

Phase fluctuations, dissipation, and superfluid stiffness in d -wave superconductors

L. Benfatto,¹ S. Caprara,¹ C. Castellani,¹ A. Paramekanti,² and M. Randeria²

¹*Università di Roma "La Sapienza" and Istituto Nazionale per la Fisica della Materia, Unità di Roma 1, Piazzale Aldo Moro, 5, 00185 Roma, Italy*

²*Tata Institute of Fundamental Research, Mumbai 400005, India*

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We study the effect of dissipation on quantum- and thermal-phase fluctuations in d -wave superconductors. Dissipation, arising from a nonzero low-frequency optical conductivity that has been measured in experiments below T_c , has two effects: (1) a reduction of zero-point phase fluctuations, and (2) a reduction of the temperature at which one crosses over to classical thermal fluctuations. For parameter values relevant to the cuprates, we show that the crossover temperature is still too large for classical phase fluctuations to play a significant role at low temperature. Quasiparticles are thus crucial in determining the linear temperature dependence of the in-plane superfluid stiffness. Thermal phase fluctuations become important at higher temperatures and play a role near T_c .

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I. INTRODUCTION

There is considerable experimental evidence for a linear temperature dependence of the superfluid density at low temperatures in high- T_c superconducting cuprates, i.e.,

$$\rho_s(T) = \rho_s(0) - \alpha T \quad (1)$$

where α is a weakly doping-dependent constant.^{1,2} However, there is still some controversy regarding the low-energy excitations responsible for this thermal suppression of ρ_s . The simplest explanation is in terms of quasiparticle excitations near the d -wave nodes.³⁻⁶ An alternative explanation is in terms of thermal fluctuations of the phase of the order parameter⁷⁻¹⁰ or other collective modes.¹¹

The existence of well-defined quasiparticles in the superconducting state of cuprates is supported both by transport¹²⁻¹⁴ and angle-resolved photoemission spectroscopy (ARPES)¹⁵ experiments (even though there are some studies questioning their Fermi liquid description^{16,17}). However, the contributions of phase fluctuations to low-temperature properties could still be important, especially in the underdoped regime where the superfluid density ρ_s becomes vanishingly small as the Mott insulator is approached. In the literature the role of quantum- and thermal-phase fluctuations has been addressed in connection with various experimental problems, ranging from c -axis transport¹⁸ to the effect of these fluctuations on quasiparticle properties in the pseudogap state.¹⁹⁻²¹

In this paper we study in particular the role of phase fluctuations in the depletion of the superfluid density. Within this context two important issues have to be addressed: (1) The form of the phase-only action for layered d -wave superconductors (SC's) taking into account the long-range Coulomb interaction; (2) the crossover between quantum and classical regimes of phase fluctuations. These questions were studied in detail in Ref. 22 that, however, did not discuss the role of dissipation. In this paper, we focus on dissipative effects and how they affect the form of the phase-only action and the quantum-to-classical crossover.

There are several reasons to believe that low-energy dissipation is important in the high- T_c cuprates. Theoretically, weak disorder within a self-consistent T -matrix calculation leads to a nonzero "universal" low-frequency quasiparticle conductivity²³ in d -wave SC's. Experiments have also measured a nonzero low-frequency conductivity,^{24,17} much larger than the "universal" value. While there is no consensus on the origin of this large conductivity, one would definitely expect this dissipation to affect the phase fluctuations in the system, as first emphasized by Emery and Kivelson (EK).⁹ Our formalism and results, however, differ from those of EK as discussed in detail in the paper.

We summarize our main results below.

(1) We derive the Gaussian-effective action for phase fluctuations in the presence of dissipation using a functional integral approach and integrating out fermionic degrees of freedom. While our effective action is derived microscopically by looking at fluctuations around a BCS mean field solution, we make contact with experiment by using parameter values relevant to the high- T_c SC's. We believe this phenomenological approach of using the derived *form* for the action, with *coefficients* taken from experiment, is valid for the SC state of the high- T_c materials, at least for $T \ll T_c$, when quasiparticles are well defined. In addition, we also present in an appendix, a hydrodynamic derivation for the phase mode based on a two-fluid model, which serves as a check on the microscopic derivation.

(2) A dissipative quantum XY model is obtained by coarse graining the Gaussian action to the scale of the coherence length and analyzed within a self-consistent harmonic approximation.

(3) We find that the magnitude of quantum fluctuations at $T=0$ is reduced by the presence of dissipation. This outcome is in agreement with similar results obtained within models of granular superconductors coupled to a dissipative Ohmic bath,^{25,26} as we shall discuss below.

(4) Dissipative phase fluctuations alone, in the absence of quasiparticle excitations, are shown to lead to a T^2 reduction⁵ of the superfluid stiffness. This behavior crosses

over to a classical linear T reduction at a scale T_{cl} , which decreases with increasing dissipation.

(5) Choosing parameters appropriate to the high- T_c cuprates, and overestimating the dissipation, we nevertheless find that the crossover scale T_{cl} is still fairly large. Thus one cannot attribute the low-temperature linear reduction of $\rho_s(T)$ to classical phase fluctuations. This T dependence must therefore arise entirely from quasiparticle excitations near the d -wave nodes.

II. EFFECTIVE PHASE ACTION

We find it convenient to express the superfluid density ρ_s in terms of a stiffness $D_s = \hbar^2 \rho_s / m^*$, which in the London limit is related to the penetration depth λ in a three-dimensional (3D) bulk system through $1/\lambda^2 = 4\pi e^2 D_s / \hbar^2 c^2$. The in-plane superfluid stiffness in a layered system, with interlayer spacing d_c , is $d_c D_s$ with dimension of energy. Henceforth, unless explicitly displayed, we set $\hbar = k_B = 1$.

