Spin accumulation and resistance due to a domain wall

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(Received 20 October 2000; published 22 May 2001)

Starting from the kinetic equation for the Wigner function, we study the spin-accumulation mechanism of excess resistance in a domain wall. The magnetization induced around the wall is calculated in local (rotated) coordinates. Compared to the spin-diffusion length of the longitudinal magnetization, that of the transverse one is greatly reduced due to the fast precession of the magnetization vector in the local exchange field. Consequently, a considerable quenching of the spin-accumulation mechanism is predicted. For a Bloch wall in cobalt, the excess resistance due to this mechanism is $10^{-3}-10^{-4}$ of the value expected for the interface with an abrupt rotation of the magnetization vector. Thus, contrary to the suggestion of Ebels *et al.* [Phys. Rev. Lett. **84**, 983 (2000)], the spin accumulation mechanism cannot explain the large excess resistance observed on thin cobalt wires.

DOI: 10.1103/PhysRevB.63.224412 PACS number(s): 75.70.−i, 73.40.Cg, 73.50.Bk

I. INTRODUCTION

A number of transport experiments on ferromagnetic metals with electric currents traversing domain walls have shown that the walls act as a source of extra resistance.[1–3] Theoretical studies of this effect began with the works of Cabrera and Falicov.[4] Using the classical Boltzmann approach, they calculate the local conductivity tensor from the local effective mean free path. Assuming a relatively pure specimen, such as iron whisker,[1] they investigate first the electron scattering due to the twisting exchange field in the wall (paramagnetic effect). The main result is that as long as the wall is thick compared to the electron wavelength, there is no significant reflection of the electrons by the wall and the contribution of the paramagnetic effect to the electrical resistivity is negligible. Further investigation led to the conclusion that, in metals with a few impurities, the observed wall resistance can be accounted for by the ''diamagnetic effect'' involving zigzagging electron trajectories near the wall.[4]

Compared to the resistivity of iron whiskers at low temperatures, that of thin films of cobalt studied by Gregg *et al.*[2] is three orders of magnitude larger due to inherent defects. The recently studied cobalt wires probably exhibit resistivity of the same order of magnitude.[3] Due to the short mean free path, the diamagnetic contribution[4] that is operative in a clean metal is quenched in samples of Refs. 2 and 3. Moreover, since the walls are thick compared to the electron wavelength, the paramagnetic effect involving reflections of the electron from the wall cannot explain the large extra resistance in these samples.

To interpret their measurements, Gregg *et al.*[2] advanced the idea of spin mistracking. The relevant physics is based on a theorem well known in magnetic resonance.[5] According to this theorem, the electron spin can follow adiabatically the direction of a rotating magnetic field if the precession frequency of the spin about the field is much greater than the angular velocity of the rotation. When an electron moves at Fermi velocity v_F through the wall of thickness d, it sees an exchange field rotating at angular velocity $\omega=\pi v_F/d$. Adiabatic tracking takes place when $\omega \ll \omega_e$ where $\omega_e=2J_{sd}/\hbar$, J_{sd} being the s-d exchange integral. Actually, a certain degree mistracking given by the ratio $\xi=\omega/2\omega_e$ takes place in a domain wall in cobalt. According to Gregg *et al.*,[2] this mistracking acts in combination with the spin-dependent scattering due to impurities. The latter effect is also responsible for the giant magnetoresistance (GMR) in magnetic multilayers.

Starting from the same Hamiltonian as used to explain this GMR, Levy and Zhang[6] calculate the spin-dependent scattering in the wall using electron wave functions perturbed by the twisting magnetization in the wall. Due to mistracking, the wave function of an electron with spin aligned along the local magnetization direction acquires an admixture, proportional to ξ, of a state with opposite spin. This causes the impurity potentials to mix the two-spin channels. The extra resistance of the wall is obtained since the channels exhibit unequal conductivities in a ferromagnet in the absence of the wall (bulk spin asymmetry). Levy and Zhang[6] find a good agreement with the resistivity data of Gregg *et al.*[2]

The present investigation is motivated by recent paper of Ebels *et al.*[3] These authors report transport measurements on thin epitaxial Co wires containing one or two isolated domain walls. The observed relative change of the resistivity is at least one order of magnitude larger than that deduced from the model involving mixing of the spin channels.[6] Ebels *et al.*[3] argue that the large extra resistance is due to the spin accumulation at the domain wall.

This concept was originally introduced by Johnson and Silsbee[7] and independently by Son, Kempen, and Wyder[8] to treat the electron transport through an interface between ferromagnetic and nonmagnetic metals. The idea is based on the observation that, due to the spin asymmetry of the conductivity, the electric current in a ferromagnetic metal is spin polarized. This leads to the formation of excess spin density around the interface. The chemical potential associated with this spin density gives rise to a voltage drop that is proportional to the net current density. Consequently, the interface imposes an extra resistance called the ''spin-coupled interface resistance.''[7]

The same mechanism is believed to cause the giant mag-

netoresitance in ferromagnetic multilayers with currents flowing perpendicular to the plane (CPP geometry). Valet and Fert[9] developed a theory of the CPP-GMR starting from the Boltzmann equation. They find that when the spin diffusion length is much greater than the electron mean free path, the macroscopic equations of the theory of the ''spin-coupled interface resistance'' [7,8] are recovered.

The example of an isolated interface of ferromagnetic metals with opposite magnetizations, presented in Ref. 9, is closely related to the interpretation of the domain-wall magnetoresistance proposed by Ebels *et al.*[3] In this case, the interface resistance is found proportional to the spin-diffusion length times the square of the conductivity asymmetry parameter. Arguing that the spin-diffusion length in cobalt is much larger than the width of the domain wall, Ebels *et al.*[3] replace the wall by an abrupt transition and estimate the relative change of the resistance using the result obtained by Valet and Fert[9] for an isolated interface. This estimate is in quantitative agreement with the measured wall resistance.[2]

The assumption of an abrupt transition, made in Ref. 3, deserves closer inspection for the following reason. As shown by Levy and Zhang,[6] the mistracking in the Bloch wall of cobalt is small ($\xi \approx 0.1$). In the extreme adiabatic limit, $\xi = 0$, the electron spin sees a homogeneous medium so that there is no scattering of the spin-polarized current. Hence, no spin accumulation is expected in this case. The fact that the parameter ξ in cobalt is small casts doubts on the assumption of an abrupt transition.

