Faraday rotation in a study of charged excitons in $Cd_{1-x}Mn_xTe$

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Interband Faraday rotation is usually interpreted as resulting from a difference in refractive indices caused by the Zeeman effect in a magnetic field. We show that, in the case of transitions due to the creation of excitons in semiconductor quantum wells containing charge carriers, a strong dependence of the line amplitude on the circular polarization in the presence of a magnetic field can also generate Faraday rotation. The spectral dependence of this Faraday rotation is completely different from that in the usual case. We demonstrate that the amplitude-related Faraday effect can dominate the rotation spectrum of charged excitons. We show that the Faraday rotation is particularly useful in studies of excitons in quantum wells with two-dimensional carrier gas where line intensities vary strongly with magnetic field.

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I. INTRODUCTION

Faraday rotation is a classical magneto-optical effect which has also been studied in semiconductors in relation to various types of optical transitions.[1–4] Research on the Faraday rotation is especially attractive due to possible applications in optical insulators. Usually the rotation spectra are described assuming that a magnetic field induces a splitting of the circularly polarized transitions, but changes neither the oscillator strength nor the linewidth of each component. Such an assumption was made in Ref. 4 in the case of the excitonic transition in CdTe-$Cd_{1-x}Mn_xTe$ quantum wells, and good agreement was found between transmission data and Faraday rotation data. However, under special conditions, the components of, say, an absorption line observed in the two circular polarizations can differ not only in their energy positions, but also in their intensities and linewidths.[5] Then a completely different spectrum of the resulting Faraday rotation can be expected. Such a possibility in a reflection configuration (magneto-optical Kerr effect) was considered by Buda *et al.*[6] for CdTe/$Cd_{1-x}Mn_xTe$ superlattices. However, within the approximations they used, the amplitude variation could only modify the Kerr rotation but not generate it.

A particularly interesting case of intensity-related Faraday effect has been observed for charged excitons in quantum wells in the present work. Charged excitons, predicted theoretically in the 1950s,[7] became a very hot topic in 1993 when their existence in quantum well structures was experimentally confirmed.[8] One of the important issues in studies of charged excitons is the oscillator strength of excitonic lines and its variation under influence of magnetic field, temperature changes, etc.[9] As pointed out in Ref. 9, application of a low magnetic field is essential to separate occupation effects from the influence of magnetic field on excitonic wave functions (occurring at higher fields). The Faraday rotation method used for highly sensitive, quantitative line intensity studies even at very low magnetic fields is capable of giving considerable insight into these problems. To obtain a strong influence of a magnetic field on optical transitions, we used semimagnetic semiconductors known for their giant magneto-optical effects.[10] We are convinced that Kramers-Kronig consistency between transmission and Faraday rotation (or reflectivity and Kerr rotation) is an important test of the experimental results. Such tests have been performed in transmission[4] and reflectivity[11] geometry for quantum wells without carriers. In this work we handle quantum wells with carriers that strongly modify the optical properties in the excitonic region. We find that the experimental spectra of $Cd_{1-x}Mn_xTe$-$Cd_{1-x-y}Zn_yMg_xTe$ quantum wells with an electron or hole two-dimensional gas of low density, observed in both transmission and Faraday rotation, can be reproduced only if the model incorporates both the splitting and a change of the oscillator strengths.

II. SAMPLES

Two specially designed samples were used in order to assure three different regimes of experimental conditions: a quantum well intentionally without carriers, a quantum well containing a hole gas, and one containing an electron gas. In the case of the ''empty'' quantum well the only optical transition is related to neutral exciton formation. In the two other cases the carrier concentration was chosen such that both the neutral and the charged exciton transitions were observable. The material of the quantum well ($Cd_{1-x}Mn_xTe$) was chosen because of the giant Zeeman splitting, which ensures a

significant influence of a small magnetic field on optical transitions. The quantum well width (80 Å) and the low Mn concentration ($x<0.01$) assure a very small line broadening resulting from chemical disorder in the mixed material and from the interface roughness. At the same time, the Mn molar fraction is high enough to produce a significant Zeeman splitting (over ~ 5 meV) in a magnetic field of 5 T and at a temperature of 1.5 K.

