Andreev tunneling, Coulomb blockade, and resonant transport of nonlocal spin-entangled electrons

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We propose and analyze a spin-entangler for electrons based on an *s*-wave superconductor coupled to two quantum dots, each of which is coupled to normal Fermi leads. We show that in the presence of a voltage bias and in the Coulomb blockade regime two correlated electrons provided by the Andreev process can coherently tunnel from the superconductor via different dots into different leads. The spin singlet coming from the Cooper pair remains preserved in this process, and the setup provides a source of mobile and nonlocal spin-entangled electrons. The transport current is calculated and shown to be dominated by a two-particle Breit-Wigner resonance that allows the injection of two spin-entangled electrons into different leads at exactly the same orbital energy, which is a crucial requirement for the detection of spin entanglement via noise measurements. The coherent tunneling of both electrons into the same lead is suppressed by the on-site Coulomb repulsion and/or the superconducting gap, while the tunneling into different leads is suppressed through the initial separation of the tunneling electrons. In the regime of interest the particle-hole excitations of the leads are shown to be negligible. The Aharonov-Bohm oscillations in the current are shown to contain single- and two-electron periods with amplitudes that both vanish with increasing Coulomb repulsion albeit differently fast.

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I. INTRODUCTION

The creation of nonlocal pairwise-entangled quantum states, so-called Einstein-Podolsky-Rosen (EPR) pairs,[1] is essential for secure quantum communication,[2] dense coding and quantum teleportation,[3] or more fundamental, for testing the violation of Bell's inequality.[4] Such tests already exist for photons but not yet for *massive* particles such as electrons since it is difficult to produce and to detect entangled electrons. However, there is strong experimental evidence that electron spins in a semiconductor environment show unusually long dephasing times approaching microseconds and that they can be transported phase coherently over distances exceeding 100 μm.[5–10] This makes spins of electrons in semiconductors promising candidates for carriers of quantum information (qubits).[11,12] In particular, we have recently proposed a setup[13] consisting of a spin-entangler and a beam splitter where the spin entanglement is detectable via electronic transport properties. We have shown that the current-current correlations (noise) are enhanced if the entangled electrons are spin singlets leading to bunching behavior, whereas the noise is suppressed for spin triplets leading to antibunching behavior.

In Ref. 13 we assumed the existence of an entangler, i.e., a device that generates spin singlets that are made out of two electrons that reside in different but degenerate orbital states, and we focused on the question of how to detect spin-entangled electrons via transport and noise measurements. Here, we address the problem of how to implement such an entangler in a solid state device. We have found[13] that for such noise measurements, which are based on two-particle interference effects, it is absolutely crucial that both electrons, coming from different leads, possess the *same orbital energy*. If the orbital energies of the two entangled electrons are different, the electrons cannot interfere with each other, and thus spin correlations would not be observable in the noise.[13]

In the following we propose a setup that involves a superconductor coupled to two quantum dots, which themselves are coupled to normal leads; see Fig. 1. We show that such a setup acts as an entangler that meets all the requirements needed for a successful detection of spin entanglement via noise measurements. In previous work[14] we showed that in equilibrium the spin correlations of an *s*-wave superconductor induce a spin-singlet state between two electrons, each of which resides on a separate quantum dot that both are weakly coupled to the same superconductor (but not among themselves). This nonlocal spin entanglement leads then to observable effects in a generalized Josephson junction setup.[14] In the present work we consider a *nonequilibrium* situation where an applied voltage bias drives a stationary current of pairwise spin-entangled electrons from the superconductor through the quantum dots into the leads; see Figs. 1 and 2.

II. QUALITATIVE DESCRIPTION OF THE ANDREEV ENTANGLER

We begin with a qualitative description of the entangler and its principal mechanism based on Andreev processes and Coulomb blockade effects. In subsequent sections we introduce the Hamiltonian and calculate the stationary current in detail. We consider an *s*-wave superconductor that acts as a natural source of spin-entangled electrons, since the electrons form Cooper pairs with singlet spin wave functions.[15] The superconductor, which is held at the chemical potential μ_S, is weakly coupled by tunnel barriers to two separate quantum dots D_1 and D_2, which themselves are weakly coupled to Fermi liquid leads L_1 and L_2, respectively, both held at the

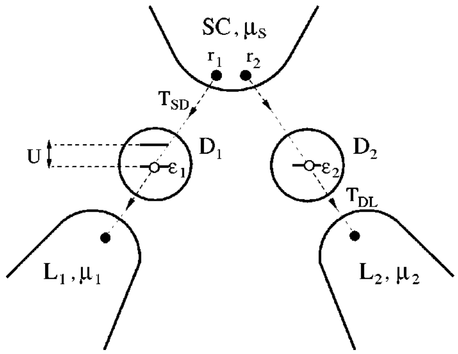

FIG. 1. The entangler setup: Two spin-entangled electrons forming a Cooper pair can tunnel with amplitude T_{SD} from points $\mathbf{r}_1$ and $\mathbf{r}_2$ of the superconductor, SC, to two dots, D_1 and D_2, by means of Andreev tunneling. The dots coupled to normal leads L_1 and L_2, with tunneling amplitude T_{DL}. The superconductor and leads are kept at chemical potentials μ_S and μ_l, respectively.

same chemical potential $\mu_1=\mu_2$. The corresponding tunneling amplitudes between superconductor and dots and between dots and leads are denoted by T_{SD} and T_{DL}, respectively (for simplicity we assume them to be equal for both dots and leads).

In general, the tunnel coupling of a superconductor to a normal region allows for coherent transport of two electrons of opposite spins due to Andreev tunneling,[15] while single-electron tunneling is suppressed.[16] In the present setup, we envision a situation where the two electrons are forced to tunnel coherently into *different* leads rather than both into the same lead. This situation can be enforced in the presence of two intermediate quantum dots that are assumed to be in the Coulomb blockade regime[17] so that the state with the two electrons being on the same quantum dot is strongly suppressed, and thus the electrons will preferably tunnel into separate dots and subsequently into separate leads (this will be quantified in the following).

By applying a bias voltage $\Delta\mu=\mu_S-\mu_l>0$ transport of entangled electrons occurs from the superconductor via the dots to the leads. The chemical potentials ϵ_1 and ϵ_2 of the quantum dots can be tuned by external gate voltages[17] such that the coherent tunneling of two electrons into different leads is at resonance, described by a two-particle Breit-Wigner resonance peaked at $\epsilon_1+\epsilon_2=2\mu_S$. In contrast, we will see that the current for the coherent tunneling of two electrons into the *same* lead is suppressed by the on-site Coulomb U repulsion of a quantum dot and/or by the superconducting gap Δ.

Next, we introduce the relevant parameters describing the proposed device and specify their regime of interest. First we note that to avoid unwanted correlations with electrons already on the quantum dots one could work in the cotunneling regime[17] where the number of electrons on the dots are fixed and the resonant levels ϵ_l, $l=1,2$ cannot be occupied. However, we prefer to work at the resonances $\epsilon_l\simeq\mu_S$, since then the total current and the desired suppression of tunneling into the same lead is maximized in this regime. Also, the desired injection of the two electrons into different leads but at the *same orbital energy* is then achieved. It turns out to be most efficient to work in the regime where the dot levels ϵ_l have vanishing occupation probability. For this purpose we require that the dot-lead coupling is much stronger than the superconductor-dot coupling, i.e., $|T_{SD}|<|T_{DL}|$, so that electrons that enter the dots from the superconductor will leave the quantum dots to the leads much faster than new electrons can be provided from the superconductor. In addition, a stationary occupation due to the coupling to the leads is exponentially small if $\Delta\mu>k_BT$, T being the temperature and k_B the Boltzmann constant. Thus in this asymmetric barrier case, the resonant dot levels ϵ_l can be occupied only during a virtual process.

