansatz Eq. (9) is given by

$$E_\pm = \pm[\varepsilon^2(\mathbf{k}) + \eta^2(\mathbf{k})]^{1/2}, \quad (11)$$

where

$$\varepsilon(\mathbf{k}) = -2J\chi(\sin k_x a + \sin k_y a), \quad (12)$$

$$\eta(\mathbf{k}) = 2J\Delta^f(\sin k_x a - \sin k_y a). \quad (13)$$

Due to our gauge choice, this dispersion is shifted by $(\pi/2, \pi/2)$ compared with the more conventional parametrization that has a maximum $\Delta_0 = J\Delta^f$ at $(0, \pi), (\pi, 0)$ and nodes at $(\pm\pi/2, \pm\pi/2)$. The boson dispersion is the same except that J is replaced by t .

To study the boson-condensed phase at low temperatures, we note that the boson band bottom is at $k = (\pi/2, \pi/2)$ if $a_0^{(l)}$ are not too large. Thus the condensed boson has a form

$$h_i = \begin{pmatrix} b_1(i) \\ b_2(i) \end{pmatrix} = \begin{pmatrix} \tilde{b}_1 \\ \tilde{b}_2 \end{pmatrix} e^{-i(i_x+i_y)\pi/2}, \quad (14)$$

where $\tilde{h} = (\tilde{b}_1, \tilde{b}_2)$ have no site dependence in the mean-field theory.

In Ref. 19 the approximation of constant \mathbf{a}_0 and \tilde{h} is improved by introducing a nonlinear σ model description in terms of a slowly varying boson field. The idea is that at low temperatures the bosons are nearly Bose-condensed to the bottom of the boson bands and are slowly varying in space and time. On the other hand, the fermions have a short coherence length $\xi_F = v_f / \Delta_0$, which is the lattice scale because $\Delta_0 \sim J/3$. Then the fermions follow the local boson field and can be integrated out after choosing an \mathbf{a}_0 field that minimizes the action locally. The result is an effective Lagrangian that depends only on the local boson field. In deriving the effective boson theory, it is convenient to choose the fermion mean field in the staggered-flux representation given by Eq. (9) because the gauge breaking pattern from SU(2) to U(1) is manifest. Since the bottom of the boson band is at $(\pi/2, \pi/2)$, $\tilde{h}_j = h_j \exp[i(j_x + j_y)\pi/2]$ is slowly varying. At low temperatures, $\tilde{h}_j^\dagger \tilde{h}_j = x$ and we write

$$\tilde{h}_j = \sqrt{x} \begin{pmatrix} z_{j1} \\ z_{j2} \end{pmatrix}, \quad (15)$$

where $\sum_\alpha |z_{j\alpha}|^2 = 1$ and are parametrized by

$$z_{j1} = e^{i\alpha} e^{-i\phi/2} \cos \frac{\theta}{2}, \quad (16)$$

$$z_{j2} = e^{i\alpha} e^{i\phi/2} \sin \frac{\theta}{2}. \quad (17)$$

The phase α is the overall U(1) phase that couples to the electromagnetic field. The angles θ and ϕ are best visualized by the isospin quantization axis \mathbf{I} ,

$$\mathbf{I} = z_\alpha^* \boldsymbol{\tau}_{\alpha\beta} z_\beta = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta), \quad (18)$$

i.e., θ and ϕ are the polar angles of the quantization axis. The physical state depends on the orientation of the vector \mathbf{I} as shown in Fig. 2. \mathbf{I} pointing along \hat{z} corresponds to the staggered-flux state in the U(1) formulation. This state breaks translational symmetry and is characterized by a staggered pattern of *physical* hole-current distribution as shown in Fig. 1. We shall refer to this state as the staggered-flux state. \mathbf{I} pointing along $-\hat{z}$ describes the same physical state except that the current pattern is shifted by a unit cell. On the other hand, \mathbf{I} in the x - y plane corresponds to a d -wave superconductor state (with a finite chemical potential) that does not break translational symmetry. These different phases are discussed in greater details in the Appendix.

