Localized spin excitations in an anisotropic Heisenberg ferromagnet with Dzyaloshinskii-Moriya interactions

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We investigate the nonlinear spin dynamics of an anisotropic Heisenberg ferromagnetic spin chain with Dzyaloshinskii-Moriya interactions in the semiclassical limit. We have identified four completely integrable spin models with soliton spin excitations for specific parametric choices and also constructed perturbed solitons. We also obtain solitary spin excitations for a linearly perturbed integrable model at higher order.

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I. INTRODUCTION

In addition to dominant magnetic interactions, such as exchange, anisotropy, etc., which involve integrable spin models with soliton spin excitations, there exist certain magnetic interactions that are less spoken about in the literature of nonlinear dynamics due to the mathematical complexity of their representations in the Hamiltonian and in the governing dynamical equations. Notable among them, which has been reexamined by several authors in recent times,[1–4] is the Dzyaloshinskii-Moriya (DM) interaction, which is essentially an antisymmetric spin coupling that occurs when the symmetry around the magnetic ions is not high enough, thus leading to the mechanism of weak ferromagnetism which is due to the combined effect of spin-orbit coupling and spin-spin exchange interactions. Weak ferromagnetism plays an important role in describing insulators, spin glasses, low-temperature phases of copper oxide superconductors, phase transitions, etc.[3,4] In spite of these developments, not much is known about nonlinear excitations in weak ferromagnets. Under small-angle approximations certain specific isotropic and anisotropic one-dimensional weak-ferromagnetic models and an inhomogeneous radially symmetric weak ferromagnet with specific inhomogeneity in the classical continuum limit have been found to be integrable and to exhibit soliton spin excitations.[5] In the present paper, we investigate the nonlinear spin dynamics of weak ferromagnets in the general case using a semiclassical approach. We identify a few integrable spin models which exhibit nonlinear spin excitations in the form of solitons (Sec. II) and also construct perturbed soliton spin excitations (Sec. III) and solitary spin excitations (Sec. IV).

II. NONLINEAR DYNAMICS OF ANISOTROPIC FERROMAGNETS WITH DM INTERACTION

The Heisenberg model of the Hamiltonian for an anisotropic (both exchange and crystal field) ferromagnetic spin system with DM interaction in dimensionless form is given by

$$H=-\sum_i\Bigg[\frac{J_1}{2}[\hat{S}_i^+\hat{S}_{i+1}^-+\hat{S}_i^-\hat{S}_{i+1}^+]+J_2[\hat{S}_i^z\hat{S}_{i+1}^z]-i\frac{J_3}{2}\{D^+[\hat{S}_i^z\hat{S}_{i+1}^- -\hat{S}_i^-\hat{S}_{i+1}^z]+D^-[\hat{S}_i^+\hat{S}_{i+1}^z-\hat{S}_{i+1}^+\hat{S}_i^z]+D^z[\hat{S}_i^-\hat{S}_{i+1}^+-\hat{S}_i^+\hat{S}_{i+1}^-]\}-A(\hat{S}_i^z)^2-A'(\hat{S}_i^z)^4\Bigg], \quad (2.1)$$

where $\hat{S}_i^\pm=\hat{S}_i^x\pm i\hat{S}_i^y$, $\hat{\mathbf{S}}_i=\mathbf{S}_i/\hbar$, and $\mathbf{S}_i=(S_i^x,S_i^y,S_i^z)$. Here J_1 and J_2 are the exchange integrals due to spin-spin coupling and the terms proportional to J_3 correspond to the DM interaction ($D^\pm=D^x\pm iD^y$). A and A' are anisotropy parameters. For treating the problem semiclassically at sufficiently low temperature we use the following truncated Holstein-Primakoff[6] expansion for the spin operators in terms of the bosonic operators. After introducing Glauber's coherent-state representation[7] for the Bose operators, the Heisenberg equation of motion is written in the form