Next, the quantum dots in the ground state are allowed to contain an arbitrary but even number of electrons, N_D $=$even, with total spin zero (i.e., antiferromagnetic filling of the dots). An odd number N_D must be excluded since a simple spin flip on the quantum dot would be possible in the transport process and as a result the desired entanglement would be lost. Further, we have to make sure that also spin-flip processes of the following kind are excluded. Consider an electron that tunnels from the superconductor into a given dot. Now, it is possible in principle (e.g., in a sequential tunneling process[17]) that another electron with the opposite spin leaves the dot and tunnels into the lead, and, again, the desired entanglement would be lost. However, such spin-flip processes will be excluded if the energy level spacing of the quantum dots, $\delta\epsilon$ (assumed to be similar for both dots) exceeds both temperature k_BT and bias voltage $\Delta\mu$. A serious source of entanglement loss is given by electron-hole pair excitations out of the Fermi sea of the leads during the resonant tunneling events. However, we show in the following that such many-particle contributions are suppressed if the resonance width $\gamma_l=2\pi\nu_l|T_{DL}|^2$ is smaller than $\Delta\mu$ (for

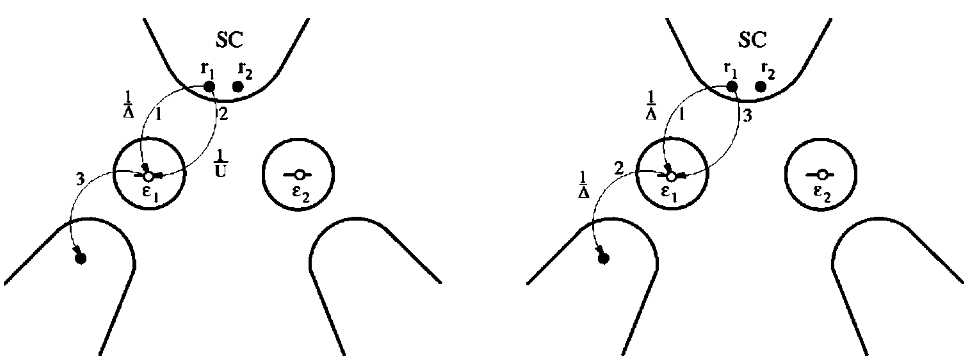

FIG. 2. Two competing virtual processes are shown when the two electrons tunnel via the same dot. The left panel shows an Andreev process leading to a double occupancy of the dot with virtual energy $1/U$ [process (I)]. The process on the right differs by the sequence of tunneling, leading to an additional virtual energy $1/\Delta$ instead of $1/U$ [process (II)].

$\epsilon_l \simeq \mu_S$), where ν_l is the density of states (DOS) per spin of the leads at the chemical potential μ_l.

Finally, an additional energy scale that enters the consideration is the superconducting gap energy Δ, which is half the minimum energy it costs to break up a Cooper pair into two quasiparticles. This gap energy also characterizes the time delay between the subsequent coherent Andreev tunneling events of the two electrons of a Cooper pair. In order to exclude single-electron tunneling where the creation of a quasiparticle in the superconductor is a final excited state we require that $\Delta > \Delta\mu, k_B T$.

To summarize, the regime of interest in this work is then given by

$$\Delta, U, \delta\epsilon > \Delta\mu > \gamma_l, k_B T, \quad \gamma_l > \gamma_S. \tag{1}$$

Some inequalities will become clear when we discuss the various processes in detail below. As regards possible experimental implementations of the proposed setup and its parameter regime, we would like to mention that typically quantum dots are made out of semiconducting heterostructures that satisfy the above-noted inequalities.[17] Furthermore, in recent experiments, it has been shown that the fabrication of hybrid structures with semiconductor and superconductor being coupled by tunnel barriers is possible.[18,19] Other candidate materials are, e.g., carbon nanotubes, which also show Coulomb blockade behavior with U and $\delta\epsilon$ being in the regime of interest here.[20] The present work might provide further motivation to implement the structures proposed here.

Our goal in the following is to calculate the stationary charge current of pairwise spin-entangled electrons for two competing transport channels, first for the desired transport of two entangled electrons into different leads (I_1) and second for the unwanted transport of both electrons into the same lead (I_2). We compare then the two competing processes and show how their ratio, I_1/I_2, depends on the various system parameters and how it can be made large. An important finding is that when tunneling of two electrons into different leads occurs, the current is suppressed due to the fact that tunneling into the dots will typically take place from different points $\mathbf{r}_1$ and $\mathbf{r}_2$ on the superconductor (see Fig. 1) due to the spatial separation of the dots D_1 and D_2. We show that the distance of separation $\delta r = |\mathbf{r}_1 - \mathbf{r}_2|$ leads to an exponential suppression of the current via different dots if $\delta r > \xi$ [see Eq. (20)], where ξ is the coherence length of a Cooper pair. In the relevant regime, $\delta r < \xi$, however, the suppression is only polynomial and $\propto 1/(k_F \delta r)^2$, with k_F being the Fermi wave vector in the superconductor. On the other hand, tunneling via the same dot implies $\delta r = 0$, but suffers a suppression due to U and/or Δ. The suppression of this current is given by the small parameter $(\gamma_l/U)^2$ in the case $U < \Delta$, or by $(\gamma_l/\Delta)^2$, if $U > \Delta$ as will be derived in the following. Thus, to maximize the efficiency of the entangler, we also require $k_F \delta r < \Delta/\gamma_l, U/\gamma_l$. Finally, we will discuss the effect of a magnetic flux on the entangled current in an Aharonov-Bohm loop, and we will see that this current contains both single- and two-particle Aharonov-Bohm periods whose amplitudes have different parameter dependences.

III. HAMILTONIAN OF THE ANDREEV ENTANGLER

We use a tunneling Hamiltonian description of the system, $H = H_0 + H_T$, where

$$H_0 = H_S + \sum_l H_{Dl} + \sum_l H_{Ll}, \quad l = 1,2. \tag{2}$$

Here, the superconductor is described by the BCS Hamiltonian[15] $H_S = \sum_{\mathbf{k},\sigma} E_{\mathbf{k}} \gamma^\dagger_{\mathbf{k}\sigma} \gamma_{\mathbf{k}\sigma}$, where $\sigma = \uparrow, \downarrow$, and the quasiparticle operators $\gamma_{\mathbf{k}\sigma}$ describe excitations out of the BCS ground state $|0\rangle_S$ defined by $\gamma_{\mathbf{k}\sigma}|0\rangle_S = 0$. They are related to the electron annihilation and creation operators $c_{\mathbf{k}\sigma}$ and $c^\dagger_{\mathbf{k}\sigma}$ through the Bogoliubov transformation[15]

$$c_{\mathbf{k}\uparrow} = u_{\mathbf{k}} \gamma_{\mathbf{k}\uparrow} + v_{\mathbf{k}} \gamma^\dagger_{-\mathbf{k}\downarrow}, \tag{3}$$

$$c_{-\mathbf{k}\downarrow} = u_{\mathbf{k}} \gamma_{-\mathbf{k}\downarrow} - v_{\mathbf{k}} \gamma^\dagger_{\mathbf{k}\uparrow},$$

where $u_{\mathbf{k}} = (1/\sqrt{2})(1 + \xi_{\mathbf{k}}/E_{\mathbf{k}})^{1/2}$ and $v_{\mathbf{k}} = (1/\sqrt{2})(1 - \xi_{\mathbf{k}}/E_{\mathbf{k}})^{1/2}$ are the usual BCS coherence factors,[15] $\xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu_S$ is the normal state single-electron energy counted from the Fermi level μ_S, and $E_{\mathbf{k}} = \sqrt{\xi^2_{\mathbf{k}} + \Delta^2}$ is the quasiparticle energy. We choose energies such that $\mu_S = 0$. Both dots are represented as one localized (spin-degenerate) level with energy ϵ_l and is modeled by an Anderson-type Hamiltonian $H_{Dl} = \epsilon_l \sum_\sigma d^\dagger_{l\sigma} d_{l\sigma} + U n_{l\uparrow} n_{l\downarrow}$, $l = 1,2$. The resonant dot level ϵ_l can be tuned by the gate voltage. Other levels of the dots do not participate in transport if $\delta\epsilon > \Delta\mu > k_B T$, where $\Delta\mu = -\mu_l$, μ_l is the chemical potential of lead $l = 1,2$, and $\delta\epsilon$ is the single-particle energy level spacing of the dots. The leads $l = 1,2$ are assumed to be noninteracting (normal) Fermi liquids, $H_{Ll} = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} a^\dagger_{l\mathbf{k}\sigma} a_{l\mathbf{k}\sigma}$. Tunneling from the dot l to the lead l or to the point $\mathbf{r}_l$ in the superconductor is described by the tunnel Hamiltonian $H_T = H_{SD} + H_{DL}$ with

$$H_{SD} = \sum_{l\sigma} T_{SD} d^\dagger_{l\sigma} \psi_\sigma(\mathbf{r}_l) + \text{H.c.},$$

$$H_{DL} = \sum_{l\mathbf{k}\sigma} T_{DL} a^\dagger_{l\mathbf{k}\sigma} d_{l\sigma} + \text{H.c.} \tag{4}$$