In the present paper, we study the contribution of spin accumulation to the wall resistance by extending the transport theory of Valet and Fert[9] to a ferromagnet with noncollinear magnetization. The conduction-electron magnetization is obtained from the kinetic equation in Wigner space.[10] For spin $\frac{1}{2}$ particles, the density matrix and the corresponding Wigner function are 2×2 matrices in spin space. The equations of motion for these matrices are transformed from the laboratory frame (X,Y,Z) to a rotated reference frame (x,y,z) where the z axis coincides with the orientation of the local magnetization vector and the rotation is about the x axis (see Ref. 6).

The components of the conduction-electron magnetization **m** in the rotated frame satisfy Bloch equations, with diffusion,[11] containing additional forcing torques proportional to the gradient of the rotation angle. These torques are present in the kinetic equation since the rotation operator does not commute with the kinetic energy operator for the conduction electron.[6] They are responsible for the mistracking and the magnetization pile up around the wall.

In the rotated reference frame, the conductivity asymmetry factor is constant (in the x direction) and the voltage drop due to spin accumulation is obtained by integrating the y component of the magnetization over x. This result follows from the transformation of the derivative of the z component of the induced magnetization to the rotated frame. The presence of the y component of the magnetization originates from the fact that $\partial/\partial x$ does not commute with the rotation operator.

We obtain $m_y(x)$ by solving the coupled equations for m_x and m_y to first order in the external electric field and to the lowest order in the gradient of the rotation angle. It is the exchange field acting along the z axis that couples m_x to m_y and causes a fast precession of the magnetization about this axis. It turns out that the precession frequency ω_e is much greater than the transverse relaxation rate of the magnetization making the spin diffusion length much shorter than that of the longitudinal component. Thus we expect that the voltage drop across the wall is strongly reduced compared to that obtained for the isolated interface.[9] Estimates made in this paper confirm this expectation.

The paper is organized as follows. Section II introduces the kinetic equation for the Wigner function. The spin-polarized current is reviewed in Sec. III. The magnetization in the rotated frame is studied in Sec. IV. The resistance due to the spin accumulation around the wall is derived in Sec. V.

II. KINETIC EQUATION FOR WIGNER FUNCTION

We follow the standpoint that most of the electrical current in a metallic ferromagnet is carried by electrons in the s band. This point of view was first introduced by Mott,[12] and adopted in recent theories of GMR.[6–9] Similar to Refs. 2 and 6, the mutual interaction between the s electrons is neglected and the interaction with the d electrons is described by an s-d exchange field acting parallel to the local magnetization. The corresponding many-body Hamiltonian is a sum of one-body Hamiltonians of the form[6]

$$\hat{h} = -\frac{\hbar^2}{2m}\nabla^2 + \frac{2}{\hbar}J_{sd}\hat{\mathbf{s}}\cdot\mathbf{M}(X) + \hat{h}^{(s)} + eV(X), \qquad (1)$$

where J_{sd}, $\hat{\mathbf{s}}$, and $\mathbf{M}(x)$ are s-d exchange integral, the s-electron spin operator, and the unit vector parallel to the local magnetization, respectively. The Hamiltonian $\hat{h}^{(s)}$ represents the scattering of the conduction by impurities, phonons, and magnons. It is responsible for the relaxation terms in the kinetic equations. The last term describes the interaction with an electric field $E_0 = -\partial V/\partial X$.

We consider a 180° wall in an infinite ferromagnet.[6] In the laboratory coordinate system (X,Y,Z), the magnetization for $X=-\infty$ is parallel to OZ, for $X=+\infty$ antiparallel to OZ. The wall is centered about the origin of the (X,Y,Z) system. The angle between the magnetization and the OZ axis is a function of X and is denoted by α. We now choose a new coordinate system $(x=X,y,z)$, such that the OZ axis coincides with the direction of the local magnetization [parallel to the vector $\mathbf{M}(x)$], and the OX axis is not changed.

In the spin space, the transformation to the rotated reference frame (x,y,z) is described by the 2×2 matrix

$$\hat{R}_\alpha = \exp\left[-\frac{i}{2}\alpha(x)\hat{\sigma}_x\right], \qquad (2)$$

where $\hat{\sigma}_x = 2/\hbar\,\hat{s}_x$ is the Pauli matrix.

If $\chi(r)$ is the spinor in rotated (local) frame, then the eigenstate of the Hamiltonian (1) is given by the spinor $\psi = \hat{R}_\alpha\chi(r)$. Let $\hat{\rho}$ be the density matrix in the representation

of ψ states. It is related to the density matrix in the χ representation $\hat{\rho}_\chi$ by the transformation

$$\hat{\rho}=\hat{R}_\alpha\hat{\rho}_\chi\hat{R}_\alpha^{-1}. \tag{3}$$

The equation of motion for $\hat{\rho}_\chi$ reads

$$\frac{\partial\hat{\rho}_\chi}{\partial t}=\frac{i}{\hbar}[\hat{\rho}_\chi,\hat{h}_\chi], \tag{4}$$

where

$$\hat{h}_\chi=\hat{R}_\alpha^{-1}\hat{h}\hat{R}_\alpha. \tag{5}$$

Using Eqs. (1) and (2), we obtain from Eq. (5) (see Ref. 6)

$$\hat{h}_\chi\approx-\frac{\hbar^2}{2m}\nabla^2-\frac{\hbar}{2m}\hat{\sigma}_x\alpha'(x)p_x+J_{sd}\hat{\sigma}_z+\hat{h}_\chi^{(s)}+eV(x). \tag{6}$$

Based on the assumption that the wall thickness d is much larger than the Fermi wavelength, we confine ourselves to the ''twist-induced'' perturbation that is first order in $\alpha'(x)=d\alpha/dx$.

