Low-frequency response of a collectively pinned vortex manifold

S. E. Korshunov

L. D. Landau Institute for Theoretical Physics, Kosygina 2, Moscow 117940, Russia

A low-frequency dynamic response of a vortex manifold in a type-II superconductor can be associated with thermally activated tunneling of large portions of the manifold between pairs of metastable states. We suggest that statistical properties of these states can be verified by using the same approach for the analysis of thermal fluctuations the behavior of which is well known. The exponent describing the frequency dependence of a linear response is found for the generic case of a vortex manifold with nondispersive elastic moduli and also for the case of thin superconducting film in which the compressibility modulus is always nonlocal.

I. INTRODUCTION

In various situations the theory of weak collective pinning (for a review see Ref. 1) treats a vortex medium in a type-II superconductor like an elastic manifold interacting with a random pinning potential. In particular, a vortex contribution to a low-frequency response to an applied current (impedance) is ascribed to the classical (thermally activated) tunneling of large portions of the vortex manifold between different minima of the uncorrelated random potential.[2–4] In the framework of this approach the pairs of states that allow for thermally activated tunneling between them are treated as current-biased two-level systems. Earlier analogous ideas were applied for the description of the dynamic response of spin glasses[5] and of randomly pinned dislocations and interfaces.[6]

The frequency dependence of the two-level systems contribution to specific impedance $z_{tl}(\omega) \equiv \rho_{tl}(\omega) + i\omega l_{tl}(\omega)$ of a type-II superconductor penetrated by external magnetic field has been found by different groups of authors[2–4] to be of the form

$$l_{tl}(\omega) \propto [\ln(1/\tau_0|\omega|)]^y; \tag{1a}$$

$$\rho_{tl}(\omega) \propto |\omega|[\ln(1/\tau_0|\omega|)]^{y-1}, \tag{1b}$$

but the conclusions of these three groups on the value of the exponent y are not compatible with each other.

Comparison shows that the discrepancy appears because the calculation of Fisher, Fisher, and Huse[2] is based on a semiphenomenological conjecture that a contribution of an active two-level system to inductance can be estimated (from above) by replacing this two-level system with an insulating hole of the same volume. This assumption does not take into account that the motion of vortices inside of a two-level system leads to a change of a phase distribution in superconductor (and of the energy of electric current in it) also outside of the limits of this particular two-level system. Accordingly, it turns out to be in direct contradiction with the explicit expression for a two-level system contribution to impedance derived by Koshelev and Vinokur[4] and used in their calculation of $z_{tl}(\omega)$, and naturally leads to a different value of y. The less pronounced descrepancy between the

results of Ref. 3 and Ref. 4 can be explained, in particular, to the difference in assumptions on statistics of metastable states.

In the present work we revisit this problem following the more reliable (as is demonstrated below) approach of Koshelev and Vinokur.[4] We start by deriving the value of the exponent y for the generic case of a vortex manifold with a single elastic modulus (but with arbitrary dimensionality). Any more complex system with different (but local) elastic moduli will be characterized by the same value of y. In Ref. 4 the value of y in nondispersive system has been found only for the particular case of a single vortex pinning.

We then suggest a new independent way to check the validity of the numerous assumptions involved in calculation of the two-level systems contribution to a linear dynamic response by using the same set of assumptions for calculation of another quantity (the amplitude of thermal fluctuations), which on the other hand can be calculated exactly, because in the framework of a random manifold description it has to be the same as in absence of disorder.[7] It turns out that application of the assumptions used in the previous calculation indeed produces an answer which is in agreement with the well-known result for the pure case.

The possibility of the additional check turns out to be very useful when we address the case of a thin superconducting film in which the compressibility modulus c_{11} of vortex manifold is always nonlocal. It allows to draw some conclusions about size and shape distribution of two-level systems which otherwise would be unavailable. Inclusion of these conclusions into calculation leads [for $c_{11}(q) \propto 1/q^2$] to very weak (double logarithmic) frequency dependence of $l_{tl}(\omega)$ corresponding to $y = 0$.

The case of a thin superconducting film is also of a special interest in relation with recent experimental investigation of $z(\omega)$ in ultrathin $YBa_2Cu_3O_7$ films.[8,9] Note that in a two-dimensional (2D) geometry with complete penetration of the magnetic field the frequency-dependent specific (sheet) impedance directly determines the response of a film to external field, whereas in the case of a bulk superconductor it has to be extracted from the surface impedance.

In all the cases we have considered $l_{tl}(\omega)$ diverges for $\omega \to 0$. Thus the important consequence of our results is that the random manifold approximation cannot be sufficient for

the description of a truly superconducting vortex glass phase[10,11] with finite superfluid density.

The outline of the article can be summarized as follows. In Sec. II the random manifold problem is briefly introduced and some situations when it is appropriate for the description of a vortex manifold in a superconductor are specified. In Sec. III the statistical properties of the metastable states which have to be taken into account in the framework of the two-level system approach are discussed. In Sec. IV the application of this approach to calculation of the vortex medium contribution to impedance is presented in the systematic form for the nondispersive case. In Sec. V the same approach is used for the analysis of thermal fluctuations. We show that (with the same set of assumptions as used earlier in calculation of a linear dynamic response) it indeed produces an answer which is consistent with expectations based on consideration of pure system.