Here, $\psi_\sigma(\mathbf{r}_l)$ annihilates an electron with spin σ at site $\mathbf{r}_l$, and $d^\dagger_{l\sigma}$ creates it again (with the same spin) at dot l with amplitude T_{SD}. $\psi_\sigma(\mathbf{r}_l)$ is related to $c_{\mathbf{k}\sigma}$ by the Fourier transform $\psi_\sigma(\mathbf{r}_l) = \sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}_l} c_{\mathbf{k}\sigma}$. Tunneling from the dot to the state $\mathbf{k}$ in the lead is described by the tunnel amplitude T_{DL}. We assume that the $\mathbf{k}$ dependence of T_{DL} can be safely neglected.

IV. STATIONARY CURRENT AND THE *T* MATRIX

The stationary current of *two* electrons passing from the superconductor via virtual dot states to the leads is given by

$$I=2e\sum_{f,i} W_{fi}\rho_i, \tag{5}$$

where W_{fi} is the transition rate from the superconductor to the leads. We calculate this transition rate in terms of a T-matrix approach,[21]

$$W_{fi}=2\pi|\langle f|T(\varepsilon_i)|i\rangle|^2\delta(\varepsilon_f-\varepsilon_i). \tag{6}$$

Here,

$$T(\varepsilon_i)=H_T\frac{1}{\varepsilon_i+i\eta-H}(\varepsilon_i-H_0)$$

is the on-shell transmission or T matrix, with η being a small positive real number that we take to zero at the end of the calculation. Finally, ρ_i is the stationary occupation probability for the entire system to be in the state $|i\rangle$. The T matrix $T(\varepsilon_i)$ can be written as a power series in the tunnel Hamiltonian H_T,

$$T(\varepsilon_i)=H_T+H_T\sum_{n=1}^{\infty}\left(\frac{1}{\varepsilon_i+i\eta-H_0}H_T\right)^n, \tag{7}$$

where the initial energy is $\varepsilon_i=2\mu_S\equiv 0$. We work in the regime defined in Eq. (1), i.e., $\gamma_l>\gamma_S$, and $\Delta,U,\delta\epsilon>\Delta\mu>\gamma_l,k_BT$, and around the resonance $\epsilon_l\simeq\mu_S$. Further, $\gamma_S=2\pi\nu_S|T_{SD}|^2$ and $\gamma_l=2\pi\nu_l|T_{DL}|^2$ define the tunneling rates between superconductor and dots and between dots and leads, respectively, with ν_S and ν_l being the DOS per spin at the chemical potentials μ_S and μ_l, respectively. We will show that the total effective tunneling rate from the superconductor to the leads is given by γ_S^2/γ_l due to the Andreev process. In the regime (1) the entire tunneling process becomes a two-particle problem where the many-particle effect of the reservoirs (leads) can be safely neglected and the coherence of an initially entangled Cooper pair (spin singlet) is maintained during the transport into the leads as we shall show below. Since the superconducting gap satisfies $\Delta>\Delta\mu,k_BT$, the superconductor contains no quasiparticle initially. Further, in the regime (1), the resonant dot levels ϵ_l are mostly empty, since in the assumed asymmetric case, $|T_{DL}|>|T_{SD}|$ (or $\gamma_l>\gamma_S$), the electron leaves the dot to the lead much faster than it can be replaced by another electron from the superconductor. In addition, we can neglect any stationary occupation of the dots induced by the coupling of the dots to the leads. Indeed, in the stationary limit and for given bias $\Delta\mu$ this occupation probability is determined by the grand canonical distribution function $\propto\exp(-\Delta\mu/k_BT)<1$, and thus $\rho_i\simeq 0$ for any initial state where the resonant dot level is occupied. In this regime, the initial state $|i\rangle$ becomes $|i\rangle=|0\rangle_S|0\rangle_D|\mu_l\rangle_l$, where $|0\rangle_S$ is the quasiparticle vacuum for the superconductor, $|0\rangle_D$ means that both dot levels ϵ_l are unoccupied, and $|\mu_l\rangle_l$ defines the occupation of the leads that are filled with electrons up to the chemical potential μ_l. We remark that in our regime of interest no Kondo effects appear that could destroy the spin entanglement, since our dots contain each an even number of electrons in the stationary limit.

V. CURRENT DUE TO TUNNELING INTO DIFFERENT LEADS

We now calculate the current for simultaneous coherent transport of two electrons into different leads. The final state for two electrons, one of them being in lead 1 the other in lead 2, can be classified according to their total spin S. This spin can be either a singlet (in standard notation) $|S\rangle=(|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle)/\sqrt{2}$ with $S=0$, or a triplet with $S=1$. Since the total spin is conserved, $[\mathbf{S}^2,H]=0$, the singlet state of the initial Cooper pair will be conserved in the transport process and the final state must also be a singlet. That this is so can also be seen explicitly when we allow for the possibility that the final state could be the $S_z=0$ triplet $|t_0\rangle=(|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle)/\sqrt{2}$. (The triplets $|t_+\rangle=|\uparrow\uparrow\rangle$ and $|t_-\rangle=|\downarrow\downarrow\rangle$ can be excluded right away since the tunnel Hamiltonian H_T conserves the spin component σ and an Andreev process involves tunneling of two electrons with different spin σ.) Therefore we consider final two-particle states of the form $|f\rangle=(1/\sqrt{2})[a^\dagger_{1\mathbf{p}\uparrow}a^\dagger_{2\mathbf{q}\downarrow}\pm a^\dagger_{1\mathbf{p}\downarrow}a^\dagger_{2\mathbf{q}\uparrow}]|i\rangle$, where the $-$ and $+$ signs belong to the singlet $|S\rangle$ and triplet $|t_0\rangle$, respectively. Note that this singlet/triplet state is formed out of two electrons, one being in the $\mathbf{p}$ state in lead 1 and with energy $\epsilon_{\mathbf{p}}$, while the other one is in the $\mathbf{q}$ state in lead 2 with energy $\epsilon_{\mathbf{q}}$. Thus, the two electrons are entangled in spin space while separated in orbital space, thereby providing a nonlocal EPR pair. The tunnel process to different leads appears in the following order. A Cooper pair breaks up, where one electron with spin σ tunnels to one of the dots (with empty level ϵ_l) from the point of the superconductor nearest to this dot. This is a virtual state with energy deficit $E_{\mathbf{k}}>\Delta$. Since $\Delta>\gamma_l$, the second electron from the Cooper pair with spin $-\sigma$ tunnels to the other empty dot level *before* the electron with spin σ escapes to the lead. Therefore, both electrons tunnel almost simultaneously to the dots (within the uncertainty time $\hbar/\Delta$). Since we work at the resonance $\epsilon_l\simeq\mu_S=0$ the energy denominators in Eq. (7) show divergences $\propto 1/\eta$, indicating that tunneling between the dots and the leads is resonant and we have to treat tunneling to all orders in H_{DL} in Eq. (7), eventually giving a finite result in which η will be replaced by $\gamma_l/2$. Tunneling back to the superconductor is unlikely since $|T_{SD}|<|T_{DL}|$. We can therefore write the transition amplitude between initial and final state as

$$\langle f|T_0|i\rangle=\frac{1}{\sqrt{2}}\langle a_{2\mathbf{q}\downarrow}a_{1\mathbf{p}\uparrow}T'd^\dagger_{1\uparrow}d^\dagger_{2\downarrow}\rangle\langle[d_{2\downarrow}d_{1\uparrow}\pm d_{2\uparrow}d_{1\downarrow}]T''\rangle, \tag{8}$$

where $T_0=T(\varepsilon_i=0)$, and the partial T matrices T' and T'' are given by

$$T''=\frac{1}{i\eta-H_0}H_{SD}\frac{1}{i\eta-H_0}H_{SD} \tag{9}$$

and

$$T'=H_{DL}\sum_{n=0}^{\infty}\left(\frac{1}{i\eta-H_0}H_{DL}\right)^{2n+1}. \tag{10}$$

In Eq. (8) we used the fact that the matrix element containing T' is invariant under spin exchange $\uparrow\leftrightarrow\downarrow$, and the abbreviation $\langle\cdots\rangle$ stands for $\langle i|\cdots|i\rangle$. The part containing T'' describes the Andreev process, while the part containing T' is the resonant dot $\leftrightarrow$ lead tunneling.