Both samples were grown by molecular beam epitaxy (MBE). Sample A was grown in a MBE chamber equipped with a home-designed electron cyclotron resonance plasma cell as a nitrogen source for *p*-type doping.[12] The modulation *p*-doped structure studied consists of a single quantum well (QW) of $Cd_{1-x}Mn_xTe$ ($x=0.0018$) embedded between $Cd_{0.66}Mg_{0.27}Zn_{0.07}Te$ barriers grown pseudomorphically on a (100) $Cd_{0.88}Zn_{0.12}Te$ substrate. Such a layout provides a large confinement energy for the holes in the quantum well, at the same time minimizing the effects of the lattice mismatch. Nitrogen doped regions in both barriers are located at a distance of 700 Å from the QW. Furthermore, in order to reduce depletion effects, two additional nitrogen doped layers reside at a distance of 1000 Å from the QW on both sides. The equilibrium hole concentration in the doped structure is of the order of 10^{11} cm^{-2}, as estimated from the nominal structure parameters. The concentration of the hole gas was controlled by additional illumination with light of energy higher than the energy gap of the barrier material. Such illumination decreases the concentration of the hole gas in the quantum well. The mechanism of the suppression effect is based on the diffusion of electrons excited by the light to the conduction band of the barrier. These electrons neutralize the hole gas and under continuous illumination the system approaches a steady state with a reduced hole gas concentration (see Ref. 9 for details). This process is so efficient that under moderate illumination by a halogen lamp no transition related to charged exciton formation is observed.

Negatively charged excitons X^- were studied in sample B. In this sample, the quantum well made of $Cd_{1-x}Mn_xTe$ was placed between $Cd_{1-y}Mg_yTe$ barriers ($x=0.0064$, $y=0.24$). Remote *n*-type doping in the barrier (separated from the QW by a 400 Å spacer layer) with iodine ions supplied electrons to the well. This sequence (the well and the doped layer) was repeated six times in the sample. The steplike profile of the width of the doped layer resulted in discrete changes of carrier concentrations along the sample. Such a design allowed us to pick up regions with different electron concentration by moving the light beam along the sample.[13,14] The electron concentration in the region studied in this work was of the order of 10^{10} cm^{-2}, as estimated from the nominal structure parameters.

III. EXPERIMENT

The optical transmission was measured in a conventional setup with a halogen lamp and a 0.25 m grating monochromator with a charge-coupled device (CCD) detector. Samples were mounted strain-free and immersed in superfluid helium. Two linear polarizers were placed in front of and behind the sample (outside the cryostat). Spectra for different relative angles between the polarizers (close to the angle of minimum transmission) were recorded. Usually we measured nine spectra with an angular step of 1°. The minimum transmission angle for each wavelength was determined by fitting a parabola (approximating cosine squared dependence close to its extremum) to the angular dependence of transmission. This procedure is not sensitive to differences in the sensitivity between the columns of the CCD matrix corresponding to different wavelengths. The cold windows of the cryostat were mounted stress-free, assuring a low level of stress-induced birefringence, corresponding to a phase shift below 3×10^{-2} rad between the linear polarizations parallel and perpendicular to the stress direction. We checked that this low stress-induced birefringence did not introduce any significant perturbation of the measured rotation angle. Nevertheless, all the measurements were performed for light polarization close to the strain direction, to obtain the highest ratio of intensities between the crossed and parallel polarizers (about 5×10^{-5}). For best accuracy, the Faraday rotation angle for each wavelength was determined as one-half of the difference between the minimum transmission angles in the case of two opposite directions of the magnetic field. In this way we also get rid of possible sample birefringence, which was not very important for the present samples but can be strongly wavelength dependent in the excitonic region for axial crystals. The presented procedure allows us to obtain a relative accuracy of Faraday rotation angle better than $(10^{-2})°$ (10^{-4} rad) even though the accuracy of a single positioning of a polarizer was about 0.15°. That is comparable with the typical sensitivity of the modulation technique (see, e.g., Krenn *et al.*[15]). This high relative accuracy of the present method comes from the fact that at a given polarizer angle we collect the whole spectrum, so any mispositioning of the polarizer causes systematic errors that are approximately the same for the whole spectrum. We prefer this ''absolute'' method to modulation techniques, which are based on intensity measurements and require a calibration.