We begin with the Gaussian-phase action for a 3D isotropic SC,

$$S_G[\theta] = \frac{a^3}{8T} \sum_{\mathbf{q}, \omega_n} [\omega_n^2 \chi + D(i\omega_n) \mathbf{q}^2] \theta(\mathbf{q}, i\omega_n) \theta(-\mathbf{q}, -i\omega_n). \quad (2)$$

For a derivation in the s -wave case see Refs. 27 and 28 and for the d -wave case see Ref. 22. We show in Appendix A that the above action, and its generalization to layered systems, can be also derived from hydrodynamic considerations within a two-fluid model. In the above action (2), the compressibility $\chi(\mathbf{q} \rightarrow 0) \approx 1/V_{\mathbf{q}}$ where $V_{\mathbf{q}}$ is the Coulomb interaction and a is the lattice spacing. On continuing to real frequency $D(\omega)$ is the mean-field stiffness, which is related to the mean-field complex conductivity $\sigma(\omega)$ through $D(\omega) = [-i\omega\sigma(\omega)/e^2]$. In arriving at the above action, we have assumed that $\sigma(\mathbf{q}, \omega) \approx \sigma(0, \omega)$, and ignored the \mathbf{q} dependence of the conductivity for $\mathbf{q} \leq \pi/\xi_0$, which we expect to be small for frequencies of interest.²⁹

We use the spectral representation for σ , and find that

$$D(i\omega_n) = D_s^0 + \frac{1}{e^2} \int_0^\infty \frac{d\omega}{\pi} \frac{2\omega_n^2}{(\omega^2 + \omega_n^2)} \text{Re} \sigma_{reg}(\omega), \quad (3)$$

where we have used $\text{Re} \sigma(\omega) = \pi D_s^0 e^2 \delta(\omega) + \text{Re} \sigma_{reg}(\omega)$. For a frequency-independent $\text{Re} \sigma_{reg}(\omega) = \sigma_{DC}$, this simplifies to

$$D(i\omega_n) = D_s^0 + \frac{\sigma_{DC}}{e^2} |\omega_n|. \quad (4)$$

Unless indicated otherwise, we will use this simplified form of the conductivity below, and use $\bar{\sigma} = \sigma_{DC} d_c / (e^2/h)$ as a dimensionless measure of the dissipation.³⁰

It is straightforward to generalize the above results to a layered system with an in-plane stiffness D_{\parallel}^0 and a c -axis

stiffness D_{\perp}^0 . Further, the Coulomb interaction in a system with layer spacing d_c gets modified to³¹

$$V(\mathbf{q}) = \frac{2\pi e^2 d_c}{q_{\parallel} \epsilon_{\infty}} \left[\frac{\sinh(q_{\parallel} d_c)}{\cosh(q_{\parallel} d_c) - \cos(q_{\perp} d_c)} \right] \quad (5)$$

where q_{\parallel}, q_{\perp} are the in-plane and c -axis components of \mathbf{q} , respectively.

To investigate the contribution of the phase fluctuations to the depletion of superfluid density it is necessary to go beyond the Gaussian approximation. The simplest model that allows for such an analysis is the quantum XY model, in which the phase field is defined on a coarse-grained lattice, with an in-plane lattice constant of ξ_0 and layer spacing d_c . The coherence length enters as a short distance cutoff since the mean-field assumption of a constant amplitude breaks down at shorter distances.

Following exactly the same procedure of coarse-graining used in Ref. 22 (for the non-dissipative case) we now obtain the dissipative quantum XY action:

$$\begin{aligned} S_{XY}[\theta] = & \frac{1}{8T} \sum'_{\mathbf{Q}, \omega_n} \left(\frac{\omega_n^2 \xi_0^2 d_c}{\tilde{V}(\mathbf{Q})} + \frac{\bar{\sigma}}{2\pi} \left| \omega_n \right| \gamma_{\parallel}(\mathbf{Q}) \right) |\theta(\mathbf{Q}, \omega_n)|^2 \\ & + \frac{D_{\parallel}^0 d_c}{4} \int_0^{1/T} d\tau \sum_{\mathbf{R}, \alpha=x,y} \{1 - \cos[\theta(\mathbf{R}, \tau) \\ & - \theta(\mathbf{R} + \alpha, \tau)]\} + \frac{D_{\perp}^0 d_c}{4} \left(\frac{\xi_0}{d_c} \right)^2 \int_0^{1/T} d\tau \\ & \times \sum_{\mathbf{R}} \{1 - \cos[\theta(\mathbf{R}, \tau) - \theta(\mathbf{R} + \hat{\mathbf{z}}, \tau)]\}. \quad (6) \end{aligned}$$

Here $\gamma_{\parallel}(\mathbf{Q}) = (4 - 2 \cos Q_x - 2 \cos Q_y)$ with \mathbf{Q} being the dimensionless momentum, and the scaled interaction³² $\tilde{V}(\mathbf{Q}) \equiv V(\mathbf{Q}_{\parallel}/\xi_0, \mathbf{Q}_{\perp}/d_c)$. While all momenta with $|Q_x|, |Q_y|, |Q_z| \leq \pi$ contribute in Eq. (6) above, the prime on the summation denotes a Matsubara frequency cutoff discussed below (see also Ref. 22).

In this derivation we have promoted the gradient terms in the Gaussian action arising from the superfluid stiffness to the cosine form, while the dissipative terms have still been retained at Gaussian level. A more sophisticated approach would probably end up with a τ nonlocal kernel within the cosine term; we will however continue to work with the simplest action above. This action (6) is well known in the literature as the resistively shunted Josephson junction (RSJJ) model and its phases and quantum-phase transitions have been extensively studied.^{25,26} Here we are interested in the effect of dissipation on quantum-phase fluctuations and the classical crossover temperature, in the superconducting state.

We now discuss the differences between our action (6) and that considered by EK.⁹ EK included the effects of screening by replacing the $V(\mathbf{Q})$ appearing in Eq. (2) with the screened interaction $V_s(\mathbf{Q}) = V(\mathbf{Q})/\epsilon(\omega)$. Here $\epsilon(\omega) = 1 + 4\pi i \sigma_L(\omega)/\omega$ is the dielectric function at $\mathbf{Q}=0$, and $\sigma_L(\omega)$ is the longitudinal optical conductivity. Considering

the isotropic 3D Coulomb interaction, $4\pi e^2/\epsilon_\infty \mathbf{Q}^2$, the dynamical term in the EK analysis reduces to the form $\omega_n^2 \epsilon_\infty \epsilon(\omega) \mathbf{Q}^2/4\pi e^2$. This expression has been shown to be correct^{33,28} when the screening in the superconductor is produced by some external degrees of freedom with conductivity $\sigma_L(\omega)$. An example is provided by a coupled system consisting of a superconductor interacting via Coulomb interactions with a normal metal.