Section VI is devoted to the discussion of a thin superconducting film in which the nonlocality of the compressibility modulus is always important. We show that if one assumes that the dominant two-level systems in this case are strongly anisotropic (as suggested by the energy balance estimates used in the analysis of the nonlinear creep[11–14]), the expression for thermal fluctuations amplitude turns out to be convergent in contrast to its logarithmic divergence in the pure system. The only way to resolve this contradiction consists in assuming that the statistics of metastable states is dominated by the presence of hierarchical sequence of quasi-isotropic two-level systems. The same distribution is then used for the calculation of the components of $z_{tl}(\omega)$. In Sec. VII the results are summarized and discussed.

II. RANDOM MANIFOLD PROBLEM

An elastic manifold (with internal dimension D) interacting with a random pinning potential can be described by the Hamiltonian

$$
\begin{aligned}
H &= H_{\mathrm{el}} + H_d \\
&= \frac{1}{2}\int d^D\mathbf{x}_1 \int d^D\mathbf{x}_2 [G_0^{-1}(\mathbf{x}_1-\mathbf{x}_2)]^{ab} u^a(\mathbf{x}_1) u^b(\mathbf{x}_2) \\
&\quad + \int d^D\mathbf{x}\ v[\mathbf{x},\mathbf{u}(\mathbf{x})],
\end{aligned}
\tag{2}
$$

where the N-dimensional vector $\mathbf{u}(\mathbf{x})\equiv u^a(\mathbf{x})$ is the displacement of the manifold. The first term in Eq. (2) describes (in a most general form) the elastic energy of the manifold and the second the energy of its interaction with a random pinning potential $v(\mathbf{x},\mathbf{u})$.

The simplest assumption would consist in assuming that the random potential $v(\mathbf{x},\mathbf{u})$ has a Gaussian distribution with

$$
\langle v(\mathbf{x},\mathbf{u})\rangle_d = 0,
\tag{3}
$$

$$
\langle v(\mathbf{x}_1,\mathbf{u}_1) v(\mathbf{x}_2,\mathbf{u}_2)\rangle_d = \delta(\mathbf{x}_1-\mathbf{x}_2) w(\mathbf{u}_1-\mathbf{u}_2)
\tag{4}
$$

Here and further on the angular brackets with subscript d stand for the average over disorder, and with subscript th for the thermal average. We discuss only the case of a short-ranged random potential correlation function $w(\mathbf{u})$. In the simplest situation (which would imply, in particular, full isotropy and absence of dispersion) the first term in Eq. (2) can be chosen in the form

$$
H_{\mathrm{el}} = \frac{J}{2}\int d^D\mathbf{x}\left(\frac{\partial u^a}{\partial x^\beta}\right)^2
\tag{5}
$$

with a single elastic modulus J.

The physical systems which can be described by the Hamiltonian of the form (2) include, in particular, a domain wall in a 2D or 3D Ising type ferromagnet/antiferromagnet ($D=1,2;N=1$); a dislocation in a crystal ($D=1,N=2$); a single vortex line in a large area Josephson junction ($D=1,N=1$) or bulk superconductor ($D=1,N=2$); a vortex medium in superconducting film ($D=2,N=2$) or bulk superconductor ($D=3,N=2$), a layered superconductor with in-plane field ($D=3,N=1$) or a large area Josephson junction with in-plane field ($D=2,N=1$). In all these cases the random pinning potential is automatically provided by impurities present in any solid or/and by geometrical inhomogeneities.

Note, however, that the random manifold approximation assumes the energy of the interaction with the inhomogeneities to be uncorrelated for different displacements, whereas the energy of the interaction of an ideal vortex crystal with the inhomogeneities does not change if the vortex crystal is shifted as a whole by one lattice constant. Thus, the area of applicability of the random manifold approximation for the description of vortex crystal pinning is resticted. One can use this approach if the relevant displacements do not exceed the lattice period (Larkin regime[15]) or when the ordering in the vortex crystal is destroyed by the presence of defects whose motion with respect to the vortex manifold is dynamically frozen in comparison with the motion of the manifold itself.

Recent experiments of the Neuchâtel group on ultrathin YBa$_2$Cu$_3$O$_7$ films[9] have demonstrated a crossover to the regime in which the contribution to resistivity associated with the motion of pointlike defects is negligible in comparison with the contribution which can be ascribed to collective pinning behavior.

III. LOW-ENERGY METASTABLE STATES

The low-frequency dynamics of a weakly pinned elastic manifold can be associated with the thermally activated tunneling of large domains of the manifold between different minima of the random potential.[6] In a simple system with a discrete spectrum only the tunneling between the ground state and the first excited state is of importance at low temperatures, thus it can be reduced to a two-level system. For an infinite manifold one should take into account that such two-level systems appear at all scales and form a hierarchical structure, i.e., if some domain of the manifold can tunnel between some states one also has to consider the possibility of tunneling of smaller domains inside this area between different pairs of states.

