Electron spin-flip relaxation by one magnon processes: Application to the gadolinium surface band

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(Received 10 January 2001; published 7 May 2001)

The ''*s-f* model,'' also known as the ferromagnetic Kondo lattice, contains a description of band electrons coupled to localized spins that is an appropriate description of the magnetic part of the low-energy physics of Gd metal. Here the model is used to estimate the lifetime broadening of the minority-spin component of the surface-electron band in ferromagnetic gadolinium metal at temperatures below the Curie temperature. The low-temperature result $1/\tau \approx 0.1$ eV agrees nicely with a measurement by Fedorov *et al.*

DOI: 10.1103/PhysRevB.63.214410 PACS number(s): 75.50.Cc, 72.10.Di, 72.15.Lh, 73.20.−r

I. INTRODUCTION

Fedorov *et al.*[1] have recently measured the line shape $A(\vec{k},\omega)$ of photoemitted electrons in the ferromagnetic metal Gd. For photoelectrons associated with a photohole in the (001) surface band, they find somewhat different lifetime broadening depending on whether the emitted electron has up (majority) or down (minority) spin. They interpret the source of lifetime broadening to be electron-phonon scattering for the majority-spin component of the photohole, and electron-magnon scattering for the minority-spin component of the photohole. Although the arguments given by Fedorov *et al.* seem perfectly sensible, nevertheless, this interesting diversity suggests a need for theoretical inquiry. The magnitude of electron-phonon scattering in Gd has previously[2] been estimated, with results roughly agreeing with the assigned majority-spin equilibration rate. Electron-magnon scattering has not previously been estimated.

Here I argue that the ''*s-f*'' or ''ferromagnetic Kondo lattice'' model allows reasonable estimates without free parameters. I suggest an extreme model for the nature of surface-electron and surface-magnon states: surface-electron states have amplitude 1 on the top layer and zero elsewhere, while magnon states at the surface[3] are simply the bulk Bloch states, ignoring surface boundary conditions. Using this model, and the measured mass $m^* \approx 1.2$ of the surface-hole band, the zero-temperature equilibration rate of minority-spin holes more than 25 meV from the top of the hole band is predicted to be $1/\tau = 0.10$ eV, agreeing with the experiment.

II. FORMULA FOR RELAXATION RATE

The generic Hamiltonian for the ''*s-f*'' or ''ferromagnetic Kondo lattice'' model couples electron bands $\vec{k}n$ with energy $\epsilon(\vec{k}n)$ (independent of spin, so far) to localized spins $\vec{S}_i$ located on atoms at lattice sites $\vec{R}_i$:

$$\mathcal{H} = \sum_{\vec{k}n\sigma} \epsilon(\vec{k}n) c^\dagger(\vec{k}n\sigma) c(\vec{k}n\sigma) - J \sum_{im\alpha\beta} \vec{S}_i \cdot c^\dagger(im\alpha) \vec{\sigma}_{\alpha\beta} c(im\beta). \quad (1)$$

The electron bands derive from the outer atomic orbitals $|im\alpha\rangle$ with wave functions $\psi_m(\vec{r}-\vec{R}_i)\chi_\alpha$, where χ_α is the spin part. This model should describe the low-energy spin-related physics of a metal like Gd, with three conduction electrons per atom in orbitals derived from atomic *s* and *d* states of Gd. These three orbitals have a magnetic interaction with the well-localized $S=7/2$ half-filled $4f$ shell. The exchange parameter J comes from the atomic Hund's rule trying to keep ''electron'' spins parallel to ''core'' spins. The same Hamiltonian, in the $J \to \infty$ limit, is known as the ''double-exchange Hamiltonian''[4] and is very popular right now[5] for discussions of hole-doped $LaMnO_3$.