All the different phases can be summarized by an effective Lagrangian as derived in Ref. 19. It takes the form of an anisotropic $O(4)$ σ model ($|z_1|^2 + |z_2|^2 = 1$) coupled to gauge fields. For the purpose of this paper we restrict our attention to time independent variation and the Lagrangian takes the simplified form

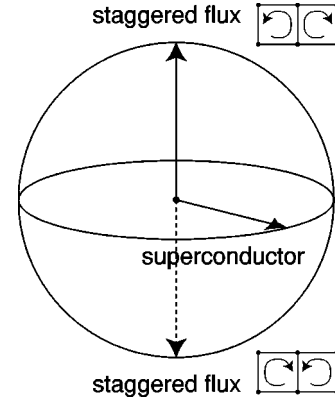


FIG. 2. The isospin quantization axis \mathbf{I} represents different states depending on its orientation. In the north and south poles, it represents the staggered-flux states. These are two degenerate states with the current pattern shifted by one lattice constant. In the equator it represents the d -wave superconductor. Vectors connected by rotation around the \hat{z} axis are gauge equivalent and represent the same physical state.

$$L_{\text{eff}} = xt |D_j z|^2 + \frac{x^2 J}{2} \left[\frac{4}{c_1} |z_1 z_2|^2 + \frac{1}{c_3} (|z_1|^2 - |z_2|^2)^2 \right] + \frac{1}{2} a_j^{(3)} \Pi_{jk} a_k^{(3)}, \quad (19)$$

where

$$D_j = \frac{\partial}{\partial r_j} + i a_j^{(3)} \tau^{(3)} - i \frac{e}{c} A_j \quad (20)$$

is the covariant spatial derivative ($j=x,y$), $a_j^{(3)}$ is the spatial component of the $\mathbf{a}^{(3)}$ gauge field, and c_1 and c_3 are numerical constants of order 1. Since the SU(2) gauge structure has been broken down to U(1) gauge structure by Eq. (9), the $\mathbf{a}^{(1)}$ and $\mathbf{a}^{(2)}$ gauge fields are massive by the Higgs mechanism and have been ignored. The first term in Eq. (14) is the boson kinetic energy minimally coupled to the remaining $\mathbf{a}^{(3)}$ gauge field and the electromagnetic field \mathbf{A} . The second term is a phenomenological term introduced to describe the difference in energy between the superconducting state and the staggered flux state so that the quantization axis prefers to lie in the x - y plane. The third term comes from integrating out the fermion degrees of freedom where Π_{jk} is the fermion polarization bubble. In momentum space it is given by^{12,13}

$$\Pi_{jk} \approx \sqrt{J\Delta_0} \left(\delta_{jk} - \frac{q_j q_k}{q^2} \right) |q|, \quad (21)$$

i.e., it does not take the Maxwell form that would have been proportional to q^2 .

At higher temperatures the anisotropy term (second term) in Eq. (14) is unimportant and the quantization axis \mathbf{I} is disordered. This corresponds to the spin-gap phase. At low temperature, the quantization axis picks out a direction in the x - y plane and at the same time the U(1) $\mathbf{a}^{(3)}$ gauge field is in a Higgs phase and obtain a finite energy gap. This corresponds to the d -wave superconductor.

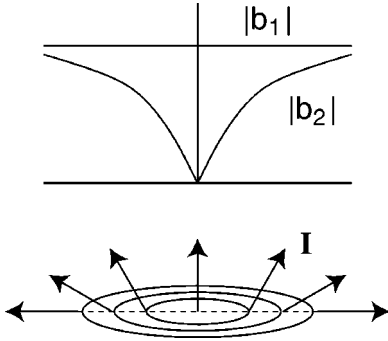


FIG. 3. Structure of the superconducting vortex. Top: b_1 is constant while b_2 vanishes at the center and its phase winds by 2π . Bottom: The isospin quantization axis points to the north pole at the center and rotates towards the equatorial plane as one moves out radially. The pattern is rotationally symmetric around the \hat{z} axis.

