Critical fluctuation effects near the normal-metal–superconductor phase transition at low temperatures

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The effects of the fluctuations on the conductivity in the normal state near normal-metal–superconductor phase transition for temperatures close to zero are studied. The generic phase diagram for the classical and quantum critical fluctuation regime is derived.

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In recent years a variety of materials have been studied where a normal-metal-to-superconductor transition occurs when parameters such as pressure or carrier concentration are varied. These transitions can happen even at zero temperature where they are driven by quantum fluctuations of superconducting phase nuclei, i.e., quantum phase transition. The heavy fermion systems $CePd_2Si_2$ and $CeIn_3$ that show superconductivity in a narrow interval of high pressure close to a magnetic quantum critical point belong to this class of materials.[1] Similar behavior with some distinctive differences are found in various low-dimensional organic superconductors, such as the $(TMTSF)_2X$ group ($X = PF_6, AsF_6, \dots$).[2] A further example of a quantum phase transition of a Fermi liquid to a superfluid for ^{3}He in an aerogel has been discovered.[3] The best-known case is found in overdoped high-temperature superconductors where superconductivity disappears upon excessive carrier doping.

At finite temperatures the superconducting phase transition is accompanied by the appearance of inhomogeneous configurations of the superconducting order parameter, i.e., thermal fluctuations of the superconducting order parameter. The typical energies of the fluctuation modes are much smaller than T_c so that the classical statistics is appropriate[4] ($k_B = \hbar = 1$). The influence of thermal critical fluctuations on the metallic properties such as the conductivity or diamagnetic susceptibility has been investigated for a long time.[5] Generally we expect that as zero temperature is approached the energy of fluctuations becomes larger than the temperature. The configurational or thermal fluctuations play then a diminishing role giving way to the dynamical or quantum fluctuations.[4] Within the framework of a weak-coupling BCS model we analyze here the necessary conditions for the quantum critical fluctuations to gain importance and present the corresponding generalization of the Aslamazov-Larkin theory[6] for fluctuation corrections to the dc conductivity. The problem of the normal-metal–superconductor quantum phase transition has been studied recently also by Ramazashvili and Coleman for exotic superconductors with "odd-pairing"[7] and for a quantum critical point driven by pair-breaking disorder.[8] Our study has the same starting point as Ref. 8. We intend, however, to analyze critical behavior in the entire region around the phase transition. For the weak-coupling BCS limit in two-dimensional (2D) case in the "classical" region our results coincide with those of Ref. 8 under special conditions discussed below. In 3D case our derivation leads to results different from theirs.

We begin our discussion by introducing the nonstationary Ginzburg-Landau equation

$$\left\{ a\frac{\partial}{\partial t} + \xi_0^2(-i\nabla)^2 + \epsilon \right\} \psi(\mathbf{r},t) = 0, \tag{1}$$

where a, the coherence length ξ_0, and the distance ϵ from the phase-transition line are the parameters that depend on temperature and purity. The term with (nondissipative) second-order time derivatives that has been analyzed in Ref. 9 is omitted here, and we take only the dissipative linear time derivative into account, which is most important in the low-frequency region. Note that in the absence of particle-hole symmetry at the Fermi level there is a propagative component to the time derivative.[10] Here we consider only the dissipative part, because its influence clearly dominates the conductivity behavior. The frequency of the order-parameter fluctuations is obtained immediately from the equation

$$i\omega_{\mathbf{k}} = \frac{\epsilon + \xi_0^2 k^2}{a}, \tag{2}$$

which is equivalent to the inverse relaxation time for a given wave vector $\mathbf{k}$. This frequency is the key quantity that distinguishes between the classical and the quantum regime. The situation with final critical temperature studied usually for the ordinary superconductors either clean or doped by the impurities not causing the depairing effects is characterized by $\epsilon = T - T_c/T_c$ and $a = \pi/8T_c$ where T_c is the eventual transition temperature. The basic frequency proportional to $(T - T_c)$ is much smaller than the temperature $T \geqslant T_c$. Consequently, the fluctuations of the order parameter can be considered as the quasistatic configurations with positive energy $|\omega_{\mathbf{k}}|$, that couple with the other degrees of freedom of the thermodynamic system, fast fluctuating with frequencies of the order of temperature T.