The zero-temperature phase diagram of the *s-f* model (for a single *s* band) as a function of filling and J/bandwidth ratio has been computed approximately.[6] Ferromagnetic order occurs over a wide range of parameters, with a Curie temperature ($T_c = 292$ K for Gd) proportional to J. Electron bands acquire a spin splitting proportional to J. Lindgard *et al.*[7] used the model to calculate (in random-phase approximation) the spin-wave dispersion, which was measured for Gd by Koehler *et al.*[8], and that has been studied using spin-density functional theory by Perlov *et al.*[9]

In lowest-order spin-wave approximation, we replace the Fourier-transformed spin operator $\vec{S}_Q$ in Eq. (1) by spin-wave creation and destruction operators $a_Q^\dagger$ and a_Q using $S_{Qz} = S\delta_{Q,0} - a_Q^\dagger a_Q$, $(S_{Qx} + iS_{Qy})/2 = S_Q^+ = \sqrt{2S}a_Q$, and $(S_{Qx} - iS_{Qy})/2 = S_Q^- = \sqrt{2S}a_Q^\dagger$. The $Q=0$ term in lowest order gives spin splitting $2JS$, lowering the energy of bands with spin parallel to the localized spin $\vec{S}_i \sim S\hat{z}$ and raising the other bands equally. The $S^\pm$ terms give rise to spin-flip scattering events.

In this paper I estimate the rate $1/\tau$ at which a single out-of-equilibrium hole in a surface band relaxes back toward equilibrium by spin-flip processes. The rate can be found from $\hbar/\tau = -2\,\mathrm{Im}\,\Sigma$, where the leading Feynman diagram for the self-energy Σ is shown in Fig. 1. Only the one-magnon process is considered. This approximation can be questioned on the grounds of fallibility of the ''Migdal approximation'' for electron-magnon processes.[10] However, it is a proper first estimate, the only one that can be reliably computed, and seems to me unlikely to make a large error when $T \ll T_c$. Two magnon processes have been considered by Lutovinov and Reizer.[11]

FIG. 1. Feynman graph for electron self-energy from electron-magnon coupling. The double solid line is the renormalized electron Green's function; the wavy line is the renormalized magnon Green's function; circles are the effective electron-magnon interaction matrix element.

The actual states $(\vec{k},n)$ of interest are surface states of spin primarily but not exclusively up. The probability $p(\vec{k},n,\uparrow)$ [or $p(\uparrow)$ for short] that the spin state is up, is less than one. The amplitude of the up-spin component of the wave function is $\sqrt{p(\uparrow)}$ and is close to 1. The corresponding probability $1-p(\uparrow)=p(\vec{k},n,\downarrow)$ [or $p(\downarrow)$ for short] that the spin state is down, is nonzero for two reasons. First, the spin-orbit interaction is not small and mixes spin states. Second, *f* spins on Gd atoms may deviate from perfect alignment by quantum and thermal fluctuations, and the conduction states are locally locked by Hund's rule in the same spin orientation as the *f* spins. The second process is a renormalization of electron bands by magnon processes. A combination of the two effects is seen experimentally[1] as a small minority-spin component in the photoemitted electron. This component is sometimes referred to as a ''shadow band'' and has received recent theoretical treatments.[12]

An elementary derivation of the ''Migdal'' result follows from the standard ''Golden-rule'' rate equations of Boltzmann theory. Suppose the down-spin component of state $\vec{k}n$ has population $p(\downarrow)F(\vec{k},n,\downarrow)$. If this deviates from the equilibrium Fermi-Dirac population $p(\downarrow)f(\vec{k},n,\downarrow)$, then it will evolve back toward equilibrium according to

$$\frac{dp(\downarrow)F(\vec{k},n,\downarrow)}{dt}=-\frac{2\pi}{\hbar}\sum_{\vec{Q}n'b}|M_b(\vec{k}n\downarrow,\vec{k}+\vec{Q}n'\uparrow)|^2 \times\{L(\text{emission})+L(\text{absorption})\}. \quad (2)$$

$$L(\text{emission})=\delta(\epsilon-\epsilon'-\omega') \times[(N'+1)F(1-F')-N'F'(1-F)], \quad (3)$$

$$L(\text{absorption})=\delta(\epsilon-\epsilon'+\omega) \times[NF(1-F')-(N+1)F'(1-F)]. \quad (4)$$