However, as discussed in Ref. 22, the EK effective action is *not* obtained for a single homogeneous SC. The longitudinal conductivity σ_L of the SC does not explicitly appear in the expression of the density-density correlation function χ . Instead dissipation appears through the transverse current-current correlation function, and affects the gradient term in the action (2), so that $D_\parallel \rightarrow D_\parallel - i\omega\sigma_T(\omega)/e^2$, where σ_T is the transverse conductivity.

If we would *assume* that physical (i.e., gauge-invariant) correlation functions appear as the coefficients in the phase action, *then*, using the equality of the physical longitudinal and transverse conductivities, it is easy to see that our action (2) is identical to the EK action,⁹ for the specific case of a SC with isotropic 3D Coulomb interactions. In this 3D case one could associate the σ with either the gradient term, as we do, or with the time derivative term as done by EK. The action used by EK is then formally the same as our action, and one could argue that dissipation should appear in the same way whether it is from an external bath (EK) or from internal degrees of freedom (our case).

The above assumption of gauge-invariant coefficients is, however, not valid in general for a single-homogeneous SC where the screening arises from the (low energy) internal degrees of freedom. The coefficients in the phase action are then mean-field correlation functions. The *longitudinal* mean-field correlations are not gauge invariant since the phase variable is yet to be integrated out. It is only upon integrating out the phase variable at Gaussian level that one restores gauge invariance²² for these. By contrast, Gaussian-phase fluctuations do not affect the *transverse* mean-field correlation functions.²² Thus, the optical conductivity, which originates from the mean-field (transverse) current-current correlation function and appears in the phase-only action (6). This will permit us below, to use the experimentally measured conductivity to estimate the dissipation in the phase action.

III. QUANTUM AND CLASSICAL PHASE FLUCTUATIONS

A. Variational analysis

We analyze the quantum XY action within the self-consistent harmonic approximation (SCHA). We believe that this is adequate to calculate the effects of phase fluctuations at low temperatures, where longitudinal (spin-wave) fluctuations dominate and transverse (vortex) excitations are exponentially suppressed given their finite core energies. To examine the low temperature in-plane properties, we assume $D_\perp^0=0$ in Eq. (6) since it is very small in highly anisotropic systems with a large λ_\perp . For parameter values appropriate to

Bi2212, we have numerically checked that setting $D_\perp^0=0$ does not affect our in-plane results.

The SCHA (Refs. 34, 26 and 7) is carried out by replacing the above action by a trial harmonic theory with the renormalized stiffness D_\parallel chosen to minimize the free energy of the trial action. This leads to

$$D_\parallel = D_\parallel^0 \exp(-\langle \delta\theta_\parallel^2 \rangle/2), \quad (7)$$

where $\delta\theta_\parallel \equiv (\theta_{\mathbf{r},\tau} - \theta_{\mathbf{r}+\alpha,\tau})$ with $\alpha=x,y$ and the expectation value evaluated in the renormalized harmonic theory is given by

$$\begin{aligned} \langle \delta\theta_\parallel^2 \rangle &= 2T \int_{-\pi}^{\pi} \frac{d^3 \mathbf{Q}}{(2\pi)^3} \\ &\times \sum_{n=-n_c}^{n_c} \frac{\gamma_\parallel(\mathbf{Q})}{\omega_n^2 \xi_0^2 d_c / \tilde{V}_\mathbf{Q} + \left(D_\parallel d_c + \frac{\bar{\sigma}}{2\pi} |\omega_n| \right) \gamma_\parallel(\mathbf{Q})}. \end{aligned} \quad (8)$$

As mentioned earlier, the dynamical phase distortions should have a frequency cutoff for the simple action we have considered. In our numerics, we use a cutoff²² n_c corresponding to $\omega_n \leq \sqrt{(D_\parallel d_c)(2\pi e^2/\epsilon_\infty \xi_0)}$, but we have checked that the presence of a finite n_c has only a minor quantitative effect on the results for $\langle \delta\theta_\parallel^2 \rangle$ in the presence of dissipation, and one may set $n_c \rightarrow \infty$ to obtain qualitatively correct results.

B. Analytical estimates of quantum and thermal fluctuations

We first present estimates of the magnitude of quantum fluctuations and the thermal crossover scale making certain simplifying assumptions. The in-plane quantum fluctuations are seen to be dominated by relatively large Q_\parallel from phase-space considerations and the form of the integrand in Eq. (8). In this case, we may set $\tilde{V}(\mathbf{Q} \sim 1)/\xi_0^2 d_c \approx 2\pi e^2/\epsilon_\infty \xi_0$. Restricting ourselves to low T , we ignore the Matsubara cutoffs and set $n_c \rightarrow \infty$. With these simplifications, we work in the limiting cases of small and large dissipation. We report further analytical results in Appendix B. In particular, we calculate the renormalization of the superfluid stiffness for an anisotropic 3D Coulomb interaction (instead of the Coulomb interaction in layered systems used in the paper) that permits us to analyze the case of arbitrary $\bar{\sigma}$.

First recall the nondissipative case²² where the problem involves only two energy scales: the Coulomb energy ($2\pi e^2/\epsilon_\infty \xi_0$) and the layer stiffness $D_\parallel d_c$. The quantum zero point fluctuations of the phase are given by the dimensionless combination $\sqrt{(2\pi e^2/\epsilon_\infty \xi_0)/D_\parallel d_c}$, while the crossover to classical fluctuations takes place at a temperature $T_{cl} \sim \sqrt{(D_\parallel d_c)(2\pi e^2/\epsilon_\infty \xi_0)}$. Taking into account the temperature dependence of the bare stiffness, a better estimate of the crossover temperature is $T_{cl} \sim T_c$.

For the dissipative case at $T=0$ we convert the Matsubara sum to an integral. For large $\bar{\sigma}$, ignoring $D_\parallel d_c$ in the

integrand, and introducing a lower-frequency cutoff, $2\pi D_{\parallel} d_c / \bar{\sigma}$, it is easy to show that the magnitude of quantum fluctuations may be estimated as $\langle \delta\theta_{\parallel}^2 \rangle \approx (8/\bar{\sigma}) \ln[(\bar{\sigma}/2\pi) \sqrt{(2\pi e^2/\epsilon_{\infty} \xi_0)/(D_{\parallel} d_c)}]$. This is similar to the result obtained by Chakravarty *et al.*²⁶ for a RSJJ model with short range charging energies. Increasing dissipation thus leads to a decrease in quantum fluctuations as the system becomes more classical.