$$iu_t+\epsilon^2\{J_1u_{xx}+2\delta_1|u|^2u-i\sqrt{2}J_3(D^+u^*+D^-u)u_x\}-i\epsilon^3\{20\delta_2(u_{xxx}-6|u|^2u_x)\}+\epsilon^4\Bigg\{\frac{J_1}{12}[u_{xxxx}-6u^*u_x^2+\delta_3(u^2u_{xx}^*+2|u_x|^2u)+\delta_4|u|^2u_{xx}-i\delta_5\{D^+(u^*u_{xxx}+3u_xu_{xx}^*+3u_x^*u_{xx})+D^-(|u_x|^2uu_x+uu_{xxx})\}+\delta_6|u|^4u]\Bigg\}-i\epsilon^5\delta_2\{u_{xxxxx}+\cdots\}+O(\epsilon^6)=0, \quad (2.2)$$

where u corresponds to the coherent amplitude and the coefficients $\delta_1,\delta_2,\ldots,\delta_6$ are given by $\delta_1=(J_2-J_1-A-3A')$, $\delta_2=\frac{1}{30}J_3D^z$, $\delta_3=6(2J_2-J_1)/J_1$, $\delta_4=\delta_3-6$, $\delta_5=2\sqrt{2}J_3/J_1$, and $\delta_6=72A'/J_1$.

While writing Eq. (2.2) we have introduced a dimensionless time and transformed away linear terms proportional to u and u_x using the transformations $u\to u\exp[2i(J_2-J_1+A+A')t]$ and $x\to x+2J_3D^z\epsilon t$, respectively. A close inspection of Eq. (2.2) reveals that it contains several well-known completely integrable models admitting soliton solutions for specific choices of parameters at different orders of ϵ. For example, at $O(\epsilon^2)$, we have

$$iu_t+\epsilon^2\{J_1u_{xx}+2\delta_1|u|^2u-i\sqrt{2}J_3[D^+u^*u_x+D^-uu_x]\}=0. \tag{2.3}$$

After rescaling x and t as $x\rightarrow(\sqrt{\delta_1/J_1})x$ and $t\rightarrow\delta_1\epsilon^2t$ and by assuming that the DM interaction is restricted to the z direction ($D^+=D^-=0$), Eq. (2.3) becomes the well-known completely integrable cubic nonlinear Schrödinger (NLS) equation $iu_t+u_{xx}+2|u|^2u=0$, for which the N-soliton solutions can be found in more than one way.[8–12] It is worth pointing out at this point the limits of isotropic and anisotropic continuum Heisenberg models. Though in the classical limit the isotropic continuum Heisenberg model can be mapped onto the completely integrable cubic NLS equation, under the semiclassical approximation, the contribution towards the nonlinear term vanishes. However, this nonlinear term ($|u|^2u$) is generated from the single-ion uniaxial anisotropic energy. At $O(\epsilon^3)$, we have

$$iu_t+\epsilon^2\{J_1u_{xx}+2\delta_1|u|^2u-i\sqrt{2}J_3[D^+u^*u_x+D^-uu_x]\}-20i\epsilon^3\delta_2\{u_{xxx}-6|u|^2u_x\}=0. \tag{2.4}$$

We assume that $J_1\ll J_2$, so that the spins point along the S^z direction and $D^+=D^-=0$. Now, upon choosing $A+3A'=J_2-J_1$, Eq. (2.4) for real u can be written as $u_t+u_{xxx}-6u^2u_x=0$, which is the completely integrable modified Korteweg–de Vries (MKdV) equation which also possesses N-soliton solutions.[13] However, if we relax two of the above conditions, namely, $J_1\ll J_2$ and u: real, it leads to another completely integrable model given by $iu_t+\frac{1}{2}u_{xx}-|u|^2u-i\delta_7[u_{xxx}-6|u|^2u_x]=0$, where $\delta_7=J_3D^z\epsilon/6J_1$. The N-soliton solutions for the above equation were found by Hirota using a direct method after bilinearizing it.[12] However, at $O(\epsilon^4)$, from Eq. (2.2), we obtain a generalized fourth-order NLS equation only when the DM interaction is completely absent:

$$iu_t+\epsilon^2\{J_1u_{xx}+2\delta_1|u|^2u\}+\epsilon^4\{(J_1/12)[u_{xxxx}-6u^*u_x^2+\delta_3(u^2u_{xx}^*+2|u_x|^2u)+\delta_4|u|^2u_{xx}+\delta_6|u|^4u]\}=0. \tag{2.5}$$