Here $M_b()=J[(2S/2N)p(\downarrow)p'(\uparrow)]^{1/2}$ is the matrix element for the process $(\vec{k},n,\downarrow)$ scattering to $(\vec{k}+\vec{Q}n'\uparrow)$ by emission of the magnon $(-\vec{Q}b)$ or absorption of the magnon $(\vec{Q}b)$. The factor $1/\sqrt{2N}$, where N is the number of unit cells and $2N$ the number of atoms in the sample, comes from normalization of the spin-wave eigenvector. There are two branches of spin waves, with amplitude $\pm 1/\sqrt{2}$ on each atom. This is discussed in the Appendix. A shorthand is used that ϵ and $p(\downarrow)F$ stand for the energy and occupancy of the quasiparticle state $(\vec{k}n\downarrow)$, ϵ' and $p(\uparrow)F'$ stand for the energy and occupancy of the quasiparticle state $(\vec{k}+\vec{Q}n'\uparrow)$, ω and N stand for the energy and occupancy of the magnon state $(\vec{Q}b)$, and ω' and N' stand for the energy and occupancy of the magnon state $(-\vec{Q}b)$. At temperature $T>0$, depletion of an excess population toward equilibrium occurs both by emission and absorption of thermal magnons. Each process (emission or absorption) has a time-reversed process that enhances the population, the ''scattering-in'' terms with opposite sign. In thermal equilibrium, scattering out and in occur at equal rates. This ''principle of detailed balance'' guarantees that the two parts of L(emission) cancel each other when the distributions N and F become the equilibrium distributions n and f, and similarly for L(absorption).

Now make the assumption that all quasiparticles are in equilibrium except for a particular state $(\vec{k}n\downarrow)$ of interest, whose population (F) deviates from equilibrium (f) by $\delta F(\vec{k}n\downarrow)$. Then the rate equation (2) takes the form

$$\frac{dF(\vec{k},n,\downarrow)}{dt}=-\delta F(\vec{k}n\downarrow)/\tau(\vec{k},n,\downarrow), \quad (5)$$

$$1/\tau(\vec{k},n,\downarrow)=\frac{2\pi}{\hbar N}\sum_{\vec{Q}n'b}J^2Sp'(\uparrow) \times\{\delta(\epsilon-\epsilon'-\omega)[n+1-f'] +\delta(\epsilon-\epsilon'+\omega)[n+f']\}. \quad (6)$$

Except for the factor $p'(\uparrow)=p(\vec{k}+\vec{Q}n'\uparrow)$, this magnon-limited scattering rate is a perfect analog of the usual phonon-limited quasiparticle relaxation rate from Migdal theory. Equation (6) can equally well be derived by evaluation of the Feynman diagram Fig. 1, analytic continuation of Matsubara frequencies $i\omega_\nu$ to complex frequency z, and use of $1/\tau=-2\,\mathrm{Im}\,\Sigma(z\rightarrow\epsilon+i\delta)$. The energies ϵ and ω used to evaluate Eq. (6) are the experimental quasiparticle and magnon energies, which is mandated by the occurrence of the full, not bare, Green's functions in the self-energy graph of Fig. 1.

III. EVALUATION OF RELAXATION RATE

Reasonable simplifying assumptions yield a ''zero-parameter'' estimate of Eq. (6) for $1/\tau$. First, we specialize to a state $(\vec{k},n)$ that is in an occupied surface band. This state at low T is dominantly spin up, but as seen experimentally, it has a spin-down component, $p(\downarrow)\approx 0.13$. The Green's function G and self-energy are 2×2 matrices in spin space. Eigenstates of the 2×2 matrix G^{-1} are the spin-split quasiparticles, of which only the lower (primarily majority spin) state is relevant. The energy of this state will be denoted ϵ_k, with no band or spin index needed, and only a two-dimensional wave vector k rather than a three-dimensional $\vec{k}$. Spin-resolved photoemission selectively depopulates a single-spin component of this state. When the photoemitted