Recently, quasi-long-range correlations in the staggered current have been found in the Gutzwiller projected d -wave BCS wave function²¹ and in the exact ground state of small samples.²² Such current fluctuations are very natural in the SU(2) theory and are a consequence of fluctuations of the quantization axis \mathbf{I} towards the north and south poles. We have suggested that these staggered-current correlations may characterize the pseudogap state, but experimental detection of such fluctuating currents seems to be very difficult. Now we are ready to use this picture to describe the vortex in the superconducting state and show that the staggered-current fluctuations may slow down inside the vortex core, making its detection more hopeful.

III. MODEL OF THE VORTEX CORE

Our model of the vortex is the following. Far away from the core $|b_1| = |b_2|$, but $b_2 = \sqrt{x} z_2$ changes its phase ($\alpha + \phi/2$) by 2π as we go around the vortex while $b_1 = \sqrt{x} z_1$ does not change its phase. The vortex contains $hc/2e$ flux for the \mathbf{A} field and $h/2$ flux for the $\mathbf{a}^{(3)}$ field. From Eq. (15), b_2 sees the sum of $\mathbf{a}^{(3)}$ and \mathbf{A} , i.e., a unit total flux while b_1 sees no net flux, so the winding we suggested is consistent. Note that the average phase α [see Eqs. (11) and (12)] has a winding of π , as appropriate for an $hc/2e$ vortex.

As we approach the vortex core, the amplitude of $|z_2|$ must vanish to avoid a divergent kinetic energy from the first term in Eq. (14). Thus the center of the vortex core is represented by $(\tilde{h} = \sqrt{x}, 0)$ and is just the staggered-flux state. As shown in Fig. 3, the quantization axis \mathbf{I} provides a nice way to visualize this structure. It points to the north pole in the center of the vortex and lies in the equator far away, but its azimuthal angle winds by 2π as we go around the vortex. This is sometimes referred to as the ‘‘Meron’’ configuration or half of a hedgehog. It is important to recall that \mathbf{I} parametrizes only the internal gauge degrees of freedom θ and ϕ and the winding of ϕ by 2π has nothing to do with the winding of the overall phase α by π around the vortex. To visualize the winding of both α and ϕ , it is necessary to go back to the (b_1, b_2) representation.

We can make a rough estimate of the vortex energy. As-

sume that the core size (size of the Meron) is l_c and the size of the $\mathbf{a}^{(3)}$ flux is l_a . There are four contributions to the energy. The first is the energy difference between the superconducting state and the staggered-flux state. The main energy cost comes from the Fermi pockets. Assuming the area of the pockets to be x , we estimate an energy cost of $l_c^2 x^{3/2} \sqrt{J\Delta_0}$. On the other hand, the Meron size cannot be smaller than $1/\sqrt{x}$ without costing too much kinetic energy. (In fact, the effective action is valid only for momentum $q \leq \sqrt{x}$ since we kept only the first quadratic term.) The second term comes from the electronic supercurrent and is of order $xt \ln(\lambda/l_c)$ where λ is the London penetration depth. The third term comes from the supercurrent associated with the $\mathbf{a}^{(3)}$ gauge field, which is of order $xt \ln(l_a/l_c)$ assuming $l_a > l_c$. Finally, the fourth contribution is from the gauge field action, the last term in Eq. (14). Setting $q = l_a^{-1}$ in Eq. (16), we estimate this contribution to be $l_a^2 |a^{(3)}|^2 \sqrt{J\Delta_0}/l_a \approx \sqrt{J\Delta_0}/l_a$. The important point is that unlike the U(1) case, the gauge field is not confined to a flux tube, but can spread over a distance l_a . We note that the supercurrent contributions depend logarithmically on l_a and l_c so that the main dependence comes from the first and fourth contributions. The staggered-flux core size l_c would like to be as small as possible while the size of the gauge flux l_a would like to be large. However, our estimate of the gauge-flux energy should be cutoff for $q < x$ because bosonic contributions will enter Eq. (16). Thus we conclude that the staggered-flux core occupies a radius of $x^{-1/2}$ while the gauge field occupies a radius of x^{-1} . The above estimate is very crude. The main purpose is to show that a standard $hc/2e$ vortex is possible with a staggered-flux core that does not cost too much energy as $x \rightarrow 0$.