Each of such two-level systems can be characterized by its (linear) size L, its volume $V\sim L^D$ (until specified we dis-

cuss the simplest isotropic case with a single nondispersive elastic modulus), the typical vortex displacement between the two states u, which, for example, can be defined by

$$u^2 = \frac{1}{V} \int d^D \mathbf{x} \, \mathbf{u}^2(\mathbf{x}), \tag{6}$$

the energy difference between the two states Δ and the energy barrier U which has to be overcome for moving the manifold (the vortex bundle) from one of the two states to the other.

The universality hypothesis introduced by Ioffe and Vinokur[6] suggests that for each length scale L there should exist only one relevant energy scale $E(L)$ such that, in particular, the typical values of Δ and U for a system of size L are proportional to $E(L)$. The magnitude of $E(L)$ can be then estimated by estimating the elastic energy associated with the displacement of the vortex bundle of the size L:

$$E(L) \sim J V(L) \frac{u^2(L)}{L^2}. \tag{7}$$

If the scale dependence of the typical displacement $u(L)$ is given by $u(L) \propto L^\zeta$ (where ζ is usually called the wandering exponent) Eq. (7) leads to $E(L) \propto L^\chi$ with

$$\chi = 2\zeta + D - 2. \tag{8}$$

On the other hand Fisher, Fisher, and Huse[2] have suggested that the scale dependence of the typical energy barrier $U(L)$ can be described by another exponent ψ not necessariliy coinsiding with χ ($\psi \gtrless \chi$). For the sake of generality in the following we will keep (for a while) the separate notation for ψ, although we will assume that the typical value of Δ (for the given length scale L) can be estimated with the help of Eq. (7).

Various properties of the manifold depend also on the form of the size distribution function $\nu(L)$ of the two-level systems. The hierarchical distribution implies that any two-level system can include smaller two-level systems whose size differ from that of the ''parent'' two-level system by some numerical factor of the order of one. Thus the ratio of the typical ''neighboring'' length scales has to be more or less constant across the whole length-scale range involved.[5] This is compatible with a uniform distribution of the logarithms of the length scales. However, for each length scale one should also include the factor $1/V(L)$ proportional to the largest possible concentration of nonoverlapping two-level systems of size L. Thus $\nu(L)$ has to be of the form

$$dL \nu(L) \propto \frac{dL}{L} \frac{1}{V(L)}. \tag{9}$$

Koshelev and Vinokur[4] have introduced the first factor (imposed by the hierarchical structure) in the right-hand side of Eq. (9) in the form dU/U (without any explanation), which for $U(L)$ algebraically dependent on L is equivalent to dL/L, whereas in Ref. 3 the hierarchical nature of the size distribution of two-level systems has not been taken into account.

IV. TWO-LEVEL SYSTEMS CONTRIBUTION TO IMPEDANCE

The contribution of the two-level systems to the specific impedance of a superconductor is given[4] by

$$z_{tl}(\omega) = n \frac{\gamma}{T} \left\langle \frac{(V\bar{u})^2}{\cosh^2(\Delta/2T)} \frac{i\omega}{1 + i\tau\omega} \right\rangle_d, \quad \gamma = \frac{B^2}{4c^2}, \tag{10}$$

where n is the concentration of such systems, B is the magnetic induction, and T is the temperature, whereas V, $\bar{u}$, Δ, and τ are parameters characterizing a particular two-level system: V is the volume in which the vortices are displaced (or the area in the case of a 2D superconductor), $\bar{u}$ is the average displacement of the vortices inside the bundle in the direction of current-induced force and Δ is the difference in energy between the two states. The relaxation time τ describing the rate of the thermally activated (incoherent) tunneling between the two states depends exponentially

$$\tau = \tau_0 \exp(U/T) \tag{11}$$

on the barrier U separating them.

If one splits $z_{tl}(\omega)$ into real and imaginary parts

$$z_{tl}(\omega) = \rho_{tl}(\omega) + i\omega l_{tl}(\omega) \tag{12}$$

and assumes that the average over disorder can be estimated by taking for each length scale the typical (scale dependent) values of all the parameters involved, the expression for the two-level systems contribution to specific impedance $l_{tl}(\omega)$ is reduced to

$$l_{tl}(\omega) \sim \frac{\gamma}{T} \int_{L_c}^{L_\omega} dL \; \nu(L) V^2(L) \bar{u}^2(L) \langle \cosh^{-2}(\Delta/2T) \rangle_d(L). \tag{13}$$

Due to the exponentially fast increase of τ with L, instead of including in Eq. (13) the factor

$$\frac{1}{1 + [\tau(L)\omega]^2} \tag{14}$$

the integration in it is cut off (at the upper limit) at a frequency-dependent length scale L_ω defined by the relation $\tau(L_\omega)\omega \sim 1$. For $U(L) \propto \epsilon(L/L_c)^\psi$

$$L_\omega \propto L_c \left(\frac{T}{\epsilon} \ln \frac{1}{\tau_0|\omega|} \right)^{1/\psi}. \tag{15}$$

The integration interval in Eq. (13) is limited from below by the (temperature dependent) collective pinning length L_c (Ref. 1) which determines the boundary between the different regimes of fluctuations. At length scales lower than L_c the manifold can be considered as fluctuating within one of the minima of the (thermally renormalized) random potential, whereas for larger scales only the jumps between different valleys of the potential are of importance. The contribution to $l(\omega)$ from length scales smaller than L_c has a finite limit for $\omega \to 0$.