On choosing $J_2=J_1/3$, $A=A'=J_1/12$, Eq. (2.5) reduces to the completely integrable fourth-order NLS equation which admits N-soliton solutions,[9] $iu_t+u_{xx}+2|u|^2u+\gamma_1[u_{xxxx}+8|u|^2u_{xx}+2u^2u_{xx}^*+4|u_x|^2u+6u^*u_x^2+6|u|^4u]=0$, where $\gamma_1=\epsilon^2\delta_1/12J_1$. Among the models only in Eq. (2.4) does the contribution due to the DM interaction enters into the integrable model. Though Eq. (2.4) can be derived from a simple Hamiltonian, the purpose for starting from a more generalized Hamiltonian is to analyze the nature of nonlinear excitations at different orders of discreteness and due to the DM interaction even under perturbation. From the above, we observe that, though the higher-order discreteness effect introduces higher degrees of nonlinearity into the spin dynamics of the system, interestingly at each higher order of ϵ, for a specific choice of parameters, still the nonlinear dynamics is governed by the completely integrable hierarchy of models in the NLS family. Nevertheless, in general at each order of ϵ the nonlinear spin dynamics is governed by the perturbed nonlinear Schrödinger family of equations. Therefore it is natural to investigate the complete nonlinear dynamics of spins under perturbation.

III. EFFECT OF THE DM INTERACTION AS PERTURBATION ON INTEGRABLE SPIN MODELS

We now investigate the nature of nonlinear spin excitations at different orders of ϵ in the more general case (without imposing restrictions on parameter values) by carrying out a multiple-scale perturbation analysis on Eqs. (2.3) and (2.4) separately.[14] We first consider Eq. (2.3), and after rescaling t and x as $t\rightarrow\epsilon^2\delta_1t$ and $x\rightarrow(\sqrt{\delta_1/J_1})x$, respectively, we get the following perturbed cubic NLS equation:

$$iu_t+u_{xx}+2|u|^2u+i\lambda_1[D^+u^*u_x+D^-uu_x]=0, \tag{3.1}$$

where the perturbation parameter $\lambda_1=-J_3\sqrt{2/J_1\delta_1}$. When $\lambda_1=0$, Eq. (3.1) reduces to the completely integrable cubic NLS equation which possesses N-soliton solutions. The explicit form of the corresponding one soliton solution is given by

$$u=\eta\,\mathrm{sech}\,\eta(\theta-\theta_0)\exp[i\xi(\theta-\theta_0)+i(\sigma-\sigma_0)], \tag{3.2}$$

where $\partial\theta/\partial t=-2\xi$, $\partial\theta/\partial x=1$, $\partial\sigma/\partial t=\eta^2+\xi^2$, and $\partial\sigma/\partial x=0$. Here η and ξ are related to the scattering parameter of the inverse scattering analysis. Now we treat the contribution due to the DM interaction proportional to $\rightarrow D^+$ and D^-, i.e., terms proportional to λ_1 as a weak perturbation, and introduce a slow time variable $T=\lambda_1t$. The envelope soliton solution of Eq. (3.1) is now given by $u=\hat{u}(\theta,T;\lambda_1)\exp[i\xi(\theta-\theta_0)+i(\sigma-\sigma_0)]$, where η, ξ, θ_0, and σ_0 are now functions of the new time scale T. Under the assumption of quasistationarity, Eq. (3.1) takes the form

$$-\eta^2\hat{u}+\hat{u}_{\theta\theta}+2\hat{u}^2\hat{u}^*=\lambda_1F(\hat{u}), \tag{3.3}$$

