

Symmetry of the remanent-state flux distribution in superconducting thin strips: Probing the critical state

A. V. Bobyl,^{1,2} D. V. Shantsev,^{1,2} Y. M. Galperin,^{1,2} and T. H. Johansen^{1,*}

¹*Department of Physics, University of Oslo, P.O. Box 1048 Blindern, 0316 Oslo, Norway*

²*A. F. Ioffe Physico-Technical Institute, Polytekhnicheskaya 26, St. Petersburg 194021, Russia*

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The critical-state in a thin strip of $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ is studied by magneto-optical imaging. The distribution of magnetic flux density is shown to have a specific symmetry in the remanent state after a large applied field. The symmetry was predicted [Phys. Rev. Lett. **82**, 2947 (1999)] for any $j_c(B)$, and is therefore suggested as a simple tool to verify the applicability of the critical-state model. At large temperatures we find deviations from this symmetry, which demonstrates a departure from the critical-state behavior. The observed deviations can be attributed to an explicit coordinate dependence of j_c since both a surface barrier and flux creep would break the symmetry in a different way.

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I. INTRODUCTION

During the last years much attention has been paid to studies of the magnetic behavior of thin superconductors placed in a perpendicular magnetic field, the so-called perpendicular geometry. On the one hand, numerous papers have investigated theoretically the critical state of thin superconductors of regular shapes like long strips and circular disks.^{1–8} On the other hand, magnetic characterization by measuring the spatial distribution of the flux density at the surface of superconductors has become quite common and powerful. This progress has been facilitated by the development of spatially resolved techniques, such as magneto-optical imaging, Hall-probe arrays, etc.; see Ref. 9 for a review.

In spite of these massive efforts, the task of making proper interpretations of a measured flux density distribution $B(\mathbf{r})$ is still one with major difficulties. In particular, we are not aware of any simple decisive method to judge whether an observed $B(\mathbf{r})$ is consistent with the critical-state model (CSM) or not. One could here expect that fitting an observed $B(\mathbf{r})$ by profiles predicted from the CSM with some B -dependent critical current density, $j_c(B)$, would be a straightforward procedure. However, this is not so since in the perpendicular geometry *explicit* expressions for the flux density distribution are available only for the Bean model, $j_c = \text{const}$, for a thin strip^{1,2} and a thin disk,^{3–5} as well as for a strip with a special kind of $j_c(B)$.⁶ For a thin strip or disk with a general B -dependent j_c , the flux distribution can be calculated only numerically by solving a set of integral equations.^{7,8}

In a recent work,¹⁰ we considered the critical-state magnetic behavior of a thin superconducting strip with a general B dependence of j_c . It was shown that the central peak in large-field magnetization hysteresis loops of such samples always occurs at the remanent state, $B_a = 0$. An intermediate result of that derivation is the prediction of a special symmetry of the remanent-state flux density distribution. Since the symmetry is independent of the particular $j_c(B)$, it may serve as a conclusive and easily implementable test for the

applicability of the critical-state model in a given experiment. In the present paper we demonstrate using magneto-optical imaging how this symmetry in the flux density profile can be revealed and used to verify the applicability of the CSM.

II. SYMMETRY OF FLUX DISTRIBUTION

Consider a long thin superconducting strip with edges located at $x = \pm w$, the y axis pointing along the strip and the z axis normal to the strip plane; see Fig. 1. The magnetic field B_a is applied along the z axis, so screening currents are flowing in the y direction. Throughout the paper B denotes the z component of magnetic induction in the strip plane. The sheet current is defined as $J(x) = \int j(x, z) dz$, where $j(x, z)$ is the current density and the integration is performed over the strip thickness, $d \ll w$.

From the Biot-Savart law for the strip geometry, the flux density is given by^{1,2}

$$B(x) - B_a = -\frac{\mu_0}{2\pi} \int_{-w}^w \frac{J(u) du}{x-u}. \quad (1)$$

Assume that the strip is in the remanent state after a very large field was applied. Then, everywhere in the strip the current density is equal to the critical value, $J(x) = \text{sgn}(x)J_c[B(x)]$, and the flux density distribution satisfies the following integral equation:

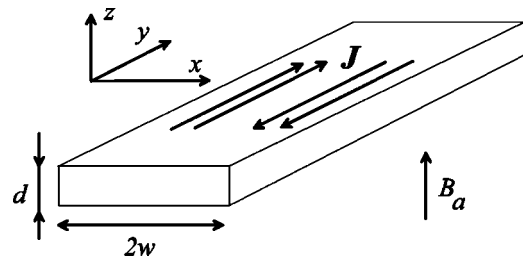


FIG. 1. Superconducting strip in a perpendicular applied magnetic field.

$$B(x) = -\frac{\mu_0}{\pi} \int_0^w \frac{J_c[B(u)]}{x^2 - u^2} u du. \quad (2)$$

By changing here the integration variable from u to $\sqrt{w^2 - u^2}$, and also replacing x by $\sqrt{w^2 - x^2}$, one obtains

$$B(\sqrt{w^2 - x^2}) = \frac{\mu_0}{\pi} \int_0^w \frac{J_c[B(\sqrt{w^2 - u^2})]}{x^2 - u^2} u du. \quad (3)$$

Clearly, this equation for $-B(\sqrt{w^2 - x^2})$ is equivalent to Eq. (2) for $B(x)$ if J_c depends only on the absolute value of the magnetic induction, $J_c(|B|)$. Thus, we conclude that

$$B(x) = -B(\sqrt{w^2 - x^2}). \quad (4)$$

This symmetry of the flux density distribution in the fully penetrated remanent state is valid for *any* $J_c(|B|)$ dependence. In particular, it also holds for the Bean model, where one finds by simple integration of Eq. (2) that

$$B(x) = \frac{\mu_0 J_c}{2\pi} \ln \frac{w^2 - x^2}{x^2}. \quad (5)$$

It follows from the general equation (4) that the flux density is always zero at $x = w/\sqrt{2}$. At this point $B(x)$ changes sign from positive in the central part of the strip to negative near the edges.

This special symmetry of the flux density profile across a thin strip has a trivial analog for the case of a long sample in a parallel field. There, the gradient in B is a function of the local value of the field. Hence, the flux distribution is always symmetric around the point x_0 where the flux density is zero, $B(x_0) = 0$. Hence, one can write $B(x) = -B(2x_0 - x)$ as long as x and $2x_0 - x$ are within the superconductor. In the perpendicular geometry the relationship between B and j is non-local, and the B profiles deviate strongly from those in the interior of long slabs and cylinders.

III. EXPERIMENT

A 200-nm-thick film of $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ was grown epitaxially on an MgO substrate using laser ablation.¹¹ The sample was patterned by chemical etching into a strip shape of width $2w = 2.5$ mm. For the measurements of flux density profiles we chose a region free of any defects visible by our magneto-optical (MO) imaging system.

The imaging system is based on the Faraday rotation of polarized light illuminating an MO-active indicator film that we mount directly on top of the superconductor's surface. The rotated Faraday angle varies locally with the value of the perpendicular magnetic field, and through a crossed analyzer in an optical microscope one can directly visualize and quantify the field distribution across the covered sample area. As Faraday-active indicator we use a Bi-doped yttrium iron garnet film with in-plane magnetization.¹² The indicator film was deposited to a thickness of 5 μm by liquid phase epitaxy on a gadolinium gallium garnet substrate. A thin layer of aluminum is evaporated onto the film allowing incident light to be reflected, thus providing double Faraday rotation

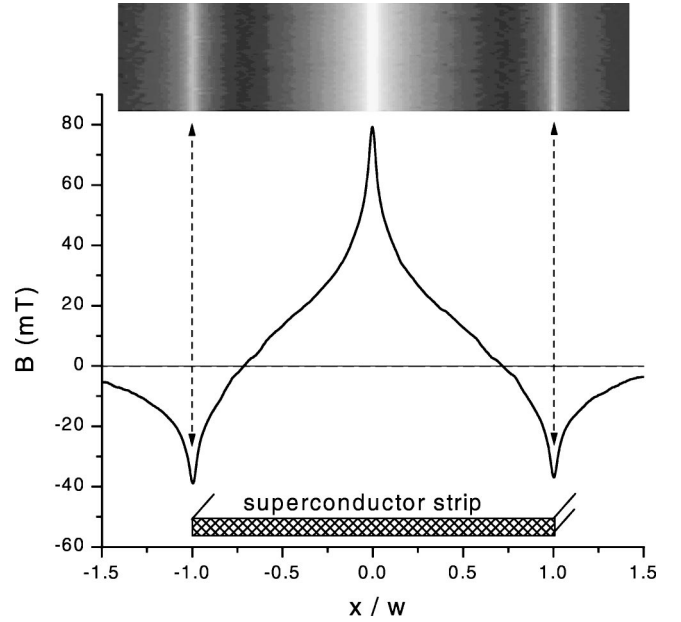


FIG. 2. Flux density profile across the $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ strip in the remanent state at 42 K. The profile is derived from the magneto-optical image shown above. The bright areas on the image correspond to large $|B|$.