The kinetic equation in Wigner space is based on the transformation[10]

$$\hat{F}(\mathbf{x},\mathbf{p})=h^{-3}\int d^3p'\,\hat{\rho}_\chi(\mathbf{p}+\tfrac{1}{2}\mathbf{p}',\mathbf{p}-\tfrac{1}{2}\mathbf{p}')\exp\left(\frac{i}{\hbar}\mathbf{p}'\cdot\mathbf{x}\right), \tag{7}$$

where the Wigner function $\hat{F}(\mathbf{x},\mathbf{p})$ is a matrix on the spin$\frac{1}{2}$ space. Applying this transformation to Eq. (4), we obtain with use of Eq. (6)

$$\frac{\partial\hat{F}}{\partial t}=-v_x\frac{\partial\hat{F}}{\partial x}+\frac{i}{2m}p_x\alpha'(x)[\hat{\sigma}_x,\hat{F}]+\frac{\hbar}{2m}\alpha'(x)\hat{\sigma}_x\frac{\partial\hat{F}}{\partial x}$$
$$-\frac{\hbar}{4m}\alpha'(x)\left[\hat{\sigma}_x,\frac{\partial\hat{F}}{\partial x}\right]-eE_0\frac{\partial\hat{F}}{\partial p_x}+\left(\frac{\partial\hat{F}}{\partial t}\right)_{\text{coll}}. \tag{8}$$

The last term on the right-hand side (RHS) of this equation originates from the scattering term $\hat{h}^{(s)}$. In what follows, we neglect the effect of the transformation (2) on the scattering probabilities. This effect has been treated in great detail in Ref. 6. Since the perturbation in Eq. (6) is independent of the coordinates y and z, the Wigner function $\hat{F}(x,\mathbf{p})$ depends only on the variables x and $\mathbf{p}$.

In the derivation of Eq. (8) we assume that $\alpha'(x)$ is a slowly varying function on the scale of the Fermi wavelength.

Since any 2×2 matrix may be expanded in terms of the three Pauli matrices and the identity matrix $\hat{I}$, we write the Wigner function as a sum

$$\hat{F}(x,\mathbf{p})=\tfrac{1}{2}[f_1(x,\mathbf{p})\hat{I}+f_x(x,\mathbf{p})\hat{\sigma}_x+f_y(x,\mathbf{p})\hat{\sigma}_y+f_z(x,\mathbf{p})\hat{\sigma}_z]. \tag{9}$$

Inserting this expansion into Eq. (8), and using the commutation relations for the Pauli matrices, we obtain the following system of equations for the four functions $f_i(x,\mathbf{p})$

$$v_x\frac{\partial f_1}{\partial x}-\frac{\hbar}{2m}\alpha'\frac{\partial f_x}{\partial x}+ev_xE_0\frac{\partial f_1}{\partial\epsilon}=\left(\frac{\partial f_1}{\partial t}\right)_{\text{coll}} \tag{10a}$$

$$v_x\frac{\partial f_x}{\partial x}-\frac{\hbar}{2m}\alpha'\frac{\partial f_1}{\partial x}+ev_xE_0\frac{\partial f_x}{\partial\epsilon}+\omega_ef_y=\left(\frac{\partial f_x}{\partial t}\right)_{\text{coll}} \tag{10b}$$

$$v_x\frac{\partial f_y}{\partial x}-v_x\alpha'f_z+ev_xE_0\frac{\partial f_y}{\partial\epsilon}-\omega_ef_x=\left(\frac{\partial f_y}{\partial t}\right)_{\text{coll}} \tag{10c}$$

$$v_x\frac{\partial f_z}{\partial x}-v_x\alpha'f_y+ev_xE_0\frac{\partial f_z}{\partial\epsilon}=\left(\frac{\partial f_z}{\partial t}\right)_{\text{coll}}. \tag{10d}$$

Since we are considering a steady state, the $\partial f_i/\partial t$ terms vanish. These equations represent the generalization of the two-component Boltzmann equation used by Valet and Fert.[9] The macroscopic transport equations forming the basis of our spin accumulation theory follow from Eqs. (10a)–(10d) by taking a trace over the momentum and spin degrees of freedom. For that purpose, we express the average magnetization density in terms of the Wigner function. Using the expansion (9), we obtain

$$m_i(x)=\frac{\mu_B}{h^3}\int d^3p\;tr[\hat{\sigma}_i\hat{F}(x,\mathbf{p})]=\frac{\mu_B}{h^3}\int d^3p\;f_i(x,\mathbf{p}), \tag{11}$$

where $i=x$, y, z, and μ_B is the Bohr magneton. For the electric current density in the x direction, we have

$$j^{(e)}=\frac{e}{h^3}\int d^3p\;v_x tr[\hat{F}(x,\mathbf{p})]=\frac{e}{h^3}\int d^3p\;v_xf_1(x,\mathbf{p}). \tag{12}$$

The magnetic current density in the x direction carrying a magnetization component m_i is expressed as

$$j_i=\frac{\mu_B}{h^3}\int d^3p\;v_x tr[\hat{\sigma}_i\hat{F}(x,\mathbf{p})]=\frac{\mu_B}{h_3}\int d^3p\;v_xf_i(x,\mathbf{p}). \tag{13}$$

III. SPIN-POLARIZED ELECTRIC CURRENT

Spin polarization of the current results mainly from conductivity asymmetry. For its description, we define the distribution function

$$f_s(x,\mathbf{p})=\tfrac{1}{2}[f_1(x,\mathbf{p})+sf_z(x,\mathbf{p})], \tag{14}$$

where $s=+1$ or -1 corresponds to a conduction electron with spin parallel or antiparallel to the vector $\mathbf{M}(x)$, respectively. Then the current density in channel s is

$$j_s^{(e)}=\frac{e}{h^3}\int d^3p\;v_xf_s(x,\mathbf{p}). \tag{15}$$

The transport equation satisfied by $f_s(x,\mathbf{p})$ is obtained by combining Eqs. (10a) and (10d) with use of the definition (14):

$$v_x\frac{\partial f_s}{\partial x}-\frac{\hbar}{2m}\alpha'\frac{\partial f_x}{\partial x}-sv_x\alpha' f_y+ev_xE_0\frac{\partial f_s}{\partial \epsilon}=\left(\frac{\partial f_s}{\partial t}\right)_{\rm coll}. \tag{16}$$