To see where and in which form the classical picture breaks down we consider now a superconductor close to a quantum critical point with pressure P as the controlling parameter. This situation can be realized if the initial pressure-dependent critical temperature $T_{c0}(P)$ is suppressed down to

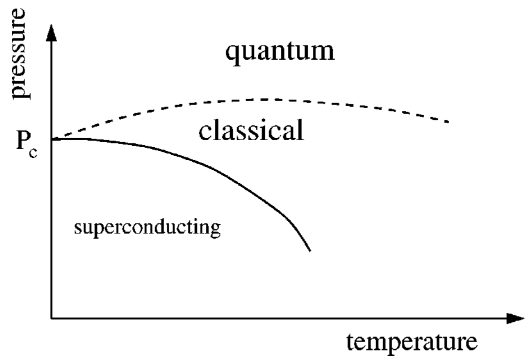

FIG. 1. Schematic phase diagram of pressure versus temperature. The solid line indicates the superconducting phase transition and the dashed line corresponds to the crossover between classical and quantum regimes.

$T_c(P)$ by the scattering of the quasiparticles at magnetic or ordinary impurities (the latter is possible for unconventional superconducting or superfluid states). For strong enough impurity scattering at some critical pressure related to the mean-free time of the quasiparticle scattering $\tau = \gamma/\pi T_{c0}(P_c)$, the superconducting transition temperature is suppressed completely and we have reached the quantum critical point ($\ln\gamma \approx 0.577\ldots$ is the Euler constant).[9] To be concrete let us assume that $T_{c0}(P)$ is a monotonically decreasing function of pressure. At pressures $P<P_c$ where the real transition temperature $T_c(P) \ll T_{c0}(P)$, it can be shown that the parameters ϵ and ξ_0 in Eq. (1) behave like[11] $\epsilon \simeq \tau[T-T_c(P)]$, $\xi_0 \simeq v_F\tau$, and $a\simeq\tau$. The last relation and the validity of the time-dependent Ginzburg-Landau equation near $T=0$ in the presence of pair-breaking scattering has been recently demonstrated by Herbut.[12] Hence we find that up to $T_c(P_c)=0$ approaching the transition point by changing temperature at fixed pressure, $|\omega_{\mathbf{k}}| \ll T$ and the classical fluctuation regime is realized. For this reason the fluctuation correction to the specific heat always originates from classic thermal fluctuations and follows standard behavior.[6]

On the other hand, we may approach the transition by changing the pressure at a fixed temperature[9] where we obtain

$$\epsilon = \frac{\partial \ln T_{c0}(P)}{\partial P}[P-P_c(T)] \tag{3}$$

and as above, $a\simeq\tau$. From this point of view the two regimes are more conveniently distinguished, as $|\omega_{\mathbf{k}}|\simeq \epsilon/a$ can be smaller as well as larger than T. In the latter situation the dynamics of the order-parameter fluctuations become important in defining the quantum regime. It is worth noting here that at $|\omega_{\mathbf{k}}|>T$ the equation based on the first-order time derivative (1) is still valid as long as the inequality $|\omega_{\mathbf{k}}|\tau \ll 1$ holds. On the other hand, the scattering rate τ^{-1} provides the necessary cutoff in infrared singularities appearing in the theory for $T\to 0$.[12] In Fig. 1 we give a schematic view of the phase diagram where the dashed line represents the crossover between the classical and quantum regimes for each $P_c(T)=P_c(0)(1-A\tau^2T^2)$ where A is of order one.