We first consider the Andreev process. We insert a complete set of single-quasiparticle (virtual) states, i.e., $\mathbb{1}=\sum_{l\mathbf{k}\sigma}\gamma^\dagger_{\mathbf{k}\sigma}d^\dagger_{l-\sigma}|i\rangle\langle i|d_{l-\sigma}\gamma_{\mathbf{k}\sigma}$, between the two H_{SD} in Eq. (9) and use the premise that the resulting energy denominator $|i\eta-E_{\mathbf{k}}-\epsilon_l|\approx|E_{\mathbf{k}}|$, since we work close to the resonance $\epsilon_l\simeq 0$ and $E_{\mathbf{k}}>\Delta$. The triplet contribution vanishes since $u_{\mathbf{k}}v_{\mathbf{k}}=u_{-\mathbf{k}}v_{-\mathbf{k}}$ for *s*-wave superconductors. For the final state being a singlet, we then get

$$\langle(d_{2\downarrow}d_{1\uparrow}-d_{2\uparrow}d_{1\downarrow})T''\rangle=\frac{4T_{SD}^2}{\epsilon_1+\epsilon_2-i\eta}\sum_{\mathbf{k}}\frac{u_{\mathbf{k}}v_{\mathbf{k}}}{E_{\mathbf{k}}}\cos(\mathbf{k}\cdot\delta\mathbf{r}),\tag{11}$$

where $\delta\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$ denotes the distance vector between the points on the superconductor from which electrons 1 and 2 tunnel into the dots. To evaluate the sum over $\mathbf{k}$ we use $u_{\mathbf{k}}v_{\mathbf{k}}=\Delta/(2E_{\mathbf{k}})$, linearize the spectrum around the Fermi level with Fermi wave vector k_F, and obtain finally for the Andreev contribution

$$\langle(d_{2\downarrow}d_{1\uparrow}-d_{2\uparrow}d_{1\downarrow})T''\rangle=\frac{2\pi\nu_S T_{SD}^2}{\epsilon_1+\epsilon_2-i\eta}\,\frac{\sin(k_F\delta r)}{k_F\delta r}\,e^{-(\delta r/\pi\xi)}.\tag{12}$$

A. Dominant contribution of resonant tunneling to different leads

Now we calculate the matrix element in Eq. (8) containing T', where tunneling has to be treated to all orders in H_T. To simplify the notation we suppress spin indices and introduce a ket notation $|12\rangle$, where 1 stands for quantum numbers of the electron on dot 1/lead 1 and similarly for 2, for example, $|pq\rangle$ stands for $a^\dagger_{1\mathbf{p}\sigma}a^\dagger_{2\mathbf{q}-\sigma}|i\rangle$, where $\mathbf{p}$ is from lead 1 and $\mathbf{q}$ from lead 2; or $|pD\rangle$ stands for $a^\dagger_{1\mathbf{p}\sigma}d^\dagger_{2,-\sigma}|i\rangle$, etc. We concentrate first on the resummation of the following dot $\leftrightarrow$ lead transitions $|DD\rangle\to|LD\rangle\to|DD\rangle$ or $|DD\rangle\to|DL\rangle\to|DD\rangle$. In this sequence, $|DD\rangle$ is the state with one electron on dot 1 and the other one on dot 2, and $|LD\rangle$ defines a state where one electron is in lead 1 and the other one on dot 2. We exclude processes of the kind $|DD\rangle\to|LD\rangle\to|LL\rangle\to|LD\rangle\to|DD\rangle$ or $|DD\rangle\to|LD\rangle\to|LL\rangle\to|DL\rangle\to|DD\rangle$, where both electrons are *virtually* simultaneously in the leads as well as the creation of electron-hole excitations out of the Fermi sea. We show in Appendixes A and B that such contributions are suppressed in the regime (1) considered here by the small parameter $\gamma_l/\Delta\mu$. The dominant contribution is then given by

$$\langle pq|T'|DD\rangle=\Bigg\{\langle pq|H_{D_1L_1}|Dq\rangle\left\langle Dq\left|\sum_{n=0}^{\infty}\left(\frac{1}{i\eta-H_0}H_{D_1L_1}\right)^{2n}\right|Dq\right\rangle\left\langle Dq\left|\frac{1}{i\eta-H_0}H_{D_2L_2}\right|DD\right\rangle+\langle pq|H_{D_2L_2}|pD\rangle$$
$$\times\left\langle pD\left|\sum_{n=0}^{\infty}\left(\frac{1}{i\eta-H_0}H_{D_2L_2}\right)^{2n}\right|pD\right\rangle\left\langle pD\left|\frac{1}{i\eta-H_0}H_{D_1L_1}\right|DD\right\rangle\Bigg\}\left\langle DD\left|\sum_{m=0}^{\infty}\left(\frac{1}{i\eta-H_0}H_{DL}\right)^{2m}\right|DD\right\rangle.\tag{13}$$

Since the sums for the transition $|DD\rangle\to|DD\rangle$ via the sequences $|DD\rangle\to|LD\rangle\to|DD\rangle$ and $|DD\rangle\to|DL\rangle\to|DD\rangle$ are independent, we can write all summations in Eq. (13) as geometric series that can be resummed explicitly. We begin with the two-particle process for which we find

$$\left\langle DD\left|\sum_{m=0}^{\infty}\left(\frac{1}{i\eta-H_0}H_{DL}\right)^{2m}\right|DD\right\rangle=\frac{1}{1-\left\langle DD\left|\left(\frac{1}{i\eta-H_0}H_{DL}\right)^{2}\right|DD\right\rangle},\tag{14}$$

where

$$\left\langle DD\left|\left(\frac{1}{i\eta-H_0}H_{DL}\right)^{2}\right|DD\right\rangle=\frac{\Sigma}{i\eta-\epsilon_1-\epsilon_2},\tag{15}$$

with Σ being the self-energy, $\Sigma=|T_{DL}|^2\sum_{l\mathbf{k}}(i\eta-\epsilon_l-\epsilon_{\mathbf{k}})^{-1}$. In the presence of a Fermi sea in the leads, we introduce a cutoff in the sum in Σ at the Fermi level $\epsilon_{\mathbf{k}}\sim-\Delta\mu$ and at the edge of the conduction band ϵ_c. Then we obtain $\Sigma=\mathrm{Re}\,\Sigma-i\gamma/2$, where $\gamma=\gamma_1+\gamma_2$, and the logarithmic renormalization of the energy level is small, i.e., $|\mathrm{Re}\,\Sigma|\sim\gamma_l\ln(\epsilon_c/\Delta\mu)<\Delta\mu$ and will be neglected. Finally, we arrive at the following expression:

$$\left\langle DD\left|\sum_{m=0}^{\infty}\left(\frac{1}{i\eta-H_0}H_{DL}\right)^{2m}\right|DD\right\rangle=\frac{\epsilon_1+\epsilon_2-i\eta}{\epsilon_1+\epsilon_2-i\gamma/2}.\tag{16}$$