If we include the effects of fluctuations, the size of the staggered flux core will very likely be bigger than the above estimate. One way to include the fluctuation effects is through the following consideration. We have shown that due to the excitation of quasiparticle, the superfluid density is reduced in the vicinity of the vortex core.²³ We have also shown that the quasiparticles carry current ev_F after including the fluctuation effects.²⁴ In this case the superfluid density vanishes inside a radius of x^{-1} , which we identify as the vortex core.²³ This argument gives a lower bound on the vortex radius, which matches the radius l_a . Inside this radius the superconducting state loses phase coherence and becomes more costly in energy. Thus our earlier estimate may have over estimated the energy difference between the staggered-flux state and the superconducting state inside the core and the staggered-flux state may expand to occupy the entire core of radius x^{-1} where the superfluid density vanishes. The important point is the topological structure of the vortex, which should be robust while the details of the structure may be model dependent.

One important consequence of the topological structure is that there are two kinds of vortices because the isospin quantization \mathbf{I} can also rotate to the south pole at the vortex core. This just expresses the fact that the staggered-flux state is doubly degenerate with the staggered flux shifted by one unit cell. In the normal state these degenerate states fluctuate be-

tween each other, being smoothly connected via the superconducting state. Inside the vortex core of the superconducting state, the topological structure of the vortex forbids such smooth fluctuations and freezes in the staggered current pattern. Thus the vortex core is closely related but not identical to the pseudogap state.

Since the degrees of freedom in a vortex core is finite, we do not expect a genuine phase transition to an ordered state. Instead, it is likely that there will be a gradual freezing into one of the two staggered-flux configurations as the temperature is lowered. Furthermore, even at $T=0$, the two possible staggered-flux states inside the core can tunnel into each other. If the staggered flux core is as small as $x^{-1/2}$, the tunneling rate can be as large as the spin gap. However, if the staggered flux core has a size of order x^{-1} (which is more likely), the tunneling will be reduced exponentially. Dissipation due to quasiparticles may further suppress the tunneling rate. Indeed, this problem is analogous to the tunneling between degenerate two-level systems coupled to a Fermi sea. There due to the orthogonality catastrophe the tunneling rate can scale to zero and the state completely frozen for strong enough coupling. In general, such exponential tunneling rate is difficult to calculate, but we are hopeful that the dynamics will slow down sufficiently for the staggered currents to be measured experimentally.

As the magnetic field is increased, the vortex cores eventually overlap at $H=H_{c2}$. The staggered-current states overlap and it is reasonable to believe that the ground state should be a long-range-ordered staggered flux state especially if the staggered-flux core has a size of order x^{-1} . The unit cell is doubled and the ground state is a Fermi liquid with small Fermi pockets with area x . We predicted that $H_{c2} \sim x^2$ since the core size scales as x^{-1} .²³ If a high-quality underdoped sample can be made, H_{c2} can be at a scale amenable to laboratory experiment. The Fermi pocket may be measurable by cyclotron resonance or Shubnikov–de Haas experiments. The cyclotron resonance has a unique signature because the Fermi surface is close to a Dirac point so that the Landau levels are not uniformly spaced. The doubling of the unit cell is difficult to measure directly because the staggered-current pattern does not couple to charge-density modulation. It does produce a small staggered magnetic field, which we estimate very crudely to be of order 10 G.^{20,21} The possibility of detecting the staggered magnetic field by neutron scattering and muon spin resonance (μ SR) was investigated theoretically by Hsu *et al.*²⁰ They estimated the neutron scattering intensity to be 1/70 of that from the ordered moments in the insulator.