To evaluate the temperature scale at which one crosses over to classical fluctuations in the presence of dissipation, we have to consider the temperature above which only the $n=0$ Matsubara frequency contributes, so that phase dynamics is unimportant. For large $\bar{\sigma}$, this may be estimated in a simple manner by setting $(\bar{\sigma}/2\pi)|\omega_n| \geq D_{\parallel} d_c$ with $n=1$, which ensures that fluctuations with $n \gg 1$ would contribute very little to the fluctuation integral in Eq. (8). This leads to $T_{cl} \geq D_{\parallel} d_c / \bar{\sigma}$. A better estimate is obtained below, which gives $T_{cl} \approx 3D_{\parallel} d_c / \bar{\sigma}$. It is clear that the classical limit emerges as the limit of infinite dissipation, $\bar{\sigma} \rightarrow \infty$, for which $T_{cl} \rightarrow 0$. The crossover scale we obtain is similar in form to the estimate, $T_{cl} \approx T_c / \bar{\sigma}$, given in Ref. 10, but is much larger in magnitude.

C. Low-temperature behavior

We next turn to the temperature dependence of the renormalized stiffness in the presence of dissipation, where we have set the bare stiffness to be independent of temperature. This is of course an unphysical assumption for a d -wave SC, but our aim is to explicitly check whether a linear T dependence can be obtained within a model of purely dissipative phase fluctuations even when temperatures are smaller than the thermal crossover scale estimated above. The fluctuation $\langle \delta\theta_{\parallel}^2 \rangle$ at low T can be evaluated analytically again by setting the cutoff $n_c \rightarrow \infty$. We can then cast the Matsubara sum in the form

$$\begin{aligned} & \sum_{n=-\infty}^{\infty} \frac{A(\mathbf{Q}, T)}{n^2 + B(\mathbf{Q}, T)|n| + C(\mathbf{Q}, T)} \\ &= 2 \sum_{n=0}^{\infty} \frac{A(\mathbf{Q}, T)}{n^2 + B(\mathbf{Q}, T)n + C(\mathbf{Q}, T)} - \frac{A(\mathbf{Q}, T)}{C(\mathbf{Q}, T)}. \end{aligned} \quad (9)$$

Rewriting the denominator of the first term in the form $(n+n_1)(n+n_2)$, we separate out the terms using partial fractions and express the resulting sums in terms of digamma functions. As $T \rightarrow 0$, $n_{1,2} \rightarrow \infty$ that allows us to use the asymptotic expansion for the digamma function. The linear T term arising from the infinite sum is precisely canceled by the linear T term from the $A(\mathbf{Q}, T)/C(\mathbf{Q}, T)$ term, leaving only a quadratic temperature dependence as was pointed out in Ref. 5. We thus finally arrive at $\langle \delta\theta_{\parallel}^2 \rangle(T) = \langle \delta\theta_{\parallel}^2 \rangle(0) + (\bar{\sigma}/3)(T/D_{\parallel} d_c)^2$ at low T , from which

$$\frac{D_{\parallel}(T)}{D_{\parallel}(0)} \approx 1 - \frac{\bar{\sigma}}{6} \left(\frac{T}{D_{\parallel}(0)d_c} \right)^2. \quad (10)$$

Thus, ignoring the effects of nodal quasiparticles, the asymptotic low-temperature stiffness decreases as T^2 in the presence of dissipation.

At high temperature, above the thermal crossover scale, we recover the classical result $\langle \delta\theta_{\parallel}^2 \rangle(T) \approx 2T/(D_{\parallel} d_c)$. An improved estimate of the thermal crossover scale T_{cl} is obtained by matching the slope of this high-temperature result for $\langle \delta\theta_{\parallel}^2 \rangle$ with the low-temperature result of Eq. (10). This gives us $T_{cl} = 3D_{\parallel} d_c / \bar{\sigma}$ as stated earlier.

At this stage, we turn to the recent results of Lemberger and co-workers,³⁵ who use a circuit analogy and model a Josephson junction as an inductance (L_0) shunted by a resistance (R) and capacitance (C). To make correspondence with this work, we note that the inductance $L_0 \sim (D_{\parallel} d_c)^{-1}$, the charging energy $e^2/2C \sim (e^2/\epsilon_b \xi_0)$ and the resistance $R \sim (1/\bar{\sigma})(h/e^2)$. Up to numerical factors of order unity, our expressions for the magnitude of quantum fluctuations and the thermal crossover scale are then in agreement. The predicted quantum to thermal crossover has also been recently observed in experiments on conventional s-wave superconducting films.³⁶

D. Numerical results

In order to obtain the various scales for the cuprates, we will choose parameters of the above action appropriate for the bilayer system Bi2212 and evaluate the above estimates. We then present results of detailed-numerical calculations that are shown to agree with these simple estimates.

In the absence of detailed information on the bilayer couplings, we make the assumption that the two layers within a bilayer are strongly coupled and phase locked. Experimentally, the in-plane penetration depth of optimally doped Bi2212 is around 2100 Å and this translates into a bilayer stiffness ≈ 75 meV. We use $\epsilon_{\infty} \approx 10$, and $d_c/a \approx 4$, with the in-plane coherence length $\xi_0/a \approx 10$. This leads to a Coulomb scale $(e^2/\epsilon_{\infty} \xi_0) \approx 35$ meV. Using the above parameters, we find large quantum fluctuations in the nondissipative case ($\bar{\sigma}=0$) with $\langle \delta\theta_{\parallel}^2 \rangle \geq 1$. The thermal crossover scale as estimated from the zero-temperature bilayer stiffness, $T_{cl} \gg T_c \sim 100$ K. A more sensible estimate is obtained by considering the temperature dependence of the bare stiffness, and this leads to a crossover scale for $T_{cl} \sim T_c$ for $\bar{\sigma}=0$.