electron has spin down, there is a hole in the down-spin component of the majority-spin hole band. This component [with distribution function $F(k\downarrow)$] decays to equilibrium with rate $1/\tau(k,\downarrow)$. We assume that $p(k\downarrow)=p(\downarrow)$ is independent of k, and similarly $p(k+Q\uparrow)=p'(\uparrow)=1-p(\downarrow)$. Then we have

$$\hbar/\tau(\vec{k},\downarrow)=\frac{\pi p'(\uparrow)}{2S}(2JS)^2\mathcal{D}, \tag{7}$$

where $\mathcal{D}$ is the relevant density of decay channels. At $T=0$, $\mathcal{D}$ is

$$\mathcal{D}=\frac{1}{N}\sum_{\vec{Q}}\{\}=\frac{\sqrt{3}a^2c/2}{(2\pi)^3}\int d^2Q dQ_z \times\{\delta(\epsilon_k-\epsilon_{k+Q}-\omega(Q,Q_z))\theta(+\epsilon_{k+Q}) + \delta(\epsilon_k-\epsilon_{k+Q}+\omega(Q,Q_z))\theta(-\epsilon_{k+Q})\}. \tag{8}$$

We assume that the spin waves retain their bulk character. Therefore, although electron energies depend only on the two-component k, the spin-wave energy $\omega(Q,Q_z)$ depends on all three components of wave vector, the z direction being normal to the surface. The first delta function in Eq. (8) is a process where an empty state $k+Q$ lying above the Fermi surface is filled from a state k that scatters into it by spin-wave emission. In other words, it refers to decay of an electron lying in a state k that is above the Fermi energy by at least a spin-wave energy. The second delta function is a process where the filled state $k+Q$ lying below the Fermi surface scatters into an empty state k by spin-wave emission; in other words, it describes decay of a hole in the state k (below the Fermi energy by at least a spin-wave energy). It is only this second process that is seen in the photoemission data. Let us also assume that the photohole state k lies below the Fermi energy by at least 25 meV, the maximum spin-wave energy, in which case $\theta(-\epsilon_{k+Q})$ is guaranteed to be 1. The Q_z integration then gives

$$\frac{c}{2\pi}\int_{-\pi/c}^{\pi/c}dQ_z\,\delta(\epsilon_k-\epsilon_{k+Q}+\omega(Q,Q_z))=\nu(k,k+Q)/s(Q) \tag{9}$$

where $s(Q)$ is the normalized spin-wave slope

$$s(Q)=\frac{2\pi}{c}\left|\frac{\partial\omega}{\partial Q_z}\right|_{\omega^*} \tag{10}$$

and $\nu(Q)$ is the number of spin-wave states (Q,Q_z) with fixed Q, but any value of Q_z, which conserve energy, i.e., which satisfy $\omega(Q,Q_z)=\omega^*=\epsilon_{k+Q}-\epsilon_k$. Examining the measured dispersion curves,[8] the normalized slope can lie between 0 and a maximum that is not very different from $\omega_{\max}=25$ meV. If $\epsilon_{k+Q}-\epsilon_k$ is not greater than $\omega_{\max}$, then there is a fairly good chance (something like 50% probability) that there are two values, $\pm|Q_z|$, such that the spin-wave state (Q,Q_z) obeys energy conservation. In other words, $\nu(Q)$ can be expected to take the value 2 or 0 with about equal probability provided $0<\epsilon_{k+Q}-\epsilon_k<\omega_{\max}$, and is definitely 0 elsewhere.