of the light beam. The images were recorded with an 8-bit Kodak DCS 420 CCD camera and transferred to a computer for processing. The conversion of gray levels into magnetic field values is based on a careful calibration; see Ref. 13. The MO imaging at low temperatures was performed by mounting the superconductor/MO indicator on the cold finger of a continuous He-flow cryostat with an optical window (Microstat, Oxford).

MO images were taken in the remanent state after applying a large field at temperatures of 42, 58, and 82 K. An MO image at 42 K and the corresponding flux density profile are shown in Fig. 2. The fact that maximum trapped flux density is observed in the center of the strip implies that the applied field had been raised to a sufficiently large value. Furthermore, one sees from the figure that the return field of the trapped flux penetrates regions near the strip edges, $x = \pm w$, where the field has negative peaks. While the intensity in the MO image does not immediately discriminate between the two field polarities, it is readily accounted for by locating the boundary where $B = 0$, also called the annihilation zone. One sees also that the flux distributions in the left and right halves of the strip are mirror images of each other, and therefore we focus only on one-half of the strip, $0 \leq x \leq w$. The flux density distribution at higher temperatures is shown in Fig. 3. Due to reduced flux pinning with increasing temperatures, the magnitude of the trapped field is reduced substantially. In addition, we also see changes in the shape of the flux profile. The spatial resolution of the method is limited by the thickness of the MO indicator film. Therefore, a few data points in the 5 μm vicinity of the peaks at the strip center and at the edge have been removed from the following analysis.

To test the symmetry property, expressed in Eq. (4), of the measured flux profiles we plot in Fig. 4 the absolute of

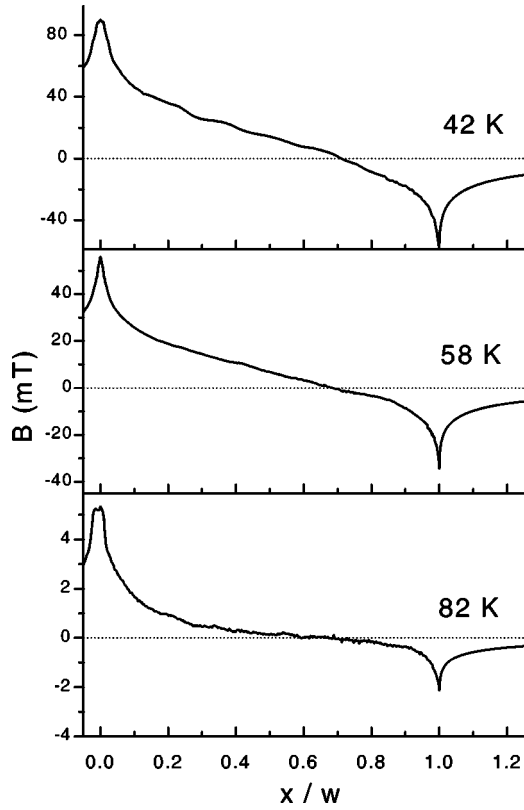


FIG. 3. Profiles of the flux density across one-half of the $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ strip at different temperatures. The strip is in the remanent state after a large field had been applied.

the flux density as a function of the new coordinate x' ,

$$x' = \begin{cases} x, & 0 < x \leq w/\sqrt{2}, \\ \sqrt{w^2 - x^2}, & w/\sqrt{2} < x < w. \end{cases} \quad (6)$$

If Eq. (4) holds, there should be a full overlap of the two branches where the measured B is positive and negative. One sees from the figure that the overlap is almost complete for the data taken at 42 and 58 K except for small deviations at large $|B|$. At 82 K, however, there is a significant splitting of the two branches over the whole range. One may therefore conclude that there is a systematic deviation from the CSM behavior at this high temperature.