To study the effect of dissipation, we use conductivity data obtained from experiments performed in the superconducting state. Consistent with our assumption of strongly coupled phases within a bilayer for Bi2212, the dissipation parameter $\bar{\sigma}$ for this system will be taken to be the dimensionless *bilayer* conductivity. Recent measurements by Corson *et al.*¹⁷ on Bi2212 films give a Drude conductivity with a large low-frequency value corresponding to $\bar{\sigma} \approx 75$ and a width of a few terahertz. Similar large conductivities have been measured in the microwave regime.²⁴ We note that the

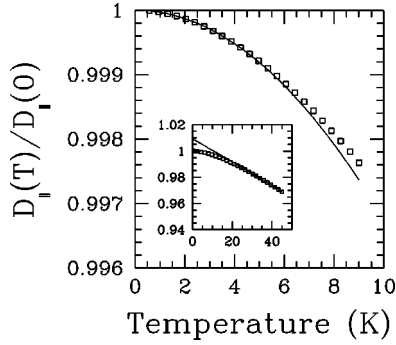


FIG. 1. Low-temperature behavior of the renormalized superfluid stiffness for the dimensionless dissipation $\bar{\sigma}=150$. The bare stiffness was chosen to be $D_{||}^0 d_c=80$ meV, independent of temperature, and the renormalized $D_{||}^0 d_c(T=0)\approx 75$ meV, corresponding to $\langle \delta\theta_{||}^2(T=0) \rangle \approx 0.1$. The values obtained from the numerics are shown as squares while the solid line is the analytical T^2 form given in Eq. (10). Linear temperature dependence sets in above a temperature ~ 20 K as seen from the asymptote in the inset, which compares well with the estimate of the thermal crossover scale $3D_{||}^0 d_c/\bar{\sigma}\approx 18$ K.

“universal” quasiparticle conductivity predicted by Lee²³ for the bilayer conductivity (at $T=0, \omega\rightarrow 0$) corresponds to³⁷ $\bar{\sigma}\approx 24$ for Bi2212. The difference between this “universal” value, and the $\bar{\sigma}\approx 150$ inferred from microwave data²⁴ may be due to vertex corrections.³⁸ In our calculations, we use a constant dissipation with $\bar{\sigma}\approx 150$ (as an *overestimate*) over the entire frequency range of interest: ω_n with $|n|\leq n_c$. This frequency range corresponds to $\omega\leq 100$ meV at $T=0$.

In the presence of dissipation, we find that quantum fluctuations are reduced to a very small value $\langle \delta\theta_{||}^2 \rangle \leq 0.2$. The thermal crossover scale is then $T_{cl}=3D_{||}^0 d_c/\bar{\sigma}\sim 18$ K. This is consistent with our numerics, where we find that linear T behavior from thermal phase fluctuations only sets in above a temperature ~ 20 K, for this magnitude of dissipation. While this is a low-temperature scale, penetration-depth measurements^{39,24} observe a smooth linear T behavior down to much lower temperatures ~ 5 K, which cannot be reconciled with this crossover scale.

Turning to YBCO, and treating this system as weakly coupled single layers, far infrared reflectance measurements⁴⁰ appear to be consistent with $\bar{\sigma}\sim 10-15$ over a wide-frequency range $\sim 5-100$ meV. This smaller value of $\bar{\sigma}$ compared to Bi2212 implies that dissipative effects are less important in YBCO. The “universal” conductivity²³ for this case³⁷ corresponds to $\bar{\sigma}\approx 9$, not inconsistent with the above data. However, low-frequency microwave measurements⁴¹ on YBCO observe a strong frequency and temperature dependent quasiparticle conductivity, of the Drude form. We have checked that using a temperature-dependent Drude conductivity in this very low frequency regime, in addition to a constant dissipation over the entire frequency range, does not significantly affect our results.

In Fig. 1 we compare our analytical results and estimates

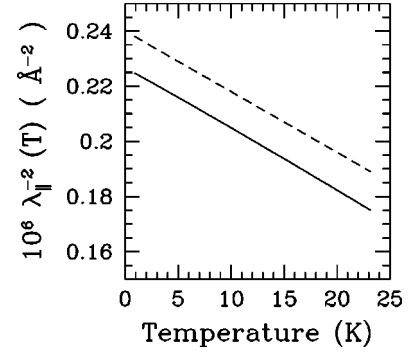


FIG. 2. The bare and renormalized $1/\lambda_{||}^2(T)$ plotted using dashed and solid line, respectively, for a dimensionless dissipation $\bar{\sigma}=150$. The bare value of $\lambda_{||,0}$ and its slope were chosen such that the renormalized values, calculated using the SCHA equations (7) and (8), $\lambda_{||}\approx 2100$ Å and its slope $d\lambda_{||}/dT\approx 10.0$ Å/K, are in agreement with experiments (Ref. 24) on Bi2212. The renormalization due to quantum fluctuations is seen to be $\approx 5\%$, much smaller than the $\approx 50\%$ renormalization obtained in the nondissipative case (Ref. 22).

obtained above, for Bi2212, with a numerical solution of the SCHA equations (7) and (8). We find that the purely quadratic dependence persists up to about 6 K for the parameters discussed above, while linear T dependence only sets in at temperatures ≥ 20 K, consistent with the above estimate for T_{cl} . Thus, it is impossible to ignore quasiparticles for understanding the smooth linear T behavior of the penetration depth that has been observed^{39,24} down to temperatures ~ 5 K.

We next include the linear T effect of quasiparticle excitations in the bare stiffness and ask how dissipative phase fluctuations renormalize this. The numerical result is plotted in Fig. 2 and shows that both the $T=0$ stiffness and its slope are renormalized by small amounts. This is completely consistent with our estimates for a small-quantum renormalization in the presence of dissipation and a temperature scale of about 20 K below which classical thermal effects are unimportant.

E. High-temperature behavior

Although quasiparticles dominate at low temperature eventually phase fluctuations do become important at higher temperatures in driving the transition to the nonsuperconducting state. The approximation used thus far (SCHA) by itself is clearly inadequate to address the problem of T_c since it only includes longitudinal phase fluctuations. We thus proceed in two steps: first we calculate the temperature dependence of the superfluid stiffness within the SCHA, and then we supplement it with the Nelson-Kosterlitz condition for the universal jump in the stiffness at a 2D Kosterlitz-Thouless transition⁴² mediated by the unbinding of vortices. We do not take into account, for simplicity, the effect of layering in this calculation, but this could be easily done using well-known results for dimensional crossover.