Similar results hold for the one-particle resummations in Eq. (13),

$$\left\langle pD\left|\sum_{n=0}^{\infty}\left(\frac{1}{i\eta-H_0}H_{D_2L_2}\right)^{2n}\right|pD\right\rangle=\frac{\epsilon_2+\epsilon_{\mathbf{p}}-i\eta}{\epsilon_2+\epsilon_{\mathbf{p}}-i\gamma_2/2},\tag{17}$$

$$\left\langle Dq \left| \sum_{n=0}^{\infty} \left(\frac{1}{i\eta - H_0} H_{D_1L_1} \right)^{2n} \right| Dq \right\rangle = \frac{\epsilon_1 + \epsilon_{\mathbf{q}} - i\eta}{\epsilon_1 + \epsilon_{\mathbf{q}} - i\gamma_1/2}. \tag{18}$$

Inserting the preceding results back into Eq. (13) we obtain

$$\langle pq|T'|DD\rangle = \frac{-T_{DL}^2(\epsilon_1+\epsilon_2-i\eta)}{(\epsilon_1+\epsilon_{\mathbf{q}}-i\gamma_1/2)(\epsilon_2+\epsilon_{\mathbf{p}}-i\gamma_2/2)}. \tag{19}$$

Thus, we see that the resummations cancel all divergences like the $\epsilon_1+\epsilon_2-i\eta$ denominator appearing in Eqs. (11) and (12), and that, as expected, the resummation of divergent terms leads effectively to the replacement $i\eta \to i\gamma_l/2$ so that the limit $\epsilon_l \to 0$ is well behaved. It is interesting to note that the two-particle resonance $(\epsilon_1+\epsilon_2-i\gamma/2)^{-1}$ occurring in Eq. (16) has canceled out in Eq. (19), and we finally obtain a product of two independent single-particle Breit-Wigner resonances. Still, we will just see that the two-particle correlation is reintroduced when we insert Eq. (19) into the expression for the current (5) due to the integrations over $\mathbf{p}$, $\mathbf{q}$, and the fact that the main contribution comes from the resonances. Indeed, making use of Eqs. (5) and (6), of energy conservation $\varepsilon_f = \varepsilon_i = 0$, i.e., $\epsilon_{\mathbf{p}} = -\epsilon_{\mathbf{q}}$, and of Eqs. (12) and (19), we finally obtain for the current (denoted by I_1) where each of the two entangled electrons tunnels into a *different* lead

$$I_1 = \frac{e\gamma_S^2\gamma}{(\epsilon_1+\epsilon_2)^2+\gamma^2/4}\left[\frac{\sin(k_F\delta r)}{k_F\delta r}\right]^2 \exp\left(-\frac{2\delta r}{\pi\xi}\right), \tag{20}$$

where, again, $\gamma = \gamma_1+\gamma_2$. We note that Eq. (20) also holds for the case with $\gamma_1 \neq \gamma_2$. The current becomes exponentially suppressed with increasing distance δr between the tunneling points on the superconductor, the scale given by the Cooper pair coherence length ξ. This does not pose severe restrictions for conventional *s*-wave material with ξ typically being on the order of micrometers. More severe is the restriction that $k_F\delta r$ should not be too large compared to unity, especially if k_F^{-1} of the superconductor assumes a typical value on the order of a few ångstroms. Still, since the suppression in $k_F\delta r$ is only power-law-like there is a sufficiently large regime on the nanometer scale for δr where the current I_1 can assume a finite measurable value. The current (20) has again a Breit-Wigner resonance form that assumes it maximum value when $\epsilon_1 = -\epsilon_2$,

$$I_1 = \frac{4e\gamma_S^2}{\gamma}\left[\frac{\sin(k_F\delta r)}{k_F\delta r}\right]^2 \exp\left(-\frac{2\delta r}{\pi\xi}\right). \tag{21}$$

This resonance at $\epsilon_1 = -\epsilon_2$ clearly shows that the current is a correlated two-particle effect (even apart from any spin correlation) as we should expect from the Andreev process involving the coherent tunneling of two electrons. Together with the single-particle resonances discussed above [see after Eq. (19)] we thus see that the current is carried by correlated pairs of electrons whose energies satisfy $|\epsilon_{\mathbf{p}}| = |\epsilon_{\mathbf{q}}| \lesssim \gamma$ if $\epsilon_1 = \epsilon_2 = 0$.

A particularly interesting case occurs when the energies of the dots, ϵ_1 and ϵ_2, are both tuned to zero, i.e., $\epsilon_1 = \epsilon_2 = \mu_S = 0$. We stress that in this case the electron in lead 1 and its spin-entangled partner in lead 2 have exactly the *same orbital energy*. We have shown previously[13] that this degeneracy of orbital energies is a crucial requirement for noise measurements in which the singlets manifest themselves in form of enhanced noise in the current (bunching), whereas uncorrelated electrons, or, more generally, electrons in a triplet state, lead to a suppression of noise (antibunching).

We remark again that the current I_1 is carried by electrons that are entangled in spin space and spatially separated in orbital space. In other words, the stationary current I_1 is a current of nonlocal spin-based EPR pairs. Finally, we note that due to the singlet character of the EPR pair we do not know whether the electron in, say, lead 1 carries an up or a down spin, this can be revealed only by a spin measurement. Of course, any measurement of the spin of one (or both) electrons will immediately destroy the singlet state and thus the entanglement. Such a spin measurement (spin readout) can be performed, e.g., by making use of the spin filtering effect of quantum dots.[22] The singlet state will also be destroyed by spin-dependent scattering (but not by Coulomb exchange interaction in the Fermi sea[13]). However, it is known experimentally that electron spins in a semiconductor environment show unusually long dephasing times approaching microseconds and can be transported phase coherently over distances exceeding 100 μm.[5–7,9,10] This distance is sufficiently long for experiments performed typically on the length scale of quantum confined nanostructures.[17]

B. Negligible tunnel contributions

We turn now to a discussion of various tunnel processes that we have not taken into account so far and show that they are negligibly small compared to the ones we have retained. As we mentioned above we exclude virtual states where both electrons are simultaneously in the leads. This is justified in the regime (1) considered here. To show this we consider the process $|DD\rangle \to |DD\rangle$. This transition occurs either in a transition sequence of the type $|DD\rangle \to |LD\rangle \to |DD\rangle$, as considered above, leading to the amplitude $A_{DL} = -i\gamma_L - \gamma_L/\pi \ln(\epsilon_c/\Delta\mu)$ [see Eq. (A1) in Appendix A)], or in a sequence of the type $|DD\rangle \to |LD\rangle \to |LL\rangle \to |DL\rangle \to |DD\rangle$, where both electrons are simultaneously in the leads ($|LL\rangle$ state), leading to the amplitude

$$A_{LL} = \frac{\gamma_L^2}{2\pi^2\Delta\mu}\left[i\pi + \ln\left(\frac{\epsilon_c}{\Delta\mu}\right)\right]$$

[see Eq. (A3) in Appendix A]. However, this amplitude A_{LL} is suppressed by a factor $\gamma_L/\Delta\mu < 1$ compared to A_{DL}. Above we used $\gamma_1 = \gamma_2 = \gamma_L$ for simplicity. Further, a process where we create an electron-hole pair out of the Fermi sea of the leads could, in principle, destroy the spin-correlation of the entangled electron pair when an electron with the ''wrong'' spin (coming from the Fermi sea) hops on the dot. But such contributions cost additional energy of at least $\Delta\mu$, and again such particle-hole processes are suppressed by a factor $(\gamma_L/\Delta\mu)^2$ as we show in detail in Appendix B.