Since Δ is the difference in energy between the two spatially separated states in the uncorrelated random potential, the distribution function $p(\Delta)$ can be expected to remain finite for $\Delta \to 0$ (the broad distribution assumption[5]). The last factor in the right-hand side of Eq. (13) (the fraction of "thermally active" two-level systems) can be then estimated[5,6] as

$$\langle \cosh^{-2}(\Delta/2T) \rangle_d(L) \sim \frac{T}{\Delta(L)}, \qquad (16)$$

where $\Delta(L)$ is the typical value of Δ for the length scale L [for example, the width of $p(\Delta)$]. Note that (for any scale) only small fraction of two-level systems is assumed to be not frozen and therefore involved in linear dynamic response (or thermal fluctuations). Thus they are expected to be well separated from each other, which justifies neglecting their interaction.

Substitution of Eq. (16) into Eq. (13) leads to

$$l_{tl}(\omega) \sim \gamma \int_{L_c}^{L_\omega} dL \, \nu(L) \frac{V^2(L)\bar{u}^2(L)}{\Delta(L)}. \qquad (17)$$

According to the universality hypothesis[6] $\Delta(L)$ has to be of the same order of magnitude as the elastic contribution to energy estimated in Eq. (7), which for $\bar{u} \sim u$ gives

$$\frac{V^2(L)\bar{u}^2(L)}{\Delta(L)} \sim V(L) \frac{L^2}{J}. \qquad (18)$$

Substitution of Eqs. (9) and (18) into Eq. (17) then leads to

$$l_{tl}(\omega) \propto \frac{\gamma}{J} \int_{L_c}^{L_\omega} \frac{dL}{L} L^2 \propto \frac{\gamma}{J} L_c^2 \left(\frac{T}{\epsilon} \ln \frac{1}{\tau_0|\omega|} \right)^y, \qquad (19)$$

where $y = 2/\psi$.

With the same assumptions that have been used for the derivation of Eq. (19) the two-level system contribution to the resistivity is given by

$$\rho_{tl}(\omega) \propto \frac{\gamma}{J} \int_{L_c}^{\infty} dL \, L \frac{\tau(L)\omega^2}{1 + [\tau(L)\omega]^2}. \qquad (20)$$

Alternatively $\rho_{tl}(\omega)$ can be restored from $l_{tl}(\omega)$ with the help of the simplified form[16,17] of the Kramers-Kronig relation

$$\rho_{tl}(\omega) \approx -|\omega| \frac{\pi}{2} \frac{d}{d \ln|\omega|} l_{tl}(\omega) \qquad (21)$$

which is applicable for $l_{tl}(\omega) = f(\ln|\omega|)$. Both methods give

$$\rho_{tl}(\omega) \propto \frac{\gamma}{J} \left(\frac{T}{\epsilon} \right)^y |\omega| \left(\ln \frac{1}{\tau_0|\omega|} \right)^{y-1}. \qquad (22)$$

It can be shown that y is equal to $2/\psi$ not only for the simplest case of a single elastic modulus, but for the general case of nondispersive moduli. In a bulk superconductor at large enough scales (which corresponds to low enough frequencies) all elastic moduli become local. The case of a thin

film in which the strong dispersion of the compressibility modulus is unavoidable is considered in Sec. VI.

For finite values of the current density j Eq. (10), which has been the starting point of our calculation, is applicable only if $V\bar{u}B/c$ is small in comparison with temperature. This determines the current dependence of the length scale $L_j \propto (T/j)^{1/(D+\zeta)}$ at which the integration in Eq. (13) should be cut off if $L_j \ll L_\omega$. In that case the growth of $l_{tl}(\omega)$ with decreasing ω saturates at $l_{tl}(\omega = 0, j) \propto (T/j)^{2/(D+\zeta)}$.