Our numerical results, obtained by solving the SCHA equations for parameter values relevant to Bi2212, are plot-

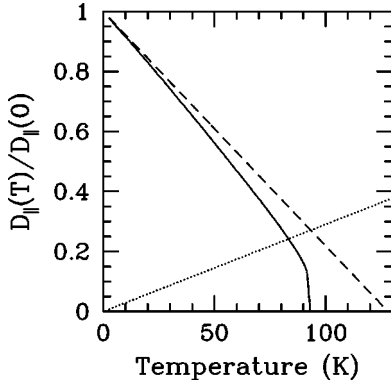


FIG. 3. The bare and renormalized stiffness plotted as a function of temperature, using dashed and solid lines, respectively, to obtain an estimate of the transition temperature T_c in Bi2212. We chose the bare bilayer stiffness $d_c D_{||}^0(T) = d_c D_{||}^0(0) - \alpha^0 T$ with parameters $d_c D_{||}^0(0) \approx 80$ meV and $\alpha^0 \approx 0.7$ meV/K, relevant to bilayer Bi2212. This leads to a renormalized-bilayer stiffness of ≈ 75 meV and a low-temperature slope ≈ 0.7 meV/K for the bilayer stiffness, consistent with low-temperature penetration depth experiments (Ref. 24) in Bi2212. The renormalized stiffness is computed using the SCHA equations (7) and (8) with a dimensionless dissipation $\bar{\sigma} = 150$. The renormalized stiffness (solid line), within the SCHA, shows a jump near $T \sim 90$ K, but that is likely an artifact of the approximation. The temperature at which *transverse excitations* drive the transition to a nonsuperconducting state is estimated from the Nelson-Kosterlitz condition, as the point at which the dotted line intersects the *renormalized stiffness* curve (see text for details). This temperature, $T_{KT} \approx 80$ K, is in reasonable agreement with experimental values (Ref. 24) for $T_c \sim 90$ K in optimal Bi2212.

ted in Fig. 3. We take the bare stiffness (the dashed line in Fig. 3) to decrease linearly with temperature, consistent with experiments, as a result of quasiparticle excitations. We extend this linear T dependence of the bare stiffness to higher temperatures assuming that the gap function is unaffected for $T < T_c$. This is probably a good assumption in underdoped systems and may not be unreasonable for optimal doping. The renormalized stiffness calculated within SCHA is shown as the full line. This stiffness shows a jump at a temperature ~ 90 K, but that is likely an artifact of the SCHA. The Kosterlitz-Thouless transition occurs at $T_{KT} = \pi D_s(T_{KT}^-) d_c / 8$, where the *fully* renormalized stiffness D_s is evaluated just below T_{KT} . We use the renormalized stiffness within the SCHA and the above condition, to obtain an approximate location of this transition. As can be seen from Fig. 3, this gives us a reasonable estimate of $T_c \approx 80$ K, when compared with experiments^{39,24} on optimal Bi2212 that give $T_c \approx 90$ K.

IV. CONCLUSIONS

In this paper we have derived and analyzed the effective action for phase fluctuations in the presence of dissipation arising from low-frequency absorption in a d -wave superconductor. We find that including dissipation reduces the magnitude of quantum-phase fluctuations. The temperature at which one crosses over to thermal phase fluctuation is also

reduced drastically. However, for parameter values relevant to the high- T_c cuprates, we find that the thermal crossover scale is still large, so that quasiparticles dominate the asymptotic low-temperature properties. In particular, they must be responsible for linear T dependence of the low-temperature penetration depth.

A similar conclusion was reached in a recent paper that used a phenomenological approach based on the fluctuation-dissipation theorem,⁴³ which however did not contain explicit estimates of the quantum-to-classical crossover temperature. Large quantum phase fluctuations have also been obtained in Ref. 44 using a continuum BCS model with d -wave symmetry and no dissipation. However, their quantitative result for the renormalization of the superfluid stiffness is smaller than our estimate, since they consider a translationally invariant model, in contrast to our dissipative quantum XY model.⁴⁵ It thus appears from various calculations that the low-temperature state of the cuprates may be well described by quasiparticle excitations around a ground state with quantum-phase fluctuations. It would be interesting to look for direct signatures of these quantum fluctuations in recently proposed Josephson junction⁴⁴ and penetration-depth⁴⁶ experiments.

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APPENDIX A: TWO-FLUID HYDRODYNAMICS AND COLLECTIVE MODES AT LOW TEMPERATURE

In this appendix we analyze the finite-temperature properties of the two-fluid model to determine the collective modes of a superconductor in the presence of dissipative processes. We first consider a 3D Galilean invariant system and later generalize the result to a layered system, still maintaining Galilean invariance in the planes. Our goal is to determine the phase action that gives rise to this collective mode, which serves as another way to arrive at the Gaussian action derived microscopically in the text.

The ordinary equations of a superfluid⁴⁷ are altered by the addition of a dissipative contribution. The longitudinal modes arising in the presence of long-range Coulomb forces obey the following set of linearized equations:

$$\omega \delta \rho = \mathbf{q} \cdot \mathbf{j}, \quad (\text{A1})$$

$$\omega \delta s + \frac{s \rho_s}{\rho} \mathbf{q} \cdot (\mathbf{v}_s - \mathbf{v}_n) = 0, \quad (\text{A2})$$

$$\mathbf{j} = \left[i \frac{e \rho_s}{m \omega} + \left(\frac{\sigma_{reg}(\omega)}{e} \right) \right] \mathbf{E} + \frac{\mathbf{q}}{\omega} \delta P, \quad (\text{A3})$$

$$i\omega\mathbf{v}_s - i\mathbf{q}\delta\mu + \frac{e\mathbf{E}}{m} = 0, \quad (\text{A4})$$

$$i\mathbf{q} \cdot \mathbf{E} = \frac{4\pi e}{\epsilon_\infty} \delta\rho. \quad (\text{A5})$$

Here the symbols have their usual meanings: ω, \mathbf{q} are the frequency and wave vector; the subscripts s, n refer to the superfluid or normal component; ρ, \mathbf{j} are the particle density and particle-current density; \mathbf{v} indicates a velocity; \mathbf{E} is the internal longitudinal electric field, $\sigma_{reg}(\omega)$ is the regular part of the complex conductivity; and ϵ_∞ is the background-dielectric constant. Further the thermodynamic variables, the pressure P , the entropy per particle s , and the chemical potential μ , are related by the identity

$$\delta\mu = -s\delta T + \frac{1}{\rho}\delta P. \quad (\text{A6})$$

Note that dissipative processes due to thermal conductivity, which should appear on the right-hand side of Eq. (A3), can be neglected at low temperature and anyway, affect only second sound, which is decoupled from the density mode, as we shall see below. Moreover, the Lorentz force and the Joule heating are second order in the fluctuations and do not affect the linearized equations that we are investigating.