We will now investigate the difference of the two regimes in the behavior of the paraconductivity. For this purpose we generalize the calculation of the Aslamazov-Larkin part of the paraconductivity[6] to be valid both in the static and the dynamic case. This can be done based on the formulation by Aronov *et al.*[10] and leads to

$$\sigma_{\rm fl}^{xx} = (2e\xi_0^2)^2 \int \frac{k_x^2 d^d k}{(2\pi)^d} \int_{-\infty}^{\infty} \frac{dz}{2\pi} [\Im L(z,\mathbf{k})]^2 \frac{\partial}{\partial z} \coth\frac{z}{2T}, \tag{4}$$

where

$$L(z,\mathbf{k}) = \frac{1}{\epsilon + \xi_0^2 k^2 - iaz} \tag{5}$$

is the Green's function of the Ginzburg-Landau Eq. (1). The expression above is valid when the current vertex corrections (blocks containing three normal-metal Green functions)[6,10] are ω and $\mathbf{k}$ independent. Our calculations show that this is fulfilled under the condition $\omega\tau<1$ and $v_F\tau k<1$. We use expression (4) to study the behavior of paraconductivity in superconductors of different geometry: a bulk superconductor, a thin film of thickness $l \ll \xi = \xi_0/\epsilon^{1/2}$, and a wire of cross section $S \ll \xi^2$.

In the static limit corresponding to $\epsilon \ll aT$ this equation reduces to the known Aslamazov-Larkin form,[6]

$$\sigma_{\rm fl}^{xx} = 2(2e\xi_0^2)^2 aT \int \frac{k_x^2 d^d k}{(2\pi)^d (\epsilon + \xi_0^2 k^2)^3}, \tag{6}$$

from which we immediately obtain the standard results,

$$\sigma_{\rm fl} = \frac{e^2 aT}{4\pi\xi_0\epsilon^{1/2}} \quad \text{(bulk)}, \tag{7}$$

$$\sigma_{\rm fl} = \frac{e^2 aT}{2\pi\epsilon l} \quad \text{(film)}, \tag{8}$$

$$\sigma_{\rm fl} = \frac{e^2 aT\xi_0}{2\epsilon^{3/2} S} \quad \text{(wire)}, \tag{9}$$

representing the classical behavior.

In the dynamical or quantum limit $\epsilon > aT$ the evaluation of Eq. (4) is more difficult. However, we may obtain the parametric dependence of the paraconductivity in leading order by considering $(aT/\epsilon) \ll 1$ as a small parameter. Then we find, up to a numerical factor of order unity, the following behavior deep in the quantum regime (Fig. 1):

$$\sigma_{\rm fl} \simeq \frac{e^2 (aT)^2}{6\pi^2 \xi_0 \epsilon^{3/2}} \quad \text{(bulk)}, \tag{10}$$

$$\sigma_{\rm fl} \simeq \frac{2e^2 (aT)^2}{3\pi^2 \epsilon^2 l} \quad \text{(film)}, \tag{11}$$

$$\sigma_{\rm fl} \simeq \frac{2e^2(aT)^2\xi_0}{\pi\epsilon^{5/2}S} \quad (\text{wire}). \tag{12}$$

We see that in the region of importance of dynamic fluctuations $aT<\epsilon$ the Aslamazov-Larkin corrections to conductivity (10)–(12) are aT/ϵ times smaller than in the classic region $aT>\epsilon$.

We may now discuss our result in comparison with the work of Ramazashvili and Coleman.[7,8] These authors studied the low-temperature properties for the case $P=P_c(T=0)$, which lies within the classical regime in our phase diagram. Using Eq. (7) and (8) with $\epsilon\propto T^2$ we get agreement with their weak-coupling result for the two dimensional ($\sigma_{\rm fl}\propto T^{-1}$), but we differ in the three-dimensional case (our $\sigma_{\rm fl}\propto$const versus their $\sigma_{\rm fl}\propto T^{1/2}$).

In summary, we have studied the fluctuation correction to the conductivity near the phase transition to the superconducting state at low temperatures within the framework of a weak-coupling BCS theory. We determined the regimes where thermal and quantum order-parameter fluctuations are important. Finally we found that the dynamical or quantum fluctuations are less efficient in the augmentation of the conductivity than thermal fluctuations.

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