V. COMPARISON OF TWO APPROACHES TO CALCULATION OF THERMAL FLUCTUATIONS AMPLITUDE

In the present work we suggest an independent way to check the validity of the two-level system approach for the description of the linear dynamic response of a collectively pinned manifold. This can be done because in the random manifold problem the static irreducible correlation function

$$\langle\langle u^a(\mathbf{x}_1) u^b(\mathbf{x}_2)\rangle\rangle \equiv \langle\langle [u^a(\mathbf{x}_1) - \langle u^a(\mathbf{x}_1)\rangle_{th}] \\ \times [u^b(\mathbf{x}_2) - \langle u^b(\mathbf{x}_2)\rangle_{th}]\rangle_{th}\rangle_d \quad (23)$$

(which can be associated with thermal fluctuations) according to Shultz *et al.*[7] should be exactly the same as in the absence of disorder

$$\langle\langle u^a(\mathbf{x}_1) u^b(\mathbf{x}_2)\rangle\rangle = T G_0^{ab}(\mathbf{x}_1 - \mathbf{x}_2). \qquad (24)$$

(a brief derivation can be found in the Appendix). An analogous relation for the case of periodic behavior of $w(\mathbf{u})$ with respect to displacement has been suggested by Dotsenko and Feigel'man.[18]

In the presence of disorder the long-distance behavior of the correlation function (23) has to be mediated by the two-level systems. Therefore, the investigation of thermal fluctuations in terms of the two-level system approach and comparison of the result with well known result for the pure system allows us to check the validity of different conjectures involved in the calculation of a linear dynamic response. Instead of considering the dependence of $\langle\langle u^a(\mathbf{x}_1) u^b(\mathbf{x}_2)\rangle\rangle$ on $|\mathbf{x}_1 - \mathbf{x}_2|$ one can alternatively investigate the dependence of $\langle\langle u^2 \rangle\rangle$ on the size of the system L_0, which also has to be the same as in the pure case.

For a single two-level system with the energy gap Δ the amplitude of the thermal fluctuations of the displacement is given by

$$\langle\langle u^2 \rangle\rangle \equiv \langle (u - \langle u \rangle_{th})^2 \rangle_{th} = \frac{(u_1 - u_2)^2}{4 \cosh^2(\Delta/2T)}. \qquad (25)$$

For a collectively pinned manifold the dominant large-scale contribution to $\langle\langle u^2(\mathbf{x})\rangle\rangle$ should come from the two-level systems which include the point $\mathbf{x}$ and [on the same assumptions as have been used while calculating $l_{tl}(\omega)$ and $\rho_{tl}(\omega)$] can be estimated as

$$\langle\langle u_{tl}^2 \rangle\rangle \sim T \int_{L_c}^{L_0} dL \, \nu(L) \frac{V(L)u^2(L)}{\Delta(L)}, \qquad (26)$$

where the upper limit of integration is now imposed by the size of the system and [in accordance with Eq. (16)] $\langle \cosh^{-2}(\Delta/2T) \rangle_d$ has been already replaced with $T/\Delta(L)$.

Substitution of Eqs. (7) and (9) into Eq. (26) then gives

$$\langle\langle u_{tl}^2 \rangle\rangle \propto \frac{T}{J} \int_{L_c}^{L_0} \frac{dL}{L^{D-1}} \tag{27}$$

which, if compared with the trivial result for the case without disorder (for which $\langle\langle u^2\rangle\rangle \equiv \langle u^2\rangle_{th}$):

$$\langle\langle u^2\rangle\rangle \approx \frac{T}{J} \int_{|q^\alpha| > \pi/L_0} \frac{d^D\mathbf{q}}{(2\pi)^D} \frac{1}{q^2} \tag{28}$$

reproduces all its important features. Namely, the right-hand side of Eq. (27) (i) contains the correct prefactor T/J, (ii) demonstrates the correct dependence on the size of the system

$$\langle\langle u^2\rangle\rangle \propto \begin{cases} L_0^{2-D}, & D<2, \\ \ln(L_0), & D=2, \\ L_c^{2-D} - L_0^{2-D}, & D>2, \end{cases} \tag{29}$$

and (iii) the system-size dependent contribution to $\langle\langle u_{tl}^2\rangle\rangle$ does not depend on the unknown disorder-related parameters L_c and ϵ.

This allows us to conclude that different assumptions which have been used while calculating $l_{tl}(\omega)$, $\rho_{tl}(\omega)$, and $\langle\langle u_{tl}^2\rangle\rangle$ [the universality hypothesis, the hierarchical distribution of two-level systems, the broad distribution assumption for $p(\Delta)$] were indeed chosen in a reasonable way.

VI. THIN SUPERCONDUCTING FILM IN PERPENDICULAR MAGNETIC FIELD

The long-range interaction of vortices makes the compressibility modulus c_{11} of a bulk superconductor strongly nonlocal for the wavelengths smaller than the magnetic field penetration depth λ. In a thin superconducting film the penetration depth Λ is strongly increased in comparison with that of a bulk superconductor: $\Lambda = 2\lambda^2/d$,[19] where $d \ll \lambda$ is the thickness of the film. Therefore, in a thin film the dependence $c_{11}(q) \approx \bar{c}_{11}/q^2$ (where $\bar{c}_{11} \approx B^2/2\pi\Lambda \propto c_{66}a^{-2}$) resulting from a nonscreened vortex-vortex interaction holds in a much wider range of length scales than in a bulk superconductor. Here $c_{66} \approx \Phi_0 B/32\pi^2\Lambda$ is the shear modulus of the film (which in contrast to c_{11} is always local), $\Phi_0 = hc/2e$ is the flux quantum and $a^{-2} = B/\Phi_0$ is the vortex density.