The results from the symmetry analysis of B profiles are now compared with direct evaluation of j in regions with equal $|B|$. For this purpose the current density distributions were calculated from the measured B profiles by the inversion scheme proposed in Ref. 13 and developed elsewhere.¹⁴ The latter procedure is, in general, much more complicated than the analysis of B profiles and it requires knowledge of B distributions across rather large regions also outside the strip. Furthermore, the inversion procedure involves filtering of short-wavelength noise in experimental data.

The current profile at 42 K is shown as a solid line in the inset of Fig. 5. One can see an enhancement in j in the region near $B=0$. In the main figure the data are replotted as j versus $|B|$. Again we see that the two branches corresponding to positive and negative B collapse, which proves exist-

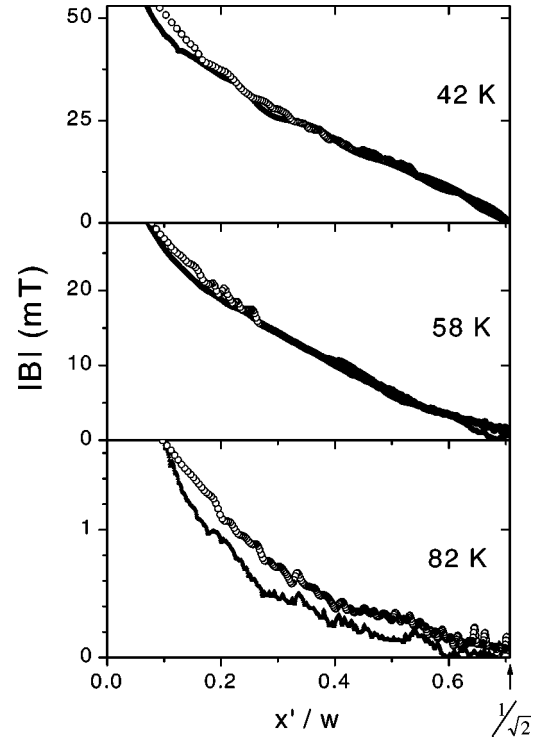


FIG. 4. Profiles of the flux density from Fig. 3 replotted with new coordinate x' defined by Eq. (6). Open and solid symbols correspond to $x > w/\sqrt{2}$, and $x < w/\sqrt{2}$, respectively. A large splitting of the two branches at 82 K indicates a deviation from the CSM.

tence of the critical state in the strip. The unified curve characterizes the $j_c(B)$ dependence. Also in this plot, like in Fig. 4, there are small deviations from data collapse at the largest fields. We thus see that the analysis based on j inversion gives similar results as the symmetry analysis of B profiles.

IV. DISCUSSION

The symmetry condition, Eq. (4), was derived within the framework of the CSM, and is not expected to hold when one or more of the basic assumptions of this very successful model is violated. The most probable reasons for lack of this symmetry are therefore the following.

(i) The presence of a *surface barrier* for vortex entry and exit. There are vast experimental observations of such a barrier in high- T_c superconductor crystals. The barrier leads to an excessive current density in the vicinity of edges, which then destroys the symmetry. In particular, it shifts the point x_0 where $B=0$ towards the strip edge. For the geometrical barrier which arises from a rectangular shape of the cross sectional area, the excessive current density is of the order of H_{c1}/d .¹⁵ For the Bean-Livingston barrier it should be larger and scale with temperature as the thermodynamic critical field H_c .¹⁶ In both cases, the temperature dependence can differ from that of the bulk pinning j_c , suggesting that deviation from the symmetry can be temperature dependent. Moreover, it is known from experiment¹⁷ that the surface barrier dominates at the higher temperatures, while bulk pin-