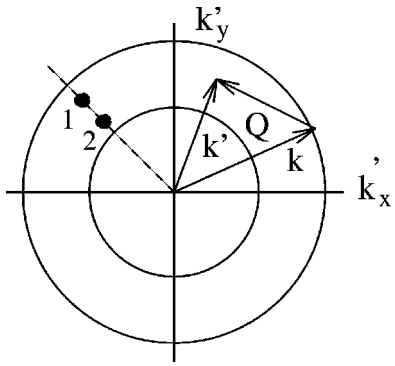

FIG. 2. For a given hole state k, the annular region shows possible lower-energy holes (higher-energy filled states) that the hole can scatter into by spin-wave emission. The outer circle denotes states with energy $-\epsilon_0-\hbar^2k^2/2m^*$, and the inner circle denotes states higher in energy by $\omega_{\max}$.

The remaining integral is

$$\mathcal{D}=\frac{\sqrt{3}a^2/2}{(2\pi)^2}\int d^2Q\,\nu(k,k+Q)/s(Q). \tag{11}$$

This integral should be evaluated using the correct dispersion relations for the surface-electron state and for the spin-wave states, with allowance for boundary conditions and wave function amplitudes altering the matrix elements. However, this requires a large ''first-principles'' calculation of uncertain reliability. Therefore, a slightly cavalier estimate is in order. Assume that the surface state has parabolic dispersion $\epsilon_k=-\epsilon_0-\hbar^2k^2/2m^*$. Experimentally this state is seen[1] to disperse downwards in energy with effective mass $m^*\sim1.2m$, where m is the electron mass.

The inequality $0<\epsilon_{k+Q}-\epsilon_k<\omega_{\max}$ is obeyed in an annular region shown in Fig. 2. The area of this region is $\pi(k^2-k'^2)$ and $(\hbar^2/2m^*)(k^2-k'^2)=\omega_{\max}$. Inside this region we assume ν has an average value of 1 and the normalized slope has an average value $\omega_{\max}/2$. Then the density of decay channels is

$$\mathcal{D}=\frac{\sqrt{3}a^2/2}{(2\pi)^2}\pi\frac{2m^*}{\hbar^2}\omega_{\max}\frac{2}{\omega_{\max}}=\frac{\sqrt{3}}{2\pi}\frac{m^*a^2}{\hbar^2}\sim0.57\ \ (\mathrm{eV})^{-1}. \tag{12}$$

This is ϵ independent because of the two-dimensionality of the surface band. A more careful treatment, yielding exactly the same result, is in the Appendix.

Finally we evaluate the decay rate Eq. (7) by choosing $2JS$ to be the spin splitting of the surface state, ~0.65 eV,[13] S to be 7/2, and the fractional up-spin probability $p'\sim0.87$.[1] These assumptions yield $\hbar/\tau\sim0.10$ eV for all photohole states k that are not too much closer than 25 meV to the top of the surface-state band. For states closer to the top, the decay rate should diminish because of reduction of the number of decay channels $\mathcal{D}$. It is also assumed that T is fairly low. At higher T, the rate is enhanced by thermal spin-wave population, but diminished by diminishing fractional up-spin probability $p'(\uparrow)$. The estimate $\hbar/\tau\sim0.10$ eV agrees with

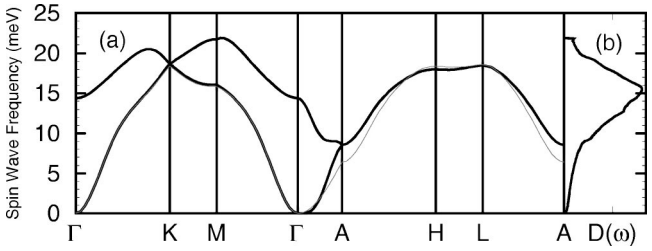

FIG. 3. Spin-wave dispersion in Gd, calculated from Lindgard's parameters (Ref. 7). The magnon density of states is shown in part (b) of the panel. The thin line represents the dispersion curve obtained by truncating farther than first neighbors in the *z* direction.

experiment[1] to greater precision than the uncertainties of the model. The calculation supports the interpretation that the source of minority-spin line broadening is spin-flip decay.