VI. TUNNELING VIA THE SAME DOT

The two electrons of a Cooper pair can also tunnel via the *same* dot into the same lead. In this section we calculate the current induced by this process. We show that we obtain a suppression of such processes by a factor $(\gamma_l/U)^2$ and/or $(\gamma_l/\Delta)^2$ compared to the process discussed in the preceding section. However, in contrast to the previous case, we do not get a suppression resulting from the spatial separation of the Cooper pair on the superconductor, since here the two electrons tunnel from the same point either from $\mathbf{r}_1$ or $\mathbf{r}_2$ (see Fig. 2). As before, a tunnel process starts by breaking up a Cooper pair followed by an Andreev process with two possible sequences, see Fig. 2. (I) In the first step, one electron tunnels from the superconductor to, say, dot 1, and in a second step the second electron also tunnels to dot 1 (see left panel in Fig. 2). There are now two electrons on the *same* dot, which costs additional Coulomb repulsion energy U; thus this virtual state is suppressed by $1/U$. Finally, the two electrons leave dot 1 and tunnel into lead 1. (II) There is an alternative competing process that avoids the double occupancy (see right panel in Fig. 2). Here, one electron tunnels to, say, dot 1, and then the same electron tunnels further into lead 1, leaving an excitation on the superconductor that costs additional gap energy Δ (instead of U), before finally the second electron tunnels from the superconductor via dot 1 into lead 1.

We first concentrate on the tunneling process (II), and note that the leading contribution comes from the processes where both electrons have left the superconductor so that the system has no energy deficit anymore. We still have to resum the tunnel processes from the dot to the lead to all orders in the tunnel Hamiltonian H_{DL}. In what follows we suppress the label $l=1,2$ since the setup is assumed to be symmetric and tunneling into either lead 1 or lead 2 gives the same result. The transition amplitude $\langle f|T_0|i\rangle$ including only leading terms is

$$\begin{aligned}\langle f|T_0|i\rangle=&\sum_{\mathbf{p}''\sigma}\langle f|H_{DL}|D\mathbf{p}''\sigma\rangle\\&\times\left\langle D\mathbf{p}''\sigma\left|\sum_{n=0}^{\infty}\left(\frac{1}{i\eta-H_0}H_{DL}\right)^{2n}\right|D\mathbf{p}''\sigma\right\rangle\\&\times\left\langle D\mathbf{p}''\sigma\left|\frac{1}{i\eta-H_0}H_{SD}\frac{1}{i\eta-H_0}H_{DL}\right.\right.\\&\times\left.\left.\frac{1}{i\eta-H_0}H_{SD}\right|i\right\rangle,\end{aligned}\tag{22}$$

where again $|f\rangle=(1/\sqrt{2})(a^\dagger_{\mathbf{p}\uparrow}a^\dagger_{\mathbf{p}'\downarrow}\pm a^\dagger_{\mathbf{p}\downarrow}a^\dagger_{\mathbf{p}'\uparrow})|i\rangle$, with $\pm$ denoting the triplet $(+)$ and singlet $(-)$, respectively, and the intermediate state $|D\mathbf{p}''\sigma\rangle=d^\dagger_{-\sigma}a^\dagger_{\mathbf{p}''\sigma}|i\rangle$. There are some remarks in order regarding Eq. (22). The electron that tunnels to the state $|\mathbf{p}''\sigma\rangle$ does not have to be resummed further since this would lead either to a double occupancy of the dot that is suppressed by $1/U$ or to the state with two electrons simultaneously in the lead with a *virtual* summation over the state $\mathbf{p}''$. But we already showed that the latter process is suppressed by $\gamma_l/\Delta\mu$. Making then use of Eq. (18), we obtain for the dot-lead resummation in Eq. (22)

$$\begin{aligned}&\left\langle f\left|H_{DL}\sum_{n=0}^{\infty}\left(\frac{1}{i\eta-H_0}H_{DL}\right)^{2n}\right|D\mathbf{p}''\uparrow\right\rangle\\&\quad=-\frac{T_{DL}}{\sqrt{2}}\frac{\epsilon_l+\epsilon_{\mathbf{p}''}-i\eta}{\epsilon_l+\epsilon_{\mathbf{p}''}-i\gamma_l/2}(\delta_{\mathbf{p}''\mathbf{p}}\mp\delta_{\mathbf{p}''\mathbf{p}'}),\end{aligned}\tag{23}$$

$$\begin{aligned}&\left\langle f\left|H_{DL}\sum_{n=0}^{\infty}\left(\frac{1}{i\eta-H_0}H_{DL}\right)^{2n}\right|D\mathbf{p}''\downarrow\right\rangle\\&\quad=\frac{T_{DL}}{\sqrt{2}}\frac{\epsilon_l+\epsilon_{\mathbf{p}''}-i\eta}{\epsilon_l+\epsilon_{\mathbf{p}''}-i\gamma_l/2}(\delta_{\mathbf{p}''\mathbf{p}'}\mp\delta_{\mathbf{p}''\mathbf{p}}),\end{aligned}\tag{24}$$

where again in Eqs. (23) and (24) the upper sign belongs to the triplet and the lower sign to the singlet. For the superconductor-dot transitions in Eq. (22) we obtain

$$\begin{aligned}&\left\langle D\mathbf{p}''\uparrow\left|\frac{1}{i\eta-H_0}H_{SD}\frac{1}{i\eta-H_0}H_{DL}\frac{1}{i\eta-H_0}H_{SD}\right|i\right\rangle\\&\quad=-\left\langle D\mathbf{p}''\downarrow\left|\frac{1}{i\eta-H_0}H_{SD}\frac{1}{i\eta-H_0}H_{DL}\frac{1}{i\eta-H_0}H_{SD}\right|i\right\rangle\\&\quad=\frac{T_{DL}T_{SD}^2\nu_S}{\Delta(\epsilon_l+\epsilon_{\mathbf{p}''}-i\eta)}.\end{aligned}\tag{25}$$

Combining the results (23)–(25), we obtain for the amplitude (22)

$$\langle f|T_0|i\rangle=-\frac{2^{3/2}\nu_S(T_{SD}T_{DL})^2(\epsilon_l-i\gamma_l/2)}{\Delta(\epsilon_l+\epsilon_{\mathbf{p}}-i\gamma_l/2)(\epsilon_l+\epsilon_{\mathbf{p}'}-i\gamma_l/2)}\tag{26}$$

for the final state $|f\rangle$ being a singlet, whereas we get again zero for the triplet.

Next we consider the process (I) where the tunneling involves a double occupancy of the dot (see left panel in Fig. 2). In this case the transition amplitude can be written as

$$\begin{aligned}\langle f|T_0|i\rangle=&\sum_{\mathbf{p}''\sigma}\langle f|H_{DL}|D\mathbf{p}''\sigma\rangle\\&\times\left\langle D\mathbf{p}''\sigma\left|\sum_{n=0}^{\infty}\left(\frac{1}{i\eta-H_0}H_{DL}\right)^{2n}\right|D\mathbf{p}''\sigma\right\rangle\\&\times\left\langle D\mathbf{p}''\sigma\left|\frac{1}{i\eta-H_0}H_{DL}\frac{1}{i\eta-H_0}H_{SD}\right.\right.\\&\times\left.\left.\frac{1}{i\eta-H_0}H_{SD}\right|i\right\rangle.\end{aligned}\tag{27}$$

As before, the transition amplitude $\langle f|T_0|i\rangle$ is only nonzero for the final lead state $|f\rangle$ being a singlet state. Repeating a similar calculation as before we find that the amplitude is given by Eq. (26) but with Δ being replaced by U/π. We note that the two amplitudes (26) and (27) have the same initial and same final states. Thus, to obtain the total current due to processes (I) and (II) we need to add these two amplitudes. Then, using Eq. (5) we find for the total current I_2 in case of tunneling of two electrons into the same lead,

$$I_2=\frac{2e\gamma_S^2\gamma}{\mathcal{E}^2},\quad \frac{1}{\mathcal{E}}=\frac{1}{\pi\Delta}+\frac{1}{U}. \tag{28}$$