Our measurements were limited to a low magnetic field range $B\leqslant 50$ mT. We carefully checked that within the range used in the measurements Faraday rotation was precisely linear with changing magnetic field.

IV. MODEL

To describe the Faraday rotation at energies close to the energy of the excitonic transition we use a simple model assuming a Lorentzian energy dependence of the complex dielectric function ϵ:

$$\epsilon(E)=\epsilon_\infty+\frac{A}{E_0-E-i\Gamma}, \tag{1}$$

where E_0 is the resonance energy and A and Γ are the amplitude and the width of resonance, respectively. The constant ϵ_∞ was determined from the low energy value of the refraction coefficient in CdTe,[16] equal to 2.7 at $\lambda=2.5$ μm, and yielding $\epsilon_\infty=7.29$. The real refractive index n and the extinction coefficient κ were calculated to satisfy the well

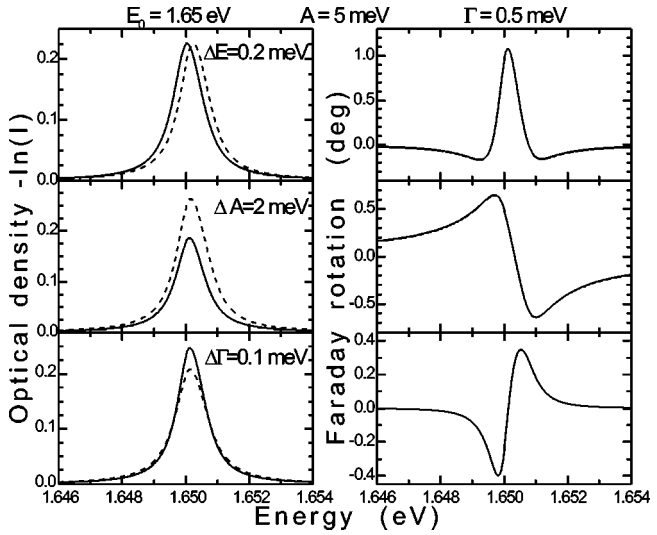

FIG. 1. Theoretical Faraday rotation spectra (right panel) and corresponding absorption spectra for two circular polarizations (left panel), resulting from the Zeeman splitting (upper plots), the amplitude variation (middle plots), and the linewidth variation (lower plots). For the two upper plots realistic parameters were used (for magnetic fields of the order of 0.05 T). The range of $\Delta\Gamma$ in the lowest plot is exaggerated.

known relation between the complex refractive index $n+i\kappa$ and the dielectric function ϵ,

$$\epsilon(E)=[n(E)+i\kappa(E)]^2. \tag{2}$$

The electronic bands split in an applied magnetic field into different spin subbands, resulting in the splitting of the absorption lines observed in two circular polarizations σ^+ and σ^-. In this case one obtains Faraday rotation of the polarization plane by an angle

$$\Theta(E)=\frac{Ed}{2\hbar c}[n_+(E)-n_-(E)], \tag{3}$$

where $n_\pm$ is the refractive index for circular polarizations σ^+ and σ^-, respectively, and d is the layer thickness. In this work we adopt an approximation neglecting multiple reflections in the sample. A correct description of transmission and/or reflectivity including interference effects requires a detailed knowledge of the sample structure[17] including thickness of all the layers of the sample and their optical constants. Such parameters are not available with sufficient precision for real structures. We justify this approach in the spirit of general linear response theory[18] by noting that the logarithm of transmission T is linked to the doubled phase Φ of the transmitted light by Kramers-Kronig (KK) relations independently of the interference effects in the sample. This comes from the fact that the logarithm of transmission and the doubled phase are real and imaginary parts of the doubled logarithm of the amplitude transmission coefficient t. The precise form of the KK relations will depend on the symmetry of the $\log t(\omega)$ function, which in the case of a linear response is routinely deduced from the causality principle.[19] However, in the case of a narrow spectral structure this symmetry has little practical importance (we do not respect it in the Lorentzian model for the sake of simplicity). Summarizing, there is no doubt that the transmitted light is often strongly influenced by multiple reflections. However, the relation between the transmission coefficient and the phase of the transmitted light (and therefore the Faraday rotation, equal to one-half of the phase difference) should not depend significantly on the details of the sample structure, at least for narrow spectral structures. This justifies the assumed approximation, which will be tested experimentally in what follows.