We assume that the external electric field produces only a small deviation from equilibrium and write

$$f_s(x,\mathbf{p})=f_s^{(0)}(\epsilon)+\tilde{f}_s^{(1)}(x)+f_s^{(1)}(x,\mathbf{p}). \tag{17}$$

In the relaxation time approximation, the collision term in Eq. (16) takes the form[9]

$$\left(\frac{\partial f_s}{\partial t}\right)_{\rm coll}=-\left(\frac{1}{\tau_{\rm sf}}+\frac{1}{\tau_s}\right)f_s^{(1)}(x,\mathbf{p})-\frac{1}{\tau_{\rm sf}}[\tilde{f}_s^{(1)}(x)-\tilde{f}_{-s}^{(1)}(x)], \tag{18}$$

where τ_s is the relaxation time in the s channel without spin flip, and $\tau_{\rm sf}$ is the (s independent) spin-flip relaxation time. Introducing Eqs. (17) and (18) into Eq. (16) and linearizing, we have

$$f_s^{(1)}(x,\mathbf{p})=-v_xT_s\left(eE_0\frac{\partial f_s^{(0)}(\epsilon)}{\partial\epsilon}+\frac{\partial\tilde{f}_s^{(1)}(x)}{\partial x}\right), \tag{19}$$

where

$$T_s^{-1}=\tau_s^{-1}+\tau_{\rm sf}^{-1}. \tag{20}$$

The quantity $f_s^{(0)}$ represents the equilibrium distribution in an exchange polarized electron gas. It can be expressed in terms of the unpolarized Fermi-distribution function $f^{(0)}$ as follows

$$f_s^{(0)}(\epsilon)=f^{(0)}(\epsilon-sJ_{sd}). \tag{21}$$

Compared to the asymmetry of T_s, this spin-polarization effect is small and can be neglected in the calculation of the bulk electrical conductivity. The current density j_s is obtained from Eqs. (15) and (19), with the use of the relation

$$\frac{\partial f_s^{(0)}}{\partial\epsilon}\simeq\frac{\partial f^{(0)}}{\partial\epsilon}=-\frac{\delta(v-v_F)}{mv_F}. \tag{22}$$

Performing the momentum integration in Eq. (15), we obtain with use of Eq. (19) the electric current density in the s channel in terms of the gradient of the electrochemical potential

$$j_s^{(e)}=\frac{\sigma_s}{e}\frac{\partial}{\partial x}[\mu_s(x)-eV(x)]=\frac{\sigma_s}{e}\frac{\partial\overline{\mu_s}(x)}{\partial x}, \tag{23}$$

where $\sigma_s=(1/2m)ne^2T_s$ is the channel conductivity, n being the total conduction electron density. The chemical potential $\mu_s(x)$ is defined by[9]

$$\tilde{f}_s^{(1)}(x)=\frac{\partial f_s^{(0)}}{\partial\epsilon}[\mu_s(x)-\mu^{(0)}]. \tag{24}$$

Following Ref. 9, we introduce the bulk-spin asymmetry coefficient β

$$\sigma_s=\frac{\sigma_0}{2}(1-s\beta)^{-1}, \tag{25}$$

where $\sigma_0(1-\beta^2)^{-1}$ is the conductivity due to both spin channels. The spin-polarized electric current density $j_z^{(e)}$ is obtained from the magnetization current density of Eq. (13) by replacing the Bohr magneton by the electron charge

$$j_z^{(e)}=\frac{e}{\mu_B}j_z=\frac{e}{h^3}\int d^3p\,v_xf_z(x,\mathbf{p}). \tag{26}$$

Using the relation (14), we express f_z in Eq. (26) in terms of f_s and find in the absence of spin accumulation

$$j_z^{(e)}=\frac{e}{h^3}\int d^3p\,v_xs(f_s-f_{-s})=s(\sigma_s-\sigma_{-s})E_0. \tag{27}$$

With the use of relation (25), we obtain from Eq. (26) the bulk-spin-polarized electric current density

$$j_z^{(e)}=\frac{\beta E_0}{(1-\beta^2)\rho_F^*}, \tag{28}$$

where $\rho_F^*=\sigma_0^{-1}$ is the resistivity of a ferromagnet introduced in Ref. 9. For cobalt, the coefficient $\beta\simeq0.5$. Then a considerable spin polarization of the electric current is predicted by formula (28) in this metal. In combination with the magnetization twist, the spin-polarized current acts as a source of the spin accumulation around the domain wall.

IV. SPIN ACCUMULATION

In theories of GMR of ferromagnetic multilayers, the spin accumulation is obtained by solving the spin-diffusion equation with the appropriate boundary condition at the interface.[7–9] For a domain wall, the continuous rotation of the magnetization endows the spin-diffusion equation with an additional torque and turns it into a kind of Bloch equation with the diffusion.[11] The Bloch equations for the components of $\mathbf{m}(x)$ then follow by momentum averaging of the equations (10a)–(10d).

A. Diffusion equation for $m_z(x)$

To establish the diffusion of equation for $m_z(x)$ we need a continuity equation and the magnetic version of Ohm's law. The continuity equation follows from the momentum average of Eq. (10d), while the Ohm's law is given by the average of the same equation multiplied by v_x. The collision term in Eq. (10d), that is, consistent with Eqs. (14) and (18), is of the form

$$\left(\frac{\partial f_z}{\partial t}\right)_{\rm coll}=-\frac{2}{\tau_{\rm sf}}\tilde{f}_z^{(1)}(x)-\tfrac{1}{2}\left(\frac{1}{T_s}+\frac{1}{T_{-s}}\right)f_z^{(1)}(x,\mathbf{p})-\frac{s}{2}\left(\frac{1}{T_s}-\frac{1}{T_{-s}}\right)f_1^{(1)}(x,\mathbf{p}). \tag{29}$$

Introducing this result into Eq. (10d), multiplying the latter by μ_B, and integrating over the momentum variable, we obtain

$$\frac{\partial j_z(x)}{\partial x}-\alpha'(x)j_y(x)=-\frac{2}{\tau_{\rm sf}}[m_z(x)-m_z^{(0)}]. \tag{30}$$

This is a continuity equation modified by coupling to $j_y(x)$ as a result of the magnetization twist.