If one tries to shift a vortex bundle in such a system (in search of the next potential minimum), it turns out that the optimal shape of the bundle is strongly anisotropic[11-14] with the size in the direction of the displacement L much larger than the size in the perpendicular direction $L_\perp$. The optimal relation between L and $L_\perp$ can be found by minimizing the total elastic energy for the given area of a bundle $S \propto LL_\perp$. Minimization (for fixed S) of $E_{com} + E_{sh}$, where[14,20]

$$E_{com} \sim \bar{c}_{11} S^2 \left(\frac{\bar{u}}{L}\right)^2 \tag{30}$$

and

$$E_{sh} \sim c_{66} S \left(\frac{u}{L_\perp}\right)^2 \tag{31}$$

or a simple comparison of E_{com} with E_{sh} for $\bar{u} \sim u$ gives

$$L \propto \frac{L_\perp^3}{a^2} \gg L_\perp. \tag{32}$$

Note, however, that when the energy scale defined by Eq. (31) is used as an estimate for Δ, the factor Su^2/Δ in the 2D version of Eq. (26) is reduced to

$$\frac{Su^2}{\Delta} \sim \frac{L_\perp^2}{c_{66}} \tag{33}$$

for arbitrary relation between L and $L_\perp$.

The calculation of Sec. V has confirmed that the form of the size distribution of two-level systems can be correctly estimated by assuming that they do not overlap with each other, but can be situated inside each other forming hierarchical structures. Such estimate is consistent with the conjecture that the number of metastable states to which a particular domain of a manifold can tunnel (without moving a much larger part of the manifold) is always of the order of 1.[6] In what follows we assume that the same property holds also in presence of dispersion.

The most optimistic estimate for $\nu(L)$ can be then obtained by assuming that for all scales the strongly anisotropic two-level systems are arranged in the most advantageous way to cover all the area available, which corresponds to

$$dL\,\nu(L) \Rightarrow \frac{dL}{L} \frac{1}{LL_\perp}. \tag{34}$$

The more realistic estimate should probably take into account that independent anisotropic two-level systems are likely to have uncorrelated orientations, so the requirement of nonoverlapping will lead to $\nu(L) \propto L^{-3}$.

After substitution of Eq. (33) into the integral [of the form (26)] defining $\langle\langle u_{tl}^2\rangle\rangle$ one obtains that for $L \propto L_\perp^3$ it is convergent at the upper limit

$$\langle\langle u_{tl}^2\rangle\rangle \propto \frac{T}{c_{66}} \int_{L_c}^{\infty} \frac{dL}{L}\left(\frac{L_\perp}{L}\right) < \infty \tag{35}$$

even for the optimistic form of $\nu(L)$ given by Eq. (34). On the other hand, we know that in a pure system the contribution of the transverse modes (which depends only on the shear modulus which is local) leads to the logarithmic divergence of $\langle\langle u^2\rangle\rangle \equiv \langle u^2\rangle_{th}$. It follows from the results of Schulz et al.[7] that in the presence of disorder the same behavior should be mediated by the large-scale two-level systems.

A plausible way to explain the logarithmic divergence of $\langle\langle u_{tl}^2\rangle\rangle$ consists in assuming that the film should contain not only anisotropic two-level systems with $L \gg L_\perp$, but also a hierarchical sequence of quasi-isotropic two-level systems in which $L_\perp$ is of the same order as L. For such two-level

systems the requirement of the balance between the different contributions to the elastic energy ($E_{com} \sim E_{sh}$) leads to $\bar{u} \propto (a/L)u \ll u$, which means that the displacement of the vortices in quasi-isotropic bundles is mostly of rotational type.

The linear dynamic response has to be associated with the same degrees of freedom as are taken into account in the calculation of the static thermal fluctuations. However, $z_{tl}(\omega)$ cannot be obtained by direct application of the fluctuation-dissipation theorem to $\langle\langle u_{tl}^2 \rangle\rangle$, since these two quantities include different linear combinations of the degrees of freedom involved. Nonetheless, when calculating $z_{tl}(\omega)$ one should take into account the same set of two-level systems as for the calculation of $\langle\langle u_{tl}^2 \rangle\rangle$, in contrast to the case of the nonlinear creep for which the shape of the moving vortex bundles is imposed by the applied current.[12–14]

Application of the expression (30) for E_{com} as an estimate for Δ shows that the factor $S^2\bar{u}^2/\Delta$ in the 2D version of Eq. (17) does not depend on $L_\perp$ and can be estimated as $L^2/\bar{c}_{11} \sim L^2 \Lambda/B^2$. For the hierarchical sequence of quasi-isotropic two-level systems with $\nu(L) \propto L^{-3}$ this leads to the extremely weak (double logarithmic) frequency dependence of

$$l_{tl}(\omega) \propto \frac{\gamma\Lambda}{B^2} \int_{L_c}^{L_\omega} \frac{dL}{L} \frac{\Lambda}{c^2} \propto \frac{\Lambda}{c^2} \ln\left(\frac{T}{\epsilon} \ln\frac{1}{\tau_0|\omega|}\right), \quad (36)$$

which can hardly be expected to be resolvable from the background superfluid contribution l_0 in the experiments probing the low-frequency response of thin films.