where $F(\hat{u})=i[-\hat{u}_T+D^+\hat{u}^*\hat{u}_\theta+D^-\hat{u}\hat{u}_\theta]+[(\theta-\theta_0)\xi_T-\xi\theta_{0T}-\sigma_{0T}-D^+\hat{u}^*\xi-D^-\hat{u}\xi]\hat{u}$. Making the first-order perturbation $\hat{u}(\theta,T;\lambda_1)=\hat{u}_0(\theta,T)+\lambda_1\hat{u}_1(\theta,T)$ and substituting $\hat{u}_1=\hat{\phi}_1+i\hat{\psi}_1$ ($\hat{\phi}_1$ and $\hat{\psi}_1$ are real) in Eq. (3.3) we get

$$L_1\hat{\phi}_1\equiv-\eta^2\hat{\phi}_1+\hat{\phi}_{1\theta\theta}+6\hat{u}_0^2\hat{\phi}_1=\mathrm{Re}\;F_1(\hat{u}_0), \tag{3.4}$$

$$L_2\hat{\psi}_1\equiv-\eta^2\hat{\psi}_1+\hat{\psi}_{1\theta\theta}+2\hat{u}_0^2\hat{\psi}_1=\mathrm{Im}\,F_1(\hat{u}_0), \tag{3.5}$$

where L_1 and L_2 are self-adjoint operators and $\mathrm{Re}\,F_1(\hat{u}_0)=[(\theta-\theta_0)\xi_T-\xi\theta_{0T}-\sigma_{0T}+D^+\hat{u}_0^*\xi-D^-\hat{u}_0\xi]\,\widehat{u}_0$, $\mathrm{Im}\,F_1(\hat{u}_0)=[-\hat{u}_{0T}+D^+\hat{u}_0^*\hat{u}_{0\theta}+D^-\hat{u}_0\hat{u}_{0\theta}]$. As $\hat{u}_{0\theta}$ and $\hat{u}_0$ are solutions of the homogeneous parts of Eqs. (3.4) and (3.5), respectively, we have $\int_{-\infty}^{\infty}\hat{u}_{0\theta}\,\mathrm{Re}\,F_1d\theta=0$, $\int_{-\infty}^{\infty}\hat{u}_0\,\mathrm{Im}\,F_1d\theta=0$. On substituting the values of $\hat{u}_{0\theta}$, $\hat{u}_0$, $\mathrm{Re}\,F_1(\hat{u}_0)$, and $\mathrm{Im}F_1(\hat{u}_0)$ and after evaluating the integrals we obtain $\xi_T=\eta_T=0$ which implies that the velocity and amplitude of the soliton remain unchanged during propagation under perturbation.

The perturbed solution $\hat{u}_1$ can be constructed by solving Eqs. (3.4) and (3.5). The homogeneous part of

Eq. (3.4) admits two particular solutions $\hat{\phi}_{11}$ and $\hat{\phi}_{12}$ given by $\hat{\phi}_{11}=\operatorname{sech}\eta(\theta-\theta_0)\tanh\eta(\theta-\theta_0)$, $\hat{\phi}_{12}=(-1/\eta)\times[\operatorname{sech}\eta(\theta-\theta_0)-\frac{3}{2}\eta(\theta-\theta_0)\operatorname{sech}\eta(\theta-\theta_0)\tanh\eta(\theta-\theta_0)-\frac{1}{2}\tanh\eta(\theta-\theta_0)\sinh\eta(\theta-\theta_0)]$. Knowing two particular solutions, the general solution can be constructed using the form $\hat{\phi}_1=C_1\hat{\phi}_{11}+C_2\hat{\phi}_{12}-\hat{\phi}_{11}\int_{-\infty}^{\theta}\hat{\phi}_{12}\,\mathrm{Re}\,F_1 d\theta'+\hat{\phi}_{12}\int_{-\infty}^{\theta}\hat{\phi}_{11}\mathrm{Re}\,F_1 d\theta'$, where C_1 and C_2 are arbitrary constants. We substitute $\hat{\phi}_{11}$, $\hat{\phi}_{12}$, and $\mathrm{Re}\,F_1$ in the above expression for $\hat{\phi}_1$ and evaluate the integrals. We remove the secular terms which make the solution unbounded, by choosing the arbitrary constant C_2 as $C_2=\frac{1}{2}[\xi\theta_{0T}+\sigma_{0T}]$. Implementing the boundary conditions $\hat{\phi}_1(0)|_{\theta_0=0}=\hat{\phi}_{1\theta}(0)|_{\theta_0=0}=0$, we obtain $C_1=0$, $C_2=D\xi\eta^2/3(1-\eta)$ where $D_1=(D^+-D^-)$. Thus, the final form of the general solution $\hat{\phi}_1$ becomes