We see that the effect of the quantum dots consists in the suppression factor $(\gamma/\mathcal{E})^2$ for tunneling into the *same* lead. We remark that in contrast to the previous case (tunneling into different leads) the current does not have a resonant behavior since the virtual dot states are no longer at resonance due the energy costs U or Δ in the tunneling process. Our final goal is to compare I_1 given in Eq. (21) with I_2. Thus, forming the ratio of the currents of the two competing processes, we obtain

$$\frac{I_1}{I_2}=\frac{2\mathcal{E}^2}{\gamma^2}\left[\frac{\sin(k_F\delta r)}{k_F\delta r}\right]^2\exp\left(-\frac{2\delta r}{\pi\xi}\right). \tag{29}$$

From this ratio we see that the desired regime with I_1 dominating I_2 is obtained when $\mathcal{E}/\gamma>k_F\delta r$, and $\delta r<\xi$. We would like to emphasize that the relative suppression of I_2 (as well as the absolute value of the current I_1) is maximized by working around the resonances $\epsilon_l\simeq\mu_S=0$.[23]

VII. DISCUSSION AND AHARONOV-BOHM OSCILLATIONS

We have seen that there are two competing processes of currents, one where the two electrons proceed via different dots into different leads, and one where the two electrons proceed via the same dot into the same lead. We will show now that these two processes also lead to different current oscillations in an Aharonov-Bohm loop that is threaded by an external magnetic flux ϕ. For this let us consider now a setup where the two leads 1 and 2 are connected such that they form an Aharonov-Bohm loop, where the electrons are injected from the left via the superconductor, traversing the upper (lead 1) and lower (lead 2) arm of the loop before they rejoin to interfere and then exit into the same lead, where the current is then measured as a function of varying flux ϕ. It is straightforward to analyze this setup with our results obtained so far. In particular, each tunneling amplitude obtains a phase factor, $T_{D_1L_1}\rightarrow T_{D_1L_1}e^{i\phi/2\phi_0}$ and $T_{D_2L_2}\rightarrow T_{D_2L_2}e^{-i\phi/2\phi_0}$, where $\phi_0=h/e$ is the single-electron flux quantum. For simplicity we also assume that the entire phase is acquired when the electron hops from the dot into the leads, so that the process dot-lead-dot gives basically the full Aharonov-Bohm phase factor $e^{\pm i\phi/\phi_0}$ of the loop (and only a negligible amount of phase is picked up along the path from the superconductor to the dots). Now, we repeat the calculations of the transition amplitude and find it to be of the following structure: $\langle f|T_0|i\rangle\sim T_{D_1L_1}T_{D_2L_2}+T_{D_1L_1}^2e^{i\phi/\phi_0}+T_{D_2L_2}^2e^{-i\phi/\phi_0}$. Here, the first term comes from the process via different leads [see Eq. (19)], where no Aharonov-Bohm phase is picked up. The Aharonov-Bohm phase appears in the remaining two terms, which come from processes via the same leads, either via lead 1 or lead 2 [see Eqs. (26) and (27)]. The total current I is now obtained from $|\langle f|T_0|i\rangle|^2$, giving $I=I_1+I_2+I_{AB}$, and the flux-dependent Aharonov-Bohm current I_{AB} is given by

$$I_{AB}=\sqrt{8I_1I_2}F(\epsilon_l)\cos(\phi/\phi_0)+I_2\cos(2\phi/\phi_0),$$

$$F(\epsilon_l)=\frac{\epsilon_l}{\sqrt{\epsilon_l^2+(\gamma_L/2)^2}}, \tag{30}$$

where, for simplicity, we have assumed that $\epsilon_1=\epsilon_2=\epsilon_l$, and $\gamma_1=\gamma_2=\gamma_L$. Here, the first term (different leads) is periodic in ϕ_0 like for single-electron Aharonov-Bohm interference effects, while the second one (same leads) is periodic in *half* the flux quantum $\phi_0/2$, describing thus the interference of two coherent electrons (similar single- and two-particle Aharonov-Bohm effects occur in the Josephson current through an Aharonov-Bohm loop[14]). It is clear from Eq. (30) that the h/e oscillation comes from the interference between a contribution where the two electrons travel through different arms with contributions where the two electrons travel through the same arm. Both Aharonov-Bohm oscillations with period h/e and $h/2e$, vanish with decreasing I_2, i.e., with increasing on-site repulsion U and/or gap Δ. However, their relative weight is given by $\sqrt{I_1/I_2}$, implying that the $h/2e$ oscillations vanish faster than the h/e oscillations. This behavior is quite remarkable since it opens up the possibility to damp down the unwanted leakage process $\sim I_2\cos(2\phi/\phi_0)$ where two electrons proceed via the same dot/lead by increasing U with a gate voltage applied to the dots. The dominant current contribution with period h/e comes then from the desired entangled electrons proceeding via different leads. On the other hand, if $\sqrt{I_1/I_2}<1$, which could become the case, e.g., for $k_F\delta r>\mathcal{E}/\gamma$, we are left with $h/2e$ oscillations only. Note that dephasing processes that affect the orbital part suppress I_{AB}. Still, the flux-independent current I_1+I_2 can remain finite and contain electrons that are entangled in spin space, provided that there is only negligible spin-orbit coupling so that the spin is still a good quantum number.

We would like to mention another important feature of the Aharonov-Bohm effect under discussion, namely the relative phase shift between the amplitudes of tunneling to the same lead and to different leads, resulting in the additional prefactor $F(\epsilon_l)$ in the first term of the right-hand side (rhs) of the Eq. (30). This phase shift is due to the fact that there is a two-particle resonance in the amplitude (19) while there is only a single-particle resonance in the amplitudes (26) and (27) (we recall that the second resonance is suppressed by the Coulomb blockade effect). Thus, when the chemical potential μ_S of the superconductor crosses the resonance, $|\epsilon_l|\lesssim\gamma_L$, the amplitude (19) acquires an extra phase factor $e^{i\phi_r}$, where $\phi_r=\arg[1/(\epsilon_l-i\gamma_L/2)]$. Then the interference of the two amplitudes leads to the prefactor $F(\epsilon_l)=\cos\phi_r$ in the first term in the rhs of Eq. (30). In particular, exactly at the middle of the resonance, $\epsilon_l=0$, the phase shift is $\phi_r=\pi/2$, and thus the h/e oscillations vanish, since $F(0)=\cos(\pi/2)=0$. Note however, that although $F=\pm 1$ away from the resonance ($|\epsilon_l|>\gamma_L$) the h/e oscillations vanish again, now because the current $I_1\sim e\gamma_S^2\gamma_L/\epsilon_l^2$ vanishes. Thus the optimal regime for the observation of the Aharonov-Bohm effect is $|\epsilon_l|\sim\gamma_L$.

Finally, the preceding discussion shows that even if the spins of two electrons are entangled their associated charge current does not reveal this spin correlation in a simple Aharonov-Bohm interference experiment.[24] Only if we consider the current-current correlations (noise) in a beam splitter setup can we detect also this spin-correlation in the transport current via its charge properties.[13]

VIII. CONCLUSION

We have proposed an entangler device that can create pairwise spin-entangled electrons and provide coherent injection by an Andreev process into different dots that are coupled to leads. The unwanted process of both electrons tunneling into the same lead can be suppressed by increasing the Coulomb repulsion on the quantum dot. We have calculated the ratio of currents of these two competing processes and shown that there exists a regime of experimental interest where the entangled current shows a resonance and assumes a finite value with both partners of the singlet being in different leads but having the same orbital energy. This entangler then satisfies the necessary requirements needed to detect the spin entanglement via transport and noise measurements. We also discussed the flux-dependent oscillations of the current in an Aharonov-Bohm loop.