However, substitution of Eq. (36) into Eq. (21) gives

$$\rho_{tl}(\omega) \propto \frac{\Lambda}{c^2} \frac{|\omega|}{\ln(1/\tau_0|\omega|)} \quad (37)$$

which, in contrast to Eq. (36), does not exhibit any specially weak dependence on ω. Note that two unknown disorder-related parameters L_c and ϵ as well as the magnetic field dependence have dropped out from Eq. (37).

The presence of the hierarchical sequence of quasi-isotropic two-level systems still leaves enough place for more optimal anisotropic two-level systems with $L_\perp \ll L$. Although they do not contribute much to $\langle\langle u_{tl}^2 \rangle\rangle$, their contributions to the components of $z_{tl}(\omega)$ could be of importance. However, their size distribution will be forced by the presence of hierarchical sequence of quasi-isotropic two-level systems to be of the same form $\nu(L) \propto L^{-3}$ and, therefore, their contribution to $l_{tl}(\omega)$ and $\rho_{tl}(\omega)$ will be of the same form as given by Eqs. (36), (37).

In a thin superconducting film the compressibility modulus c_{11} is nonlocal not only for $\Lambda q \gg 1$ where $c_{11}(q) \approx \bar{c}_{11}/q^2$, but also for $\Lambda q \ll 1$ where $c_{11}(q) \approx B^2/2\pi q$. An analogous calculation for such form of $c_{11}(q)$ produces for the components of $z_{tl}(\omega)$ the answers of the form (1) with $y = 1/\psi$.

VII. CONCLUSION

In the present work we argue that thermal fluctuations of a collectively pinned vortex manifold are determined by the same degrees of freedom (related to thermally activated tunneling between the pairs of low-lying metastable states—two-level systems) as its low-frequency linear dynamic response. Therefore one can use the known dependence of thermal fluctuations amplitude on the size of the system (which has to be exactly the same as in absence of disorder[7]) for checking the consistency of the conjectures which are used in the calculation of the linear dynamic response. The set of the assumptions which are necessary to produce the correct answer for the thermal fluctuations amplitude includes, in particular, the conjecture on hierarchical distribution of two-level systems (which means that they can be situated inside each other) and also the universality hypothesis. If the same set of assumptions is used for the calculation of a vortex manifold contribution to impedance its frequency dependence (in absence of dispersion) is given by Eqs. (1) with $y = 2/\psi$, where ψ is the exponent (which depends both on D and N) describing the scale dependence of the typical energy barrier $U(L)$.

The same result is also applicable in the limit of small fields, when one can neglect the interaction between different vortices and treat each vortex separately as 1D manifold. In that case one should take the value of ψ corresponding to $D = 1$.

Note that in the framework of our analysis the universality hypothesis has been used only for the estimate of $\Delta(L)$. If (as suggested by Ioffe and Vinokur[6]) it is further assumed that the same energy scale can be used for the estimate of $U(L)$, the value of ψ will coinside with χ given by Eq. (8). Different approaches including scaling arguments,[11,21] functional renormalization group[22,23] and a self-consistent calculation incorporating replica symmetry breaking,[24] lead to

$$\zeta = \frac{4-D}{4+\beta N} \quad (38)$$

with $1/2 \leq \beta \leq 1$. For the case of thin superconducting film ($D=2, N=2$) or bulk superconductor ($D=3, N=2$) Eqs. (8) and (38) give the values of χ in the interval from 2/3 to 7/5, that is around 1.

Koshelev and Vinokur[4] have found the value of the exponent y only for the particular values of ζ and χ corresponding to $D=1$ and $N=2$ and not in the general form as above. In Ref. 3 the hierarchical nature of the size distribution (of the two-level systems) has not been taken into account and the estimate of $\rho_{tl}(\omega)$ has been obtained without integrating over the scales, which has led (in our notation) to $y = 2/\psi + 1$. As has been already mentioned in the Introduction the analogous calculation in Ref. 2 has been performed using the assumption which is in contradiction with Eq. (10) and therefore cannot be used for comparison.

Note that $l_{tl}(\omega)$ diverges in the limit of $\omega \to 0$, which corresponds to supression of superfluid density [inversely proportional to $\lim_{\omega\to 0} l(\omega)$]. Thus the results of this work are not in agreement with the popular point of view that

random manifold approach provides an exhaustive description of dynamic properties of a truly superconducting vortex glass phase (which is supposed to be formed at low temperatures due to pinning[10,11]), at least if superconductivity is understood as the ability to carry a superconducting (nondissipative) current and not only as the vanishing of the linear resistance. Our analysis suggests that in the framework of random manifold approach the finite value of $l_{tl}(\omega \to 0)$ is incompatible with the correct scale dependence of $\langle\langle u_{tl}^2\rangle\rangle$. Therefore one has to conclude that the accurate description of a vortex glass phase which can carry a superconducting current (if such phase exists at all) requires a more sophisticated treatment than the description of vortex medium in terms of an elastic manifold interacting with a random potential. For example, it possibly should take into account that some of the defects of a vortex lattice are generated by a disorder and cannot freely move with a vortex manifold.