Next, we integrate Eq. (10d), multiplied by $\mu_B v_x$, over the momentum variable. Besides Eqs. (11)–(13), the following relations play a role:

$$\mu_B\langle v_x^2 f_i(x,\mathbf{p})\rangle = \frac{\mu_B}{h^3}\int d^3p\, v_x^2 f_i(x,\mathbf{p}) = \tfrac{1}{3}v_F^2 m_i(x) \tag{31}$$

and

$$\left\langle v_x^2 \frac{\partial f_x^{(0)}}{\partial \epsilon}\right\rangle = -\frac{2J_{sd}}{m}D(\epsilon_F) \tag{32}$$

where $D(\epsilon_F)=3n/4\epsilon_f$ is the conduction electron density of states at the Fermi level, n being the electron density. The brackets on the left-hand side (LHS) of Eqs. (31) and (32) are shorthand for the momentum integral. Linearizing the E_0 field term in Eq. (10d), we arrive at the following macroscopic equation (Ohm's law):

$$\begin{aligned}\tfrac{1}{3}v_F^2\left(\frac{\partial m_z(x)}{\partial x}-\alpha'(x)m_y(x)\right)+\frac{3J_{sd}}{2m\epsilon_F}e\mu_B E_0 \\ =-\frac{1}{2}\left(\frac{1}{T_s}+\frac{1}{T_{-s}}\right)j_z(x)-\frac{\mu_B}{2\epsilon}\left(\frac{1}{T_s}-\frac{1}{T_{-s}}\right)j^{(e)}.\end{aligned} \tag{33}$$

Differentiating Eq. (33) with respect to x and noting that $\partial j^{(e)}/\partial x=0$, we obtain

$$\frac{\partial^2 m_z}{\partial x^2}=\alpha'\frac{\partial m_y}{\partial x}-\frac{1}{2D}\frac{\partial j_z}{\partial x}, \tag{34}$$

where D is the diffusion constant

$$D=\tfrac{1}{3}v_F^2\tilde{T} \tag{35}$$

with $\tilde{T}=(T_s^{-1}+T_{-s}^{-1})^{-1}$.

The diffusion equation for $m_z(x)$ follows from Eqs. (30) and (34). Eliminating $\partial j_z/\partial x$, we obtain

$$\frac{\partial^2 m_z}{\partial x^2}-\frac{m_z-m_z^{(0)}}{D\tau_{\rm sf}}=\alpha'\left(\frac{\partial m_y}{\partial x}-\frac{1}{2D}j_y\right). \tag{36}$$

This is a diffusion equation augmented by the torques generated by the magnetization twist. In the absence of these torques, Eq. (36) takes the form of the spin diffusion equation of Refs. 8 and 9, with the spin diffusion length $l_{\rm sf}=(D\tau_{\rm sf})^{1/2}$.

B. Diffusion equation for $m_y(x)$

Since the exchange field couples f_y to f_x, we must consider both the Eqs. (10c) and (10d) to derive a diffusion equation for $m_y(x)$. The collision term in these equations is of the form

$$\left(\frac{\partial f_i}{\partial t}\right)_{\rm coll}=-\frac{2}{\tau}\tilde{f}_i^{(1)}(x)-\frac{1}{2\tilde{T}}f_i^{(1)}(x,\mathbf{p}), \tag{37}$$

where $i=x$, y, and τ is the transverse-relaxation time that is assumed independent of i. Introducing this relation into Eq. (10b), we obtain after the momentum integration

$$\frac{\partial j_x}{\partial x}-\frac{\mu_B\hbar\alpha'}{2m}\left\langle\frac{\partial f_1}{\partial x}\right\rangle+\omega_e m_y=-\frac{2}{\tau}m_x. \tag{38}$$

The quantity $\langle\partial f_1/\partial x\rangle$ can be expressed in terms of the excess electric field. For that purpose we use the relations (14) and (24) to write

$$\left\langle\frac{\partial f_1}{\partial x}\right\rangle=\left\langle\frac{\partial f^{(0)}}{\partial\epsilon}\right\rangle\frac{\partial}{\partial x}[\mu_s(x)+\mu_{-s}(x)]. \tag{39}$$

Next, the electrochemical potential defined in Eq. (23) is written as[9]

$$\overline{\mu_s}(x)=\overline{\mu}(x)+s\Delta\mu(x), \tag{40}$$

where $\Delta\mu(x)$ is the spin accumulation part, and $\overline{\mu}(x)$ is the spin independent part the gradient of which gives the actual electric field

$$F(x)=\frac{1}{e}\frac{\partial\overline{\mu}(x)}{\partial x}. \tag{41}$$

Using Eqs. (40) and (41), we obtain from Eq. (39)

$$\left\langle\frac{\partial f_1}{\partial x}\right\rangle=2e\left\langle\frac{\partial f^{(0)}}{\partial x}\right\rangle\left[\frac{1}{e}\frac{\partial\overline{\mu}}{\partial x}-E_0\right]=-\frac{3ne}{2\epsilon_F}[F(x)-E_0], \tag{42}$$

where Eq. (22) was used to evaluate the quantity $\langle\partial f^{(0)}/\partial x\rangle$. We note that the excess electric field $F(x)-E_0$ vanishes for $\alpha'(x)=0$. Thus the second term on the LHS of Eq. (38) is of higher order in α' and will be neglected in our perturbation approach.