$$\begin{aligned}\hat{\phi}_1=D_1\xi\Bigg(\frac{\eta}{6(1-\eta)}\Bigg\{\Bigg[6\left(\frac{9}{\eta}+2\right)\\ -27\operatorname{sech}^2\eta(\theta-\theta_0)\Bigg]\operatorname{sech}\eta(\theta-\theta_0)+3\eta(\theta-\theta_0)\\ \times[2-3\operatorname{sech}^2\eta(\theta-\theta_0)]\operatorname{sech}\eta(\theta-\theta_0)\\ \times\tanh\eta(\theta-\theta_0)\Bigg\}-[2-(2+\eta)\operatorname{sech}^2\eta(\theta-\theta_0)\\ +3\eta(\theta-\theta_0)\operatorname{sech}^2\eta(\theta-\theta_0)\tanh\eta(\theta-\theta_0)]\\ \times\operatorname{sech}^2\eta(\theta-\theta_0)\Bigg).\end{aligned}\tag{3.6}$$

In a similar way, Eq. (3.5) can also be solved and ψ_1 can be constructed. The homogeneous part of Eq. (3.5) admits the following two particular solutions namely $\hat{\psi}_{11}=\operatorname{sech}\eta(\theta-\theta_0)$ and $\hat{\psi}_{12}=(1/2\eta)[\eta(\theta-\theta_0)\operatorname{sech}\eta(\theta-\theta_0)+\sinh\eta(\theta-\theta_0)]$. Following the same procedure we construct the general solution which, after removing the secular terms and applying the boundary conditions, becomes

$$\begin{aligned}\hat{\psi}_1=(D_2\eta/3)\{\eta(\theta-\theta_0)[\operatorname{sech}^3\eta(\theta-\theta_0)-\tan^{-1}\\ \times[\sinh\eta(\theta-\theta_0)]-6]\operatorname{sech}\eta(\theta-\theta_0)\\ +[\operatorname{sech}\eta(\theta-\theta_0)-3]\operatorname{sech}\eta(\theta-\theta_0)\tanh\eta(\theta-\theta_0)\},\end{aligned}\tag{3.7}$$

where $D_2=(D^++D^-)$. Knowing $\hat{\phi}_1$ and $\hat{\psi}_1$ the perturbed solution $\hat{u}_1=\hat{\phi}_1+i\hat{\psi}_1$ can be constructed using Eqs. (3.6) and (3.7).

Next, we try to construct the perturbed soliton for Eq. (2.4) using the same procedure. After making the same rescaling for t and x, Eq. (2.4) can be written as

$$iu_t+u_{xx}+2|u|^2u+i\lambda_2[\omega u_{xxx}+2|u|^2u_x]=0,\tag{3.8}$$

where $\omega=(J_1-J_2+A+3A')/3J_1$ and $\lambda_2=\epsilon J_3D^z/\sqrt{J_1(J_2-J_1-A-3A')}$ is the perturbation parameter. Following the same procedure used in the previous case, we get, for Eq. (3.8), $\mathrm{Re}\,F_1(\hat{u}_0)=[(\theta-\theta_0)\xi_T-\xi\theta_{0T}-\sigma_{0T}++3\omega\xi\hat{u}_{0\theta\theta}+2\xi|\hat{u}_0|^2\hat{u}_0-\omega\xi^3\hat{u}_0]\hat{u}_0$, $\mathrm{Im}\,F_1(\hat{u}_0)=-\{\hat{u}_{0T}+\omega\hat{u}_{0\theta\theta\theta}-3\omega\xi^2\hat{u}_{0\theta}+2|\hat{u}_0|^2\hat{u}_{0\theta}\}$. The secularity conditions in this case also lead to the same conclusion that the velocity and amplitude of the soliton remain unchanged. We construct the perturbed solution u_1 following the same procedure. Finally, the first-order perturbed soliton solution for Eq. (3.8) takes the form