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APPENDIX A: SUPPRESSION OF VIRTUAL STATES WITH BOTH ELECTRONS IN THE LEADS

We have stated in the main text that the contributions of virtual states where two electrons are simultaneously in the leads are negligible. Here we estimate this contribution and show that indeed it is suppressed by $\gamma_L/\Delta\mu<1$ (here the spin of the electrons is not important, and we set $\gamma_1=\gamma_2=\gamma_L$ for simplicity). First we consider the dominant transition from $|DD\rangle$ back to $|DD\rangle$ with the tunneling of only one electron to the lead, i.e., a sequence of the type $|DD\rangle\rightarrow|LD\rangle\rightarrow|DD\rangle$. From now on we impose the resonant condition $\epsilon_c=0$, and find for the amplitude [cf. Eqs. (14) and (15)]

$$A_{DL}=\left\langle DD\left|H_{DL}\frac{1}{i\eta-H_0}H_{DL}\right|DD\right\rangle=-i\gamma_L-\frac{\gamma_L}{\pi}\ln\left(\frac{\epsilon_c}{\Delta\mu}\right). \tag{A1}$$

We compare this amplitude A_{DL} with the amplitude A_{LL} of the lowest-order process of tunneling of two electrons via the virtual state $|LL\rangle$, where both electrons are simultaneously in the leads, i.e., the sequence $|DD\rangle\rightarrow|LD\rangle\rightarrow|LL\rangle\rightarrow|DL\rangle\rightarrow|DD\rangle$. We find for the amplitude of this process

$$\begin{aligned}A_{LL}&=\left\langle DD\left|H_{DL}\left(\frac{1}{i\eta-H_0}H_{DL}\right)^3\right|DD\right\rangle\\&=\sum_{\mathbf{kk}'}\frac{2|T_{DL}|^4}{(i\eta-\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}'})(i\eta-\epsilon_{\mathbf{k}})}\left[\frac{1}{i\eta-\epsilon_{\mathbf{k}}}+\frac{1}{i\eta-\epsilon_{\mathbf{k}'}}\right],\end{aligned} \tag{A2}$$

where the first term in the bracket results from the sequence of, say, electron 1 tunneling into lead 1, then electron 2 tunneling into lead 2, then electron 2 tunneling back into dot 2, and finally electron 1 tunneling back into dot 1. The second term in the bracket results from the sequence where the order of tunneling back to the dots is reversed, i.e., electron 1 tunnels back to its dot before electron 2 does. Note that due to these two terms in the bracket the two-particle pole in Eq. (A2) cancels.

Replacing $\Sigma_{\mathbf{k}}(\cdots)$ with $\nu_L\int_{-\Delta\mu}^{\epsilon_c}d\epsilon(\cdots)$, we can write

$$\begin{aligned}A_{LL}&=\frac{\gamma_L^2}{2\pi^2}\int_{-\Delta\mu}^{\epsilon_c}\frac{d\epsilon'}{i\eta-\epsilon'}\int_{-\Delta\mu}^{\epsilon_c}\frac{d\epsilon}{(i\eta-\epsilon)^2}\\&=\times\frac{\gamma_L^2}{2\pi^2\Delta\mu}\left[i\pi+\ln\left(\frac{\epsilon_c}{\Delta\mu}\right)\right].\end{aligned} \tag{A3}$$

Thus, comparing A_{DL} with A_{LL}, we see that indeed a virtual state involving two electrons simultaneously in the leads is suppressed by a factor of $\gamma_L/\Delta\mu$ compared to the one with only one electron in the leads.

APPENDIX B: ELECTRON-HOLE PAIR EXCITATION

In this Appendix we consider a tunnel process where the two electrons starting from the superconductor tunnel over different dots but during the process of repeated tunneling from the dots to the leads and back to the dots an electron from the Fermi sea hops on one of the dots (say dot 1) when this dot is empty. In principle, such contributions could destroy the desired entanglement since then a ''wrong'' spin can hop on the dot and the electron on the other dot (dot 2) would no longer be entangled with this electron (while the original partner electron disappears in the reservoir provided by the Fermi sea). We show now that in the regime $\Delta\mu>\gamma_l$ such electron-hole pair processes due to the Fermi sea are suppressed. We start with our consideration when the two electrons, after the Andreev process, are each on a different dot forming the $|DD\rangle$ state (we neglect spin and set $\gamma_1=\gamma_2=\gamma_L$ in this consideration for simplicity). Instead of the amplitude $\langle pq|T'|DD\rangle$ calculated in Eq. (13) we consider now the following process:

$$A_{eh}=\langle\overline{pq}|T'|\overline{DD}\rangle\left\langle\overline{DD}\right|\frac{1}{i\eta-H_0}H_{D_1L_1}\times\sum_{n=0}^{\infty}\left(\frac{1}{i\eta-H_0}H_{D_2L_2}\right)^{2n}\frac{1}{i\eta-H_0}H_{D_1L_1}\left|DD\right\rangle\times\left\langle DD\right|\sum_{m=0}^{\infty}\left(\frac{1}{i\eta-H_0}H_{DL}\right)^{2m}\left|DD\right\rangle. \quad \text{(B1)}$$

The new sequence of interest in Eq. (B1) is the amplitude containing the sum over n. For instance, let us consider the $n=0$ term,

$$\left\langle\overline{DD}\right|\left(\frac{1}{i\eta-H_0}H_{D_1L_1}\right)^2\left|DD\right\rangle,$$

where we assume that the electron-hole excitation occurs in, say, lead 1. From $|DD\rangle$, the tunnel Hamiltonian $H_{D_1L_1}$ takes the electron from dot 1 to the state $\mathbf{k}$ in lead 1. Instead of tunneling back of this electron to dot 1, an electron from the state $\mathbf{k}'$ with energy $\epsilon_{\mathbf{k}'}<-\Delta\mu$ from the Fermi sea of lead 1 hops on dot 1. Now the dot-lead system is in the state $|\overline{DD}\rangle=d_1^\dagger d_2^\dagger a_{1\mathbf{k}'}a_{1\mathbf{k}}^\dagger|i\rangle$. The sum over n resums the hopping back and forth of electron 2 from D_2 to D_2, resulting in the replacement of η in $H_{D_1L_1}(i\eta-H_0)^{-1}H_{D_1L_1}$ by $\gamma_L/2$. We perform the further resummation in Eq. (B1) with this Fermi sea electron on dot 1 and the other electron still on dot 2, assuming that electron 1 in the state $\mathbf{k}$ in lead 1 is in its final state (and not a virtual state). All the resummation processes in Eq. (B1) are similar to those already explained in the main text, except for having now an excitation with energy $\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}'}>0$. The final state $|\overline{pq}\rangle$ consists of two electrons in the lead states $\mathbf{p}$ and $\mathbf{q}$ (their multiple tunneling is resummed in T') and of the excitation with energy $\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}'}$, so $|\overline{pq}\rangle=a_{1\mathbf{p}}^\dagger a_{2\mathbf{q}}^\dagger a_{1\mathbf{k}'}a_{1\mathbf{k}}^\dagger|i\rangle$. The normalized correction to the current, I_{eh}/I_1, can be obtained by summing $|A_{eh}|^2/I_1$ over the final states $|\overline{pq}\rangle$, and thus we arrive at the following integral for $\epsilon_l=0$, retaining only leading terms in $\gamma_L/\Delta\mu$, and using energy conservation, $\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}'}+\epsilon_{\mathbf{p}}+\epsilon_{\mathbf{q}}=0$:

$$\frac{I_{eh}}{I_1}=\left(\frac{\gamma_L}{2\pi}\right)^3\int\int_{-\Delta\mu}^{+\infty}\int d\epsilon_{\mathbf{k}}\,d\epsilon_{\mathbf{p}}\,d\epsilon_{\mathbf{q}}\times\frac{1-\theta(\epsilon_{\mathbf{k}}+\epsilon_{\mathbf{p}}+\epsilon_{\mathbf{q}}+\Delta\mu)}{[\epsilon_{\mathbf{k}}^2+(\gamma_L/2)^2][\epsilon_{\mathbf{p}}^2+(\gamma_L/2)^2][\epsilon_{\mathbf{q}}^2+(\gamma_L/2)^2]}. \quad \text{(B2)}$$

We evaluate the integral in leading order and find

$$\frac{I_{eh}}{I_1}=\frac{3}{2\pi^2}\left(\frac{\gamma_L}{\Delta\mu}\right)^2\ln\left(\frac{\Delta\mu}{\gamma_L}\right). \quad \text{(B3)}$$

We see now that the current involving an electron-hole pair, I_{eh}, is suppressed compared to the main contribution I_1 [see Eq. (20)] by a factor of $(\gamma_L/\Delta\mu)^2$.

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