However, both in the case of Larkin regime and in the case of dynamically frozen thermally excited defects (frozen vortex liquid regime) the frequency range of the applicability of the random manifold description of a vortex medium in a superconductor is anyway restricted from below. Moreover, the two-level system contribution to impedance $l_{tl}(\omega)$ is only logarithmic in ω and in a practical situation may be negligible in comparison with ''bare'' impedance l_0 down to exponentially low frequencies.

On the other hand, $\rho_{tl}(\omega)$ produces in the low frequency limit the dominant contribution to the resistivity in comparison with the contributions related with the normal channel conductance and with the oscillations of manifold within each minimum of a random potential, both of which at low frequencies are proportional to ω^2.

In thin superconducting films the compressibility modulus of vortex manifold is always nonlocal. This leads to the strong anisotropy of the vortex bundles participating in the nonlinear creep.[11–14] However, our analysis has shown that the correct length-scale dependence of thermal fluctuations amplitude requires the presence of a hierarchical sequence of quasi-isotropic two-level systems. A linear dynamic response (which has to be calculated assuming that the applied current does not change the properties of the system) produced by the same set of the two-level systems corresponds to $y = 1/\psi$ for $c_{11}(q) \propto 1/q$ ($\Lambda q \ll 1$) and to $y = 0$ [with $l_{tl}(\omega)$ still diverging at $\omega \to 0$ but only as a double logarithm] for $c_{11}(q) \propto 1/q^2$ ($\Lambda q \gg 1$). The contribution from the more optimal anisotropic two-level systems can be expected to be of the same form. In both regimes [$c_{11}(q) \propto 1/q$ and $c_{11}(q) \propto 1/q^2$] the value of the magnetic field B drops out from the expression for $z_{tl}(\omega)$.

Although in thin films the dc resistivity is always finite due to thermally activated motion of pointlike defects of vortex lattice (vacancies, interstitials, dislocation pairs),[12,13] the collective pinning behavior has been found to be accessible to experimental observation[9] in the range of frequencies and temperatures where the activated contribution to resistivity is too small.

APPENDIX

In the case of a random manifold described by Eqs. (2)–(4) it is possible to show that the irreducible correlation function of the form (23) remains the same as in absence of disorder.[7] To this end one can express this correlation function through the second derivative

$$\langle\langle u^a(\mathbf{x}_1) u^b(\mathbf{x}_2)\rangle\rangle = T \left.\frac{\partial^2 \widetilde{F}}{\partial s_1^a \partial s_2^b}\right|_{s_1^a = s_2^a = 0} \tag{A1}$$

of the (disorder-averaged) free energy

$$\widetilde{F} = -T \left\langle \ln\left[\int d\mathbf{u}\, \exp\left(-\frac{\widetilde{H}}{T}\right) \right] \right\rangle_d \tag{A2}$$

with respect to the coefficients in the auxilary (source) terms added to the Hamiltonian

$$\widetilde{H} = H + s_1^a u^a(\mathbf{x}_1) - s_2^a u^a(\mathbf{x}_2). \tag{A3}$$

In order to calculate $\widetilde{F}$ it is convenient to shift the variables $\mathbf{u}(\mathbf{x})$ (over which the integration in the partition function is performed) according to

$$u^a(\mathbf{x}) \Rightarrow u_*^a(\mathbf{x}) \equiv u^a(\mathbf{x}) + G_0^{ab}(\mathbf{x}-\mathbf{x}_1) s_1^b - G_0^{ab}(\mathbf{x}-\mathbf{x}_2) s_2^b, \tag{A4}$$

which allows us to split the nonrandom contribution to $\widetilde{H}\{u^a\}$ into two terms $H_{\text{el}}\{u_*^a\} + E(s_1^a, s_2^a)$, the first of which does not depend on $s_{1,2}^a$ and the second

$$E(s_1^a, s_2^a) = -\frac{1}{2} G_0^{ab}(0) s_1^a s_1^b + G_0^{ab}(\mathbf{x}_1-\mathbf{x}_2) s_1^a s_2^b - \frac{1}{2} G_0^{ab}(0) s_2^a s_2^b \tag{A5}$$

does not depend on $u_*^a(\mathbf{x})$.

On the other hand, the distribution function of the random potential $v(\mathbf{x}, \mathbf{u})$ is not affected by the shift defined by Eq. (A4). Therefore the free energy defined by Eq. (A2) differs from its value for original problem (that is for $s_1^a = s_2^a = 0$) only by addition of the term $E(s_1^a, s_2^a)$, differentiation of which leads to Eq. (24).

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