$$\begin{aligned}\hat{u}_1=\frac{\eta}{2}\{\xi[\eta(\theta-\theta_0)\tanh\eta(\theta-\theta_0)-1]\operatorname{sech}\eta(\theta-\theta_0)\\ +i\eta(3\omega-1)[\tanh\eta(\theta-\theta_0)-(\theta-\theta_0)]\\ \times\operatorname{sech}\eta(\theta-\theta_0)\}.\end{aligned}\tag{3.9}$$

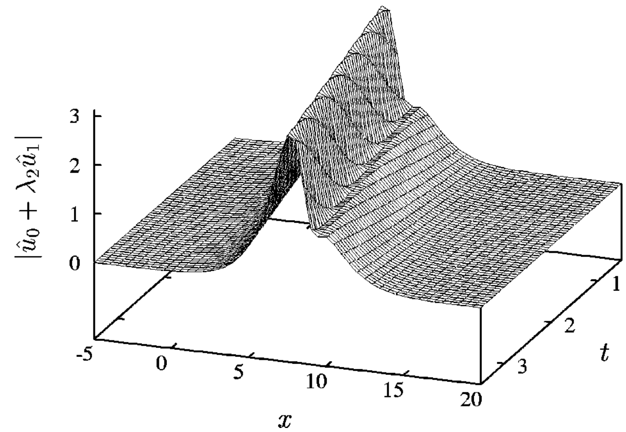

FIG. 1. The soliton ($|\hat{u}_0+\lambda_2\hat{u}_1|$) of Eq. (2.4) representing spin excitations with fluctuations in the tail due to the DM interaction.

The results of the perturbation analysis show that the DM interaction introduces a small fluctuation in the tail of the soliton. Further the elementary spin excitations are intact and the perturbation due to the DM interaction does not alter the velocity and amplitude of the soliton. To illustrate this we have plotted the perturbed soliton (with fluctuations in the tail) $|\hat{u}_0+\lambda_2\hat{u}_1|$ by choosing $\lambda_2=0.002$, $\eta=\omega=0.9$, and $\xi=1.5$ in Fig. 1. A similar perturbation analysis for a generalized fourth-order NLS equation [similar to Eq. (2.5)] can be found in Ref. 9.

IV. SOLITARY-WAVE-LIKE SPIN EXCITATIONS

Even though perturbed solutions over unperturbed solitons can be constructed, it is always useful to construct explicit localized solutions if possible (see, e.g., Ref. 15). We therefore, following the procedure of Grimshaw and Pavlov,[15] construct a solitary-wave solution to a linearly perturbed MKdV equation that describes the spin excitations of the weak ferromagnet which we obtain from Eq. (2.2) at $O(\epsilon^5)$ by considering only terms proportional to u_{xxxxx}. Choosing $J_1\ll J_2$ and $J_2-J_1=A+3A'$ and assuming that the DM interaction is restricted to the z direction ($D^+=D^-=0$) when u is real, Eq. (2.2) becomes

$$u_t+u_{xxx}+6u^2u_x+\hat{\epsilon}u_{xxxxx}=0,\tag{4.1}$$

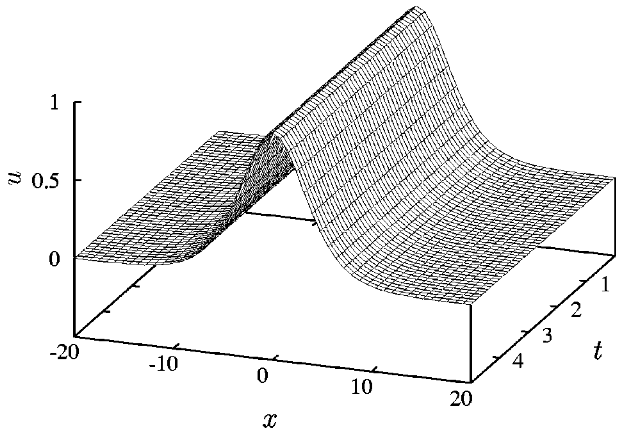

FIG. 2. Localized solitary wave solution of Eq. (4.1) (for specific parametric choices) representing spin excitations.