ACKNOWLEDGMENTS

I thank A. Abanov, I. Aleiner, and P. Johnson for discussions. This work was supported by NSF Grant No. DMR-0089492.

APPENDIX

Here I try to illuminate some of the approximations in the model and also to demonstrate the surprising reliability of the estimate

$$\hbar/\tau = \frac{\sqrt{3}}{4}\frac{p'(\uparrow)m^*}{S}\left(\frac{2JSa}{\hbar}\right)^2, \tag{A1}$$

which comes from combining Eqs. (7) and (12). Spin waves in Gd have been fitted[14] with the model

$$\begin{pmatrix} V_{11}(\vec{Q}) & V_{12}(\vec{Q}) \\ V_{12}(\vec{Q}) & V_{11}(\vec{Q}) \end{pmatrix}\begin{pmatrix} 1/\sqrt{2} \\ \pm 1/\sqrt{2} \end{pmatrix} = \omega_{\pm}(\vec{Q})\begin{pmatrix} 1/\sqrt{2} \\ \pm 1/\sqrt{2} \end{pmatrix}, \tag{A2}$$

where the spin-wave eigenfrequencies are $\omega_{\pm}(\vec{Q}) = V_{11}(\vec{Q}) \pm V_{12}(\vec{Q})$. The elements of the matrix are defined by

$$V_{11}(\vec{Q}) = S[J_{11}(0) - J_{11}(\vec{Q}) + J_{12}(0)],$$

$$V_{12}(\vec{Q}) = -SJ_{12}(\vec{Q}), \tag{A3}$$

$$J_{11}(\vec{Q}) = \mathrm{Re}\sum_{\vec{R}} J(\vec{R})e^{i\vec{Q}\cdot\vec{R}},$$

$$J_{12}(\vec{Q}) = \mathrm{Re}\sum_{\vec{R}} J(\vec{R}+\vec{\tau})e^{i\vec{Q}\cdot(\vec{R}+\vec{\tau})}, \tag{A4}$$

where $J(\vec{R})$ are fitted exchange coupling constants, $\vec{R}$ runs over the translation vectors of the hexagonal lattice, and $\vec{\tau}$ with *z*-component $c/2$ gives the position of the second atom in the two-atom basis of the hcp crystal structure. The frequencies and density of states calculated from this model fit experiment[8] well, and are shown in Fig. 3.

Notice that the two branches $\omega_{\pm}$ are degenerate at the top of the Brillouin zone $Q_z = \pi/c$ (directions $A \to H \to L \to A$ in Fig. 3). These branches can be ''unfolded'' into a zone twice as large in the *z* direction; ω_+ is the extension of ω_- under the mapping $\omega_+(Q_x, Q_y, Q_z) = \omega_-(Q_x, Q_y, 2\pi/c - Q_z)$.

An accurate analytic approximation for Eq. (11) is possible provided Lindgard's fitting parameters, Eqs. (A4) are slightly simplified by truncating off farther than first-neighbor planes in the *z* direction. The alteration of $\omega(\vec{Q})$ caused by this approximation is shown in Fig. 3. The change is fairly small, and could be largely compensated by a further tuning of the nearer-neighbor couplings. With this approximation, the one-extended magnon branch $\omega = \omega_-$ has frequency given by

$$\omega(\vec{Q}) = a(Q_x, Q_y) + b(Q_x, Q_y)\cos(Q_z c/2). \tag{A5}$$

Then the normalized slope $s(Q)$ is given by

$$s(Q_x, Q_y) = \pi\sqrt{b^2 - (\omega - a)^2}. \tag{A6}$$

The remaining integral Eq. (11) can be considered an integral over d^2k' running over the annulus of Fig. 2,

$$\mathcal{D} = \frac{\sqrt{3}a^2/2}{(2\pi)^2}\int_{k_1}^{k_2} d\phi k' dk' \frac{2}{\pi\sqrt{b^2 - (\omega - a)^2}}, \tag{A7}$$