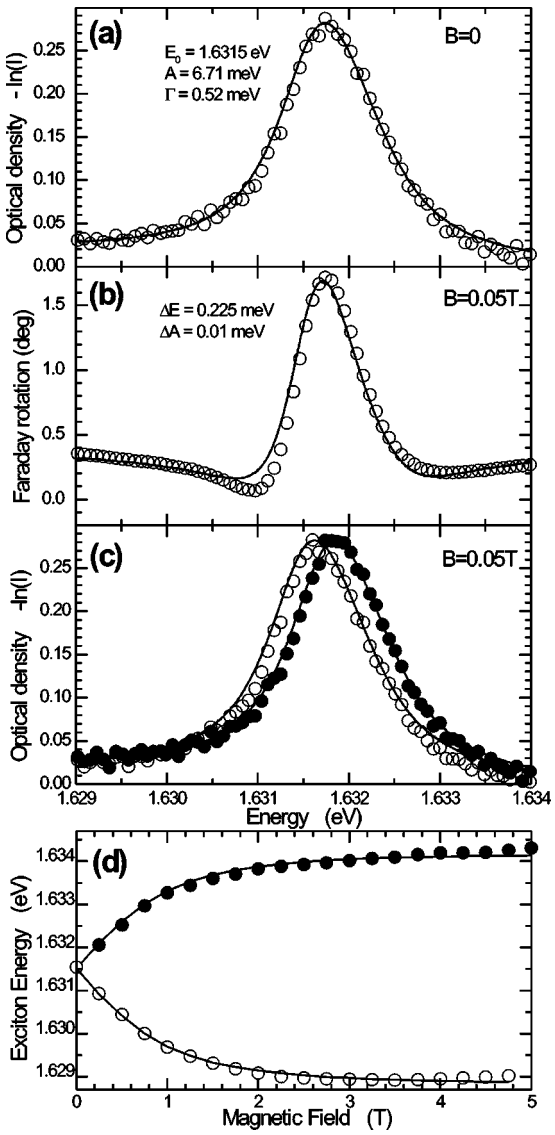

FIG. 2. Sample A under strong illumination at $T=1.6$ K. (a) Experimental absorption at $B=0$. Solid line represents the fit of the absorption model to the data (the parameters of the fit are displayed in the figure). (b) Experimental Faraday rotation (circles) at $B=0.05$ T. Fit to the data with ΔA and ΔE_0 as free parameters is displayed as the solid line. (c) Experimental absorption in two circular polarizations (σ^+ open circles; σ^- full circles) at $B=0.05$ T. Solid lines represent calculated absorption within a model described in the text with parameters obtained from fits presented in (a) and (b). (d) Field dependence of exciton energies obtained from transmission spectra in two circular polarizations. Solid lines represent a fit with the modified Brillouin function.

TABLE I. Parameters of the fit to the experimental data described in the text. Measurements on 80 Å wide QW in sample A were performed at $B=0.05$ T; the data for sample B containing six identical 80 Å wide QW's were taken at $B=0.025$ T.

Sample	E_0 (meV)	A (meV)	Γ (meV)	ΔE_0 (meV)	ΔA (meV)
			Neutral exciton		
A (strong illumination)	1631.5	6.71	0.52	0.225	0.01
A (moderate illumination)	1631.6	4.35	0.74	0.246	−1.19
B	1652.2	7.75	3.60	0.259	−0.03
			Charged exciton		
A (strong illumination)					
A (moderate illumination)	1629.1	1.55	0.66	0.082	1.46
B	1646.6	3.61	1.68	0.149	0.22

If the dielectric constant remains Lorentzian in the magnetic field, Faraday rotation may be caused by a change of the resonance energy (Zeeman splitting), a change of the resonance amplitude, or a change of its linewidth. Examples of spectra resulting from different contributions are presented in Fig. 1. Most characteristic is the shape of the spectrum of Faraday rotation resulting from the Zeeman splitting. The spectrum is symmetric and dominated by a strong line with a maximum at the resonance energy. The two other mechanisms lead to asymmetric shapes. We assumed further that the dielectric constant for both polarizations remains Lorentzian, and only the resonance energy and the amplitude of resonance may change. As we show below, changes of the linewidth can be neglected, which reduces the number of parameters.