where $\hat{\epsilon}=\epsilon^2/20$. The perturbed MKdV equation (4.1) possesses the Hamiltonian structure described by the Hamiltonian $H=\int[\frac{1}{2}u_x^2-u^4/12-\frac{1}{2}\hat{\epsilon}u_{xx}^2]dx$. The traveling-wave solutions for the perturbed soliton equation can be obtained from the Lax-Novikov (LN) equation $\delta(H-\alpha P-\beta Q)=0$, where the momentum integral P and Casimir Q for Eq. (4.1) take the form $P=\int\frac{1}{2}u^2dx$ and $Q=\int u\,dx$, respectively. Here α is the speed of the wave and β is the integration constant. Now the LN equation corresponding to our perturbed MKdV equation (4.1) can be written as $-\hat{\epsilon}u_{xxxx}-u_{xx}-\sigma u^3/3-\alpha u-\beta=0$, where $\sigma=\pm 1$. The Hamiltonian structure of the equation helps us to rewrite the LN equation as $\hat{\epsilon}[u_xu_{xxx}-\frac{1}{2}u_{xx}^2]+\frac{1}{2}u_x^2+\sigma u^4/12+\frac{1}{2}\alpha u^2+\beta u+\gamma=0$ where γ is a new constant of integration. Assuming $u_x^2=y(u)$, the LN equation can be written as

$$\hat{\epsilon}[yy''-y'^2/4]+y+u^4/6+\alpha u^2+2\beta u+2\gamma=0. \quad (4.2)$$

Now we seek a solution[15] for y in the form $y(u)=u_x^2=Au^3+Bu^2+Cu+D$, which on substituting in Eq. (4.2) and collecting the coefficients of different powers of u gives on solving them $A^2=-2\sigma/45\hat{\epsilon}$, $B=-1/5\hat{\epsilon}$, $AC=2\alpha/9\hat{\epsilon}+8/225\hat{\epsilon}^2$, $D=-10\gamma/3+5\hat{\epsilon}C^2/12$, and $\beta=3\hat{\epsilon}AD+2C/5$. In order that the above two parameter (α and β) families of solutions exist, $\sigma\hat{\epsilon}<0$. On choosing $\beta=\gamma=0$, $C=D=0$, and $\alpha=-4/25\hat{\epsilon}$ we obtain $u_x^2=Au^3-\hat{\epsilon}u^2/5$, which when $\hat{\epsilon}=-1$ gives the localized solitary-wave solution $u=(\pm 3/\sqrt{10})\mathrm{sech}^2(x/2\sqrt{5})$, which is plotted for the upper sign in Fig. 2.

V. CONCLUSIONS

In this paper, we investigated the nonlinear spin dynamics of one-dimensional anisotropic continuum Heisenberg weak-ferromagnetic spin chains in the semiclassical limit using the Holstein-Primakoff transformation combined with Glauber's coherent-state representation. The spin dynamics is then found to be governed by a generalized nonlinear model containing at least four completely integrable nonlinear models at different orders of ϵ for particular parametric choices. In all these specific cases, the nonlinear spin excitations are governed by N solitons. In the more general case it was found that the addition of discreteness and DM interaction does not alter the velocity and amplitude of the envelope soliton. We also constructed perturbed soliton solutions at $O(\epsilon^2)$ and $O(\epsilon^3)$ using a multiple-scale perturbation analysis. The results show that the perturbation due to discreteness effect and DM interaction adds only a fluctuation in the tail of the soliton without affecting the coherent structure of it. At $O(\epsilon^5)$ we constructed localized solitary-wave solutions to a linearly perturbed equation of the system.

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