IV. EXPERIMENTAL PROBE OF THE STAGGERED CURRENT

If it is not possible to reach $H > H_{c2}$, the topological aspect of the vortex offers us an opportunity to test the staggered-current picture. It is difficult to probe the staggered-current pattern in the normal state because of spatial and temporal fluctuations. One of the few possible techniques is x-ray scattering, which couples to chirality fluctuations at (π, π) .²⁵ However, according to our analysis, the

dynamics of the staggered-current pattern slow down inside the vortex core. Depending on the time scale, it may be possible to measure the small staggered magnetic field created by the circulating current. The field distribution in the vortex state is remarkably uniform as expected for an extreme type II superconductor. From μ SR measurements, the field distribution has a width of roughly 5 G at $H=0.5$ T.²⁶ It should be even narrower at higher fields. If the dynamics is slower than the μ SR scale, the field distribution inside the vortex core is detectable. For even slower dynamics, a more sensitive experiment is NMR. In YBCO, the Y ion is ideally placed to detect this current because it sits at the center of the plaquette. The weak magnetic field generated by the circulating current will produce side bands in the Y NMR line with a splitting independent of H but with weight proportional to H . However, there remains one complication with this proposed experiment. YBCO is a bilayer material with Y sitting between the bilayers. It is likely that the staggered pattern on the bilayers are out of phase in which case the magnetic field at the Y site exactly cancels. A way out of this difficulty is to study the 2-4-7 structure where the two layers are asymmetric because they are connected to different charge reservoirs (single chain vs double chain). It should be possible to have one plane of the bilayer optimally doped while the other plane (next to the double chain) remains underdoped. Obviously, this proposal is quite a challenge (but a rewarding one) for the experimentalists.

If it is possible to reach $H > H_{c2}$, NMR, μ SR, neutron, cyclotron resonance and Shubnikov–de Haas experiments can all be performed to look for the staggered flux state.

V. CONCLUSION

In summary, the SU(2) formulation of the t - J model leads naturally to a picture of the staggered-flux phase above H_{c2} and a stable $hc/2e$ vortex with a staggered-flux core in the superconducting state. The basic physical picture is that the staggered-flux state is nearly degenerate in energy to the d -wave superconducting state. The pseudogap state is described by fluctuations between the staggered-flux state and the superconductor. It has no long-range order, but may be characterized by short-range staggered currents. There may be short-range superconducting fluctuations as well, but these are not described by conventional phase fluctuations alone. As the temperature is raised above T_c , the fluctuations initially resemble conventional phase fluctuations but gradually crossover to fluctuations into the metallic staggered-flux state all the while maintaining the energy gap at $(0, \pi)$. The picture may reconcile the rather conventional x - y model behavior observed 30 K above T_c (Ref. 27) with the surprising persistence of a few vortexlike excitations up to 150 K.²⁸

Inside the vortex core, these fluctuations are almost frozen out. The core consists mostly of the staggered-flux phase and the tunneling rate between the two kinds of vortex can be very small. The small energy difference between the staggered-flux state and the superconductor in the limit of small doping renders this vortex stable. This picture suggests a (H, T) phase diagram shown in Fig. 4. Below a relatively high-temperature scale (of order $\Delta_0 \approx J/3$), the spin-gap

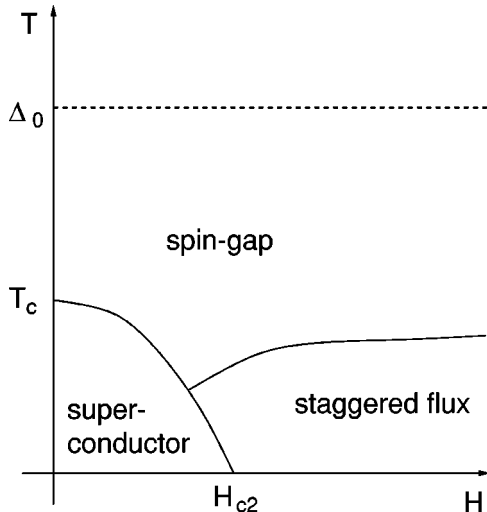


FIG. 4. Schematic phase diagram in the (H, T) plane for underdoped cuprates. The pseudogap phase onsets below an energy scale Δ_0 . This is described by the spin-gap phase where the vector in Fig. 2 is disordered. The dashed line is a crossover temperature. The superconducting state appears below $T_c \approx xt$. Its vortex core contains ordered staggered currents. For H exceeding H_{c2} , the vortex cores overlap and the staggered-flux state is stabilized.