The equations for first and second sound, i.e., for density and entropy fluctuations, respectively, are in principle coupled via the pressure variations

$$\begin{aligned} \omega^2 \delta\rho &= \left[\frac{4\pi e^2 \rho_s}{m\epsilon_\infty} - i \frac{4\pi\omega\sigma_{reg}(\omega)}{\epsilon_\infty} \right] \delta\rho + q^2 \delta P, \\ \omega^2 \delta s &= q^2 \frac{\rho_s s^2}{\rho_n} \delta T - \left[\frac{4\pi e^2 \rho_n}{m\epsilon_\infty} + i \frac{4\pi\omega\sigma_{reg}(\omega)}{\epsilon_\infty} \right] \frac{\rho_s s}{\rho_n \rho} \delta\rho. \end{aligned} \quad (\text{A7})$$

However, observing that we can rewrite

$$\delta P = \left(\frac{\partial P}{\partial \rho} \right)_T \delta\rho + \left(\frac{\partial P}{\partial T} \right)_\rho \delta T, \quad (\text{A8})$$

and using $(\partial P/\partial T)_\rho = 0$ as $T \rightarrow 0$ as a consequence of Nernst's theorem,⁴⁸ we can conclude that, at low temperature, second sound does not mix with the longitudinal density modes. From Eqs. (A7) and (A8), we then deduce the dispersion for density fluctuations, given by

$$\omega^2 = \left[\frac{4\pi e^2 \rho_s}{m\epsilon_\infty} - i \frac{4\pi\omega\sigma_{reg}(\omega)}{\epsilon_\infty} \right] + c_p^2 q^2, \quad (\text{A9})$$

where c_p is a constant. From now on we neglect the term $c_p^2 q^2$ that is unimportant in the long-wavelength limit. This is the plasmon dispersion and coincides with result of the microscopic derivation in Ref. 22. [To make connection with Eq. (29) of that reference, note that the real part of the right-hand side of Eq. (A9) above is $4\pi\omega \text{Im} \sigma(\omega)$, where $\sigma(\omega)$ is the *total* conductivity.]

Since we are interested in layered systems, Eq. (A9) must be modified. For simplicity we consider the case of carriers confined to stacked (Galilean invariant) planes interacting via an anisotropic Coulomb potential $V(\mathbf{q})$ defined in Eq.

(5). Referring to the components in-plane and across planes with a subscript \parallel and \perp , respectively, we then assume that $m_\perp = \infty$, $m_\parallel = m$, $\sigma_\perp = 0$ and denoting $\sigma_\parallel^{reg} = \sigma_\parallel$. The longitudinal electric field E has components, $E_\parallel = -iq_\parallel \phi(\mathbf{q})$ and $E_\perp = -iq_\perp \phi(\mathbf{q})$, where the electrostatic potential $\phi(\mathbf{q})$ for a density disturbance $\delta\rho(-\mathbf{q})$ is given by $\phi(\mathbf{q}) = [V(\mathbf{q})/e] \delta\rho$. As $m_\perp = \infty$ and $\sigma_\perp = 0$, only the in-plane component E_\parallel enters in the linear response Eq. (A3) and determines the ballistic motion of the superfluid electrons, Eq. (A4). Rewriting Eqs. (A1)–(A5) and decoupling again the first and second sound as before, we obtain the density mode

$$\omega^2 = \left[\frac{\rho_s}{m} - \frac{i\omega\sigma_\parallel(\omega)}{e^2} \right] q_\parallel^2 V(\mathbf{q}). \quad (\text{A10})$$

Equation (A10) allows us to deduce the correct expression for the phase-only Lagrangian in the layered case. Indeed, according to the previous equation, in the presence of dissipation, the superfluid stiffness must be transformed as $\rho_s/m \rightarrow [\rho_s/m - i\omega\sigma_\parallel(\omega)/e^2]$. We thus obtain, at the Gaussian level, the Lagrangian density

$$\mathcal{L}(\mathbf{q}, \omega) = \frac{1}{8} \left[V^{-1}(\mathbf{q}) \omega^2 - \left(\frac{\rho_s}{m} - \frac{i\omega\sigma_\parallel(\omega)}{e^2} \right) q_\parallel^2 \right] |\theta(\mathbf{q}, \omega)|^2, \quad (\text{A11})$$

which is the same as the Gaussian action used in the paper.

APPENDIX B: ANALYTICAL RESULTS FOR PHASE FLUCTUATIONS WITH ARBITRARY $\bar{\sigma}$

It is useful to calculate the fluctuation $\langle \delta\theta^2 \rangle$ for arbitrary dissipation $\bar{\sigma}$, even approximately. To make progress we work with the anisotropic Coulomb interaction

$$V(\mathbf{q}) = \frac{4\pi e^2}{\epsilon_\parallel q_\parallel^2 + \epsilon_\perp q_\perp^2}. \quad (\text{B1})$$

which is different from the Coulomb potential (5) for layered systems used in the paper. In particular, one cannot take a 2D limit of Eq. (B1) unlike in the layered case. However, it permits us a simple evaluation of the integrals appearing for $\langle \delta\theta^2 \rangle$ in Eq. (8).