where $\omega = \epsilon(k) - \epsilon(k')$ is the energy of the spin wave with wave vector $(k_x - k_x', k_y - k_y', Q_z)$ that scatters the hole out of state k into state k'. For a given two-vector $(Q_x, Q_y) = k - k'$, one searches over Q_z to find whether there is an energy conserving solution. Either there are no energy-conserving spin-wave states, or else there are $\nu = 2$ such solutions at $\pm Q_z$. Consider a path in k' space at fixed azimuth ϕ shown as a dotted line in Fig. 2. The outer circle is states k' that are degenerate in energy with the starting state k. Unless k' is the same as k, there is no zero-energy spin wave that can couple these states. Moving down the dotted line to higher-energy electron states that can fall into the hole at k by magnon emission, one finds the state labeled 1 that has just enough energy difference that an energy-conserving magnon transition is found. The magnon has $Q_z = 0$ because it is the least-energy magnon allowed to couple on the dotted line. Moving farther down the dotted line, one comes eventually to the state labeled 2 that is the highest-energy electron state that can fall into the hole in state k by magnon emission. The magnon has $Q_z = 2\pi/c$ because it is the highest-energy magnon. For both these extreme states, $\omega = a \pm b$, the slope s is zero (the magnon energy is quadratic in Q_z near $Q_z = 0$ and $2\pi/c$). The integrand of Eq. (A7) diverges at the two end points. However, it is an integrable divergence, and in fact, almost exactly independent of the variables a and b. We observe that in going from states 1 to 2 along the dotted line, the components Q_x and Q_y are not changing much, which allows us to set $a(Q_x, Q_y)$ and $b(Q_x, Q_y)$ to constants during the k' integration. Then we have

$$\mathcal{D}=\frac{\sqrt{3}a^2/2}{(2\pi)}\frac{m^*}{\hbar^2}\int_{\epsilon-a-b}^{\epsilon-a+b}d\epsilon'\frac{2/\pi}{\sqrt{b^2-(\epsilon-\epsilon'-a)^2}},$$

$$=\frac{\sqrt{3}a^2}{(2\pi)}\frac{m^*}{\hbar^2}. \tag{A8}$$

This is exactly Eq. (12).

Finally it is appropriate to mention the hidden assumptions about surface and bulk states. It is implicitly assumed that the electron-surface state has a wave function of unit amplitude on the surface layer and zero amplitude elsewhere. This kind of state will be absolutely insensitive to the z component of the magnon wave vector, and the electron-magnon matrix element will be $J[(2S/2N)p(\uparrow)p(\downarrow)]^{1/2}$ for all k and k'. Suppose instead a surface-electron wave function of amplitude $1/\sqrt{2}$ on each of the top two layers. This state will couple to the ω_- branch of magnons with the full-matrix element $J(2S/2N)^{1/2}$, but will not see the ω_+ branch. Conversely, a surface wave function that has amplitude $\pm 1/\sqrt{2}$ on the top two layers, with a sign change, will couple only to the ω_+ branch and not to the ω_- branch. It is not reasonable that the net coupling to magnons should depend much on the depth or details of the surface-electron wave function; therefore we should ask where has the missing-magnon coupling gone when the electron wave function extends two layers down instead of one. The answer is in the orthogonal electron wave function on the top two layers, with opposite phase relation between layer 1 and 2. Since by assumption there is only one surface state, the orthogonal state is not an eigenstate but a superposition of bulk states. The missing-magnon coupling is from the surface state into these bulk states. For the actual Gd-surface state, how much of the magnon-induced scattering is to bulk and how much is to surface states is an unknown element. The extreme model used here hides this problem. The justification is belief that the net-scattering probability should have a tendency to be conserved, i.e., to be weakly dependent on depth. Similarly, we have not asked what is the nature of the magnon states near the surface, but instead assumed that we can use bulk magnon states. Instead, it might be that a surface band of magnon states grabs all the spectral weight. Then details would be quite different, but over-all coupling strength should be similar.

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