phase is formed as described above. [This was called the staggered-flux phase in the SU(2) mean-field theory.¹⁶] Its onset is a crossover, not a phase transition. Due to the high-energy scale, this onset is insensitive to magnetic field, consistent with experimental findings.²⁹ Superconductivity onsets below a coherence temperature $\approx xt$. In a magnetic field, the vortex has a core of radius x^{-1} . The state inside the core forms staggered currents on some slow time scales. At $H_{c2} \approx x^2$, these cores overlap forming a truly long-range ordered staggered-flux state. This state has a doubled unit cell and its Fermi surface consists of small pockets of area x , consistent with Fermi liquid theory. Thus the metallic state generated by a high magnetic field is a Fermi liquid state. This state is connected to the pseudogap phase by an Ising-like phase transition. The long-range order of the staggered-flux state requires coherence among the holes and we expect its transition temperature to be xt (i.e., comparable to the superconducting T_c) as well. This phase diagram is in contrast to a recent proposal by Chakravarty *et al.*,³⁰ who suggested that the onset of the pseudogap is a genuine transition. In their picture the staggered-flux state will extend up to the energy scale Δ_0 . The experimental test of staggered currents that we proposed should, in principle, be capable of distinguishing their proposal from ours.

We emphasize that the zero-temperature ground state in the x - H phase is entirely conventional consisting of a d -wave superconductor, antiferromagnetic insulators, and Fermi liquids. At some critical x_c there is a transition between the staggered-flux state with Fermi pockets to a Fermi liquid state with a large Fermi surface of area $(1-x)$. The $x_c(H)$ line should terminate at the superconducting $H_{c2}(x)$ boundary. Our picture of the zero-temperature phase diagram is the same as that proposed by Chakravarty *et al.* However, Chakravarty *et al.* asserted that the transition between the

two Fermi liquids involves a violent change of Fermi surface topology and, by implication, of physical properties such as transport measurements. In contrast, we believe that a line of continuous transitions with a change of the translation symmetry is possible and in fact likely, in view of the smooth crossover observed at $H=0$ above T_c as a function of x . The Fermi pockets are elongated and may merge to form a single Fermi surface in the reduced Brillouin zone in the staggered-flux phase for $x < x_c$. The restoration of translational symmetry and a large Fermi surface can take place continuously by the disappearance of the Fermi surface shifted by (π, π) that lies outside the first reduced Brillouin zone. The scenario of a continuous evolution from small to large Fermi surface via the “shadow band” was described by one of us some time ago.³¹

Finally, a third alternative exists, i.e., the staggered-flux state is never stable (or in other words, it is destroyed by strong quantum fluctuations even at $T=0$). In this case, something resembling the spin-gap state becomes the ground state in a high magnetic field and inside the vortex core. If true, this will be, to our knowledge, the first example of a non-Fermi-liquid ground state apart from superconductivity in dimensions higher than one. Our proposed phase diagram offers a very natural route to avoid this exotic possibility.

We would like to stress that even when the vortex core is described by the spin-gap state, the $hc/2e$ vortex still has a small core energy that vanishes in the $x \rightarrow 0$ limit in the mean-field theory. Hence the $hc/2e$ vortex is still stable. In fact, the SU(2) vortex is the only mean-field theory at present that gives a stable $hc/2e$ vortex with a pseudogap in the core. Whether there exists a quasistatic staggered current inside the core is a question that is difficult to treat theoretically and which is best settled by experiments.

We end by making a comment on the experiments proposed by Senthil and Fisher³² to test for electron fractionalization. They propose trapping a vortex in a hole in a superconductor. When the temperature is raised above T_c , the magnetic field escapes, but the Z(2) vortex (vison) is trapped. Then when the temperature is cooled down below T_c , the vison must capture a magnetic flux to spontaneously form a $hc/2e$ vortex of either sign. We would like to point out that our model of the vortex does not exhibit the Senthil-Fisher effect. While our vortex is also a bound state of a magnetic flux with half a flux quantum of the gauge field $a^{(3)}$, the important difference is that the gauge vortex has a finite extent and is not a flux tube. Above T_c , the size of this gauge flux will expand to infinity at the same time the size of the magnetic vortex does, i.e., the penetration depth of the $a^{(3)}$ field and the A field both diverge in the normal state. They allow the gauge vortex to escape the hole in the normal state.