To determine the term $\partial j_x/\partial x$, we consider the momentum average of Eq. (10a), multiplied by ev_x

$$\begin{aligned}e\left\langle v_x^2\frac{\partial f_1}{\partial x}\right\rangle-\frac{e\hbar}{2m}\alpha'\left\langle v_x\frac{\partial f_x}{\partial x}\right\rangle+e^2E_0\left\langle v_x^2\frac{\partial f_1^{(0)}}{\partial\epsilon}\right\rangle \\ =-\frac{e}{T_s}\langle v_x f_s^{(1)}(x,\mathbf{p})\rangle-\frac{e}{T_{-s}}\langle v_x f_{-s}^{(1)}(x,\mathbf{p})\rangle.\end{aligned} \tag{43}$$

On the LHS of this equation we use [see Eq. (39)]

$$\left\langle v_x^2\frac{\partial f_1}{\partial x}\right\rangle=-\frac{n}{2m}\frac{\partial}{\partial x}[\mu_s(x)+\mu_{-s}(x)] \tag{44}$$

and

$$\left\langle v_x^2\frac{\partial f_1^{(0)}}{\partial\epsilon}\right\rangle=-\frac{n}{m}. \tag{45}$$

On the RHS of Eq. (43) we have

$$-\frac{1}{T_s}j_s-\frac{1}{T_{-s}}j_{-s}=-\frac{ne}{2m}\left(\frac{\partial\mu_s}{\partial x}+\frac{\partial\mu_{-s}}{\partial x}+2eE_0\right). \tag{46}$$

Introducing Eqs. (44)–(46) into Eq. (43), we see that all terms cancel out except the second on the LHS. This yields a constraint

$$\frac{\partial j_x}{\partial x}(x)=0. \tag{47}$$

With this constraint Eq. (38) yields

$$m_x \simeq -\frac{\omega_e \tau}{2} m_y . \tag{48}$$

Let us turn to Eq. (10c) and take the momentum average

$$\mu_B \left\langle v_x \frac{\partial f_y}{\partial x} \right\rangle - \omega_e \mu_B \langle f_x \rangle = -\frac{2\mu_B}{\tau} \langle \tilde{f}_y^{(1)} \rangle + \mu_B \alpha' \langle v_x f_z \rangle . \tag{49}$$

Using Eqs. (11) and (13), this equation reads

$$\frac{\partial j_y}{\partial x} - \omega_e m_z = -\frac{2}{\tau} m_y + \alpha' j_z . \tag{50}$$

This is a continuity equation from m_y, modified by coupling to m_x and j_z.

Proceeding to the Ohm's law for the magnetization current density j_y, we multiply Eq. (10c) by $\mu_B v_x$, and take the momentum average. Noting that $\partial f_y^{(0)}/\partial y = 0$, we have

$$\tfrac{1}{3} v_F^2 \frac{\partial m_y}{\partial x} - \tfrac{1}{3} \alpha' v_F^2 m_z - \omega_e j_x = -\frac{1}{2\tilde{T}} j_y . \tag{51}$$

Differentiating this equation with respect to x, and eliminating $\partial j_y/\partial x$ with use of Eq. (50), we obtain to order α'

$$2D \frac{\partial^2 m_y}{\partial x^2} = \frac{2}{\tau} m_y - \omega_e m_x - \alpha' j_z = \left(\frac{2}{\tau} + \frac{1}{2} \omega_e^2 \tau \right) m_y - \alpha' j_z , \tag{52}$$

where the second equality follows by applying the relation (48).

Generally, we expect that $\omega_e \tau \gg 1$. In fact, assuming that the transverse relaxation time $\tau \approx \tau_{\rm sf}$, we obtain $\omega_e \tau \approx 10^3$ for cobalt (see Sec. V). In view of this, Eq. (52) can be simplified to

$$\frac{\partial^2 m_y(x)}{\partial x^2} - \frac{m_y(x)}{l_y^2} = -\frac{\alpha'(x)}{2D} j_z , \tag{53}$$

where l_y is given by

$$l_y^2 = \frac{4D}{\omega_e^2 \tau} = \frac{4\tau}{(\omega_e \tau)^2 \tau_{\rm sf}} l_{\rm sf}^2 . \tag{54}$$

We see that compared with the spin-diffusion length $l_{\rm sf} = (D\tau_{\rm sf})^{1/2}$, the diffusion length for the m_y component is drastically reduced due to the rapid precession of **m** about the exchange field.

Seeking a solution for $m_y(x)$ that is first order in α', we need the quantity j_z for a homogeneous ferromagnet ($\alpha' = 0$). The latter is given by Eqs. (26) and (28). Thus, denoting the RHS of Eq. (53) by $b(x)$, we have

$$b(x) = -\frac{\alpha'(x) \mu_B \beta E_0}{2eD(1-\beta^2)\rho_F^*} . \tag{55}$$

The solution of the inhomogeneous equation (53) can be obtained using the Green's function

$$m_y(x) = \int_{-\infty}^{\infty} dx' G(x-x') b(x') , \tag{56}$$

where

$$G(x) = -\tfrac{1}{2} l_y \exp\left(-\frac{|x|}{l_y} \right) . \tag{57}$$

As $l_y \to 0$, the Green's function $G(x) \to -l_y^2 \delta(x)$, and the Eq. (56) yields

$$m_y(x) \simeq -l_y^2 b(x) = \frac{\alpha'(x) \mu_B \beta E_0 l_y^2}{2eD(1-\beta^2)\rho_F^*} . \tag{58}$$

This approximation is applicable to a domain wall in cobalt, since the function $\alpha'(x)$ varies slowly on the scale of l_y.