On using the appropriate scaled Coulomb interaction and simplifying $\gamma_\parallel(\mathbf{Q}) \simeq Q_\parallel^2$, the renormalized-Gaussian action (with $D_\perp = 0$) takes the form

$$\begin{aligned} S_G[\theta] &= \frac{1}{8T} \sum_{\mathbf{Q}, \omega_n} \left[\frac{\epsilon_\parallel \omega_n^2 d_c}{4\pi e^2} \left(1 + \eta \frac{Q_\perp^2}{Q_\parallel^2} \right) \right. \\ &\quad \left. + \left(D_\parallel d_c + \frac{\bar{\sigma}}{2\pi} |\omega_n| \right) \right] Q_\parallel^2 |\theta(\mathbf{Q}, n)|^2 \end{aligned} \quad (\text{B2})$$

with $\eta \equiv (\epsilon_\perp \xi_0^2)/(\epsilon_\parallel d_c^2)$. Setting $\omega_c \equiv 4\pi e^2/d_c$, we then obtain the fluctuation

$$\langle \delta\theta_{\parallel}^2 \rangle = 2T\omega_c \int_{-\pi}^{\pi} \frac{dQ_{\perp} d^2 Q_{\parallel}}{(2\pi)^3} \times \sum_{\omega_n} \left[\varepsilon_{\parallel} (1 + \eta\zeta_{\mathbf{Q}}) \omega_n^2 + \omega_c \left(D_{\parallel} d_c + \frac{\bar{\sigma}}{2\pi} |\omega_n| \right) \right]^{-1}, \quad (\text{B3})$$

with $\zeta_{\mathbf{Q}} \equiv Q_{\perp}^2 / Q_{\parallel}^2$.

The integrand in Eq. (B3) depends on \mathbf{Q} only through $\zeta_{\mathbf{Q}}$. The \mathbf{Q} integral can therefore be transformed to an integral over the variable ζ , with density $N(\zeta) = 1/3\sqrt{\zeta}$ for $\zeta \leq 1$ and $N(\zeta) = 1/3\zeta^2$ for $\zeta > 1$, and

$$\langle \delta\theta_{\parallel}^2 \rangle = \int_0^{\infty} d\zeta N(\zeta) \Phi[(1 + \eta\zeta)\varepsilon_{\parallel}] \equiv \Phi(\bar{\varepsilon}) \int_0^{\infty} d\zeta N(\zeta) = \Phi(\bar{\varepsilon}), \quad (\text{B4})$$

where

$$\Phi(\varepsilon) = 2T\omega_c \sum_{\omega_n} \left[\varepsilon \omega_n^2 + \omega_c \left(D_{\parallel} d_c + \frac{\bar{\sigma}}{2\pi} |\omega_n| \right) \right]^{-1}, \quad (\text{B5})$$

and $\bar{\varepsilon}$ is a suitable average value of $(1 + \eta\zeta)\varepsilon_{\parallel}$. We have written Eq. (B4) such that the effect of the \mathbf{Q} integral appears as a renormalization of the bare in-plane dielectric constant ε_{\parallel} to a larger value $\bar{\varepsilon}$. Of course, the value of $\bar{\varepsilon}$ is temperature dependent. However, the leading temperature dependence of $\langle \delta\theta_{\parallel}^2 \rangle$ is, in most cases, $\bar{\varepsilon}$ independent (with the noticeable exception of the dissipation-less case $\bar{\sigma} = 0$). Therefore, to proceed further analytically, we take $\bar{\varepsilon}$ to be a constant henceforth.

The quantum corrections can be calculated analytically, expressing $\langle \delta\theta_{\parallel}^2 \rangle$ by means of the spectral representation for the Matsubara phase propagator deduced from Eq. (B5), so that

$$\langle \delta\theta_{\parallel}^2 \rangle(T) = \frac{2\omega_c}{\bar{\varepsilon}} \int_{-\infty}^{+\infty} \frac{dz}{2\pi} \times \frac{z\omega_c \bar{\sigma} / 2\pi \bar{\varepsilon}}{(z^2 - \omega_p^2)^2 + z^2 (\bar{\sigma}\omega_c / 2\pi \bar{\varepsilon})^2} \coth\left(\frac{z}{2T}\right), \quad (\text{B6})$$

where $\omega_p^2 = (D_{\parallel} d_c) (\omega_c / \bar{\varepsilon}) = 4\pi e^2 \rho_s / \bar{\varepsilon} m^*$.

The above expression allows us to explicitly calculate the value of $\langle \delta\theta_{\parallel}^2 \rangle(T=0)$. Introducing the dimensionless parameter $s = (\bar{\sigma}/4\pi) (\omega_c / \bar{\varepsilon} \omega_p) = (\bar{\sigma}/4\pi) \sqrt{\omega_c / \bar{\varepsilon} D_{\parallel} d_c}$, and evaluating the above integral at zero temperature we obtain the zero-point-motion contribution

$$\langle \delta\theta_{\parallel}^2 \rangle(0) = \frac{2\omega_c}{2\pi \bar{\varepsilon} \omega_p \sqrt{s^2 - 1}} \ln\left(\frac{s + \sqrt{s^2 - 1}}{s - \sqrt{s^2 - 1}}\right) \quad (\text{B7})$$

in the case $s > 1$, which is physically relevant for the cuprates, and

$$\langle \delta\theta_{\parallel}^2 \rangle(0) = \frac{2\omega_c}{\pi \bar{\varepsilon} \omega_p \sqrt{1 - s^2}} \arctan\frac{\sqrt{1 - s^2}}{s} \quad (\text{B8})$$

in the case $s < 1$. For $s \rightarrow 1$ both the above expressions (B7) and (B8) reduce to $2\omega_c / \pi \bar{\varepsilon} \omega_p$. For $s \gg 1$, from Eq. (B7), $\langle \delta\theta_{\parallel}^2 \rangle(0) \simeq (8/\bar{\sigma}) \ln\{(\bar{\sigma}/2\pi) \sqrt{\omega_c / (\bar{\varepsilon} D_{\parallel} d_c)}\}$ in agreement with the expression used in the text, except the relevant energy scale, $\omega_c = 4\pi e^2 / d_c$, for the anisotropic Coulomb potential appears instead of $2\pi e^2 / \xi_0$, the scale appropriate when large Q_{\parallel} contributes and we are closer to a 2D limit in the fluctuation integral. The quantum fluctuations are large for small s and decrease monotonically with increasing s . For $s \rightarrow \infty$, $\langle \delta\theta_{\parallel}^2 \rangle \rightarrow 0$, and the classical limit is recovered.

At finite temperature and for a bare stiffness that is independent of temperature, the analytical result for the quadratic temperature dependence of $\langle \delta\theta_{\parallel}^2 \rangle(T)$ for arbitrary $\bar{\sigma}$, given in Eq. (10), has been derived in the text. The same result also follows from a low-temperature analysis of Eq. (B6) above.

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