In principle, the Senthil-Fisher effect, the electron fractionalization (or the true spin-charge separation), and other physics of the Z(2) theory can be readily obtained from the SU(2) slave boson theory if one assumes the SU(2) gauge symmetry is broken down to Z(2) gauge symmetry [which can be achieved by noncollinear SU(2) flux through different

plaquettes].³³ With this understanding, the difference between the $Z(2)$ approach and our $SU(2)$ approach is clear. In the $Z(2)$ approach, one assumes that the $SU(2)$ gauge symmetry is broken down to $Z(2)$. While in our $SU(2)$ approach, the $SU(2)$ is only broken down to $U(1)$ in the normal state [by a collinear $SU(2)$ flux]. The $Z(2)$ and our $SU(2)$ approaches correspond to different choices of mean-field states of the same $SU(2)$ slave boson theory.

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APPENDIX: SYMMETRIES OF THE MEAN-FIELD STATES

In this appendix, we study the symmetries of several mean-field states discussed in this paper. The mean-field ansatz described by (U_{ij}, h_i, a_0) in Eq. (9) and Eq. (14) is not translationally invariant. But the breaking of translation symmetry is a gauge artifact. Due to the $SU(2)$ gauge structure, (U_{ij}, h_i, a_0) can describe a translation-invariant physical state in the t - J model. In the following we would like to discuss the physical symmetries of the above ansatz for different boson condensations.

Note that under translation by one lattice spacing followed by a gauge transformation $U_{ij} \rightarrow g_i^\dagger U_{ij} g_j$ with $g_i = -i\tau^1$, our ansatz U_{ij} transforms as

$$\begin{pmatrix} U_{i,i+\hat{x}} \\ U_{i,i+\hat{y}} \end{pmatrix} \rightarrow \begin{pmatrix} -i\chi + (-)^i \tau^3 \eta \\ -i\chi - (-)^i \tau^3 \eta \end{pmatrix} \rightarrow \begin{pmatrix} -i\chi - (-)^i \tau^3 \eta \\ -i\chi + (-)^i \tau^3 \eta \end{pmatrix}. \quad (\text{A1})$$

Thus U_{ij} is invariant under a combination of the translation and the gauge transformation. But the two transformations do change $a_0^{(l)}$ and $\vec{h}^T = (\vec{b}_1, \vec{b}_2)$:

$$\begin{aligned} (a_0^{(1)}, a_0^{(2)}, a_0^{(3)}) &\rightarrow (a_0^{(1)}, -a_0^{(2)}, -a_0^{(3)}), \\ (\vec{b}_1, \vec{b}_2) &\rightarrow (\vec{b}_2, \vec{b}_1). \end{aligned} \quad (\text{A2})$$

Similarly, under 90° rotation about a lattice site followed by a gauge transformation $U_{ij} \rightarrow g_i^\dagger U_{ij} g_j$ with $g_i = -i\tau^1 (-)^{i_x}$, the ansatz U_{ij} transforms as

$$\begin{pmatrix} U_{i,i+\hat{x}} \\ U_{i,i+\hat{y}} \end{pmatrix} \rightarrow \begin{pmatrix} +i\chi - (-)^i \tau^3 \eta \\ -i\chi - (-)^i \tau^3 \eta \end{pmatrix} \rightarrow \begin{pmatrix} -i\chi - (-)^i \tau^3 \eta \\ -i\chi + (-)^i \tau^3 \eta \end{pmatrix}. \quad (\text{A3})$$

Thus U_{ij} is invariant under a combination of the 90° rotation and the gauge transformation. The two transformations transform $a_0^{(l)}$ and \vec{h} into

$$(a_0^{(1)}, a_0^{(2)}, a_0^{(3)}) \rightarrow (a_0^{(1)}, -a_0^{(2)}, -a_0^{(3)}),$$

$$(\vec{b}_1, \vec{b}_2) \rightarrow i(\vec{b}_2, \vec{b}_1). \quad (\text{A4})$$

Now we are ready to discuss some basic physical properties of our ansatz for different orientation of the condensate \vec{h} . It is convenient to introduce a unit vector $\mathbf{I} = \vec{h}^\dagger \vec{\tau} \vec{h} / x$ to describe the orientation of the boson condensates.