V. WALL-INDUCED RESISTANCE

The extra resistance due to the domain wall is obtained by calculating the excess voltage drop ΔV_I from the relation

$$\Delta V_I = -\int_{-\infty}^{\infty} dx [F(x) - E_0] . \tag{59}$$

The excess electric field, $F(x) - E_0$, can be expressed in terms of the gradient of $\Delta\mu(x)$. Using Eqs. (40) and (41) in Eq. (23), the electric current density in the s channel becomes

$$j_s^{(e)} = \sigma_s \left(F(x) + \frac{s}{e} \frac{\partial \Delta\mu(x)}{\partial x} \right) . \tag{60}$$

From this relation, we obtain the total electric current density

$$j^{(e)} = (\sigma_+ + \sigma_-) F(x) + \frac{1}{e} (\sigma_+ - \sigma_-) \frac{\partial \Delta\mu(x)}{\partial x} . \tag{61}$$

As $x \to \infty$, $F(x) \to E_0$, since the spin accumulation decays to zero as one moves away from the wall. In this limit, we have

$$j^{(e)} = (\sigma_+ + \sigma_-) E_0 . \tag{62}$$

Noting that $j^{(e)}$ is independent of x, we obtain from Eqs. (61) and (62) with use of Eq. (25)

$$F(x) - E_0 = \frac{\beta}{e} \frac{\partial \Delta\mu(x)}{\partial x} . \tag{63}$$

This relation was derived in Ref. 9 in the laboratory frame. It can also be used to calculate the excess voltage in the rotated frame as long as the RHS is treated carefully. First of all, the conductivity asymmetry parameter β is independent of x in the rotated frame. Second, the gradient of $\Delta\mu(x)$ on the RHS of Eq. (63) must be transformed to the rotated frame before substituting into the integral (59). In the Appendix, we derive the following expression for this quantity:

$$\left(\frac{\partial \Delta\mu(x)}{\partial x}\right)_{\chi} = -\frac{2\epsilon_F}{3n\mu_B}\left(\frac{\partial m_z(x)}{\partial x} + \alpha'(x) m_y(x)\right). \quad (64)$$

Actually, only the second term on the RHS of this expression contributes to the integral in Eq. (59). The reason is that the solution of the inhomogeneous differential equation (36) has the property

$$\lim_{|x|\to\infty} m_z(x) = 0. \quad (65)$$

This is because the forcing term on the RHS of Eq. (36) is localized over the width of the wall, and the solution decays exponentially away from the wall. Hence, as long as the sample length is much greater than the spin diffusion length, the limit (65) is applicable and the integral of $\partial m_x(x)/\partial x$ vanishes. This result is consistent with the example of an isolated interface considered in Ref. 9. In this case it is possible to obtain the excess voltage in a rotated coordinate system by integrating the quantity $[\partial \Delta\mu(x)/\partial x]_{\chi}$, where $\Delta\mu(x)$ is given by Eq. (22) of Ref. 9. The quantity $m_z(x)$ exhibits a discontinuity at $x=0$ leading to a δ-function term in $\partial m_z/\partial x$. This term causes the vanishing of the integral of the latter function.

Using Eqs. (63), (64), and (58) in the integrand of Eq. (59), we get

$$\Delta V_I = -\frac{\beta}{e}\int_{-\infty}^{\infty} dx \left(\frac{\partial \Delta\mu}{\partial x}\right)_{\chi} = \frac{2\epsilon_F l_y^2 \beta^2 E_0}{6ne^2 D \rho_F^*(1-\beta^2)} \int_{-\infty}^{\infty} dx [\alpha'(x)]^2. \quad (66)$$

We consider a 180° wall with $\alpha'(x)=\pi/d$, for $-\frac{1}{2}d<x<\frac{1}{2}d$, and zero otherwise (see Ref. 6). For the purpose of computing the resistance per unit area, we express the electric field E_0 in terms of the electric current density $j^{(e)}$. Using Eqs. (25) and (62), we get

$$E_0 = \rho_F^*(1-\beta^2) j^{(e)}. \quad (67)$$

The formula on the RHS of Eq. (66) can be further rearranged with use of the relation $ne^2/m=(2\tilde{T}\rho_F^*)^{-1}$. Substituting Eq. (54) for l_y^2, we obtain the following expression for the extra resistance per unit area due to the wall,

$$r_w = \frac{\Delta V_I}{j^{(e)}} = 2\beta^2 \rho_F^* l_{\mathrm{eff}} \quad (68)$$

with

$$l_{\mathrm{eff}} = \frac{2\pi^2 v_F^2 \tilde{T}}{3d\omega_e^2 \tau} = \frac{8}{3}\xi^2 d \frac{\tilde{T}}{\tau}, \quad (69)$$

where ξ is the mistracking parameter equal to $\pi v_F/2d\omega_e$.[6] We note that for the isolated interface considered by Valet and Fert,[9] the spin-coupled interface resistance per unit area, r_{SI}, has the same form as Eq. (68) except that l_{eff} is replaced by l_{sf}. Then the suppression of the spin accumulation effect in the domain wall can be characterized by the ratio

$$\frac{r_w}{r_{\mathrm{SI}}} = \frac{l_{\mathrm{eff}}}{l_{\mathrm{sf}}} = \frac{8\tilde{T}\xi^2 d}{3\tau l_{\mathrm{sf}}}. \quad (70)$$

We will now estimate this ratio for a domain wall in cobalt. In this case we have $d \approx 1.5\times10^{-6}$ cm, $\omega_e \approx 1.5\times10^{15}$ s^{-1}, and $v_F \approx 1.4\times10^8$ cm/s yielding $\xi \approx 0.1$. A reliable estimate of the ratio $\tilde{T}/\tau$ is more difficult since little is known about transverse spin relaxation for the conduction electrons in a domain wall. If we assume that $\tau \approx \tau_{\mathrm{sf}}$, a value of $\tilde{T}/\tau \approx 10^{-2}$ is expected at liquid helium temperatures.[9] Actually, this number should be regarded only as a lower limit for $\tilde{T}/\tau$ since τ can be shorter than τ_{sf}. This is because, in contrast with the longitudinal decay, the transverse relaxation process conserves magnetic energy.[5] This argument is particularly relevant at low temperatures in view of the large value of the exchange splitting. Nevertheless, it is doubtful that $\tilde{T}/\tau$ would be larger than 10^{-1}.

Since $l_{\mathrm{sf}} \approx 6\times10^{-6}$ cm for cobalt at 77 K, we estimate that the ratio (70) is $10^{-3}-10^{-4}$ for a Bloch wall in this material.