V. RESULTS

The first experiment was designed to check whether a fit to the Faraday rotation given by Eq. (3) renders a correct value of the Zeeman splitting. We chose experimental conditions, such that only the line related to neutral exciton formation was observed in transmission spectra. In such a case no amplitude variation is expected in magnetic fields.[4] Figure 2(a) shows a zero-field absorption spectrum of sample A under strong above-the-barrier illumination, when most of the *p*-type carriers were neutralized. Only a neutral exciton absorption line is observable. A fit of the absorption model

$$\alpha(E)d=\frac{2Ed}{c\hbar}\kappa(E) \tag{4}$$

shown in the figure describes the experimental data reasonably well. A linear background was additionally included in the fitting function. The line amplitude, the energy, and the width (parameters A, E_0, and Γ, respectively) obtained from the transmission spectra (see Table I) were used for the fit of the Faraday rotation. Opposite circular polarizations are equivalent to opposite signs of the magnetic field. Therefore within the linear approximation (tested experimentally in our case) variation of the line parameters is equal in magnitude and opposite in sign for the two polarizations. Thus the only remaining free parameters fitted to the experimental Faraday rotation [Fig. 2(b)] were the Zeeman splitting ($\Delta E=E_{0-}-E_{0+}$), the intensity variation ($\Delta A=A_{-}-A_{+}$), and a constant background, the last resulting from sample absorption unrelated to the absorption line considered as well as due to the Faraday rotation by the cryostat windows. Figure 2(c) compares the experimental absorption in two circular polarizations and the absorption model calculated with the parameters obtained from the zero-field transmission and the Faraday rotation fit (no additional adjustments were made). Our calculation reproduces precisely both absorption spectra even though the model does not reproduce perfectly the shape of the Faraday rotation spectrum.

The Zeeman splitting obtained from the Faraday spectra was precisely checked by performing a transmission measurement at higher fields (up to 5 T). In the case of a neutral exciton transition in the $Cd_{1-x}Mn_xTe$ quantum well studied the Zeeman splitting is described by a modified Brillouin function[20] in the whole range of magnetic field. The results of transmission measurements are presented in Fig. 2(d). A fit of high field data gives a Zeeman splitting at $B=0.05$ T equal to 0.238 ± 0.002 meV, which corresponds well to the parameter of the Faraday fit (0.225 meV). The difference of 13 μ eV between the two values can be regarded as an estimate of the precision of the Faraday rotation method. In the case of direct absorption measurements at the same low field the accuracy of the Zeeman splitting cannot be comparable, being a difference in energetic positions of two lines at least 1000 μ eV wide. Similarly, the analysis of Faraday rotation is superior with respect to the determination of amplitude variation in a magnetic field. We are able to demonstrate this high precision of the Faraday rotation method only for experiments performed on a single quantum well.

The observed symmetric shape of the Faraday rotation spectrum is characteristic for the case when the resonance energy splits but there is no change of the absorption amplitude. The procedure allows us to determine the energy splitting using the zero-field absorption and the Faraday rotation spectrum. We would like to point out that our simple model produces correct results for Zeeman splittings and relative intensity changes without taking into account interference effects in the structure (see Ref. 21 for details). This means that these effects modify the transmission and the Faraday rotation spectra in a very similar way. Our test has been