(1) When $\vec{b}_1 \neq 0$ and $\vec{b}_2 = 0$, we have $\mathbf{I} \propto \hat{z}$, $a_0^{(3)} \neq 0$, and $a_0^{1,2} = 0$. The ansatz, despite the boson condensation, does not correspond to a superconducting state since the superconducting order parameter $\langle \epsilon_{\alpha\beta} c_{\alpha,i} c_{\beta,j} \rangle = 0$. It instead describes a Fermi liquid, which corresponds to the staggered-flux state in the $U(1)$ formulation. The nonzero $a_0^{(3)}$, generated by the polarization of the boson field, corresponds to a shift in the chemical potential for the fermions. This in turn converts the nodes at $(\pi/2, \pi/2)$ into a small Fermi surface pocket. This state breaks translational symmetry and is characterized by a staggered pattern of *physical* hole-current distribution as shown in Fig. 1. We shall refer to this state as the staggered-flux state.

(2) When \mathbf{I} is in the x - y plane (i.e., when $|\vec{b}_1| = |\vec{b}_2|$), we have $a_0 \propto \mathbf{I}$ and the ansatz describes a translation and rotation-invariant state. We first note that \mathbf{I} pointing anywhere in the x - y plane is gauge equivalent. This is a consequence of the residual $U(1)$ symmetry of ansatz Eq. (9) and any vector \mathbf{I} related by a rotation in z direction represents states that are gauge equivalent. After a translation or a rotation [Eqs. (A2) and (A4)], we see that \mathbf{I} is transformed into another vector in the x - y plane [i.e., $(I_x, I_y) \rightarrow (I_x, -I_y)$]. Since the two \mathbf{I} 's correspond to the same physical wave function, the physical state is invariant under the translation and/or the rotation. This state describes a d -wave superconducting state of the t - J model since the superconducting order parameter $\langle \epsilon_{\alpha\beta} c_{\alpha,i} c_{\beta,j} \rangle \neq 0$ and has a d -wave symmetry.

(3) When $|\vec{b}_1| \neq |\vec{b}_2|$ and $|\vec{b}_1 \vec{b}_2| \neq 0$ [i.e., when \mathbf{I} is pointing somewhere in between the north pole and the x - y plane], we have $a_0^{(3)} \neq 0$ and $a_0^{(1,2)} \neq 0$. The ansatz describes a superconducting state of the t - J model since the superconducting order parameter $\langle \epsilon_{\alpha\beta} c_{\alpha,i} c_{\beta,j} \rangle \neq 0$. The state also breaks the translation symmetry. This is because the translation changes $(I_z, a_0^{(3)})$ to $(-I_z, -a_0^{(3)})$. Two ansatz with different $(I_z, a_0^{(3)})$ are not gauge equivalent and correspond to different physical wave functions. The quasiparticle excitations have a finite gap except at the four isolated points near $(\pm \pi/2, \pm \pi/2)$.

(4) At high enough temperatures, the thermal fluctuations make $\langle \vec{b}_1 \rangle = \langle \vec{b}_2 \rangle = \langle a_0^{(l)} \rangle = 0$. In this case, the ansatz describes a translation- and rotation-invariant metallic state, which is just the spin-gap phase (or the staggered-flux phase studied in Ref. 16). Note that the $U(1)$ gauge fluctuations are gapless since there is no boson condensation. Those gapless gauge fluctuations lead to long-range interactions between quasiparticles at low energies. Thus the metallic spin-gap phase is not a Fermi liquid.

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