VI. CONCLUSION

That some suppression of the spin-accumulation effect in domain walls takes place, due to spin tracking, was pointed out by Ebels *et al.*[3] These authors also argue that, since the spin density at the domain wall decays over distances of order l_{sf}, the spin accumulation mechanism can be strong enough to explain the large excess resistance in cobalt wires.

In contrast, the present analysis shows that the length that is relevant for the calculation of r_W is much shorter than l_{sf} as a result of spin tracking. Analyzing the transport equations in rotated frame of reference, we show that the quantity that contributes to r_W is the transverse magnetization m_y. It is the rapid precession of this component about the exchange field that causes the drastic reduction of the relevant spin-diffusion length and the quenching of the spin-accumulation mechanism. An estimate made for a Bloch wall in cobalt shows that the strength of this mechanism is three to four orders magnitude weaker than required to explain the data of Ref. 3. In fact, it is at least one to two orders of magnitude weaker than the mechanism of Ref. 6. As for a possible explanation of the enhanced excess resistance in cobalt wires,[3] it should be pointed out that the domain wall configuration in a thin wire differs from that of the planar Bloch wall considered in Ref. 6 as well as in the present work. Possible consequences of such nonplanar configurations for electron transport remain to be investigated.

ACKNOWLEDGMENT

I thank Professor B. Heinrich for inspiring correspondence.

APPENDIX: DERIVATION OF EQ. (64)

We start by establishing a relationship between the transformed derivative of $\hat{\rho}$ and the derivative of the density matrix in the χ representation. Differentiating Eq. (3) with re-

spect to x, and using Eq. (2), we get

$$\left(\frac{\partial\hat{\rho}}{\partial x}\right)_{\chi}=\hat{R}_{\alpha}^{-1}\frac{\partial\hat{\rho}}{\partial x}\hat{R}_{\alpha}=\frac{\partial\hat{\rho}_{\chi}}{\partial x}+\frac{i}{2}\alpha'(x)[\hat{\rho}_{\chi},\hat{\sigma}_{x}]. \quad \text{(A1)}$$

Since the magnetization components are defined in Eq. (11) in terms of the Wigner function, we need an analog of the relation (A1) for this function. It is convenient to express the Wigner function in terms of the density matrix in the coordinate representation

$$\hat{F}(\mathbf{x},\mathbf{p})=\int d^3x'\,\hat{\rho}_{\chi}(\mathbf{x}+\tfrac{1}{2}\mathbf{x}',\mathbf{x}-\tfrac{1}{2}\mathbf{x}')\exp\left(\frac{-i}{\hbar}\mathbf{p}\cdot\mathbf{x}'\right). \quad \text{(A2)}$$

We note that this definition is equivalent to Eq. (7) and can be derived from it. Let us also define a Wigner function $\hat{F}_A$ in the absolute (laboratory) coordinate system

$$\hat{F}_{A}(\mathbf{x},\mathbf{p})=\int d^3x'\,\hat{\rho}(\mathbf{x}+\tfrac{1}{2}\mathbf{x}',\mathbf{x}-\tfrac{1}{2}\mathbf{x}')\exp\left(\frac{-i}{\hbar}\mathbf{p}\cdot\mathbf{x}'\right). \quad \text{(A3)}$$

Multiplying Eq. (A1) by $\exp(-i\mathbf{p}\cdot\mathbf{x}'\hbar)$ and integrating over x', we get with the use of Eqs. (A2) and (A3)

$$\left(\frac{\partial\hat{F}_{A}(x,\mathbf{p})}{\partial x}\right)_{\chi}=\frac{\partial\hat{F}(x,\mathbf{p})}{\partial x}+\frac{i}{2}\alpha'(x)[\hat{F}(x,\mathbf{p}),\hat{\sigma}_{x}]. \quad \text{(A4)}$$

The commutator on the RHS of Eq. (A4) can be evaluated with use of the expansion (9) yielding

$$[\hat{F}(x,\mathbf{p}),\hat{\sigma}_{x}]=-i[f_{y}(x,\mathbf{p})\hat{\sigma}_{z}-f_{z}(x,\mathbf{p})\hat{\sigma}_{y}]. \quad \text{(A5)}$$

In the absolute coordinate system we have a relation similar to Eq. (11)

$$m_{Z}(x)=\frac{\mu_{B}}{h^3}\int d^3p\;tr[\hat{\sigma}_{z}\hat{F}_{A}(x,\mathbf{p})]. \quad \text{(A6)}$$

Using Eqs. (A4)–(A6), we get

$$\left(\frac{\partial m_{Z}(x)}{\partial x}\right)_{\chi}=\frac{\mu_{B}}{h^3}\int d^3p\;tr\times\left\{\hat{\sigma}_{z}\left(\frac{\partial\hat{F}}{\partial x}+\frac{1}{2}\alpha'(x)(f_{y}\hat{\sigma}_{z}-f_{z}\hat{\sigma}_{y})\right)\right\}=\frac{\partial m_{z}(x)}{\partial x}+\alpha'(x)m_{y}(x) \quad \text{(A7)}$$

As a final step, we relate the magnetization increment Δm_z to $\Delta\mu$. From Eqs. (14), (24), and (40) we get

$$\Delta f_{z}=\frac{\partial f^{(0)}}{\partial\epsilon}s(\overline{\mu}_{s}-\overline{\mu}_{-s})=2\frac{\partial f^{(0)}}{\partial\epsilon}\Delta\mu. \quad \text{(A8)}$$

Multiplying this equation by μ_B, and taking the momentum average, we find

$$\Delta m_{Z}=2\mu_{B}\left\langle\frac{\partial f^{(0)}}{\partial\epsilon}\right\rangle\Delta\mu=-\frac{3\mu_{B}n}{2\epsilon_{F}}\Delta\mu. \quad \text{(A9)}$$

Eqs. (A7) and (A9) then yield

$$\left(\frac{\partial\Delta\mu(x)}{\partial x}\right)_{x}=-\frac{2\epsilon_{F}}{3n\mu_{B}}\left(\frac{\partial m_{z}(x)}{\partial x}+\alpha'(x)m_{y}(x)\right) \quad \text{(A10)}$$

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