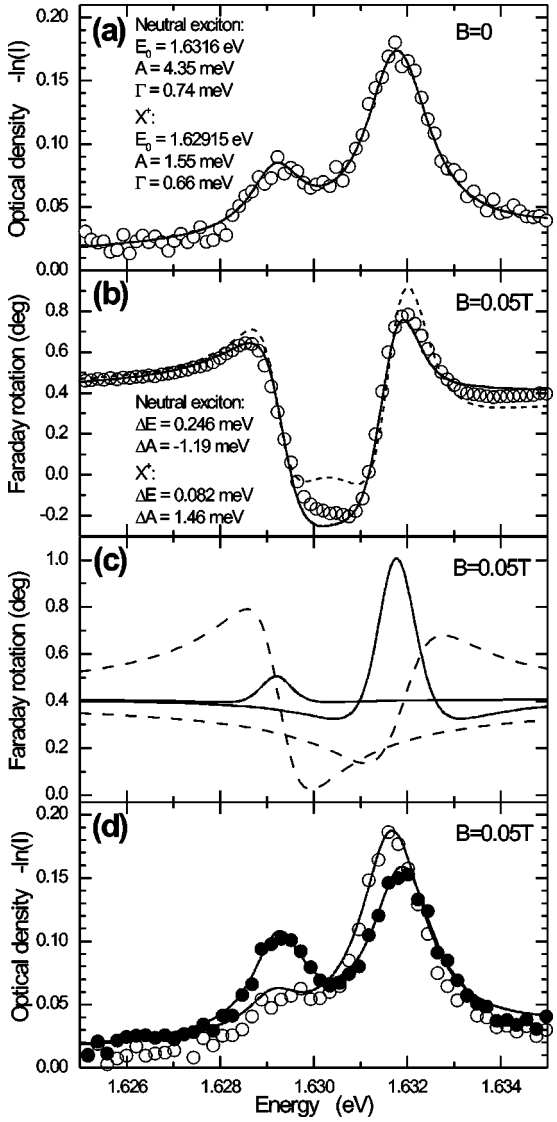

FIG. 3. Sample A under moderate illumination at $T=1.6$ K. (a) Experimental absorption (circles) at $B=0$. The solid line represents a fit of the absorption model to the data (parameters of the fit are displayed in the figure). (b) Experimental Faraday rotation (circles) at $B=0.05$ T. Solid line represents fit to the data with ΔA andΔE as free parameters for both lines. Dashed line represents fit with all ΔA, ΔE, and $\Delta\Gamma$ parameters free except ΔA for neutral exciton line which was set to zero. (c) Components of the Faraday rotation resulting from the intensity changes (dashed lines) and the Zeeman effect (solid lines) for both absorption lines. (d) Experimental absorption in two circular polarizations (σ^+, open circles; σ^-, full circles) at $B=0.05$ T. Solid lines represent absorption calculated within a model described in the text with parameters obtained from fits presented in (a) and (b).

successfully applied to structures where interference effects are known to modify the apparent line intensities by a factor from 0.5 to 1.5.

The Faraday rotation method was used to study the behavior of positively charged excitons in coexistence with neutral excitons. Figure 3(a) shows the zero-field absorption of sample A under intermediate above-the-barrier illumination, when some holes remain in the quantum well and allow formation of X^+ complexes. The two observed lines are related to X^+ (at the lower energy) and the neutral exciton (at the higher energy). Figure 3(a) shows also a fit of two absorption peaks to the data [the second absorption line was added in the dielectric function Eq. (1)]. Figure 3(b) shows the Faraday rotation spectrum obtained from the sample. The shape of the Faraday rotation spectrum is essentially different from that expected for a sum of two symmetric structures resulting only from the splitting of the transition energy. As in the case of a single line, the parameters resulting from a fit to the zero-field transmission (energies, amplitudes and widths of both lines) were further used to fit the Faraday rotation. The solid line in Fig. 3(b) represents a fit with the energy splitting and the amplitude change for each of the lines as four free parameters. We stress here that it is not possible to describe the experimental data without assuming changes of the neutral exciton intensities (oscillator strength), and even combined effects related to the linewidth change and the Zeeman splitting are not sufficient. To demonstrate this fact we present in Fig. 3(b) as a dashed line the results of a fit with all ΔA, ΔE, and $\Delta\Gamma$ parameters set free except for the amplitude changes (ΔA) for the neutral exciton line, which was arbitrary set to zero. Clearly the dashed line is a much worse fit to the experimental data much worse than the solid one. In further discussion we assume that the parameters describing our data have to include ΔA and ΔE for both lines and that the linewidth changes are negligible. Figure 3(c) demonstrates the four components of the Faraday rotation resulting from each of the fitted parameters separately (either from the energy splitting or from the amplitude change). Each component was calculated assuming that only one of the four parameters describing the absorption variation differs from zero. It is clearly visible that the contribution from the ''amplitude'' components is significant in this case. Figure 3(d) displays results of the absorption model calculations for the two circular polarizations compared to the experimental absorption values. The very good agreement between the model and the experiment indicates that the method allows us to derive exact quantitative information about the energy splitting and the amplitude change from a Faraday rotation experiment. It is worth noting that the neutral exciton splitting does not depend on the concentration of the charge carriers.[9] Indeed, the neutral exciton splittings obtained for both illumination intensities differ only by about 3%, which means that a fit of the Faraday rotation even for two close lines that both split and change their amplitude is very reliable. It is clearly visible that, unlike to the case when the charged excitons were absent, there is a strong change of the amplitudes of both lines. This can be conveniently seen in Fig. 3(d). One can also see the opposite sign of the amplitude change for the X^+ line and for the neutral exciton line, in agreement with the idea of intensity ''stealing.''[8]