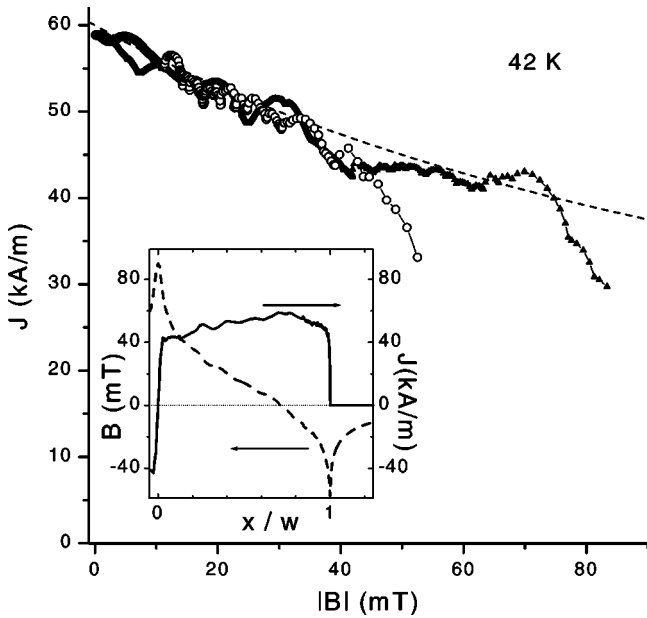


FIG. 5. Sheet current vs the absolute value of the local flux density at 42 K. The data are obtained by replotting the $B(x)$ and $j(x)$ profiles shown in the inset. A collapse of the two branches, which correspond to positive and negative B (solid and open symbols), demonstrates that the critical state is established in the strip. The dashed line shows a fitted Kim model $j_c(B) \propto (1 + B/B_0)^{-1}$ with $B_0 = 150$ mT.

ning is more important at low temperatures.

(ii) *Thermally activated creep* of vortices leading to a slow time relaxation of the flux distribution. The flux creep problem for a thin strip with a B -independent $E(j)$ law has been considered in Refs. 18 and 19. Under constant applied magnetic field the space and time dependence of the electric field is shown to decouple as $E(x,t) = f(x)g(t)$, where $f(x)$ can be found numerically. It is also argued that during relaxation of $E(x,t)$, starting from some initial $E(x,0)$, the electric field will approach the profile given by $f(x)$. From $\dot{B} = -\partial E/\partial x$ it follows that if an initial remanent flux profile $B(x)$ crossed zero at $x_0 = w/\sqrt{2}$, then during relaxation x_0 will shift towards the point where $f(x)$ has the maximum. For the voltage-current law $E = E_c(j/j_c)^n$, the maximum in f is always located at $x_0 > w/\sqrt{2}$, namely, at $x_0 = 0.735w$ for $n = 1$, and approaches $w/\sqrt{2}$ as $n \rightarrow \infty$.¹⁹ This means that at smaller n , i.e., at larger temperatures, the deviations from symmetry due to relaxation are stronger.²⁰

Thus, both a surface barrier and flux creep predict a stron-

ger deviation from the symmetry in $B(x)$ at higher temperatures. However, neither of them can explain the deviation found in our experiment. Indeed, while the CSM predicts that in the remanent state $x_0 = w/\sqrt{2}$, both surface barrier and flux creep lead to larger x_0 . Such a shift of x_0 would result in the negative- B branch being below the positive- B branch in the $|B|(x')$ plot. However, that is just the opposite to what is shown in Fig. 4.

(iii) The last possible reason for the deviation from the symmetry is *inhomogeneity of the strip*, which leads to an explicit coordinate dependence of the critical current density, $j_c(x, B(x))$. It may be caused, e.g., by a nonuniform chemical composition.²¹ The kind of deviation shown in Fig. 4 can be explained by a suppressed j_c near the strip edge. The fact that strong deviations are found only at the highest temperature can be related to the existence of two mechanisms controlling j_c with different temperature dependences. If so, only the mechanism dominant at high T has to produce an inhomogeneous j_c . An example of two such mechanisms can be bulk and intergrain pinning, which are known to have different T dependences.²²⁻²⁴ In thin $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ films the second mechanism can be realized on any planar defect such as a boundary between microblocks with slightly different crystal axis orientation,²⁵ a twin boundary, or a microcrack.²⁶

V. CONCLUSIONS

The flux density distribution in a superconducting thin strip with a general $j_c(B)$ is shown to have a special kind of symmetry in the remanent state after large applied field. Probing the symmetry of measured flux distributions is suggested as a simple method to test applicability of the critical-state model without *a priori* knowledge of $j_c(B)$. The procedure is simpler than calculation of the current distributions because it requires knowledge only of the field inside the strip and it is also weakly sensitive to “noise” in the experimental data. The method has been applied to a thin $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ strip which exhibited a fairly good CSM behavior well below T_c , but large deviations from the symmetry were observed at 82 K. Our analysis shows that the deviations can be attributed to an explicit coordinate dependence of j_c since both a surface barrier, and strong flux creep would break the symmetry in a different way.

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*Electronic address: t.h.johansen@fys.uio.no

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