Sample B with a high electron concentration revealed a strong absorption related to negatively charged excitons (X^-). Figure 4(a) displays results of the Faraday rotation experiment together with a fit of the model (parameters describing the resonance energy, the amplitude, and the width of both lines were obtained from a fit to zero-field absorp-

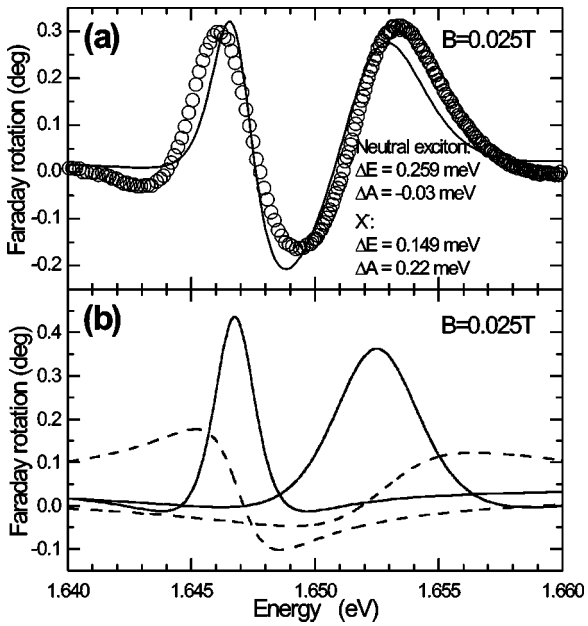

FIG. 4. (a) Experimental Faraday rotation (circles) in sample B at $T=1.8$ K and $B=0.025$ T. The fit to the data is displayed as the solid line. (b) Components of the Faraday rotation resulting from intensity changes (dashed lines) and the Zeeman effect (solid lines) for both absorption lines.

tion). The Zeeman splitting obtained of the neutral exciton (0.259 meV) is in reasonable agreement with the value of 0.307 ± 0.005 meV independently obtained from a fit to high field data. This quantitative agreement is achieved although the shape of the theoretical curve does not match the experimental data as perfectly as in the case of X^+. The observed discrepancy may originate from the fact that sample B contains a multiple quantum well with slightly different parameters of individual quantum wells. These differences do not allow us to exploit the full precision of the method in this case. The four components of the Faraday rotation spectrum are presented in Fig. 4(b). In this case, the components related to the amplitude change are not as important as in the case of X^+. This reflects a difference in the conduction and valence band splittings (the ratio of exchange constants for the conduction and the valence band $|\alpha/\beta|=1/4$; Ref. 22) that are essential for the spin polarization of carriers contributing to the formation of charged excitons.

VI. CONCLUSIONS

We have shown that Faraday rotation measurements on semiconductor quantum wells provide quantitative information not only on the Zeeman splitting values but also on the line intensity variation with a magnetic field. In particular we confirm a significant intensity variation of both neutral and charged exciton lines in doped quantum wells.[9] The precision of the results obtained is much better than in the case of direct absorption measurements performed at the same (low) value of the magnetic field. In this paper we presented data obtained in relatively high magnetic fields, which allowed us to compare the results obtained from the Faraday rotation experiment with directly measured polarized absorption. We have shown that the quantitative accuracy of the determination of contributions due to energy splitting and line intensity changes on the basis of the Faraday rotation fits allows studies in magnetic fields lower by an order of magnitude (low fields may be of great importance in charged exciton studies). The Faraday rotation method presented can be also applied for materials in which the actual splitting or amplitude changes are very small (very dilute or nonmagnetic systems).

ACKNOWLEDGMENTS

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